Fisher's Zeros in SU(2) Lattice Gauge Theory

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Overview:

- Reweighting method and the zeros of the partition function on the complex $\beta(= 2N/g^2)$ plane (Fisher's zeros).

- Direct calculations cannot predict the correct zeros of the partition function due to the small region of confidence.
  The quasi-Gaussian distribution makes it possible to approximate the probablity density with an exponential function(qausi-Gaussian fitting).

- Applications to the $SU(2)$ gauge fields on L^4 lattices and comparison with existing results.

Reweighting method

$Z(\beta)$ is the Laplace transform of density of states :

$$Z(\beta) = \int_0^\infty dS \; n(S) \exp(-\beta S) \tag{1}$$

Reweighting (Falcioni et al. 82):

$$Z(\beta_0 + \Delta\beta) = Z(\beta_0) < \exp(-\Delta\beta S) >_{\beta_0} \quad . \tag{2}$$

It has the same zeros as the following function

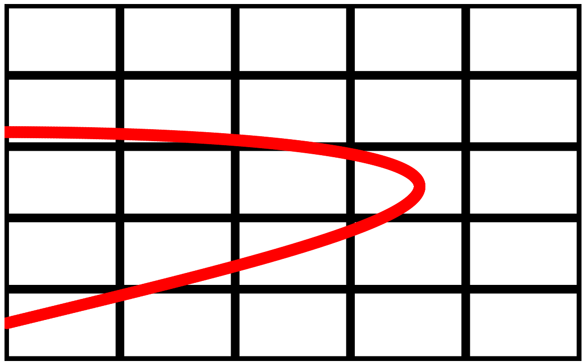
$$\begin{aligned} < \quad & \exp \quad [-\Delta\beta(S- < S >_{\beta_0})] >_{\beta_0} \\ = \quad & \exp \quad [\Delta\beta < S >_{\beta_0}] Z(\beta_0 + \Delta\beta)/Z(\beta_0) \ , \end{aligned} \tag{3}$$

Zeros of the partition function

one can then search for the zeros of the Real and Imaginary parts of $\langle \exp(-\Delta\beta S') \rangle$ directly:

$$\langle \mathsf{Re}\exp[-\Delta\beta S'] \rangle_{\beta_0} = 0, \quad \langle \mathsf{Im}\exp[-\Delta\beta S'] \rangle_{\beta_0} = 0$$

Searching the zeros on the complex plane: $\Delta\beta = \beta_R + i\beta_I$



Circle of confidence

Criterion to determine a region of confidence for MC zeros (Alves and Berg 91, based on the Gaussian approximation):

$$\frac{\sigma(|\exp(-\Delta\beta S')|)}{\sqrt{N_{conf}}} \leq |\exp(-\Delta\beta S')|$$

When the fluctuation in $\exp(-\Delta\beta S')$ becomes of the same size of the function itself, we say the region beyond that cannot be trusted. For an exact Gaussian distribution, this defines a radius of confidence $\sqrt{\ln(N_{conf.}+1)}/\sigma_S V_p$.

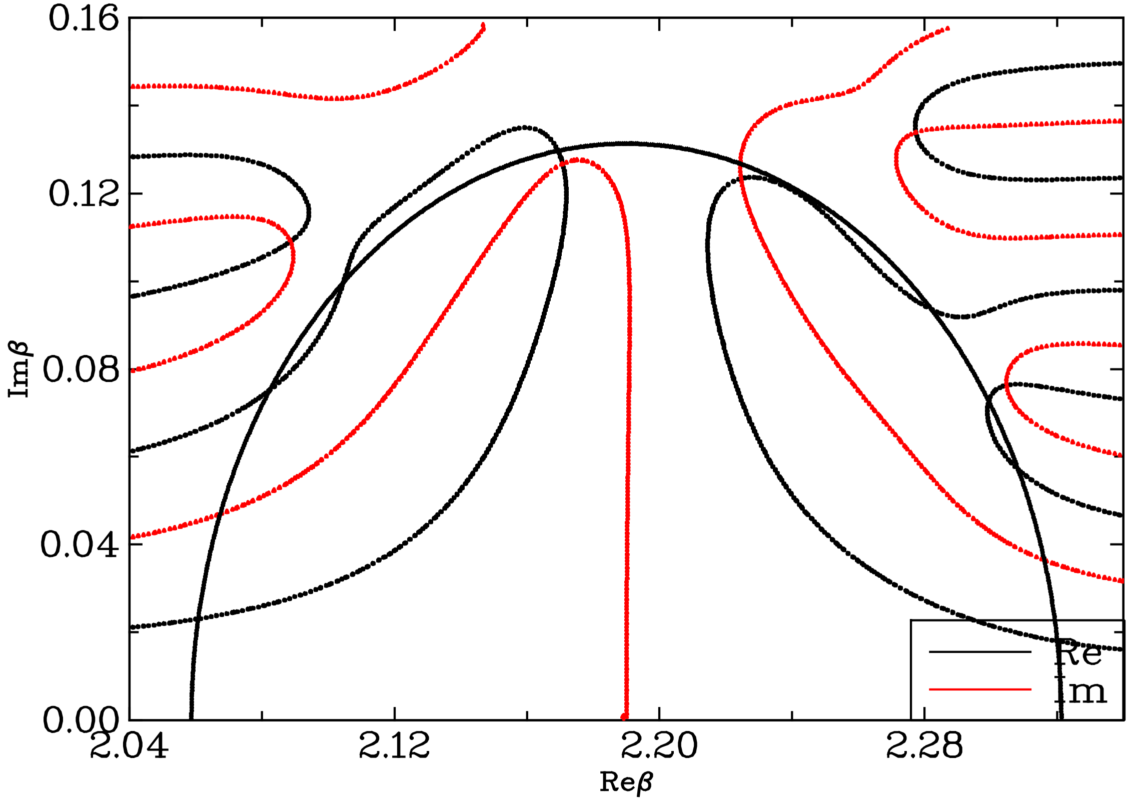


Figure 1: Zeros are calculated with 40,000 configurations of the SU(2) gauge field in 4^4 lattices. $\beta = 2.19$. The zero reported by Falcioni is at $2.23 + i0.155(1982)$.

Error Bars of the Zeros

"$Z(\beta) = 0$" is not exact. It gives rise to the errors in the zeros' locations.

$$\Delta f(\beta) = \frac{\partial f}{\partial \beta_R} \Delta\beta_R + \frac{\partial f}{\partial \beta_I} \Delta\beta_I$$

Let σ_R, σ_I be the standard deviations of Re $\left\langle e^{-\beta S} \right\rangle$, Im $\left\langle e^{-\beta S} \right\rangle$ respectively

$$a \equiv \partial_{\beta_R} Re \left\langle e^{-\beta S} \right\rangle, \; b \equiv \partial_{\beta_R} Im \left\langle e^{-\beta S} \right\rangle$$

The fluctuations of complex β are

$$\Delta\beta_R = \frac{a\sigma_R + b\sigma_I}{a^2 + b^2}, \; \Delta\beta_I = \frac{a\sigma_R - b\sigma_I}{a^2 + b^2}$$

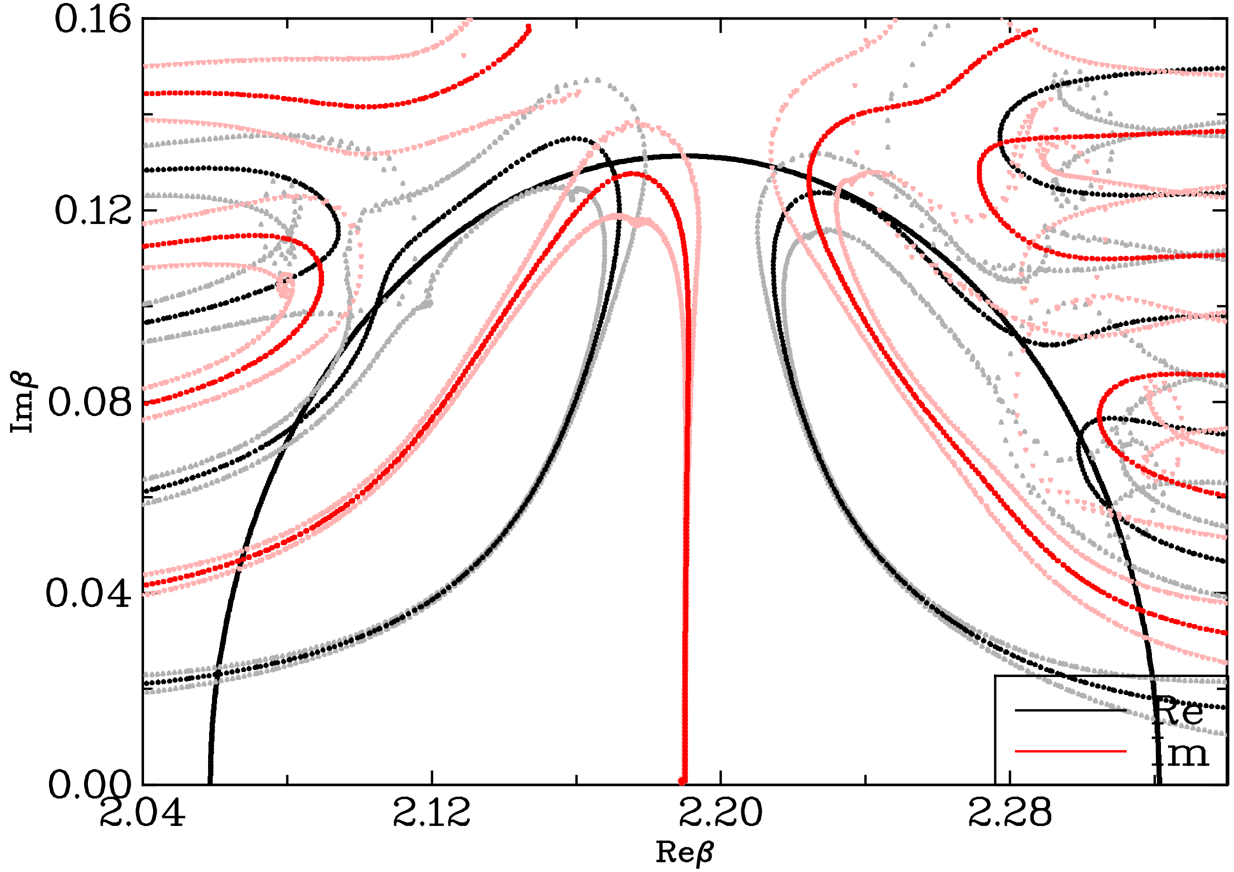


Figure 2:

Quasi-Gaussian distribution

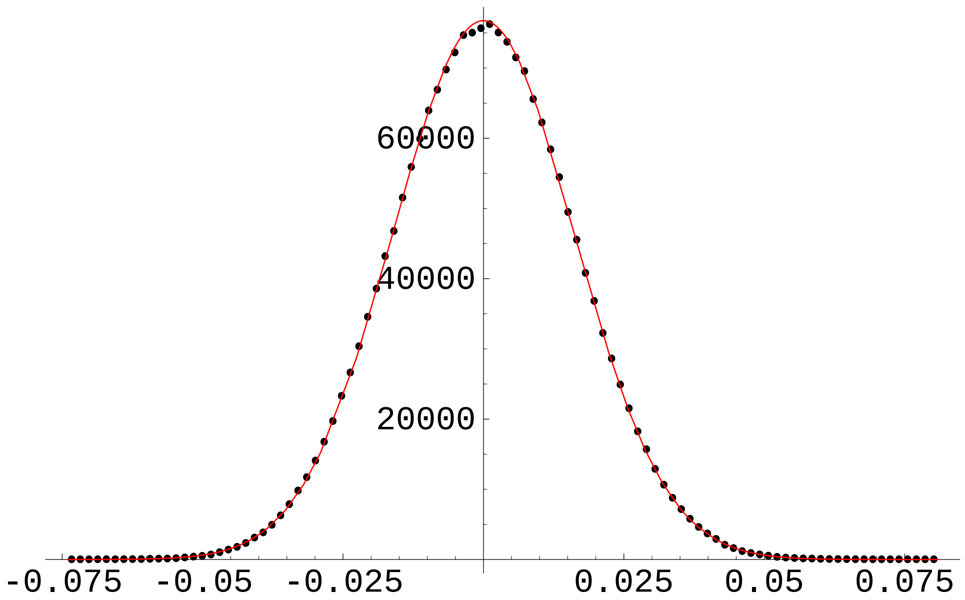


Figure 3: The histogram of bining 160,000 configurations for SU(2) gauge field at $\beta = 2.18$.

Approximate models

The small deviation of the distribution from the Gaussian distribution(for small lattices) suggests

$$P(S) \propto \exp(-\lambda_1 S - \lambda_2 S^2 - \lambda_3 S^3 - \lambda_4 S^4) \quad (4)$$

Use the standard unit $S' = (S - \langle S \rangle)/\sigma$. The unknown parameters were determined by fitting.

- Using the first four moments via Newton's methods;
- χ^2 minimization.

Very good agreement between the two methods was found on 4^4 lattices.

Testing with examples

Example 1: $\lambda_3 = 0.1,\ \lambda_4 = 0.01$. It has been chosen in such a way that we have zeros inside and outside the Gaussian region of confidence.

Example 2: $\lambda_3 = 0.01,\ \lambda_4 = 0.002$ The perturbation is much smaller and the first accurate zero is far away from the Gaussian circle of confidence.

Use MC simulation method to generate 400,000 configurations. Then make the fits and search for the zeros.

Example 1

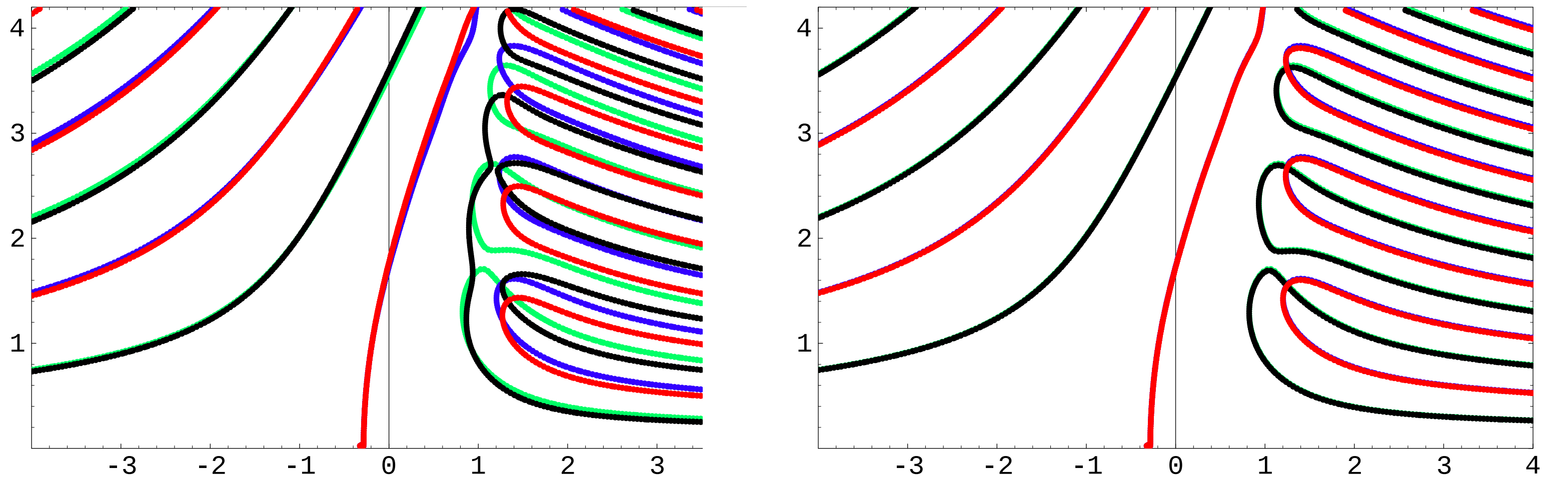


Figure 4: The result of quasi-Gaussian fitting is $\exp(0.00048x - 0.498x^2 - 0.0995x^3 - 0.00989x^4)$, compared with $\exp(-0.5x^2 - 0.1x^3 - 0.01x^4)$.

Example 2

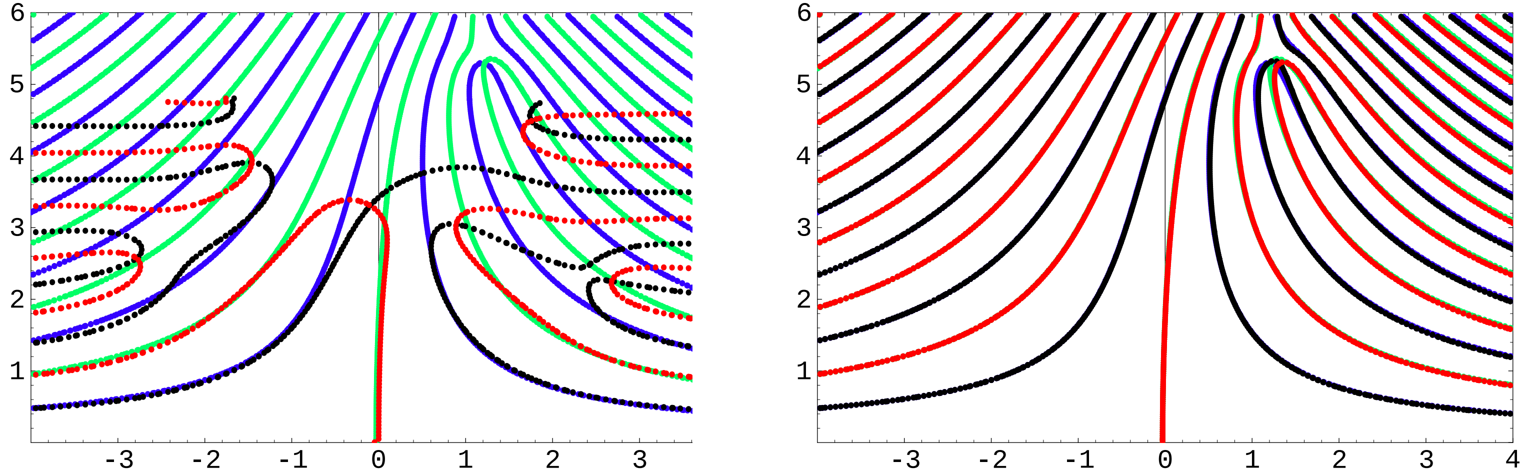


Figure 5: The result of quasi-Gaussian fitting result is $\exp(0.0011x - 0.499x^2 - 0.0106x^3 - 0.00195x^4)$, compared with $\exp(-0.5x^2 - 0.01x^3 - 0.002x^4)$

Reweighting + Parametrization on SU(2) 4^4

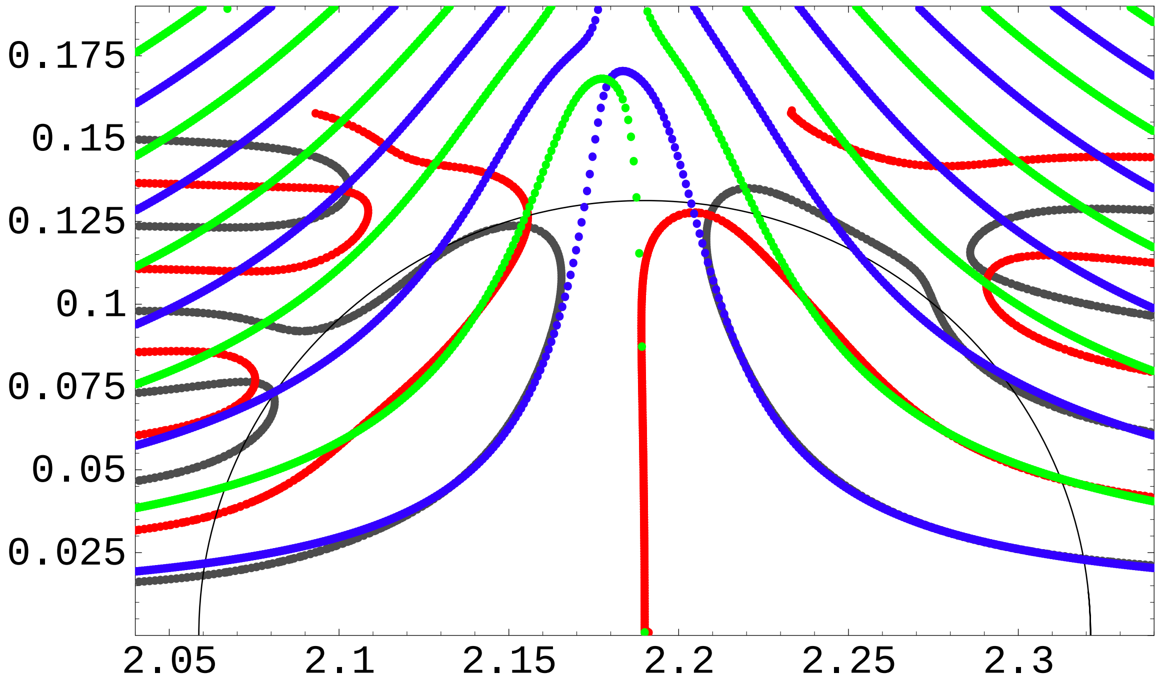


Figure 6: $\beta_0 = 2.19$.

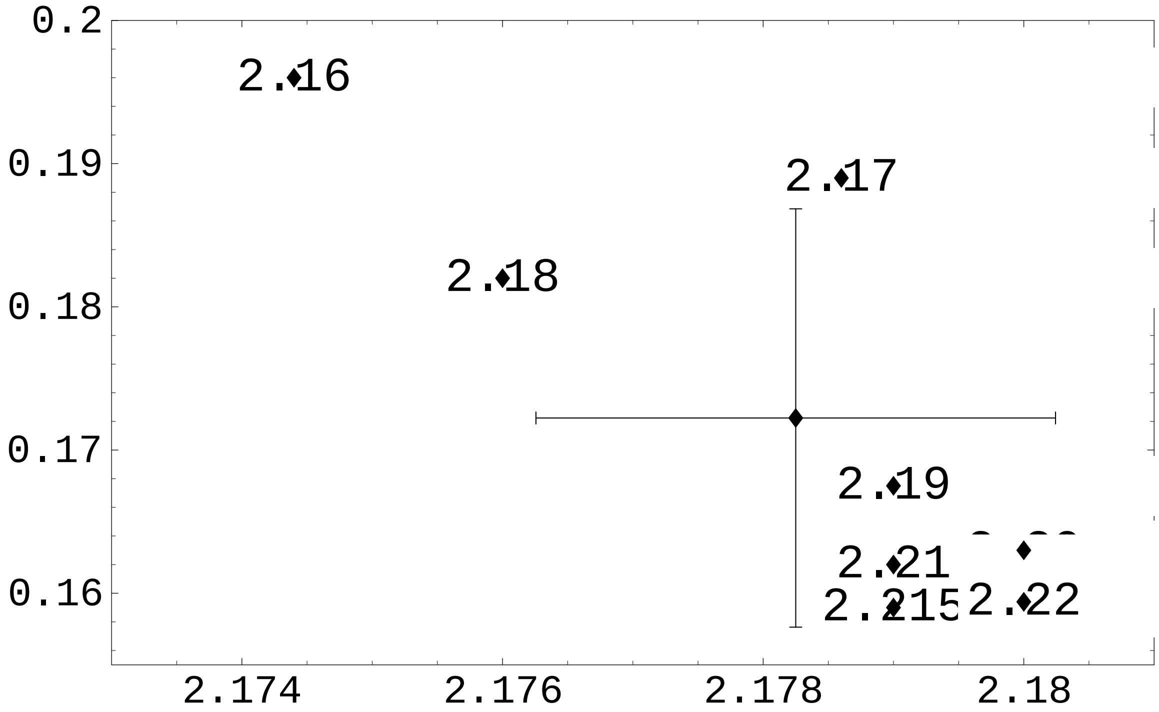


Figure 7: $\beta = 2.178(2) \pm 0.17(2)$ for $SU(2)$ (differ from Falcioni 2.23+i0.155 obtained with MC outside regions of confidence).

Summary and work in progress

- We used the reweighting method to find the Fisher's zeros in the SU(2) lattice gauge field. We showed that direct calculations could not give the reliable locations of the zeros and that quasi-Gaussian fitting worked well in finding the zeros.

- Check selfconsistency of the parametrization at different β.

- The work on 6^4, 8^4 ... is to be done to see the variation in the locations of the zeros.