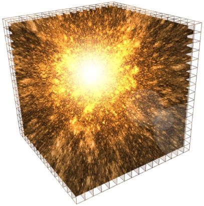

Real time lattice simulations *and* heavy quarkonia beyond deconfinement

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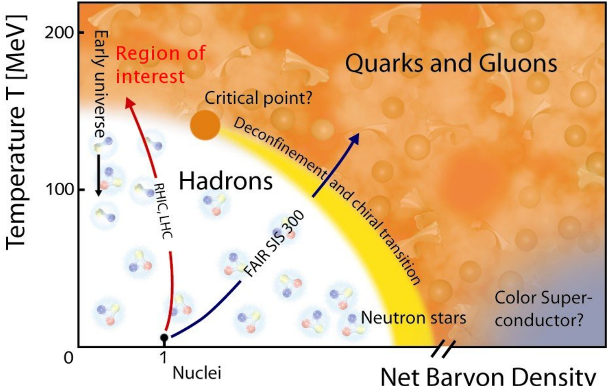


(The expected phase diagram of nuclear matter, Source: GSI)

Heavy quarkonia beyond deconfinement?

The potential of the strong interactions at T=0

Description of heavy quarkonia via a Schrödinger equation

Due to the high mass of the constituents ($m_q \gg E_{kin}$) heavy quarkonia can be described by a nonrelativistic Schrödinger equation:

$$i\partial_t \psi = (-\frac{\triangle}{2\mu} + \hat{V})\psi$$

Phenomenological potential

$$\hat{V} = k\hat{r}^{\,1)} - \frac{4}{3}\frac{\alpha_s(\hat{r})}{\hat{r}}^{\,2)}$$

1. Linear part: Due to the formation of a flux tube (predicted by strong coupling lattice QCD, k $\approx$ 1GeV/fm for $c\bar{c}$)
2. Coulomb part: similar to QED (predicted by perturbation theory, $\alpha_s \approx 0.15 - 0.25$)

Beyond deconfinement...

$$\hat{V} = \underbrace{\not{k}\not{r}}_{(1)} - \frac{Q}{r}\underbrace{e^{-m_D r}}_{(2)}$$

The Debye screened potential at $T > T_c$

1. Flux tubes are assumed to break down at $T > T_c$ (*for now*).
2. Due to the appearance of a thermal gluon mass m_D the Coulomb part of the static potential is screened, taking a Yukawa-type form.

Heavy quarkonium bound states are thus expected to survive beyond deconfinement in an increasingly screened Yukawa potential.[(Matsui, Satz, Phys.Lett.B178:416,1986)]

The heavy quarkonium spectral function

$$\rho(\omega) = \frac{1}{2}\left(1 - e^{-\beta\omega}\right)\int_{-\infty}^{\infty} dt e^{i\omega t} C_>(t,0)$$

(Definition of the heavy quarkonium spectral function)

The heavy quarkonium spectral function

Our goal will be the extraction of the heavy quarkonium spectral function ρ from the mesonic correlator

$$C_>(t,\mathbf{r}) = \int d^3x \left\langle \bar{\psi}(t,x+\frac{r}{2})\gamma^\mu W \psi(t,x-\frac{r}{2})\bar{\psi}(0,0)\gamma_\mu\psi(0,0)\right\rangle .$$

ρ is proportional to the dilepton rate from $q\bar{q}$-annihilation in the plasma. A point splitting has been introduced to the mesonic correlator $C_>(t,r)$ to facilitate a perturbative treatment. $C_>(t,r)$ *satisfies a Schrödinger-type equation.*

Infinite mass limit

Focusing on infinitely heavy quarks the correlator $C_>(t,r)$ can be obtained as the analytic continuation $C_>(t,\mathbf{r}) \sim C_E(it,\mathbf{r})$ of a Euclidean Wilson loop:

$$C_E(\tau,\mathbf{r}) = \frac{1}{N_C} Tr < W(0,\mathbf{r};\tau,\mathbf{r})W(\tau,\mathbf{r};\tau,\mathbf{0})W(\tau,\mathbf{0};0,\mathbf{0})W(0,\mathbf{0};0,\mathbf{r}) >$$

The real-time static potential

$$[i\partial_t - V(t,r)]\, C_>(t,r) = 0$$

Definition of a $q\bar{q}$-potential (Laine,Philipsen,Romatschke,Tassler,JHEP 0703 (2007) 054)

A static quarkonium potential at finite temperature can now be defined from the infinite mass Schrödinger equation shown above.

Calculation in perturbation theory

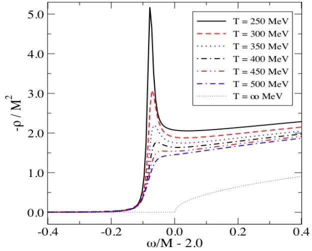
A perturbative calculation of the static Wilson loop followed by an analytic continuation yields the result

$$\lim_{t\to\infty} V(t,r) = -\frac{g^2 C_F}{4\Pi}[m_D + \frac{exp(-m_D r)}{r}] - i\frac{g^2 T C_F}{2\pi}\phi(m_D r)$$

with $\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[1 - \frac{sin(zx)}{zx}\right]$

with the real part being the usual thermal potential and an additional *imaginary part* originating from Landau damping.

The imaginary part

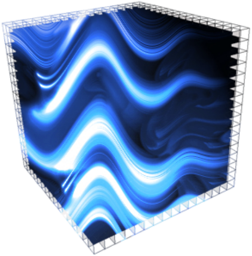


Role of the imaginary part

The potential has been reinserted into the Schrödinger equation supplemented by the appropriate mass terms to calculate the spectral function of Bottomonium via the fourier transform of $C_{>}(t, 0)$.(Laine,2007)

Note that a finite resonance width has thus been introduced to the potential model.

...but how can a dynamically defined quantity be investigated on the lattice ?



Real-time lattice simulations

Real-time lattice simulations are usually the attempt to study the long range dynamics of a quantum field theory in a semiclassical context.

In the following an appropriate framework for a semiclassical treatment of the quark gluon plasma including leading order quantum effects will be introduced.

Lets start with statistics...

The Hard-Thermal-Loop effective theory

$$Z = \int \mathcal{D}[E,B,W]\, e^{-\beta H} \quad \text{with}$$

$$H(x) = \frac{1}{2}\int d^3x \left(E^a E^a + B^a B^a + m_D^2 \int \frac{d\Omega}{4\pi} W^a(x,v) W^a(x,v) \right)$$

(Partition function of the Hard-Thermal-Loop effective theory)

The Hard-Thermal-Loop effective theory (Blaizot,Iancu,Nair,1993)

A reduction scheme is applied to the Yang-Mills theory at finite temperature by integrating out the UV degrees of freedom beyond a scale $\mu \ll T$ followed by a dimensional reduction of the partition function.
The effective field $W(\mathbf{x},\mathbf{v})$ describes the charge density of hard modes at $\mathbf{x}$ moving in the direction $\mathbf{v}$.

Dynamics: Dynamical observables can be calculated by evolving the ensemble using the classical equations of motion...

Gauge field dynamics

The original claim (Grigoriev, Rubakov, Nucl. Phys. B 299 (1988) 67)

Quantum fields can be described using classical dynamics provided the elementary excitations obey classical statistics.

Dynamical scales of the Yang-Mills theory
(Bödecker et al.,Phys. Rev. D 52 (1995) 4675)

A closer look at the classical limit using real time perturbation theory reveals the following relevant scales for the Yang-Mills theory:

- $\mathbf{k \sim T}$ (Characteristic plasma scale)
  Characteristic scale of the hard excitations.
- $\mathbf{g^2 T < k < gT}$ (Collective dynamics)
  A classical approximation of the gauge field is possible.
  Hard-thermal loop interactions should be taken into account.
- $\mathbf{k < g^2 T}$ (Nonperturbative dynamics)
  Transition to the strong coupling regime.

Hamiltonian lattice simulations

The lattice setup (Kogut, Susskind, Phys. Rev. D 11 (1975) 395)

In the following we will adopt a formalism where the spatial degrees of freedom are discretized on a three dimensional lattice while the (Minkowski-) time coordinate remains continouus. This formalism was originally introduced to study flux tube dynamics on the lattice.

Gauge fields in the Hamiltonian formalism

- The temporal gauge $A_0 = 0$ is chosen.
- Spatial gauge fields are discretized as usual as the parallel transport connecting neighbouring lattice points.
- The color electric field is defined from the dynamics of the spatial links:

$$\dot{U}_i(x) = iE_i(x)U_i(x)$$

- To preserve gauge invariance the field configurations have to satisfy constraint: $$\sum_i \left[E_i(x) - U_{-i}(x)E_i(x-i)U^{+}_{-i}(x)\right] = 0$$

  Gauss's law is recovered in the continuum limit.

The classical Yang-Mills field on the lattice

$$S = \frac{1}{2T}\int d^4x Tr F_{\mu\nu}F^{\mu\nu}$$

(The Yang-Mills action)

Wilsons action for real-time lattice simulations

$$S = \beta_3 \int dt \sum_{\mathbf{x}} \left(\frac{1}{2N} Tr \dot{U}_i^+ \dot{U}_i{}^{1)} - \sum_{i<j} \left[1 - \frac{1}{N} ReTrU_{ij} \right]^{2)} \right)$$

1. Electric part: Found by discretizing the temporal part of the action

$$S_E = \frac{1}{T}\int d^4x Tr E_i E^i, \quad (E_i = F_{0i})$$

using the definition of the electric field on the lattice.

2. Magnetic part: The remaining spatial part of the action is discretized in analogy to the 4-dimensional Wilson action.

Equations of motion

Derivation of the equations of motion for the electric field

The classical lattice equations of motion for the electric field can be derived by varying the action with respect to the link variables.

$$\delta S = 0$$

The gauge field evolution is in turn defined by the electric field.

Equations of motion (Ambjorn, Askgaard et al., Phys. Lett. B 244 (1990) 479)

The complete equations of motion:

$$\dot{U}_i(x) = iE_i(x)U_i(x) \qquad \text{(defines the electric field)}$$

$$\dot{E}_i^a = 2\sum_{|j|\neq i} ImTr(T^a U_{ij}) \qquad \text{(see above)}$$

(The Gauss constraint remains satisfied, Scaling: $a^2 g E_i^a \to E_i^a$, $a^{-1}t \to t$)

The statistical ensemble

Generation of the ensemble (Moore,Phys. Rev. D 61 (2000) 056003)

A statistical set of starting configurations respecting gauss's law has to be created:

1. Pregenerate the gauge fields by a three dimensional Monte Carlo (Moore: U=1).
2. Draw the electric fields from a gaussian distribution.
3. Project on the space of physical configurations using:

$$E_i(x) \rightarrow E_i(x) + \gamma(U_i C(x+\hat{i}))U_i^+ - C(x))$$

(C: Violation of gauss's law)

4. Evolve the fields using the EOM and repeat from (2) until the gauge fields have completely thermalized.

Note: The shown procedure to thermalize the fields is similar to the molecular dynamics used in a Hybrid Monte Carlo.

Hard thermal loop simulations

$$\dot{W}_n(x) = v_n^i(2\bar{E}_i(x) - [P_i W_n(x+i) - P_{-i} W_n(x-i)]) \qquad \textit{(EOM: HTL)}$$

$$j^i = (x)\frac{(am_D)^2}{N_p} v_n^i W_n(x) \qquad \textit{(Color current)}$$

(Lattice equations of motion)

"Discoball" discretization (Rebhan, Romatschke, Strickland, JHEP 09 (2005) 041)

To simulate the full effective theory the missing HTL equation of motion can be implemented using the above scheme, where the sphere of directions has been discretized using platonian solids. This equation is then coupled to the lattice Yang-Mills field via the current.

An alternative expansion of the HTL-fields in spherical harmonics has been implemented as well.

Used definitions:

Averaged electric field: $\bar{E}_i(x) = \frac{1}{2}(E_i(x) + P_{-i}E_i(x - \hat{i}))$
Parallel transport: $P_i\phi(x + \hat{i}) = U_i(x)\phi(x + \hat{i})U_i^{+}(x)$, Discoball vertex: v_n

Some results...

Extraction of the real time static potential

$$\left(i\Delta_t - V(t,r)\right) C_>(t,r) = 0$$

(The real time static potential)

Extraction from Wilson loop dynamics

- A Wilson loop of extent (r,t) was averaged over an ensemble of configurations using classical or HTL improved simulations:

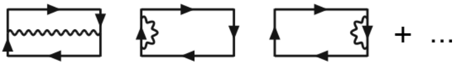
$$C_>(t,r) = \frac{1}{N}\left\langle Tr\left(W^+(t,x)W(0,x)\right)\right\rangle_{|x|=r}$$

- The potential was extracted using the 3-point derivative Δ_t:

$$V(t,r) = i\frac{\Delta_t C_>(t,r)}{C_>(t,r)}, \Delta_t F = \frac{-3F(t-\delta_t) + 4F(t) - F(t+\delta_t)}{2\delta_t}$$

- We focus on the imaginary part which originates from Landau damping and is expected to exist in the classical limit

Analytic expectations



(Some diagrams contributing to the analytic result)

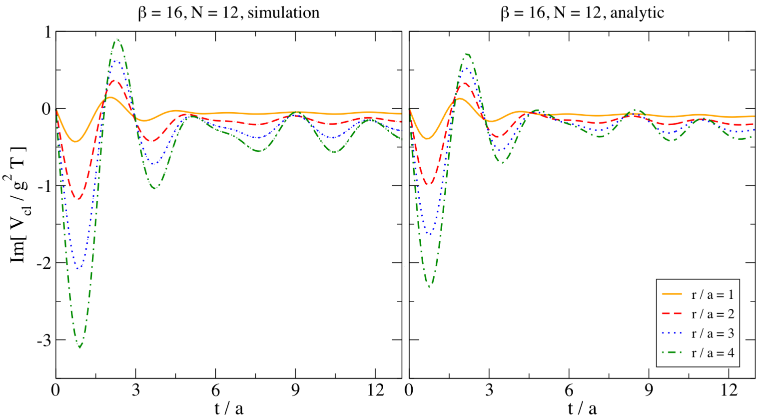
Analytical result

The Wilson Loop was also calculated using resummed perturbation theory followed by an analytic continuation leading to the following result for the real time static potential:

$$V_{>}^{(2)}(t,r) = g^2 C_F \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{2 - e^{iq_3 r} - e^{-iq_3 r}}{2} \left\{ \frac{1}{\mathbf{q}^2 + \Pi_E(0,\mathbf{q})} + \int_{-\infty}^{\infty} \frac{dq^0}{\pi} n_B(q^0) q^0 \right.$$
$$\left. \times \left(e^{\beta q^0} e^{-iq^0 t} - e^{iq^0 t} \right) \left[\left(\frac{1}{\mathbf{q}^2} - \frac{1}{(q^0)^2} \right) \rho_E(q^0,\mathbf{q}) + \left(\frac{1}{q_3^2} - \frac{1}{\mathbf{q}^2} \right) \rho_T(q^0,\mathbf{q}) \right] \right\}$$

These expressions have been evaluated using lattice regularization...

Comparison to the numerical results



...and compared to the numerical results.

Conclusion

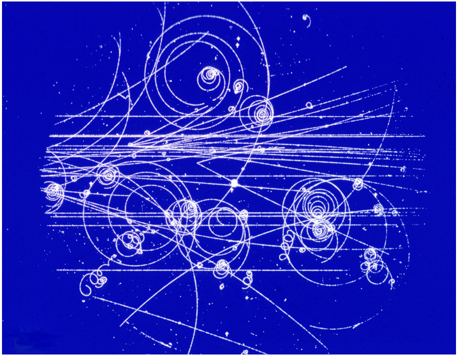
	β_3	N	am_D	r=1a	r=2a	r=3a	r=4a
Simulation* (∼ 200 Config.)	16.0	12	0.0	-0.060(2)	-0.156(8)	-0.246(26)	-0.319(56)
	16.0	16	0.0	-0.059(2)	-0.155(8)	-0.245(22)	-0.326(48)
	16.0	12	0.211	-0.059(2)	-0.147(7)	-0.229(23)	-0.297(51)
	16.0	12	0.350	-0.030(2)	-0.064(5)	-0.096(12)	-0.118(21)
	13.5	12	0.250	-0.071(2)	-0.174(10)	-0.270(33)	-0.341(97)
Analytic	16.0	∞	0.0	-0.0816	-0.1453	-0.1847	-0.2072

(Overview of the asymptotic results for $t \to \infty$)

Result (Laine, Philipsen, Tassler, Preprint: arXiv,0707 2458)

The analytic result from resummed perturbation theory for the imaginary part of the real-time static potential has been confirmed using the drastically different approach of semiclassical lattice simulations. Nonperturbative corrections from long range field dynamics where found to increase the damping of the correlator $C_>(t,r)$ by $\sim 100\%$.

Questions



Questions?