

INT Summer School on "Lattice QCD and its applications"

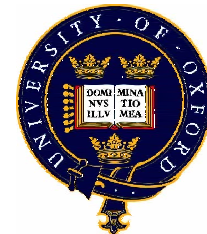
The Closed String Spectrum of $SU(N)$ gauge Theories in 2+1 Dimensions

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I. Introduction.

General question:

→ What effective string theory describes flux tubes in $SU(N)$ gauge theories?

Two cases:

→ Open string

→ Closed string

During the last decade:

→ $3D, 4D$ with $Z_2, Z_4, U(1), SU(N \leq 6)$ (Caselle and collaborators, Gliozzi and collaborators, Kuti and collaborators, Lüscher&Weisz, Majumdar and collaborators, Teper and collaborators, Meyer)

Questions to be studied in $D = 2 + 1$ dimensional $SU(N)$ theories:

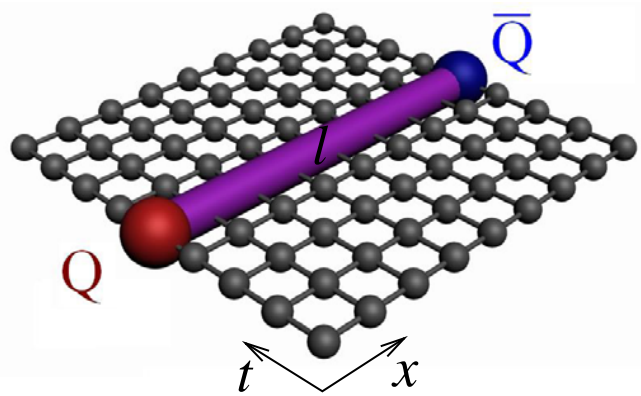
→ Calculation of excited states, and states with $p_{||} \neq 0$ and $P = \pm$

→ What is the degeneracy pattern of these states?

→ What is the leading correction in E_n^2 ?

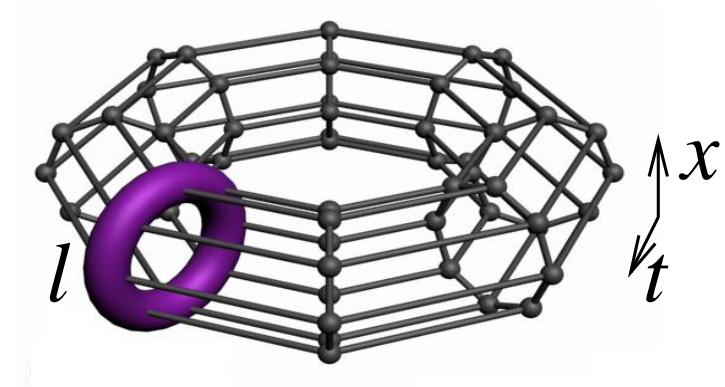
I. Introduction.

Open flux tube



$$\Phi(l, t) = \phi^\dagger(0, t)U(0, l; t)\phi(l, t)$$

Closed flux tube



$$\Phi(l, t) = \text{Tr}U(l; t)$$

periodic b.c
→

I. Introduction.

General question:

→ What effective string theory describes flux tubes in $SU(N)$ gauge theories?

Two cases:

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→ Closed string ← in $D = 2 + 1$

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II. Theoretical expectations A.

The Spectrum of the Nambu-Goto (NG) String Model

○ Action of Nambu-Goto free bosonic string leads to:

→ Spectrum given by:

$$E_{N_L, N_R, q, w}^2 = (\sigma l w)^2 + 8\pi\sigma \left(\frac{N_L + N_R}{2} - \frac{D-2}{24} \right) + \left(\frac{2\pi q}{l} \right)^2 + p_{\perp}^2.$$

→ Described by:

1. The winding number w ($w=1$),
2. The winding momentum $p_{\parallel} = 2\pi q/l$ with $q = 0, \pm 1, \pm 2$,
3. The transverse momentum p_{\perp} ($p_{\perp} = 0$),
4. N_L and N_R connected through the relation: $N_R - N_L = qw$.

$$N_L = \sum_{k>0} \sum_{n_L(k)>0} n_L(k)k \quad \text{and} \quad N_R = \sum_{k'>0} \sum_{n_R(k')>0} n_R(k')k'$$

→ String states are eigenvectors of P (In $D = 2 + 1$) with eigenvalues:

$$P = (-1)^{\sum_{i=1}^m n_L(k_i) + \sum_{j=1}^{m'} n_R(k'_j)}$$

II. Theoretical expectations B.

Effective string theory

- First prediction for $w = 1$ and $q = 0$ (Lüscher, Symanzik&Weisz. 80):

$$E_n = \sigma l + \frac{4\pi}{l} \left(n - \frac{D-2}{24} \right) + \mathcal{O}(1/l^2).$$

- Lüscher&Weisz effective string action (Lüscher&Weisz. 04):

→ Using open-closed string duality:

* For any D the $\mathcal{O}(1/l^2)$ ($1/l$) (Boundary term) is absent from $E_n(E_n^2)$

→ Spectrum in $D = 2 + 1$ (Drummond '04, Dass and Matlock '06 for any D):

$$E_n = \sigma l + \frac{4\pi}{l} \left(n - \frac{1}{24} \right) - \frac{8\pi^2}{\sigma l^3} \left(n - \frac{1}{24} \right)^2 + \mathcal{O}(1/l^4)$$

→ Equivalently:

$$E_n^2 = (\sigma l)^2 + 8\pi\sigma \left(n - \frac{1}{24} \right) + \mathcal{O}(1/l^3),$$

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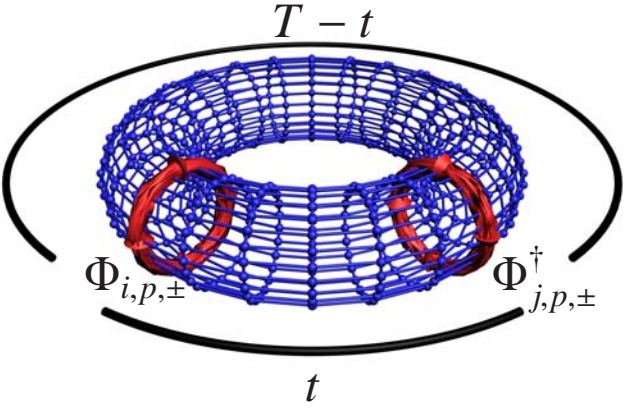
$$E_n^2 = E_{NG}^2 + \mathcal{O}(1/l^3) \longrightarrow \text{Fit : } E_{\text{fit}}^2 = E_{NG}^2 - \sigma \frac{C_p}{(l\sqrt{\sigma})^p} \quad (p \geq 3)$$

III. Lattice Calculation

Our approach:

- Construct a large basis of operators (80 – 200) with transverse deformations.
- Calculate the correlation matrix $C_{ij,p,\pm}(t) = \langle \Phi_{i,p,\pm}^\dagger(t) \Phi_{j,p,\pm}(0) \rangle$.
- Use the variational technique to extract correlators of different states.
- Fit our results using single cosh fits, and look at large t mass plateaus.

Example:

$\phi_L = \text{Tr} \left\{ \begin{array}{c} \downarrow \\ \square \\ \uparrow \end{array} \right\} \text{ and } \phi_R = \text{Tr} \left\{ \begin{array}{c} \uparrow \\ \square \\ \downarrow \end{array} \right\}$ $\Phi_{p,\pm} = \frac{1}{L_{\parallel} L_{\perp}} \sum_{x_{\parallel}, x_{\perp}} \{ \phi_L \pm \phi_R \} e^{ipx_{\parallel}}$	 <p>The diagram shows a blue lattice cylinder. Two red loops are wrapped around the cylinder. The top loop is labeled $\Phi_{i,p,\pm}$ and the bottom loop is labeled $\Phi_{j,p,\pm}^\dagger$. A black arrow above the cylinder is labeled $T - t$ and a black arrow below is labeled t.</p>
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III. Lattice Calculation

Monte-Carlo:

- We define our gauge theory on a 3D discretized periodic Euclidean space-time with $L \times L_{\perp} \times L_T$ sites.
- We choose to use the standard Wilson action:

$$S_W = \beta \sum_P \left[1 - \frac{1}{N} \text{ReTr} U_P \right]$$

with: $\beta = \frac{2N}{ag^2}$.

- Simulation:
 - We use a mixture of Kennedy-Pendelton heat bath and over-relaxation steps for updating $SU(2)$ subgroups of $SU(N)$.
 - We use Cabibbo-Marinary for updating $SU(N)$.

III. Lattice Calculation: Operators for $P = +$

$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$\times 5 bl$
$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$f(L_{\parallel}, L_{\perp}, bl)$
$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$\times 5 bl$	$\times 5 bl$

III. Lattice Calculation: Operators for $P = -$

$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$
$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$		
$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$	$\times 5 \text{ bl}$

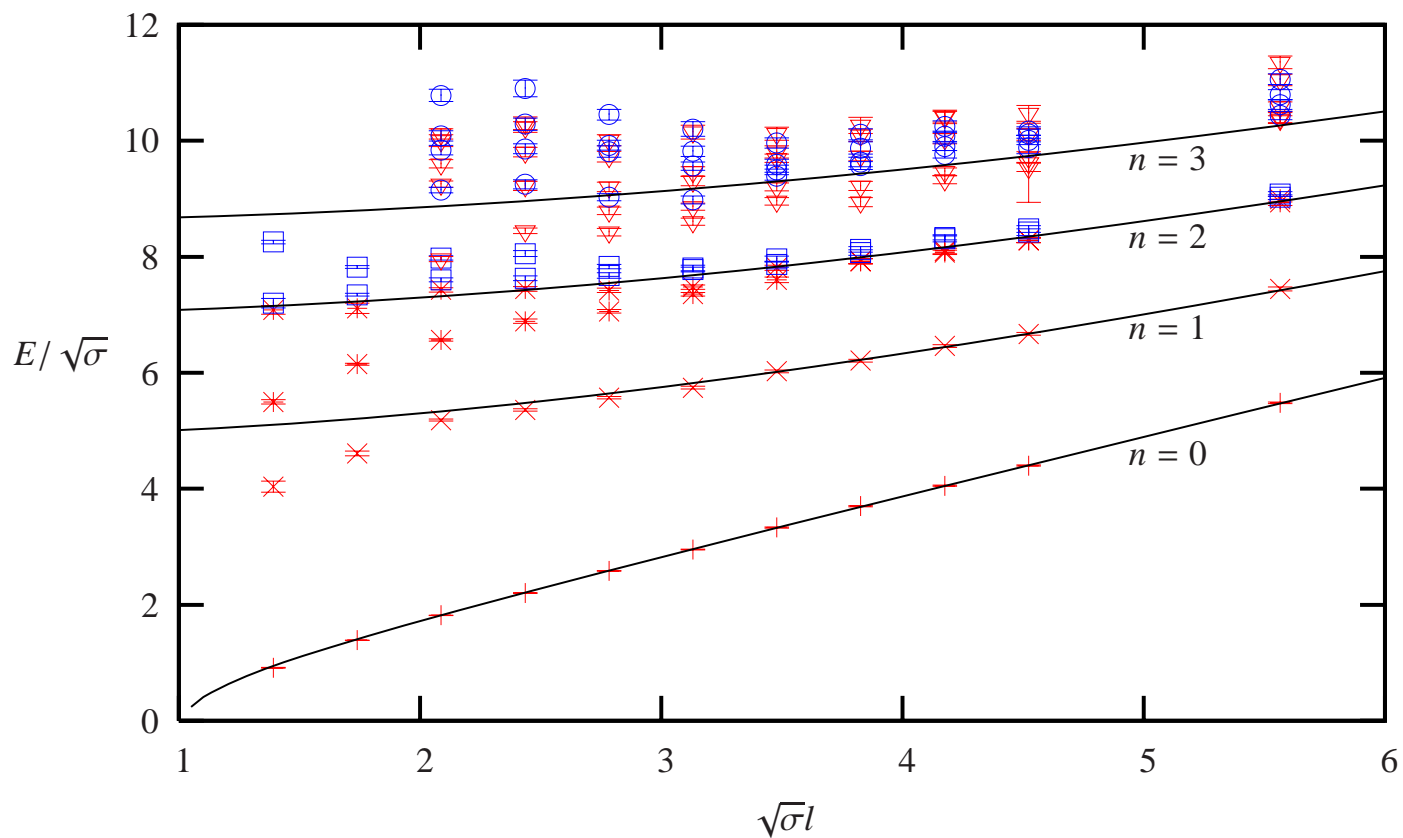
III. Lattice Calculation: Large Basis of Operators

Using this large basis of operators:

- We extract masses of **excited states**.(up to 15 states)
- It increases the **Overlaps** (using single exponential fits):
 - Ground state $\sim 99 - 100\%$,
 - First excited state $\sim 98 - 100\%$ ($\sim 90 - 95$ with just $\times 5bl$),
 - Second excited state $\sim 95 - 99\%$ ($\sim 85 - 90$ with just $\times 5bl$),
- We can extract energies of **non-zero winding momentum** states.
- We can extract energies of $P = -$ states.
- It increases **computational time** moderately.(ex. $\times 6$ for $L = 16a$)

IV. Results: Spectrum of $SU(3)$ and $\beta = 21.0$

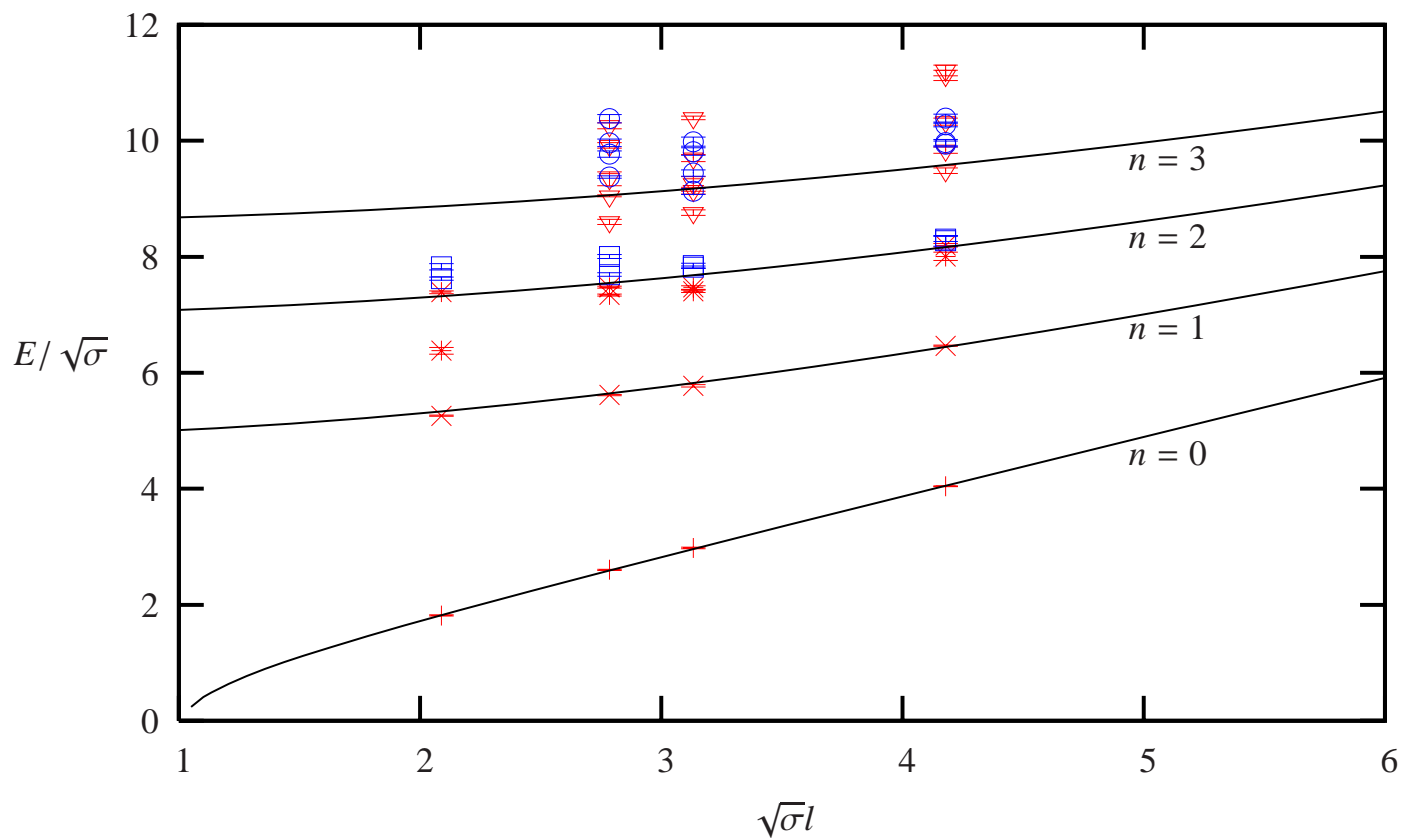
Group: $SU(3)$, $\underline{a} \simeq 0.08 fm$, Quantum Numbers: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma \left(n - \frac{1}{24} \right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(3)$ and $\beta = 40.0$

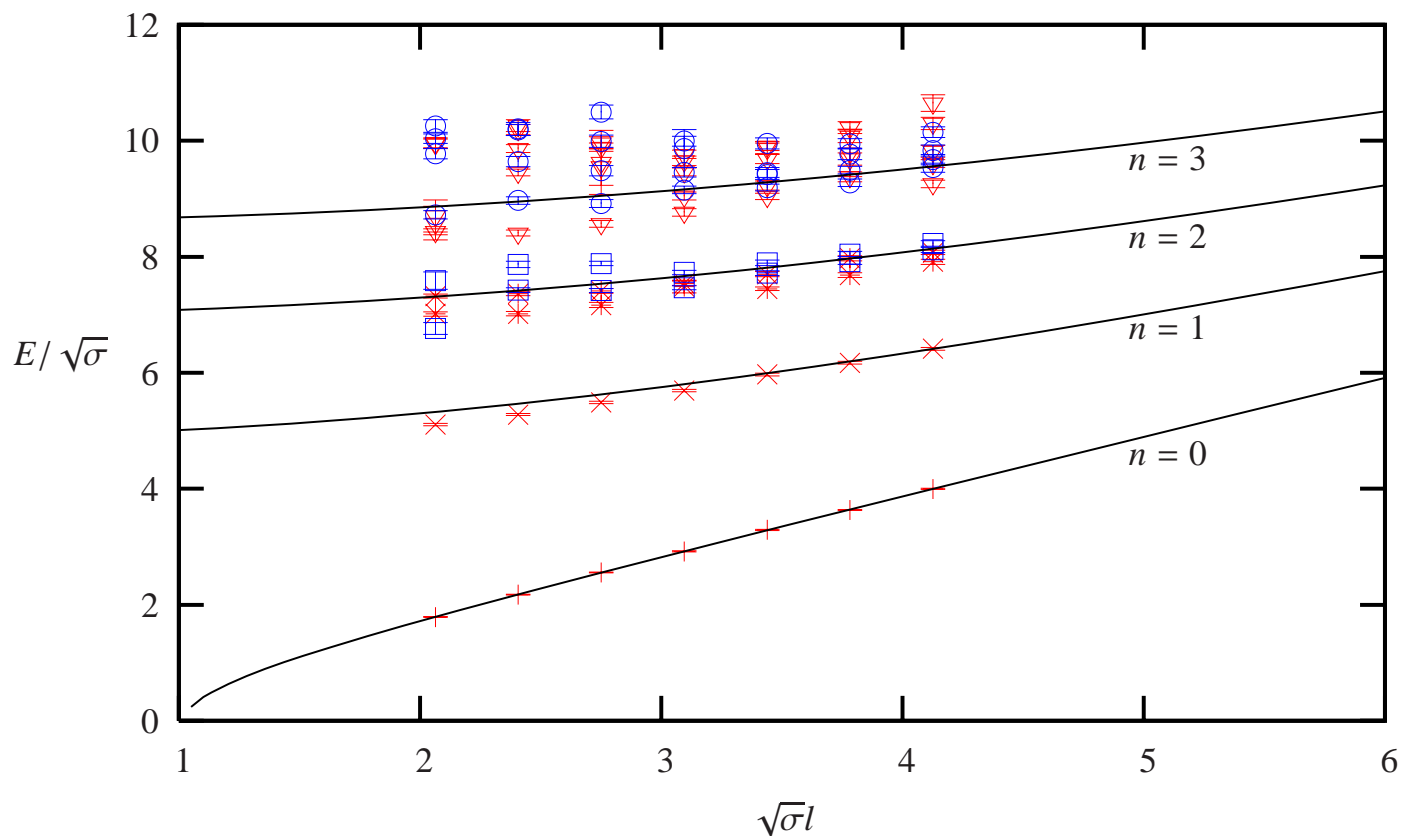
Group: $SU(3)$, $\underline{a} \simeq 0.04 fm$, Quantum Numbers: $P = +, -$ and $q = 0$



NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma \left(n - \frac{1}{24}\right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(6)$ and $\beta = 90.0$

Group: $SU(6)$, $\underline{a} \simeq 0.08 fm$, Quantum Numbers: $P = +, -$ and $q = 0$

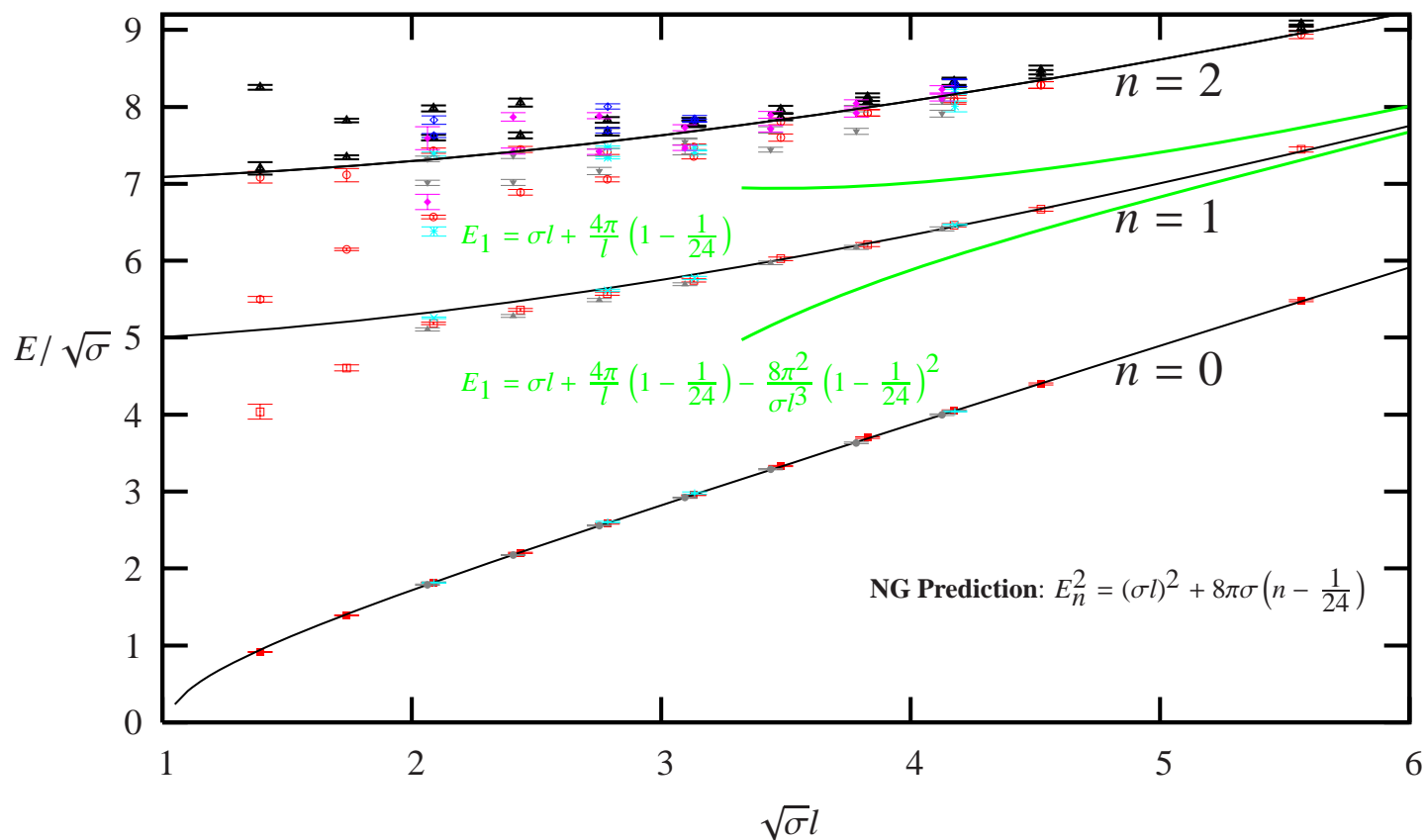


NG Prediction: $E_n^2 = (\sigma l)^2 + 8\pi\sigma \left(n - \frac{1}{24} \right)$, where $n = N_L = N_R$ since $q = 0$.

IV. Results: Spectrum of $SU(N)$

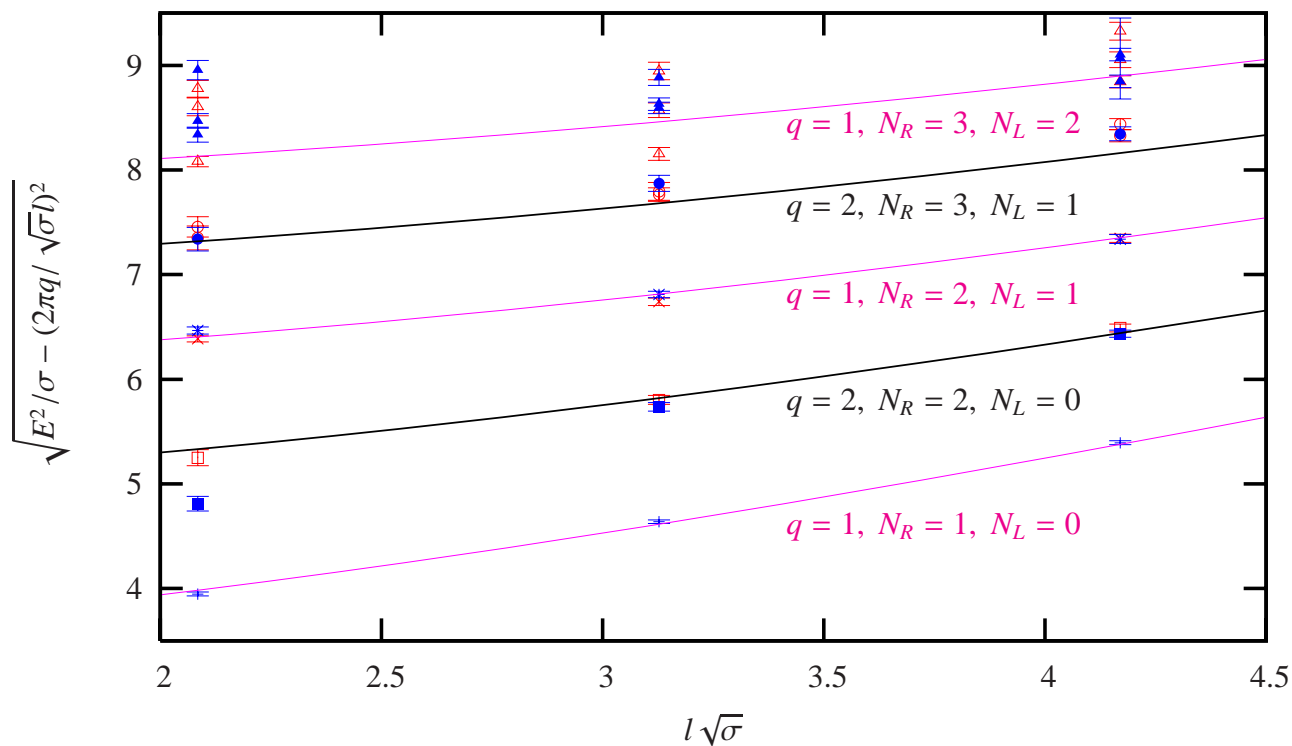
Groups: $SU(3)$ and $SU(6)$, $\underline{a} \simeq 0.04fm$ and $0.08fm$,

Quantum Numbers: $P = \pm$ and $q = 0$



IV. Results: Non-zero winding momentum.

Group: $SU(3)$, $\underline{a} \simeq 0.08 fm$, Quantum Numbers: $P = +, -, q = 1, 2$ and $w = 1$



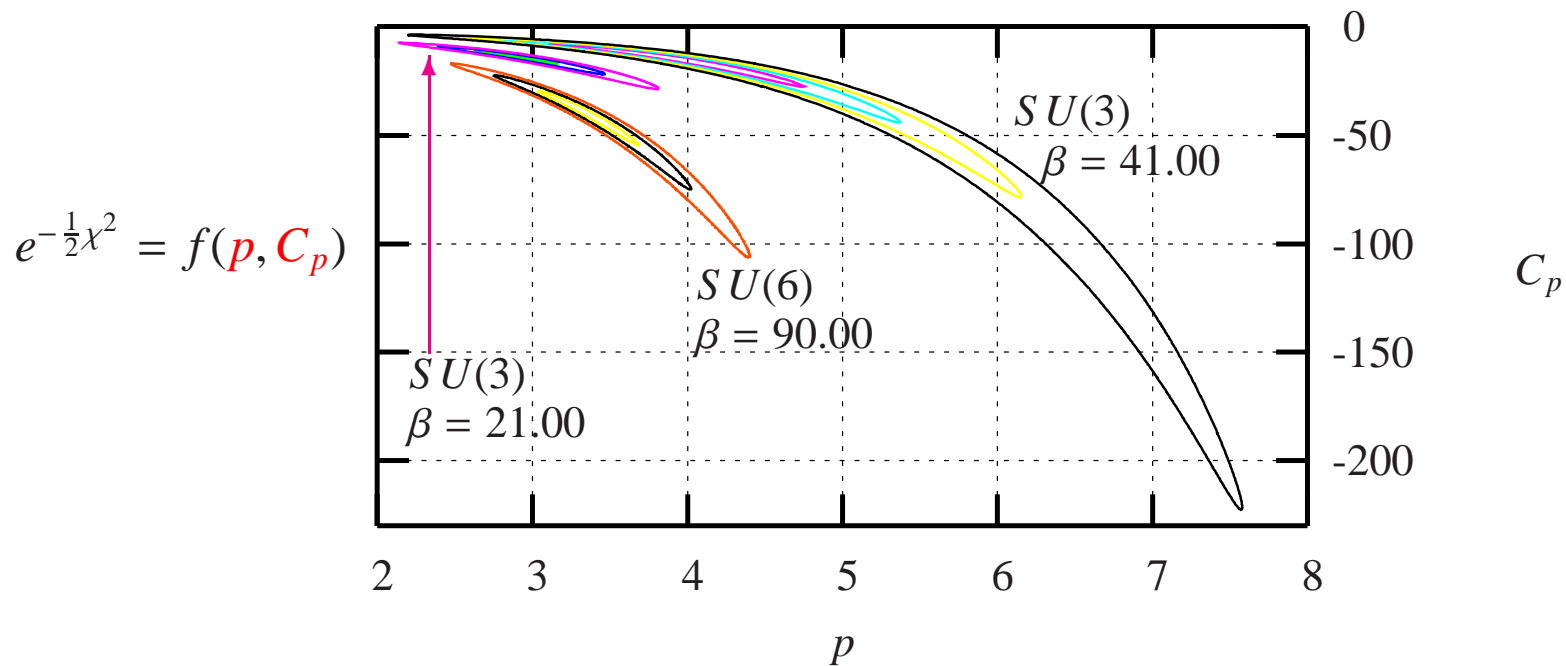
NG Prediction: $E^2 - (2\pi q/l)^2 = (\sigma l w)^2 + 8\pi\sigma \left(\frac{N_R + N_L}{2} - \frac{D-2}{24} \right)$.

Constraint: $N_R - N_L = qw$

IV. Results: Fits.

Ansatz: $E_{\text{fit}}^2 = E_{NG}^2 - \sigma \frac{C_p}{(l\sqrt{\sigma})^p},$

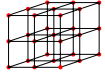
String tension: Calculated using the ground state, fixing $p = 3$



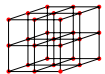
First excited states exclude $p = 1$ (Boundary term)!

Also second excited states exclude $p = 1$

V. Summary



We constructed a large basis of operators characterized by the quantum numbers of **parity P** , and **winding momentum $2\pi q/l$** ,

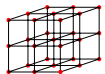


We calculated the energies of closed flux tubes in $D=2+1$ described by $P = \pm$ for:

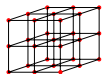
→ $SU(3)$ with $\beta = 21.0$ ($a \simeq 0.08\text{fm}$) and $q = 0, \pm 1, \pm 2$,

→ $SU(3)$ with $\beta = 40.0$ ($a \simeq 0.04\text{fm}$) and $q = 0$,

→ $SU(6)$ with $\beta = 90.0$ ($a \simeq 0.08\text{fm}$) and $q = 0$,

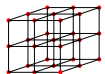


We fit our data for the ground state using $E_{\text{fit}}^2 = E_{NG}^2 - \sigma C_p / (l \sqrt{\sigma})^p$ and $p = 3$, and extract σ .



Using σ we compare our results to Nambu-Goto:

→ **Nambu-Goto is VERY good**



We fit our data for the 1^{st} and 2^{nd} excited states with E_{fit}^2 , and extract p

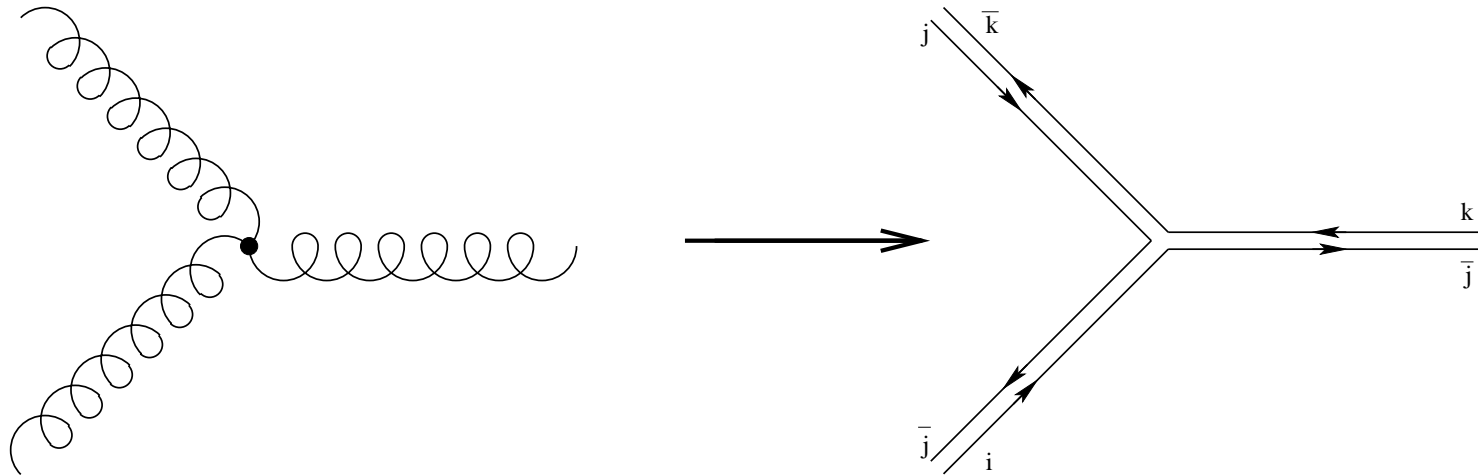
→ 1^{st} and 2^{nd} excited states exclude $p = 1$ (boundary term [Lüscher&Weisz. 04](#))

VI. Appendix 1: Large- N_c

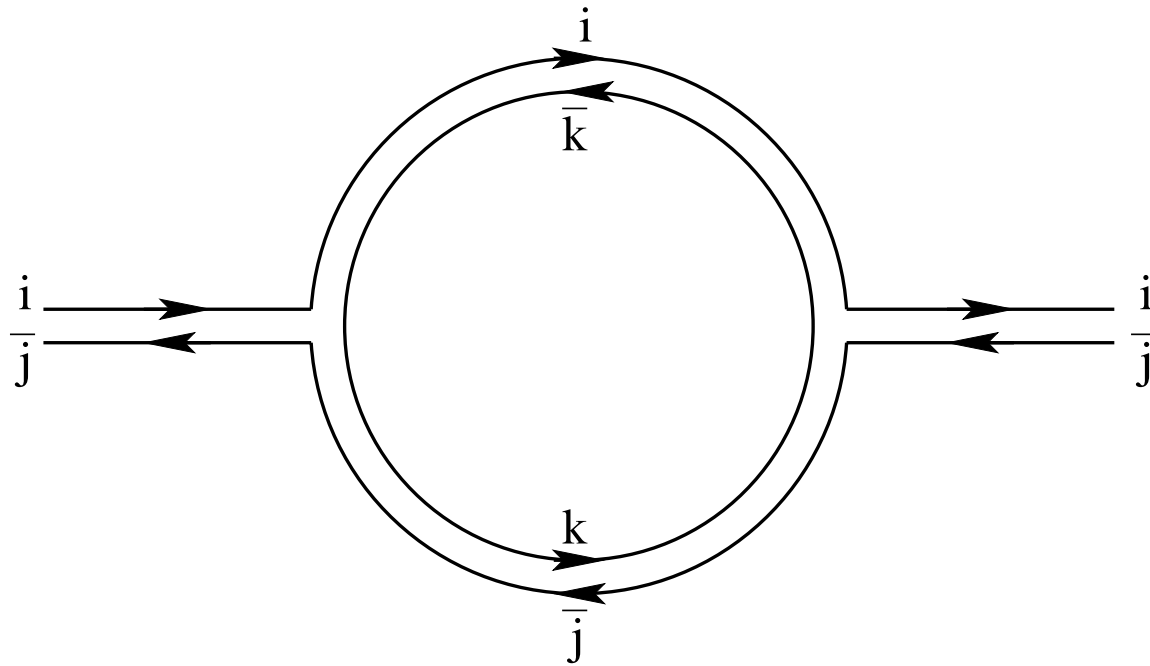
- Yet some of the essential properties of QCD including confinement, are poorly understood.
- It is useful to find a 'Neighbouring' field theory that one can analyze more simply.
- 'Neighbouring Theory' $\longrightarrow SU(N_c \longrightarrow \infty)$.
- Expansion parameter $\longrightarrow \frac{1}{N_c}$.
- Planar diagrams contribute in the Large-N limit
- Non-Planar diagrams vanish.

VI. Appendix 1: 't Hooft's coupling

- 't Hooft's coupling: $\lambda = g^2 N_c$
- 't Hooft's double line diagrammatic representation:

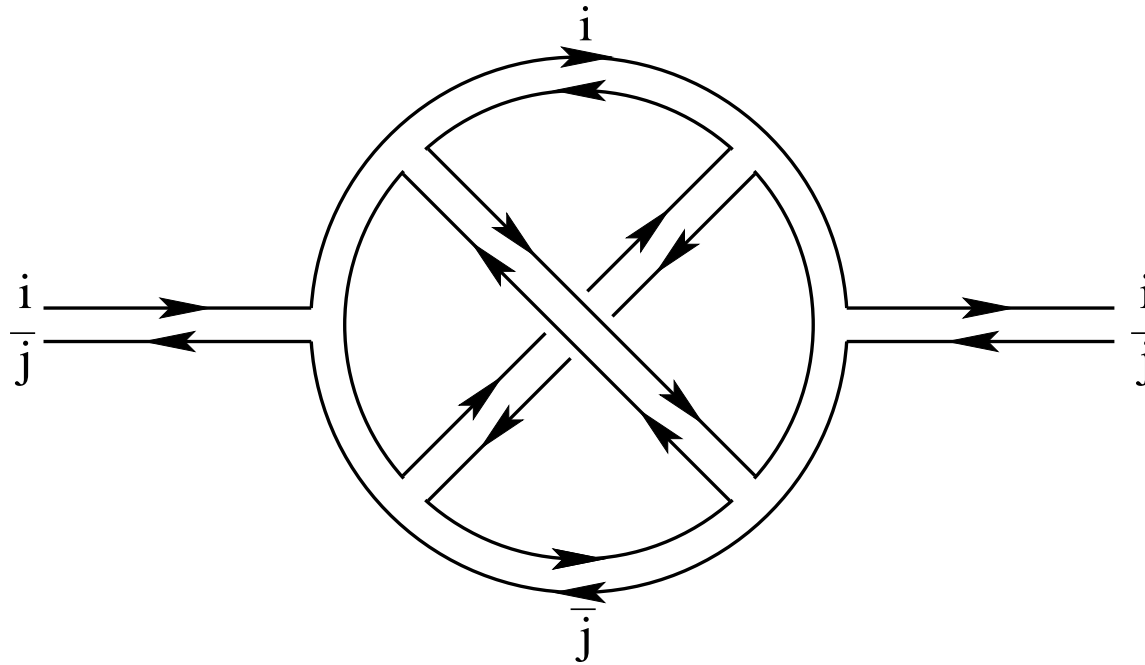


VI. Appendix 1: Planar Diagram



$$\sim g^2 \times N_c (1 \text{ closed loop}) = \frac{\lambda}{N_c} \times N_c = \lambda$$

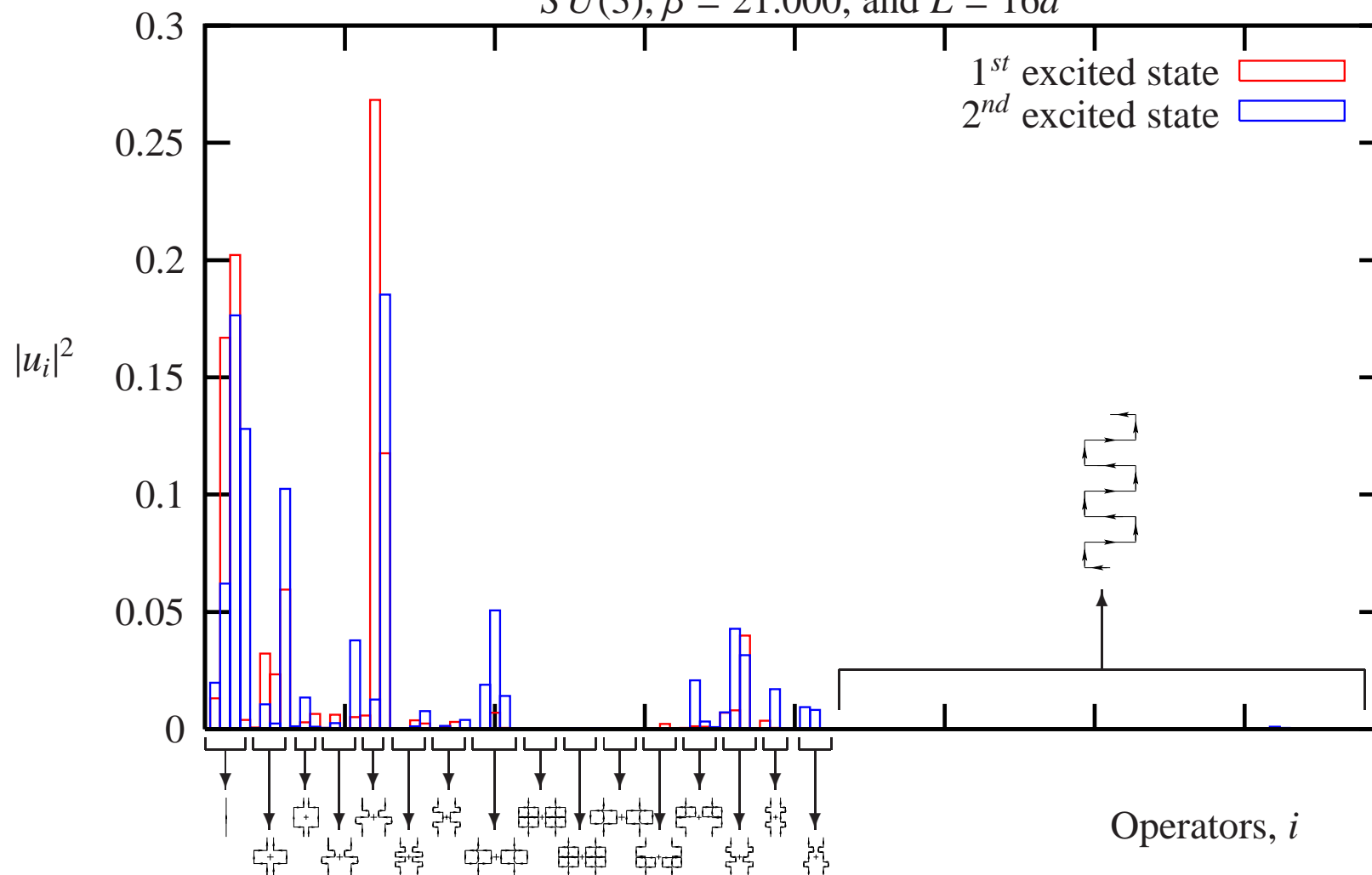
VI. Appendix 1: Non-Planar Diagram



$$\sim g^6 \times N_c (1 \text{ closed loop}) = \frac{\lambda^3}{N_c^3} \times N_c = \frac{\lambda^3}{N_c^2} \xrightarrow{N_c \rightarrow \infty} 0$$

VI. Appendix 2: Contribution of Operators

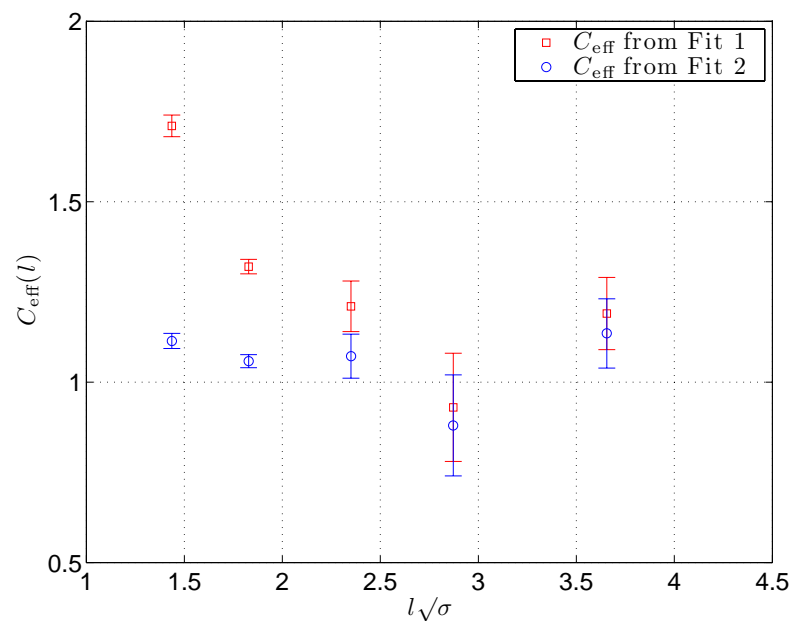
$SU(3)$, $\beta = 21.000$, and $L = 16a$



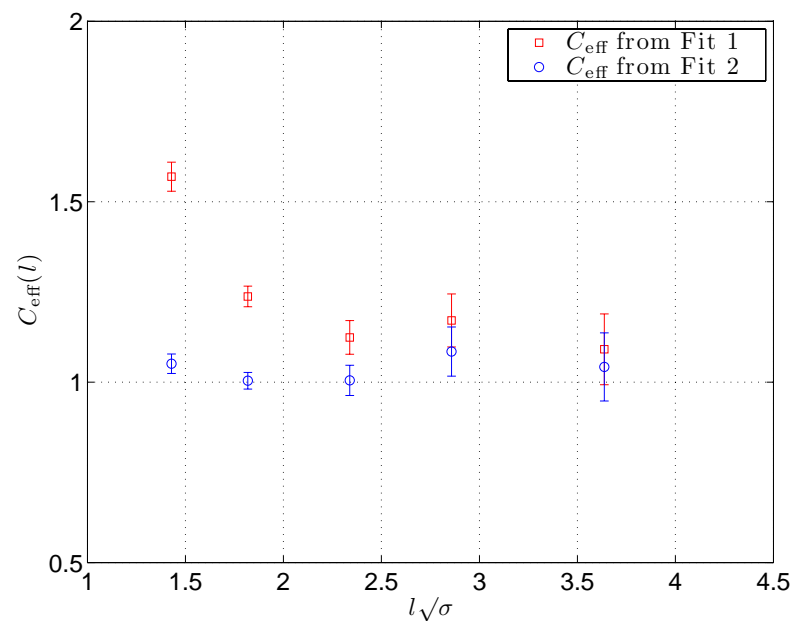
VI. Appendix 3: Why E_n^2 ?

Ground state:

$SU(3)$ and $\beta = 14.7172$



$SU(5)$ and $\beta = 80.00$

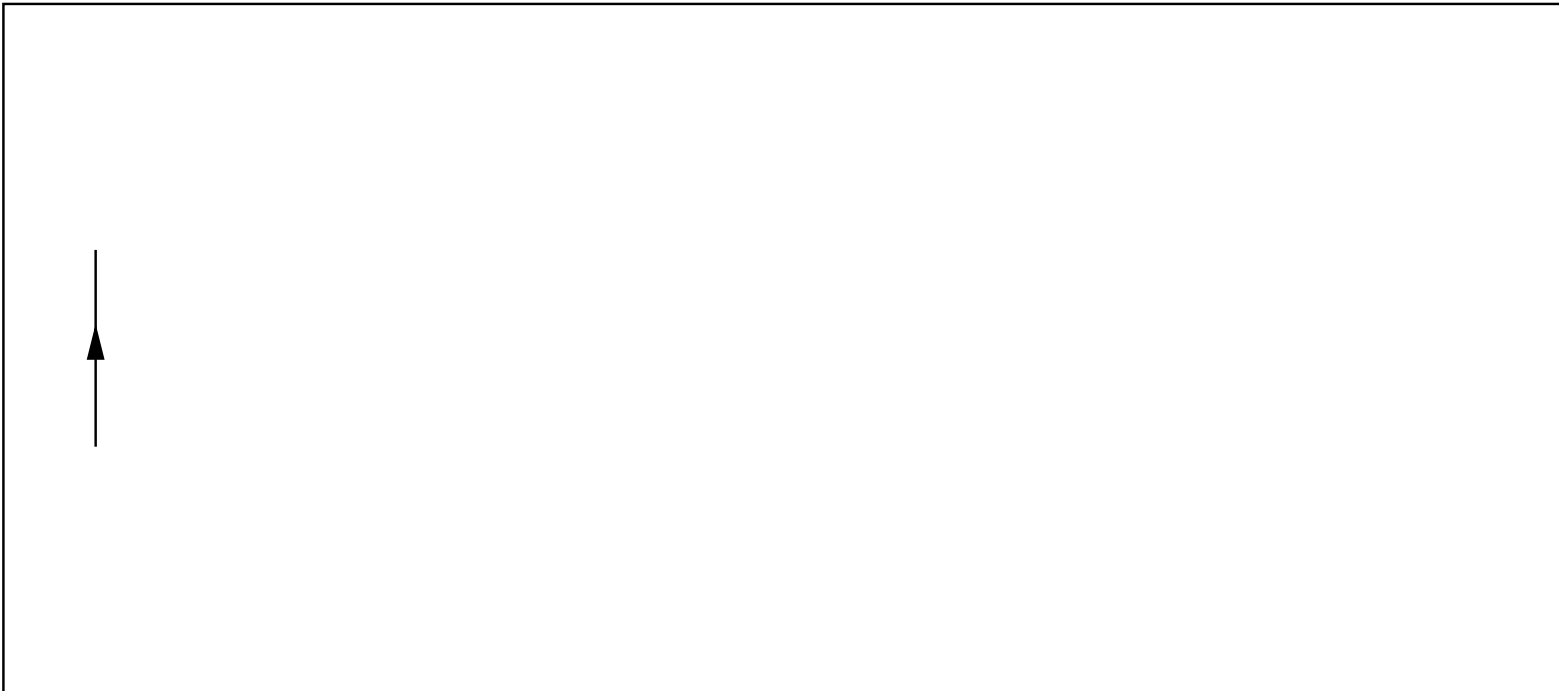


Fit 1: $E_0(l, \sigma) = \sigma l - \frac{\pi}{6l} \times C_{\text{eff}}^{(1)}(l)$

Fit 2: $E_0^2(l, \sigma) = (\sigma l)^2 - \frac{\pi\sigma}{3} \times C_{\text{eff}}^{(2)}(l).$

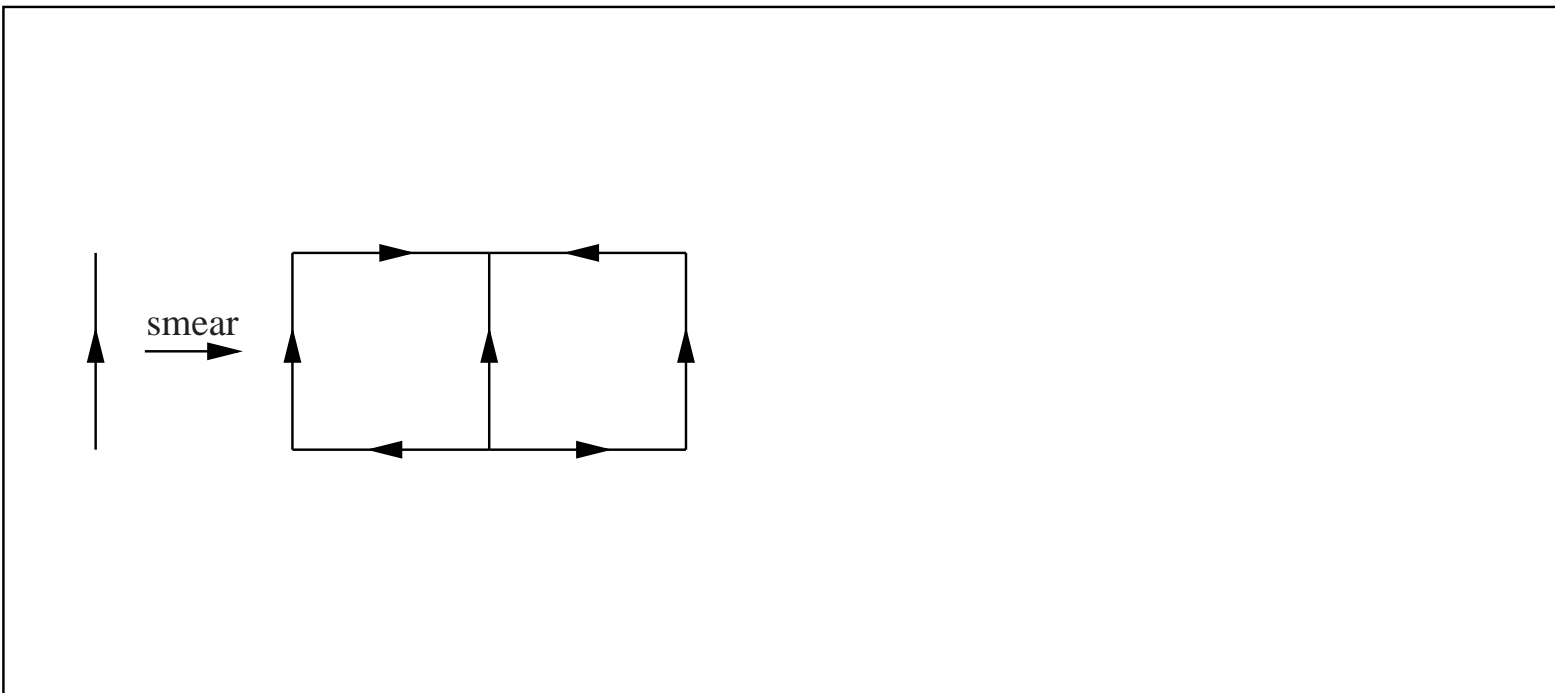
VII. Blocking-Smearing

- One needs operators that extend over physical length scales and are smooth on such scales.
- Iterative 'fuzzing' algorithm:



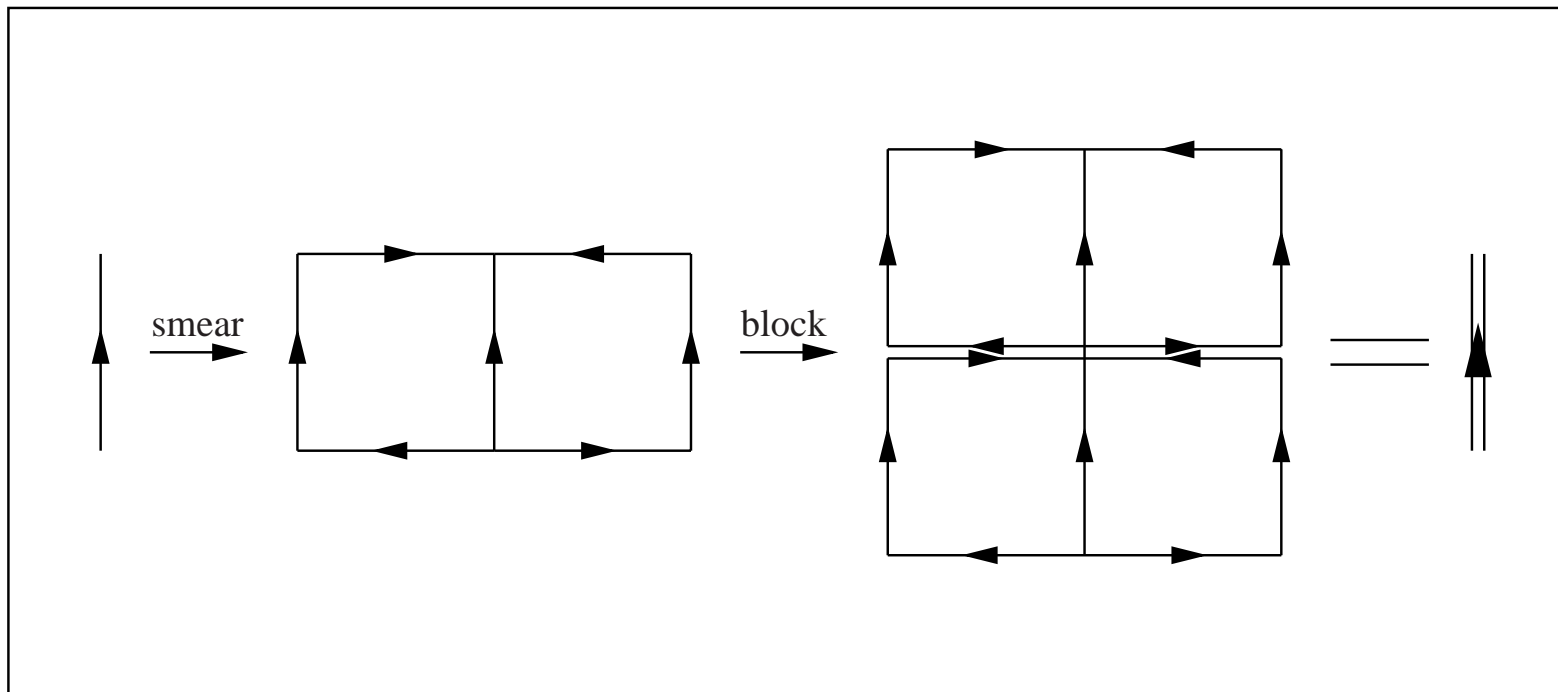
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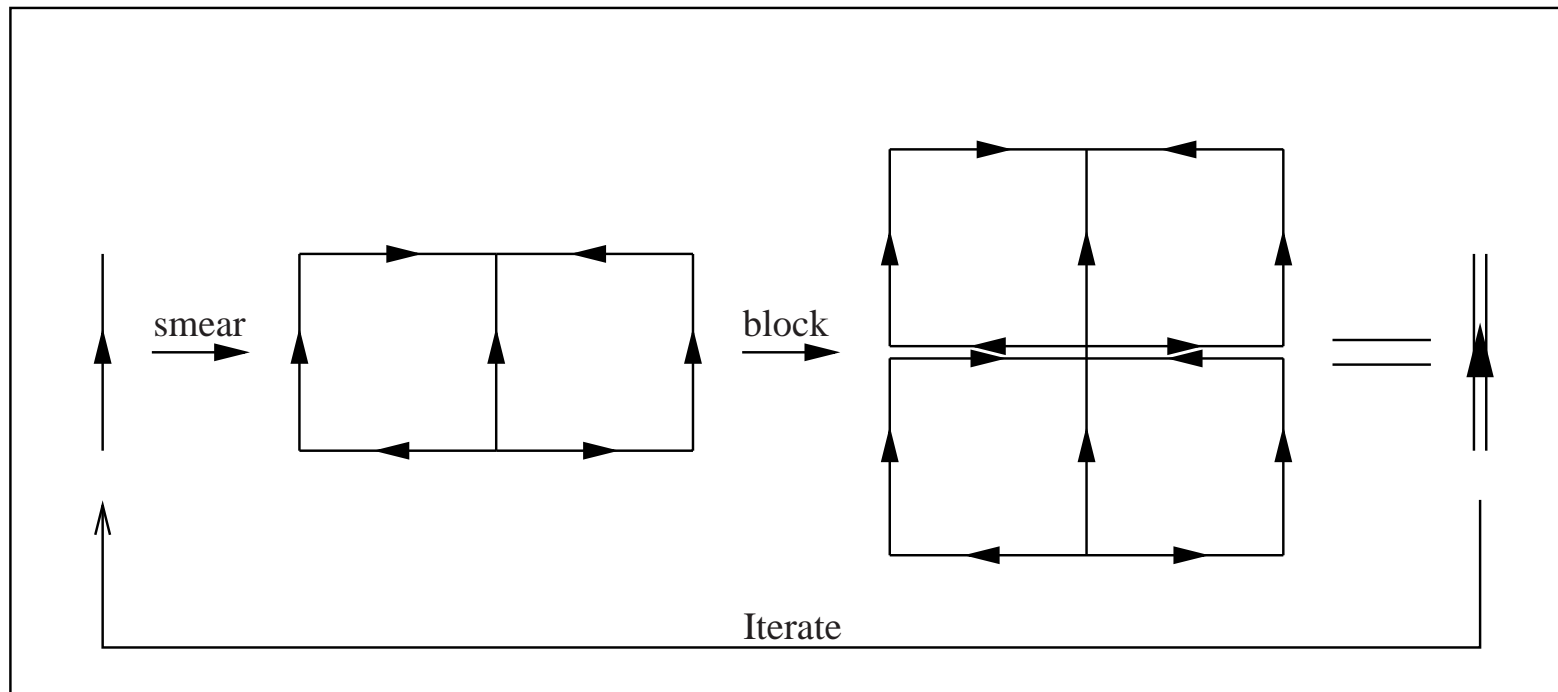
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VIII. References

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