Critical Temperature for Deconfinement in the (2+1)-d Georgi-Glashow Model

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Outline

- Definition of the theory
- Motivation
- The simple case: SU(2)
- What happens for SU(N)?
- Summary

The theory: Pure gauge SU(N) + adjoint scalars

- Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(D_\mu \phi^a\right)^2 + \lambda\left(\phi^a\phi^a - v^2\right)^2 + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$

Adjoint scalars (N^2-1)

$$\phi^a T^a \in ASU(N)$$

Covariant derivative

$$D^a_\mu = \partial_\mu + ad(A_\mu)^a$$

Field strength tensor

$$F_{\mu\nu} = F^a_{\mu\nu}T^a = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

The theory: Pure gauge SU(N) + adjoint scalars

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- Perturbatively...

	SU(2)	SU(N)	
massive bosons	$W^\pm = A^1 \pm iA^2$	N²-N	$m_W = gv$
massless photon	A^3	N-1	(for maximal sym. breaking)

In the Unitary gauge: $\phi^a = \delta^{a3}(v + \sigma(x))$

- Weak coupling... $m_W \sim m_H \gg g^2$

Motivation: why is this interesting?

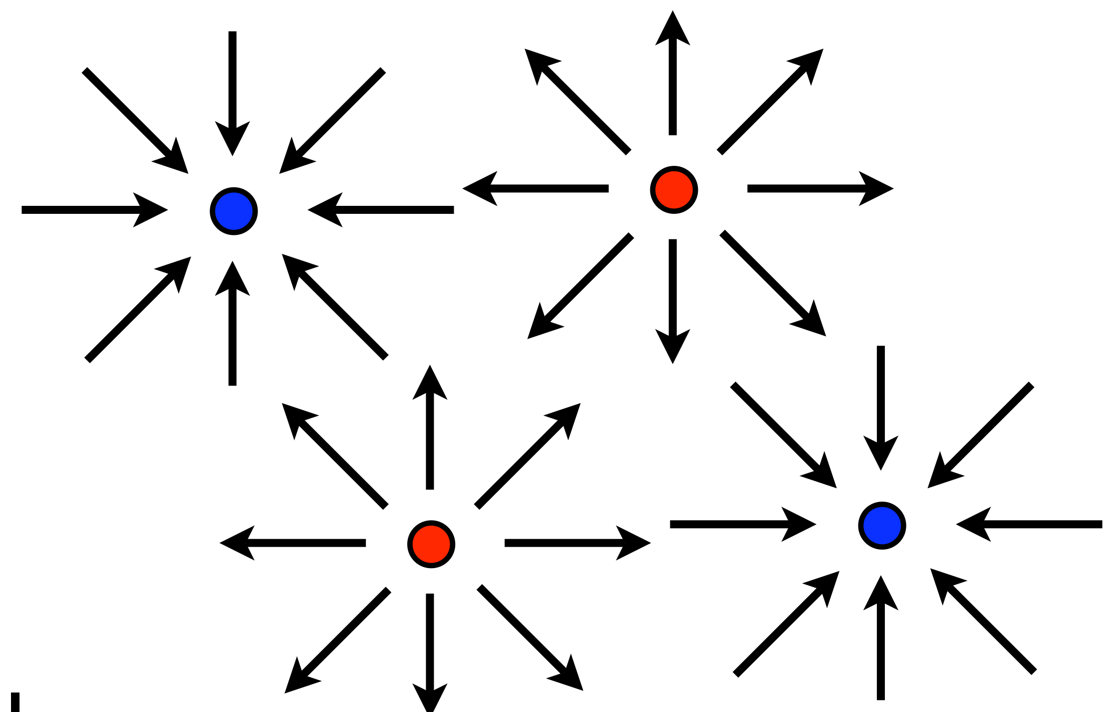
- These theories have topologically non-trivial classical solutions of finite action (a.k.a. solitons/instantons).
- In (2+1)-d, instantons provide a mechanism for confinement of charge, and this happens at weak coupling.
    - One can prove, e.g....
        - Confinement
        - T_c for Deconfinement

... analytically !

Confinement in SU(2)

1. Polyakov's proof:

'*Instantons generate a non-perturbative mass for the photon*'



Classical solutions (photon part)

$$F^3_{\mu\nu} = \frac{e}{4\pi}\epsilon_{\mu\nu\lambda}\frac{x_\lambda}{x^2} \qquad e = \frac{4\pi}{g}$$

Write the partition function in a saddle-point approximation as a sum over multi-instanton configurations.

⇨ Coulomb gas (of instantons) representation.

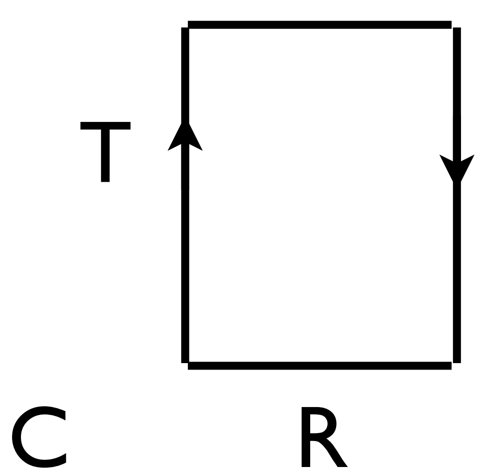
Confinement in SU(2)

➡ Map Coulomb gas onto Sine-Gordon model.

$$\mathcal{L}_{eff} \propto (\partial_\mu \varphi)^2 - M^2 \cos\varphi \qquad S_0 = \frac{4\pi v}{g}\epsilon\left(\frac{m_H}{m_W}\right)$$

$$M^2 \propto \frac{m_w^{7/2}}{g^3} e^{-S_0} \qquad \boxed{m_\gamma \sim e^{-S_0/2}}$$

➡ Compute Wilson loop



$$F(C) = e^{-E(R)T} = e^{-\gamma A} \qquad \gamma \propto g^2 M$$

$$\boxed{E(R) = \gamma R}$$

Confinement !

Deconfinement in SU(2)

2. Deconfinement transition
(Dunne et al.; Kovchegov & Son).

⇒ Again map onto Sine-Gordon model...
(plus W bosons, in dimensional reduction)

$$H = \int dx \left[\frac{1}{2}(\partial_x \Phi)^2 + \frac{1}{2}(\partial_x \Theta)^2 + 2\zeta_0 \cos \beta\Phi + 2\tilde{\zeta}_0 \cos \tilde{\beta}\Theta \right]$$

$$\zeta_0 \sim T^2 e^{-S_0}, \qquad \tilde{\zeta}_0 \sim T^2 e^{-m_W/T}$$

$$\beta = e\sqrt{T}, \quad \tilde{\beta} = \frac{g}{\sqrt{T}}$$

$$S_0 = \frac{4\pi v}{g}\epsilon\left(\frac{m_H}{m_W}\right)$$

$$e = \frac{4\pi}{g}$$

Deconfinement in SU(2)

- Study RG flow...

$$T_c = \frac{g^2}{4\pi}\frac{\epsilon+2}{2\epsilon+1}$$

$$S_0 = \frac{4\pi v}{g}\epsilon\left(\frac{m_H}{m_W}\right)$$

- Universality class

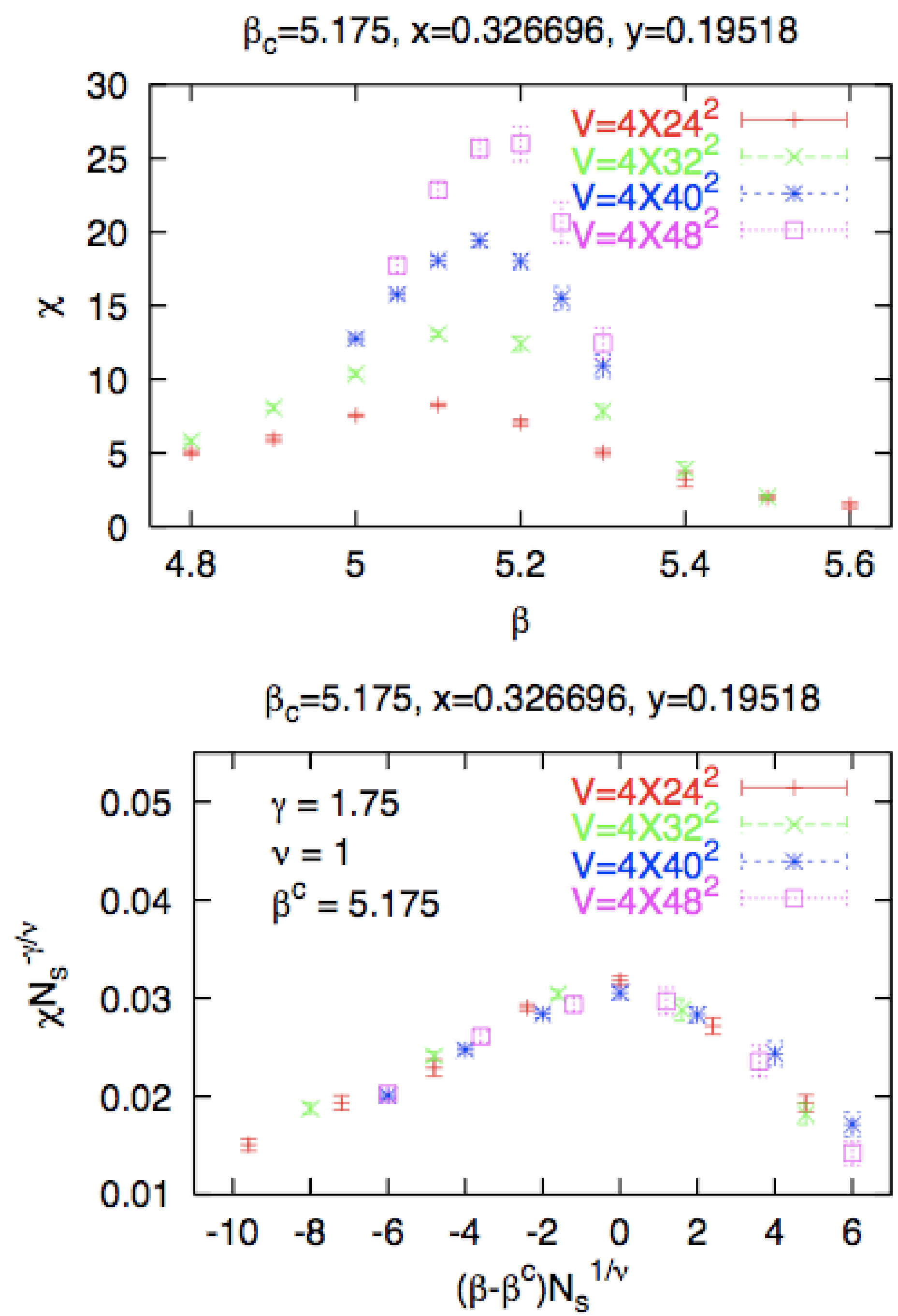
~~BKT~~ Z_2

Critical exponents are as in the 2D Ising model

Numerical results for SU(2)

$$S = \beta \sum_{x,\mu>\nu} \left(1 - \frac{1}{2}\mathrm{Tr}U_{\mu\nu}(x)\right) + 2\sum_{x} \mathrm{Tr}(\Phi(x)\Phi(x))$$
$$- 2\kappa \sum_{x,\mu} \mathrm{Tr}(\Phi(x)U_\mu(x)\Phi(x+\hat{\mu}a)U_\mu^\dagger(x))$$
$$+ \lambda \sum_{x} (2\mathrm{Tr}(\Phi(x)\Phi(x)) - 1)^2$$

Barresi et al.



$$\overline{L_F} = \frac{1}{N_s^2} \sum_{\vec{x}} \left(\mathrm{Tr}_F \left(\Pi_{t=1}^{N_\tau} U_\tau(\vec{x}, t)\right)\right)$$

$$\chi_{L_F} = N_s^2 \left(\langle \overline{L_F}^2 \rangle - \langle \overline{L_F} \rangle^2\right)$$

Z_2 Universality class
confirmed by finite size scaling

What happens for SU(N)?

Mapping onto Sine-Gordon...

$$\mathcal{H}_{\rm eff} = \frac{1}{2}\left[\left(\partial_x\vec{\Phi}\right)^2 + \left(\partial_x\vec{\Theta}\right)^2\right] - \sum_{\vec{\alpha}\epsilon\Delta_+}\left[\zeta_{\vec{\alpha}}\cos\left(\frac{4\pi\sqrt{T}}{g}\,\vec{\alpha}\cdot\vec{\Phi}\right) + \tilde{\zeta}_{\vec{\alpha}}\cos\left(\frac{g}{\sqrt{T}}\,\vec{\alpha}\cdot\vec{\Theta}\right)\right]$$

??? ???

Nature of the phase transition (Svetitsky & Yaffe; Lecheminant) Z_N

What happens for SU(N)?

- Put the theory on the lattice.
- Study Polyakov loops and their correlations
- Do finite size scaling analysis

  Critical exponents?

- Quantum phase transitions?

  Dynamical exponents?

Summary

- In the (2+1)-d SU(N) Georgi-Glashow model confinement of charge happens at weak coupling and can be proven analytically ('t Hooft & Polyakov).
- The nature of the finite temperature deconfinement transition is believed to be understood for all N (Svetitsky & Yaffe; Kogan et al.; Lecheminant). Needs numerical check.
- The critical temperature has only been computed analytically in the simplest case of SU(2) (Son & Kovchegov).
- The critical temperature for SU(N) remains unknown and is hard to compute analytically.

To be continued...

Thanks!