Hadron masses in QCD with one quark flavour

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Introduction

One-flavour QCD – a gauge theory with SU(3) colour gauge group and a fermion in the fundamental representation – is studied by Monte Carlo simulations. QCD with one flavour of quarks is an interesting theoretical laboratory to understand some aspects of the strong interaction, e. g.

- the possibility of spontaneous breakdown of parity and charge conjugation symmetry at negative quark masses [Creutz, 2004]
- a possible negative sign of the fermion determinant at negative quark masses [Creutz, 2004]
- the relation between one-flavour ($N_f = 1$) QCD and supersymmetric Yang-Mills (SYM) theory with one supersymmetry charge ($\mathcal{N} = 1$) [Armoni, Shifman, Veneziano, 2003/4]

The mass spectrum of hadronic bound states is investigated at two different lattice spacings. Properties of the theory are analyzed by making use of the ideas of partially quenched chiral perturbation theory (PQχPT). Due to the $\mathrm{U}(1)_A$ anomaly, the single-flavor QCD theory does not have a continuous chiral symmetry. However the symmetry can be artificially enhanced by adding extra valence quarks, which can be interpreted as u and d quarks. Operators in the valence pion sector can be built. Masses and decay constants are analyzed by using PQChPT formulae at next-to-leading order.

Lattice action and algorithm

For the SU(3) Yang-Mills gauge field we apply the *tree-level improved Symanzik* (tlSym) action which is a generalisation of the Wilson plaquette gauge action. It belongs to a one-parameter family of actions obtained by renormalisation group considerations and in the Symanzik improvement scheme. The fermionic part of the lattice action is the simple (unimproved) Wilson action. [Symanzik, 1983; Weisz, 1983/4]

For preparing the sequences of gauge configurations a *Polynomial Hybrid Monte Carlo* (PHMC) updating algorithm was used, which is well-suited for theories with an odd number of fermion species. This algorithm is based on multi-step (actually two-step) polynomial approximations of the inverse fermion matrix with stochastic correction in the update chain [Montvay, Scholz, 2005/6].

We measure the low lying hadron spectrum, i. e. masses of m_η, m_σ, $m_{\Delta^{++}}$, m_N and in the PQ scheme m_π and f_π. In this contribution we analyse the latter in PQχPT.

Partially quenching

In the unquenched theory the functional integral is given by

$$\int \mathcal{D}A\,\mathcal{D}[\psi\bar\psi]\; \mathrm{e}^{-S_g - \bar\psi(\gamma_\mu D_\mu + m)\psi} = \int \mathcal{D}A\; \mathrm{e}^{-S_g}\, \det(\gamma_\mu D_\mu + m)\,.$$

In the quenched version one introduces additional *valence quarks* ψ_V with a mass m_V and identifies the primary ones as *sea quarks* ψ_S with mass m_S. Further one adds for every valence quark a spin-1/2 boson $\tilde\psi$. This obviously violates the spin-statistic theorem and is *unphysical*, one calls them *ghost quarks*. The functional integral changes to

$$\int \mathcal{D}A\,\mathcal{D}[\psi_S\bar\psi_S]\;\mathcal{D}[\psi_V\bar\psi_V]\;\mathcal{D}[\tilde\psi\bar{\tilde\psi}]\;\cdot$$
$$\cdot\, \mathrm{e}^{-S_g - \bar\psi_S(\gamma_\mu D_\mu + m_S)\psi_S - \bar\psi_V(\gamma_\mu D_\mu + m_V)\psi_V - \bar{\tilde\psi}(\gamma_\mu D_\mu + m_V)\tilde\psi}$$
$$= \int \mathcal{D}A\; \mathrm{e}^{-S_g}\, \frac{\det(\gamma_\mu D_\mu + m_V)}{\det(\gamma_\mu D_\mu + m_V)}\, \det(\gamma_\mu D_\mu + m_S)$$

so that the determinants of valence and ghost quarks cancel each other. [Morel, 1987; Bernard, Golterman, 1994; Sharpe, 1997]

PQχPT point of view

For interpreting our results on the mass spectrum we find it useful to embed the $N_f = 1$ QCD theory in the partially quenched theory. This embedding is particularly useful if the additional quenched valence quark flavours have the same mass as the dynamical sea quark, i. e. $m_V = m_S$, because of the exact $\mathrm{SU}(N_F)$ flavour symmetry in the combined sea- and valence-sectors (N_F denotes here the total number of quenched and unquenched flavours). In most cases we consider the natural choice $N_F = 3$ which is closest to the situation realised in nature.

In this situation there is an exact SU(3) vector-like flavour symmetry in the valence + sea quark sector, and the hadronic bound states appear in exactly degenerate SU(3)-symmetric multiplets. For instance, there is a degenerate octet of pseudoscalar mesons – the "pions" (π^a, $a = 1, \ldots, 8$) satisfying an SU(3)-symmetric PCAC relation. With the help of the divergence of the axialvector current $A^a_{x\mu}$ and pseudoscalar density P^a_x one can define, as usual, the bare *PCAC quark mass* am_{PCAC} in lattice units:

$$am_{\mathrm{PCAC}} \equiv \frac{\langle \partial^*_\mu A^+_{x\mu} P^-_y \rangle}{2\langle P^+_x P^-_y \rangle}\,.$$

The "pions" are, of course, not physical particles in the spectrum of $N_f = 1$ QCD. Nevertheless, their properties such as masses and decay constants are well defined quantities which can be computed on the lattice. The same is true of the PCAC quark mass m_{PCAC}, which is therefore a potential candidate for a definition of a quark mass of this theory.

PQχPT fit formulae

We have calculated the masses of pseudo-Goldstone bosons in next-to-leading order of partially quenched chiral perturbation theory, including $\mathcal{O}(a)$ lattice effects [Rupak, Shoresh, 2002]. The quark masses enter the expressions in the combinations

$$\chi_V = 2B_0\, m_V, \quad \chi_S = 2B_0\, m_S, \quad \chi_{\mathrm{PCAC}} = 2B_0\, \tfrac{Z_A}{Z_P}\, m_{\mathrm{PCAC}}\,.$$

We obtain formulae similar to those of $N_f > 1$ QCD, e.g. for the pion masses

$$\begin{aligned} m_\pi^2 = \chi_{\mathrm{PCAC}} &+ \frac{\chi^2_{\mathrm{PCAC}}}{16\pi^2 F_0^2} \ln\frac{\chi_{\mathrm{PCAC}}}{\Lambda^2} \\ &+ \frac{8}{F_0^2}\Big[(2L_8 - L_5 + 2L_6 - L_4)\chi^2_{\mathrm{PCAC}} \\ &+ (W_8 + W_6 - W_5 - W_4 - 2L_8 + L_5 - 2L_6 + L_4)\chi_{\mathrm{PCAC}}\rho\Big]\,. \end{aligned}$$

The η_s singlet

The η_s can be included in the analysis by relaxing the constraint of a vanishing supertrace of the pseudo-Goldstone field, and associating it with the field

$$\Phi_0(x) = \mathrm{sTr}\,\Phi(x)\,.$$

The effective Lagrangian then contains additional terms depending on Φ_0 [Sharpe, Shoresh, 2001]:

$$\Delta\mathcal{L} = \alpha\partial_\mu\Phi_0\partial_\mu\Phi_0 + m^2_\Phi \Phi_0^2 + \mathcal{O}(\Phi_0^3)\,,$$

where α and m_Φ are free parameters in this context. We content ourselves with displaying only the leading order expression for the mass of the η_s, which reads

$$m^2_{\eta_s} = \frac{m^2_\Phi + \chi_{\mathrm{PCAC}}}{1+\alpha}\,.$$

Fits

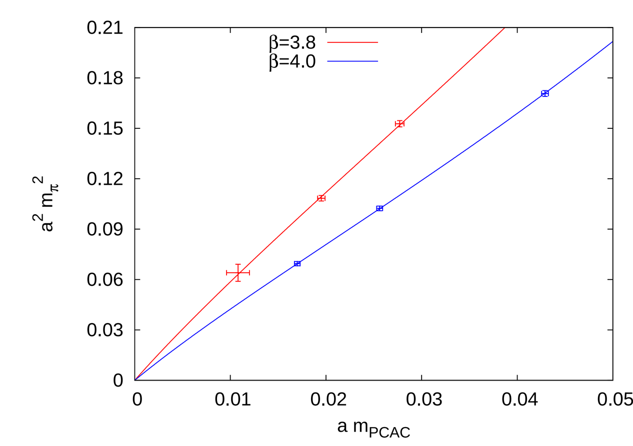
We fit $a^2 m_\pi^2$ and af_π simultaneously as a function of am_{PCAC} including the data at both values of β. There are not enough data in order to account for the lattice artifacts. Therefore the fit is done with the continuum formulae

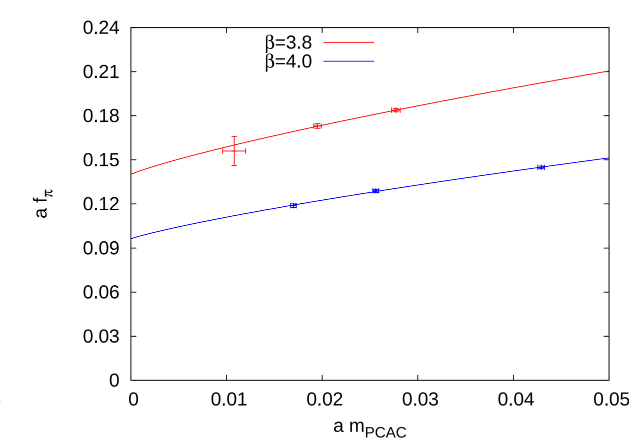
$$m_\pi^2 = \chi_{\mathrm{PCAC}} + \frac{\chi^2_{\mathrm{PCAC}}}{16\pi^2 F_0^2} \ln\frac{\chi_{\mathrm{PCAC}}}{\Lambda_3^2}\,, \qquad \frac{Z_A f_\pi}{\sqrt{2} F_0} = 1 - \frac{\chi_{\mathrm{PCAC}}}{32\pi^2 F_0^2} \ln\frac{\chi_{\mathrm{PCAC}}}{\Lambda_4^2}\,,$$

with the low-energy constants

$$\Lambda_3 = 4\pi F_0 \exp\{64\pi^2(L_4 + L_5 - 2L_6 - 2L_8)\}\,, \quad \Lambda_4 = 4\pi F_0 \exp\{64\pi^2(L_4 + L_5)\}\,.$$

The changes of the renormalisation constants Z_A, Z_P between the two β values are neglected.





Owing to the fact that the number of degrees of freedom in the fit is small, the uncertainty of the fit parameters is relatively large. The determination of the universal low-energy scales Λ_3/F_0 and Λ_4/F_0 can be improved by considering the ratios [Alpha, 2000; qq+q, 2003/4]

$$\frac{1}{\sigma}\frac{m_\pi^2}{m^2_{\pi,\mathrm{ref}}} = 1 + \zeta\frac{\sigma-1}{16\pi^2}\ln\left(\zeta\Big/\frac{\Lambda_3^2}{F_0^2}\right) + \zeta\frac{\sigma}{16\pi^2}\ln\sigma\,,$$
$$\frac{f_\pi}{f_{\pi,\mathrm{ref}}} = 1 - \zeta\frac{\sigma-1}{32\pi^2}\ln\left(\zeta\Big/\frac{\Lambda_4^2}{F_0^2}\right) - \zeta\frac{\sigma}{32\pi^2}\ln\sigma\,,$$

in which some of the coefficients cancel ($\sigma \equiv \chi/\chi_{\mathrm{ref}}$, $\zeta = \zeta_{\mathrm{ref}}$ is a dimensionless fit parameter). We consider the data on the larger lattice at $\beta = 4.0$ and take the quantities at smallest am_{PCAC} ($\kappa = 0.1615$) as reference. The fit yields

$$\frac{\Lambda_3}{F_0} = 10.0 \pm 2.6\,, \qquad \frac{\Lambda_4}{F_0} = 31.5 \pm 14.3\,,$$

which is compatible with the phenomenological values from ordinary QCD.

In order to estimate the parameters α and m_Φ, related to the mass of the η_s, we made a fit of m_π^2 and $m^2_{\eta_s}$ at $\beta = 4.0$ in leading-order ChPT. The result is

$$\alpha = -0.03(19)\,, \qquad am_\Phi = 0.18(8)\,,$$

indicating the vanishing of α. Fixing $\alpha = 0$ in the fit yields

$$am_\Phi = 0.19(2) \quad \text{or} \quad r_0 m_\Phi = 0.72(10)\,,$$

where the value of r_0/a extrapolated to vanishing PCAC quark mass is used.

Reference

[1] F. Farchioni, I. Montvay, G. Münster, E. E. Scholz, T. Sudmann and J. Wuilloud, "Hadron masses in QCD with one quark flavour," arXiv:0706.1131 [hep-lat].