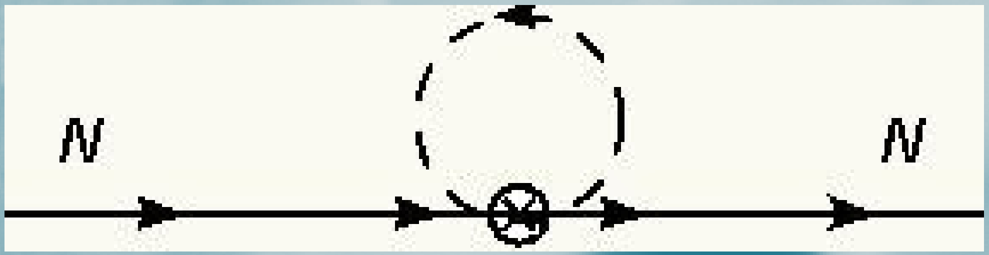

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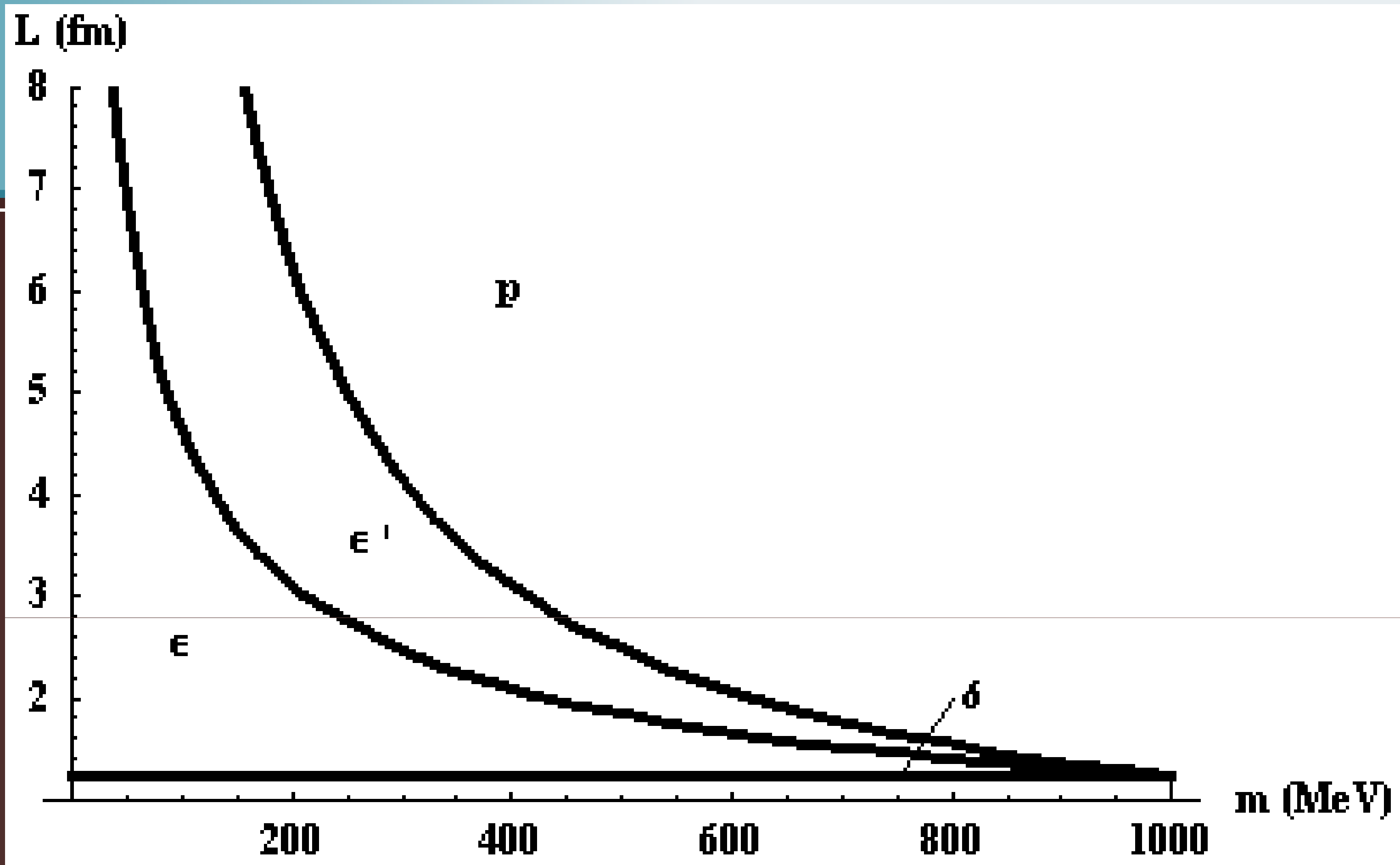
Applications of the ε-regime to Lattice QCD

or How I Learned to Stop Worrying and Love the Zero Modes

The ε-regime

- Nucleon lattice calculations occur at finite volume and unphysical quark masses. We need formulas to extrapolate to physical quark masses and infinite volume.
- To achieve this, we want to calculate observables in HBChPT (see Martin's lecture tomorrow) near the chiral limit and in small volumes. This is the ε-regime.
- Power counting in this regime is similar to the p-regime but with $m_\pi \sim \varepsilon^2$ in ε counting.
- In this regime pion zero momentum modes go as $1/m_\pi^2 V$ and dominate the path integral.
- Zero modes must be treated as $\mathcal{O}(1)$ contributions and calculated exactly.



β=L

These lines should not be interpreted as strict boundaries but rather as midpoints in the smooth transition between regimes.

Zero Modes

- In (2 flavor) ChPT pions exist within the SU(2) matrix (see lectures by C. Bernard):

$$\Sigma = Exp\left[2iM / f\right]$$

$$M = \begin{pmatrix} \pi_0 / \sqrt{2} & \pi_+ \\ \pi & -\pi_0 / \sqrt{2} \end{pmatrix}$$

- To separate the zero modes a change of variables is made (following Gasser & Leutwyler):

$$\Sigma(x) \rightarrow U\hat{\Sigma}(x)U$$

- Here the constant matrix U encapsulates the zero mode pions while $\hat{\Sigma}$ contains order ε fields.

Nucleons and Zero Modes

- Typically in HBChPT, pions and nucleons interact via (among other terms):

$$L_{\text{int}} = 2g_A \bar{N} S^\mu A_\mu N$$

$$A_\mu = \tfrac{i}{2}\left(\xi \partial_\mu \xi^+ - \xi^+ \partial_\mu \xi\right)$$

$$\xi \equiv \sqrt{\Sigma}$$

- So we must determine how ξ changes under the zero mode change of variables.
- There are two (equivalent) ways of doing this.

ξ Change of Variables

Mimic a Chiral Transformation

- The original change of variables mimics a chiral transformation:

$$\Sigma \rightarrow L\hat{\Sigma}R^{+}$$

$$L = R^{+} = U$$

$$\Rightarrow \xi \rightarrow V\hat{\xi}U \text{ or } U\hat{\xi}V^{+}$$

$$V = \sqrt{U\hat{\Sigma}U}U^{+}\hat{\xi}^{+}$$

$$V^{+} = \hat{\xi}^{+}U^{+}\sqrt{U\hat{\Sigma}U}$$

Direct Substitution

- One can put the change of variables directly into the square root:

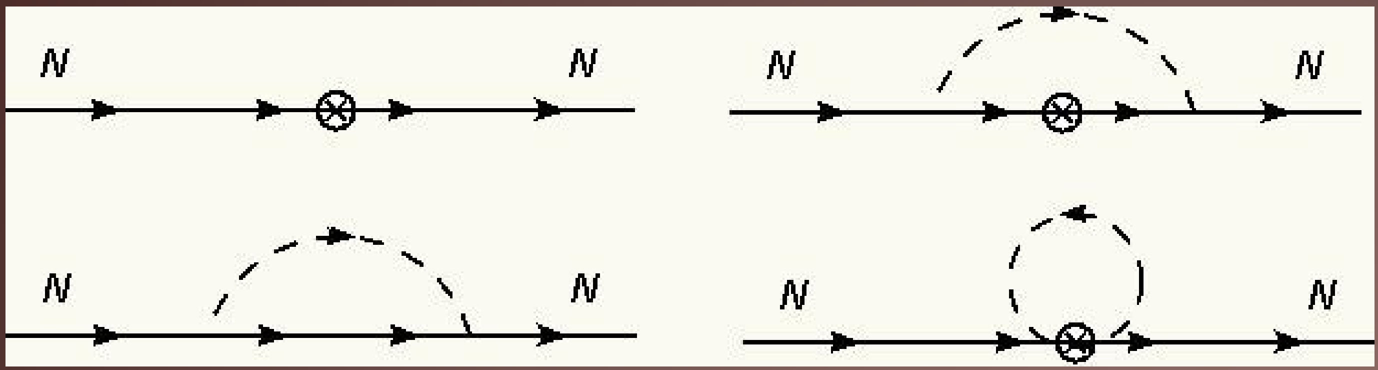
$$\xi \rightarrow \sqrt{U\hat{\Sigma}U}$$

Square Root of a Matrix

- With either method the square root of a matrix must be computed.
- No analytic method exists for doing this with an arbitrary SU(2) matrix.
- Can be done order by order in ε:
  1. Assume $\sqrt{U\hat{\Sigma}U} = U + \frac{i\varepsilon}{f}A - \frac{\varepsilon^2}{2f^2}B + \ldots$ for unknown A, B.
  2. Then $U\hat{\Sigma}U = U^2 + \frac{i\varepsilon}{f}\left(UA + AU\right) - \frac{\varepsilon^2}{2f^2}\left(UB + BU + 2A^2\right) + \ldots$
  3. Parameterize U in hyperspherical coordinates (SU(2) manifold is unit hypersphere).
  4. Expand $\hat{\Sigma}$ in ε and solve order by order in ε for unknown matrices.
- The resulting matrix can be used to get zero modes for a given Feynman diagram in HBChPT.

Example: The Axial Charge

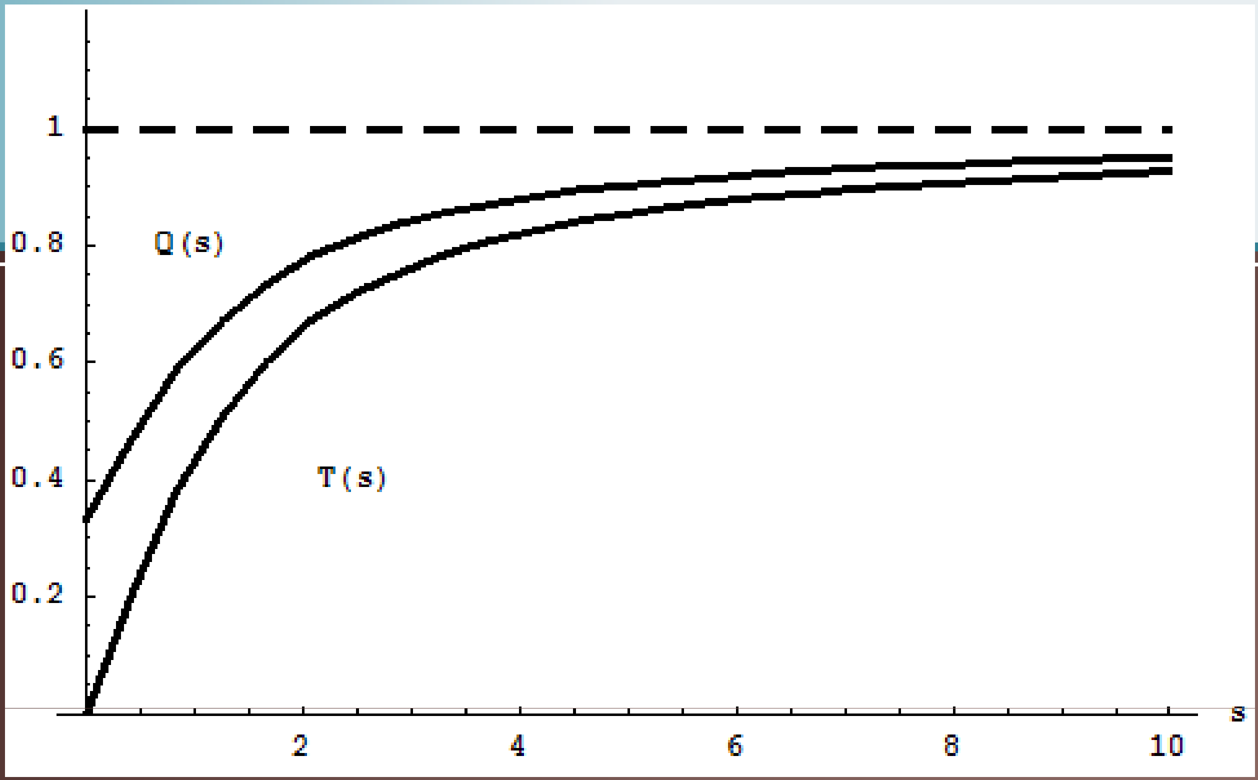
- We can use this method now to calculate the axial charge between two nucleons:



- Under the change of variables one has:

$$\bar{N}S^{\mu}A_{\mu}N \rightarrow \bar{N}S^{\mu}V\hat{A}_{\mu}V^{+}N$$

- At lowest order in ε, V=1 and so πNN vertices carry no zero mode information.
- However, zero mode information will be carried by the current insertion and the tadpole diagram.
- After integrating out the zero modes each of these graphs will be multiplied by a function containing the zero mode information.



- Q(s) is the zero mode function for the first 3 diagrams previously, T(s) is the function for the tadpole diagram.

$$s = \frac{1}{2} m_\pi^2 f^2 \beta L^3$$

- One caveat to this procedure: the group integration can only be accomplished on a diagram-by-diagram basis, and a local, zero mode free, Lagrangian cannot be formed.
- With these zero mode functions one can then proceed on to calculate the finite volume observable in the normal way.

Summary

- Lower quark mass lattice calculations are coming! The ε-regime era for nucleons is nigh! Be prepared!
- Other observables can be calculated with this method (any you can think of), to any order in ε desired.
- More details and shameless self-promotion:

Smigielski & Wasem, arXiv:0706.3731

HBChPT Lagrangian

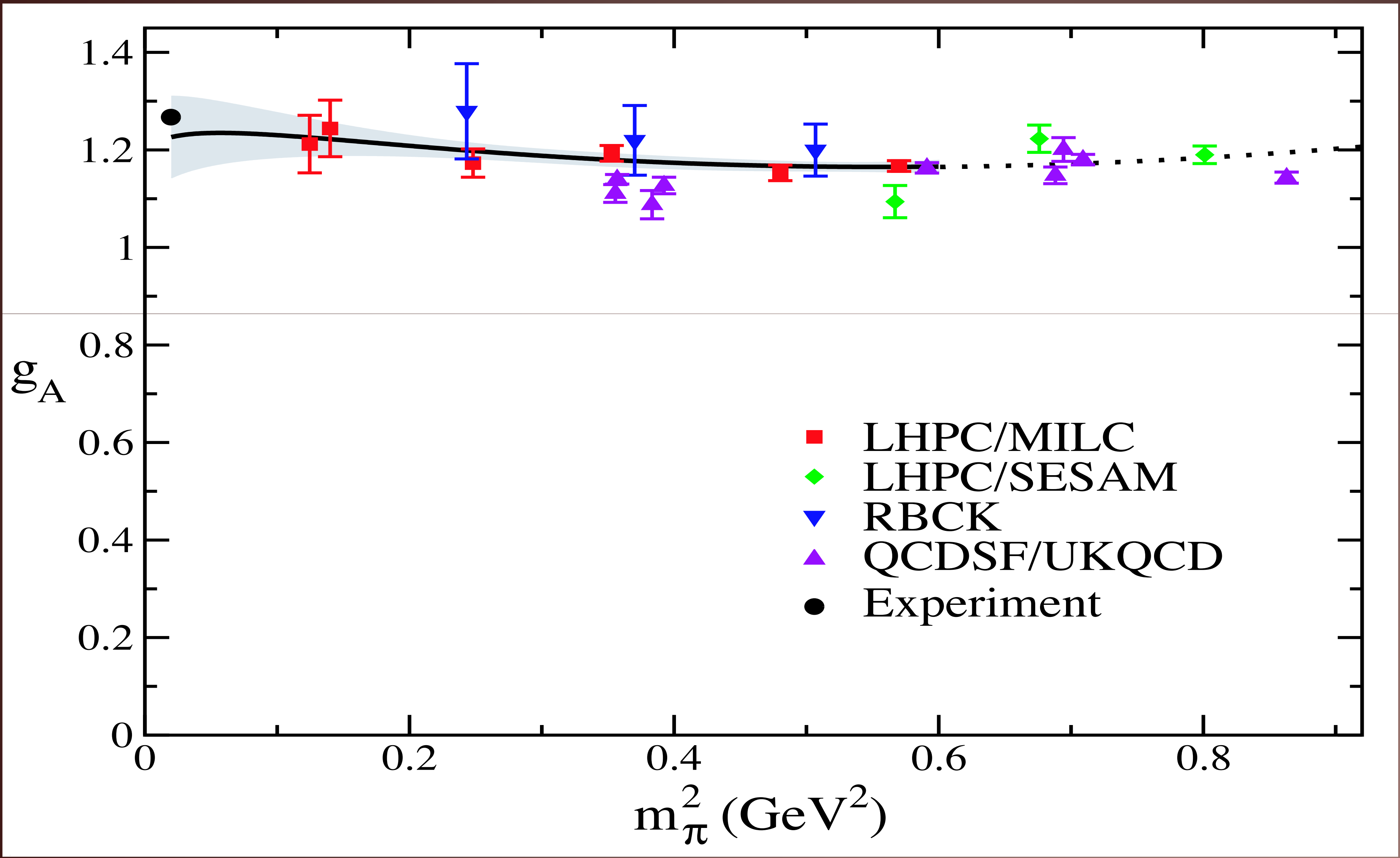
$$L = \bar{N} i v \cdot D N - \bar{T}^{\mu} i v \cdot D T_{\mu} + \Delta \bar{T}^{\mu} T_{\mu} + \frac{f^2}{8} \mathrm{Tr}\left(\partial^{\mu} \Sigma^{+} \partial_{\mu} \Sigma\right)$$

$$+ \lambda \bar{m} \frac{f^2}{4} \mathrm{Tr}\left(\Sigma + \Sigma^{+}\right) + 2 g_A \bar{N} S^{\mu} A_{\mu} N$$

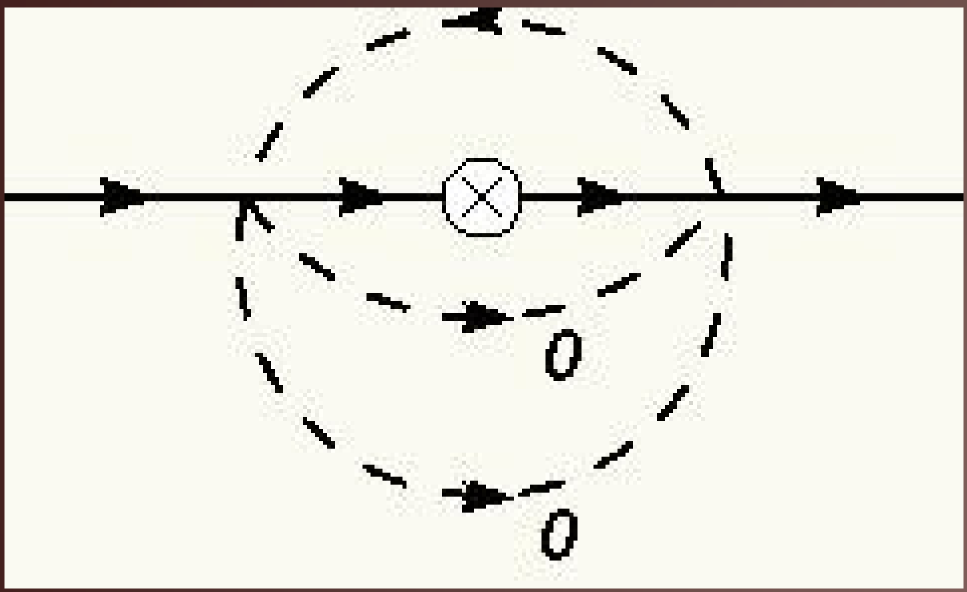
$$+ g_{\Delta N}\left(\bar{T}^{abc,\mu} A^{d}_{a,\mu} N_b \varepsilon_{cd} + \mathrm{h.c.}\right) + 2 g_{\Delta\Delta} \bar{T}^{\mu} S^{\nu} A_{\nu} T_{\mu}$$

$$A_{\mu} = \frac{i}{2}\left(\xi \partial_{\mu} \xi^{+} - \xi^{+} \partial_{\mu} \xi\right)$$

g_A Calc. (look at the masses!)



Nonlocal Lagrangian



- Which Vertex/Line do the zero modes belong to?
- Zero mode integration does not return a simple correction to a vertex or propagator.
- So no local Lagrangian after integration.