O(a) improvement & Schrödinger functional schemes (lecture V)

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O(a) improvement and twisted mass QCD

Recent interest in tmQCD mostly triggered by the observation of automatic O(a) improvement at maximal twist $\alpha = \pi/2$ [Frezzotti & Rossi '03].

The argument only relies on Symanziks effective continuum theory:

- assume that we have tuned $m_{\rm PCAC} = 0$ i.e. the renormalized standard mass vanishes (up to O(a) effects)
- Symanziks effective continuum action is then given by

$$S_{\rm eff} = S_0 + aS_1 + O(a^2), \qquad S_0 = \int \mathrm{d}^4 x\, \overline{\psi}(x)\left(\not{D} + i\mu_{\rm q}\gamma_5\tau^3\right)\psi(x)$$

where S_0 is supposed to be regularised e.g. with Ginsparg-Wilson quarks on a much finer lattice, and S_1 is given by (afer using equations of motion)

$$S_1 = \int \mathrm{d}^4 x \left\{ c\,\overline{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi + b_\mu\,\mu^2\overline{\psi}\psi \right\}$$

- Cutoff dependence of lattice correlation function:

$$\langle O\rangle = \langle O\rangle^{\rm cont} - a\langle S_1 O\rangle^{\rm cont} + a\langle \delta O\rangle^{\rm cont} + O(a^2).$$

Here δO are the O(a) counterterms to the composite fields in O.
Example:

$$O = V_\mu^1(x)P^2(y) \qquad \Rightarrow \quad \delta O = \left\{c_{\rm V}\, i\partial_\nu T_{\mu\nu}^1(x) + b_{\rm V}\,\mu A_\mu^2(x)\right\} P^2(y)$$

- Introduce a $\gamma_5\tau^1$-transformation:

$$\psi \rightarrow i\gamma_5\tau^1\psi, \qquad \overline{\psi} \rightarrow \overline{\psi}\, i\gamma_5\tau^1$$

- Under this $\gamma_5\tau^1$-transformation one has

$$\begin{array}{rclcrcl} S_0 & \rightarrow & S_0 & & S_1 & \rightarrow & -S_1 \\ O & \rightarrow & \pm O & \Rightarrow & \delta O & \rightarrow & \mp\delta O \end{array}$$

- Hence for $\gamma_5\tau^1$-even O one finds

$$\begin{aligned} \langle OS_1\rangle^{\rm cont} &= -\langle OS_1\rangle^{\rm cont} = 0 \\ \langle\delta O\rangle^{\rm cont} &= -\langle\delta O\rangle^{\rm cont} = 0 \\ \Rightarrow \quad \langle O\rangle &= \langle O\rangle^{\rm cont} + O(a^2) \end{aligned}$$

- while for $\gamma_5\tau^1$-odd O one gets

$$\begin{aligned} \langle O\rangle^{\rm cont} &= -\langle O\rangle^{\rm cont} = 0 \\ \langle OS_1\rangle^{\rm cont} &= \langle OS_1\rangle^{\rm cont} \\ \langle\delta O\rangle^{\rm cont} &= \langle\delta O\rangle^{\rm cont} = 0 \\ \Rightarrow \quad \langle O\rangle &= -a\langle OS_1\rangle^{\rm cont} + a\langle\delta O\rangle^{\rm cont} + O(a^2) \end{aligned}$$

$\Rightarrow$ $\gamma_5\tau^1$-even observables are automatically O(a) improved, while $\gamma_5\tau^1$-odd observables vanish up to O(a) terms.

Some Observations & Remarks:

- The $\gamma_5\tau^1$-symmetry corresponds to the physical flavour symmetry:

$$\psi' \to -i\tau^2\,\psi', \qquad \overline{\psi}' \to \overline{\psi}'\, i\tau^2.$$

  A similar argument based on parity has been given by Shindler

- A^a_μ and P^a have opposite $\gamma_5\tau^1$-parity!

$$\Rightarrow \qquad \langle \partial_\mu A^1_\mu(x) O_{\rm even}\rangle = 2\,\underbrace{m_{\rm PCAC}}_{\mathrm{O}(a)}\underbrace{\langle P^1(x) O_{\rm even}\rangle}_{\mathrm{O}(a)} = O(a^2)$$

  i.e. the critical mass is only defined up to O(a)

- in the O(a) improved theory one can determine $m_{\rm cr}$ up to an O(a^2) ambiguity but this requires a mixed source, and it depends on $c_{\rm A}$.
- The O(a) ambiguity in $m_{\rm cr}$ does not spoil O(a) improvement: a shift in m_cr by $a\Lambda^2$ corresponds to an insertion of the $\gamma_5\tau^1$-odd operator $\overline{\psi}\psi \Rightarrow$ O(a^2) effect in $\gamma_5\tau^1$-even correlators
- finite space time volume: there is no phase transition, no spontaneous symmetry breaking and analyticity in the quark mass parameters

$\Rightarrow$ massless Wilson quarks in a finite volume are automatically O(a) improved!

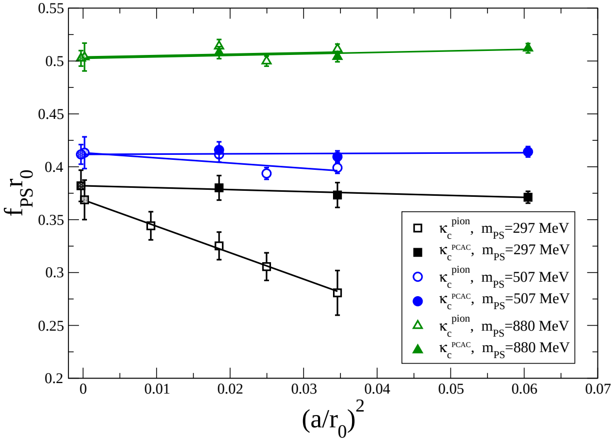
O(a) improvement in infinite volume

infinite volume:

- The twisted mass μ_{q} drives spontaneous chiral symmetry breaking. If it becomes too small, this may not be true any more and the system realigns the vacuum state $\Rightarrow$ use χPT to analyse the dynamics (cf. e.g. S. Sharpe's Nara lectures '06).
- At small μ cutoff effects may formally still be O(a^2) but can be large ("bending phenomenon"), depending on the chosen definition of m_{cr}: (Aoki & Bär, Sharpe & Wu, Sharpe, Frezzotti et al.)
- Use χPT and a "good" definition of m_{cr} (from pion physics) to control chiral extrapolations (Aoki & Bär, Sharpe & Wu, Sharpe) O(a) improvement of the action may also help

Test of O(a) improvement [Shindler, Lattice 2005]

Continuum extrapolation of F_π in quenched tmQCD:



Possible strategy for non-perturbative renormalisation

Premise: need non-perturbative renormalisation to obtain reliable error estimates; RI-MOM schemes & continuum perturbation theory from scales $\mu = \mathrm{O}(1)\mathrm{GeV}$ is difficult to control and may fail completely in some cases

Strategy:

1. use SF scheme to determine the scale evolution non-perturbatively in the continuum limit; use "cheap" regularisation (Wilson or staggered quarks) for this part
2. At a low energy scale: renormalise in the SF scheme, either
   - directly, by implementing the SF for the given regularisation; (requires extra simulations on relatively small lattices, e.g $L = 0.8\,\mathrm{fm}$, $L/a = 10$)
   - indirectly via some other physical quantity computable with periodic b.c.'s (see e.g. Hernandez et al. 2000 $Z_{\mathrm{P}}^{\mathrm{SF}}$ for overlap quarks); problems: insufficient precision, potentially sacrifice an observable for the matching procedure
3. Implementation of the SF for Ginsparg-Wilson quarks [Lüscher, Taniguchi '04, Lüscher '06, S. '06]
4. O(a) cutoff effects in the SF need to be adressed!

Sources for O(a) effects in the SF

- Renormalisability: boundary counterterms are local polynomials in the fields and their derivatives of dimension ≤ 3.
- At O(a) the same reasoning applies with terms of dimension 4;
- Possible boundary counterterms in the pure gauge theory:

$$\mathrm{tr}\,\{F_{0k}F_{0k}\}, \qquad \mathrm{tr}\,\{F_{kl}F_{kl}\},$$

- In QCD one expects fermionic terms $\overline{\psi}\gamma_0 D_0\psi$ or $\overline{\psi}\gamma_k D_k\psi$ with any regularisation,
- Which counterterms contribute depends on gauge field b.c.'s and on the observable; example: $\mathrm{tr}\,\{F_{kl}F_{kl}\} = 0$ in SF coupling calculation
- Equations of motions may be used to reduce the counterterm basis
- Typically 2-3 boundary O(a) counterterms need to be monitored/controlled
- In practice: evaluate coefficients in perturbation theory and vary them in the simulation to assess their effect numerically

The SF with Wilson quarks & O(a) improvement

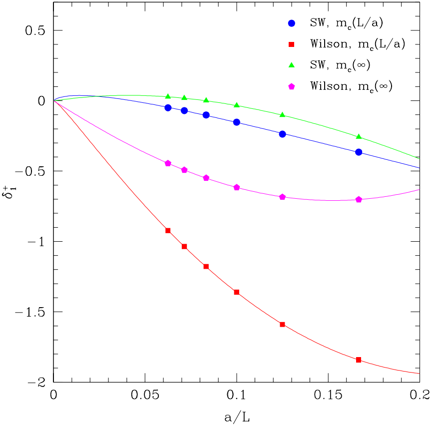
Apparent contradiction:

1. Previous discussion: $\gamma_5\tau^1$-even observables computed with Wilson quarks in a finite volume (with some type of periodic boundary conditions) are automatically O(a) improved at zero quark mass $\Rightarrow$ improvement coefficients like $c_{\rm sw}$ are irrelevant!
2. The Schrödinger functional in finite volume at zero mass was used to determine $c_{\rm sw}$, $c_{\rm A}$ and other O(a) improvement coefficients.

Distinguish 3 sources for O(a) effects in the SF:

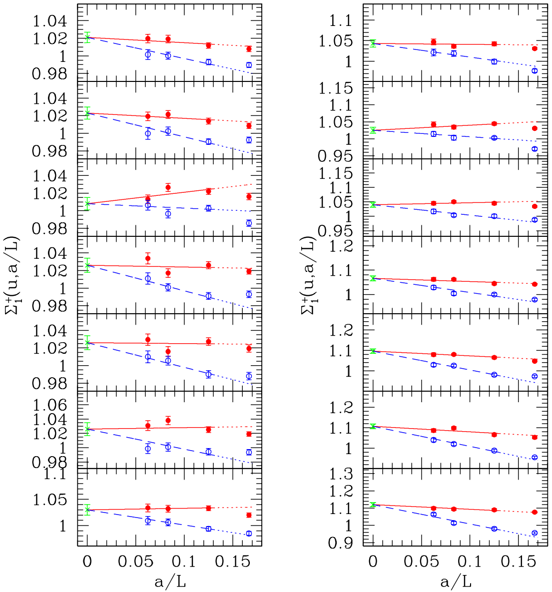
1. O(a) boundary effects (expected in any case!); can be cancelled by inclusion of boundary O(a) counterterms
2. from the bulk action; are cancelled by including the SW/clover term
3. from the composite operators; can be cancelled by including O(a) counterterms determined from chiral Ward identities; difficult for 4-quark operators!

Example: relative cutoff effects in the one-loop coefficient of the SSF for B_K (Palombi, Pena, S. '05)

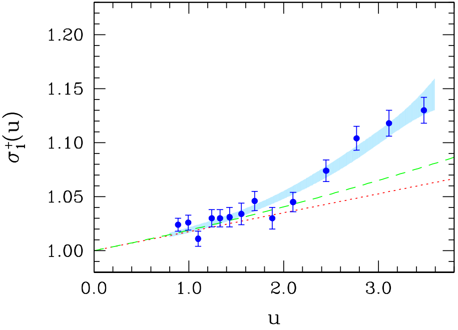


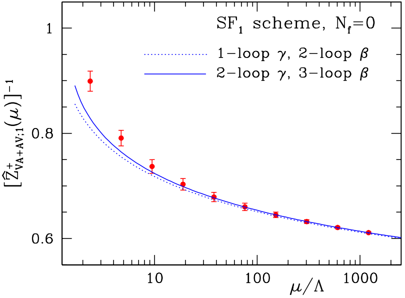
Question: Why do the bulk O(a) counterterms not vanish in the chiral limit?

SSF for B_K operator (quenched) [ALPHA '05]



Scale evolution of B_K (SF scheme) [ALPHA '05]





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- Problem: the $\gamma_5\tau^1$ field transformation switches the projectors of the quark b.c.'s:
$$P_\pm\gamma_5\tau^1 = \gamma_5\tau^1 P_\mp$$
The boundary conditions, like mass terms, break chiral symmetry and define a direction in chiral flavour space.

$\Rightarrow$ the $\gamma_5\tau^1$ transformation yields inequivalent correlation functions even in the chiral limit,
$$\langle O\rangle_{(m,\mu_q,P_\pm)} \rightarrow \langle O'\rangle_{(-m,\mu_q,P_\mp)}$$

- Possible solution: change quark boundary projectors, such that they commute with $\gamma_5\tau^1$, e.g.
$$\mathcal{P}_\pm = \tfrac{1}{2}(1\pm\gamma_0\tau^3), \qquad Q_\pm = \tfrac{1}{2}(1\pm i\gamma_0\gamma_5\tau^3),$$

- Practical problem: not obvious how to implement such boundary conditions on the lattice; solution so far only for $Q_\pm$.

SF boundary conditions and chiral rotations

Consider isospin doublets χ' and $\overline{\chi}'$ satisfying homogeneous SF boundary conditions ($P_\pm = \frac{1}{2}(1 \pm \gamma_0)$,

$$P_+\chi'(x)|_{x_0=0} = 0, \qquad P_-\chi'(x)|_{x_0=T} = 0,$$
$$\overline{\chi}'(x)P_-|_{x_0=0} = 0, \qquad \overline{\chi}'(x)P_+|_{x_0=T} = 0.$$

perform a chiral field rotation,

$$\chi' = \exp(i\alpha\gamma_5\tau^3/2)\chi, \qquad \overline{\chi}' = \overline{\chi}\exp(i\alpha\gamma_5\tau^3/2),$$

the rotated fields satisfy chirally rotated boundary conditions

$$P_+(\alpha)\chi(x)|_{x_0=0} = 0, \qquad P_-(\alpha)\chi(x)|_{x_0=T} = 0,$$
$$\overline{\chi}(x)\gamma_0 P_-(\alpha)|_{x_0=0} = 0, \qquad \overline{\chi}(x)\gamma_0 P_+(\alpha)|_{x_0=T} = 0,$$

with the projectors

$$P_\pm(\alpha) = \frac{1}{2}\left[1 \pm \gamma_0 \exp(i\alpha\gamma_5\tau^3)\right].$$

Special cases of $\alpha = 0, \pi/2$:

$$P_\pm(0) = P_\pm, \qquad P_\pm(\pi/2) \equiv Q_\pm = \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3),$$

Mapping of SF correlation functions

The chiral rotation thus introduces a mapping between correlation functions:

$$\langle O[\chi,\bar\chi]\rangle_{(m,\mu_{\rm q},P_\pm)} = \langle \tilde O[\chi,\bar\chi]\rangle_{(\tilde m,\tilde\mu_{\rm q},P_\pm(\alpha))}$$

with

$$\begin{aligned}\tilde O[\chi,\bar\chi] &= O\left[\exp(i\alpha\gamma_5\tau^3/2)\chi, \bar\chi\exp(i\alpha\gamma_5\tau^3/2\right]\\ \tilde m &= m\cos\alpha - \mu_{\rm q}\sin\alpha\\ \tilde\mu_{\rm q} &= m\sin\alpha + \mu_{\rm q}\cos\alpha\end{aligned}$$

boundary quark fields are included by replacing

$$\bar\zeta(\mathbf{x}) \leftrightarrow \bar\chi(0,\mathbf{x})P_+ \qquad \zeta(\mathbf{x}) \leftrightarrow P_-\chi(0,\mathbf{x})$$

Chirally rotating the SF

Chirally rotated SF boundary conditions would be interesting:

- bulk O(a) improvement could be automatic; remaining O(a) effects arise from a couple of boundary operators and can be monitored/eliminated; consequences:
  - O(a) improved step-scaling functions
  - Less sensitivity to the precision of the zero mass limit (determination of $m_{\rm cr}$ could be less precise)
- Decoupling of heavy quarks using the standard mass term (rather than standard SF with twisted mass term)
- devise checks of universality:
  1. between massless SF correlation functions (e.g. SF coupling)
  2. between tmQCD and standard QCD using SF correlation functions;

Technical difficulties

Standard SF b.c.'s natural for Wilson quarks due to projector structure of the Wilson-Dirac operator

$$D_W = \frac{1}{2}\left\{\left(\nabla_\mu + \nabla^*_\mu\right)\gamma_\mu - a\nabla^*_\mu\nabla_\mu\right\} = \frac{1}{2}(1-\gamma_\mu)\nabla_\mu - \frac{1}{2}(1+\gamma_\mu)\nabla^*_\mu$$

but Dirichlet boundary conditions are not always easy to implement:

- what happens with Wilson quarks and Wilson parameter $r \neq 1$?
- how does one implement SF boundary conditions for other lattice regularisations (Ginsparg-Wilson, domain-wall fermions)? ($\rightarrow$ Taniguchi '04)
- here: how do we implement the chirally rotated b.c.'s?
- in a few cases orbifold techniques can be applied ($\rightarrow$ Taniguchi '04, S. '05)

Orbifold technique

Orbifold techniques have previously been used to implemement the standard SF conditions for Ginsparg-Wilson quarks (Taniguchi '04). Here:

- start with standard lattice action for a single quark flavour

$$S_f[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\psi}(x) \left(D_W + m_0\right) \psi(x)$$

  where

$$\psi(x_0 + 2T, \mathbf{x}) = -\psi(x), \qquad \bar{\psi}(x_0 + 2T, \mathbf{x}) = -\bar{\psi}(x)$$

- introduce a reflection ($R^2 = id$)

$$R : \psi(x) \to i\gamma_0\gamma_5\psi(-x_0, \mathbf{x}), \qquad \bar{\psi}(x) \to \bar{\psi}(-x_0, \mathbf{x}) i\gamma_0\gamma_5$$

- the gauge field is extended to $[-T, T]$ and then periodically continued (cp. Taniguchi '04):

$$U_k(-x_0, \mathbf{x}) = U_k(x_0, \mathbf{x}), \qquad U_0(-x_0 - a, \mathbf{x})^\dagger = U_0(x)$$

Orbifold (2)

- Decompose fields into even and odd with respect to R,

$$R\psi_\pm = \pm\psi_\pm, \qquad R\bar{\psi}_\pm = \pm\bar{\psi}_\pm$$

- even/odd fields satisfy the boundary conditions at $x_0 = 0$

$$(1 \mp i\gamma_0\gamma_5)\psi_\pm(0,\mathbf{x}) = 0 \qquad \bar{\psi}_\pm(0,\mathbf{x})(1 \mp i\gamma_0\gamma_5) = 0$$

- and with complementary projectors at $x_0 = T$, due to antiperiodicity:

$$(1 \pm i\gamma_0\gamma_5)\psi_\pm(T,\mathbf{x}) = 0 \qquad \bar{\psi}_\pm(T,\mathbf{x})(1 \pm i\gamma_0\gamma_5) = 0$$

- consistency condition for R:

$$S_f[\psi,\bar{\psi},U] = S_f[\psi_+ + \psi_-, \bar{\psi}_+ + \bar{\psi}_-, U] = S_f[\psi_+,\bar{\psi}_+,U] + S_f[\psi_-,\bar{\psi}_-,U]$$

is indeed verified; (i.e. R commutes with D_W!)

$\Rightarrow$ the functional integral factorises!

Orbifold (3)

- interpret even and odd fields as quark flavours

$$\chi = \sqrt{2}\begin{pmatrix}\psi_- \\ \psi_+\end{pmatrix}, \qquad \bar{\chi} = \sqrt{2}\begin{pmatrix}\bar{\psi}_- & \bar{\psi}_+\end{pmatrix}$$

- functional integral:

$$\int \prod_{-T\leq x_0<T} \mathrm{d}\psi(x)\mathrm{d}\bar{\psi}(x)\mathrm{e}^{-S_f[\psi,\bar{\psi},U]} \propto \int \prod_{0\leq x_0\leq T} \mathrm{d}\chi(x)\mathrm{d}\bar{\chi}(x)\mathrm{e}^{-\frac{1}{2}S_f[\chi,\bar{\chi},U]}$$

- equivalent to theory in the interval $[0, T]$ with boundary conditions

$$Q_+\chi(x)|_{x_0=0} = 0, \qquad Q_-\chi(x)|_{x_0=T} = 0,$$
$$\bar{\chi}(x)Q_+|_{x_0=0} = 0, \qquad \bar{\chi}(x)Q_-|_{x_0=T} = 0$$

with

$$Q_\pm = P_\pm(\pi/2) = P_\mp(-\pi/2) = \tfrac{1}{2}\left(1 \pm i\gamma_0\gamma_5\tau^3\right)$$

Orbifold (4)

The dynamical field variables are

$$Q_-\chi(0,\mathbf{x}), \qquad \chi(x)|_{0<x_0<T}, \qquad Q_+\chi(T,\mathbf{x})$$

and

$$\bar\chi(0,\mathbf{x})Q_-, \qquad \bar\chi(x)|_{0<x_0<T}, \qquad \bar\chi(T,\mathbf{x})Q_+$$

The Wilson-Dirac operator in the interval is obtained by re-writing

$$S_f[\chi,\bar\chi,U] = a^4 \sum_{-T<x_0\le T} \bar\chi(x)\,(D_W+m_0)\,\chi(x) = 2a^4 \sum_{0\le x_0\le T} \bar\chi(x)\mathcal{D}\chi(x).$$

Properties of $\mathcal{D}$:

- up to modifications near the time boundaries it is just $D_W + m_0$
- hermiticity:

$$\gamma_5\tau^1\mathcal{D}\gamma_5\tau^1 = \mathcal{D}^\dagger$$

  however: not by simple “syntactic extension” of $D_W + m_0$, need to take into account b.c.’s for $\bar\chi$.

Orbifold (5)

Alternative set-up (possibly simpler due to explicit reduction to the time interval):

- Start with $2(T+a)$ anti-periodic fields ψ, $\bar{\psi}$

$$\psi(x_0+2(T+a),\mathbf{x}) = -\psi(x), \qquad \bar{\psi}(x_0+2(T+a),\mathbf{x}) = -\bar{\psi}(x),$$

- introduce a reflection ($R^2 = id$)

$$R: \psi(x) \to i\gamma_0\gamma_5\psi(-a-x_0,\mathbf{x}), \qquad \bar{\psi}(x) \to \bar{\psi}(-a-x_0,\mathbf{x})i\gamma_0\gamma_5$$

- the gauge field is extended to $[-T-a, T+a]$ and then periodically continued

$$U_k(-a-x_0,\mathbf{x}) = U_k(x_0,\mathbf{x}), \qquad U_0(-2a-x_0,\mathbf{x})^\dagger = U_0(x)$$

this implies that the boundary layer is doubled!

Orbifold (6): explicit reduction to the interval [0,T]

- decompose in even/odd fields and define doublets $\chi, \bar{\chi}$ as before
- D_W becomes block diagonal and

$$\det_{-T\le x_0\le T+a}[D_W(N_\mathrm{f}=1)] = \det_{0\le x_0\le T}[2D_W(N_\mathrm{f}=2)]$$

- Defining again $\mathcal{D}$ it is now obtained directly by "syntactic extension" and $\gamma_5\tau^1$ hermitian

$$a\mathcal{D}\chi(x) = -U(x,0)P_-\chi(x+a\hat{\mathbf{0}}) + (K\psi)(x) - U(x-a\hat{\mathbf{0}})^\dagger P_+\chi(x-a\hat{\mathbf{0}}),$$

where we have set $\chi(x)=0$ for $x_0<0$ and $x_0>T$, and

$$K = 1 + \tfrac{1}{2}\sum_{k=1}^{3}\left\{a(\nabla_k+\nabla_k^*)\gamma_k - a^2\nabla_k^*\nabla_k\right\} + \delta_{x_0,0}\, i\gamma_5\tau^3 P_- + \delta_{x_0,T}\, i\gamma_5\tau^3 P_+$$

Symmetries and Counterterms

- Symmetries ($C, P \times \tau^1$ etc.) $\Rightarrow$ possible dimension 3 counterterms at the boundaries:

$$K_1 = \overline{\chi} i\gamma_5\tau^3\chi, \qquad K_2 = \overline{\chi}\chi, \qquad K_3 = \overline{\chi} i\gamma_0\gamma_5\tau^3\chi$$

- K_1: multiplicative renormalization of ζ,ζ' and $\overline{\zeta}$,$\overline{\zeta}'$.
- $K_+ = \frac{1}{2}(K_2 + K_3) = \overline{\chi} Q_+ \chi$ only refers to Dirichlet components (at $x_0 = 0$)
  $\Rightarrow$ irrelevant for correlation functions used in practice
- $K_- = \frac{1}{2}(K_2 - K_3) = \overline{\chi} Q_- \chi$ only contains non-Dirichlet components (at $x_0 = 0$);
  if chirally rotated back to the standard SF K_- is proportional to $\overline{\chi}' i\gamma_5\tau^3 P_-\chi'$ i.e. it violates parity and flavour symmetries!
- conclude: K_- is a *finite* counterterm which can be fixed by requiring parity restauration!

Mapping of SF correlation functions

In the continuum we have:

$$\langle O[\chi,\bar\chi]\rangle_{(m,\mu_\mathrm{q},P_\pm)} = \langle \tilde O[\chi,\bar\chi]\rangle_{(\tilde m,\tilde\mu_\mathrm{q},P_\pm(\alpha))}$$

with

$$\begin{aligned}
\tilde O[\chi,\bar\chi] &= O\left[\exp(i\alpha\gamma_5\tau^3/2)\chi, \bar\chi\exp(i\alpha\gamma_5\tau^3/2\right] \\
\tilde m &= m\cos\alpha - \mu_\mathrm{q}\sin\alpha \\
\tilde\mu_\mathrm{q} &= m\sin\alpha + \mu_\mathrm{q}\cos\alpha
\end{aligned}$$

boundary quark fields are included by replacing

$$\bar\zeta(\mathbf{x}) \leftrightarrow \bar\chi(0,\mathbf{x})P_+ \qquad \zeta(\mathbf{x}) \leftrightarrow P_-\chi(0,\mathbf{x})$$

parity/flavour symmetry restoration e.g. by imposing

$$f_\mathrm{V}^{11}(x_0) = 0, \qquad f_\mathrm{P}^{12}(x_0) = 0$$

simple example for mapping: SF coupling

$$\bar g^{-2}(L) = \langle O[U]\rangle_{(0,0,P_\pm)} = \langle O[U]\rangle_{(0,0,Q_\pm)}$$

A tree-level check

At tree-level the quark propagator is an observable:
define $\gamma_5\tau^1$-even observables from tree level propagator, e.g.

$$I_1 = a^3 \sum_{\mathbf{x}} \langle \overline{\chi}(x)\gamma_0 Q_+ \chi(y) \rangle_{(0,0,Q_+)}$$

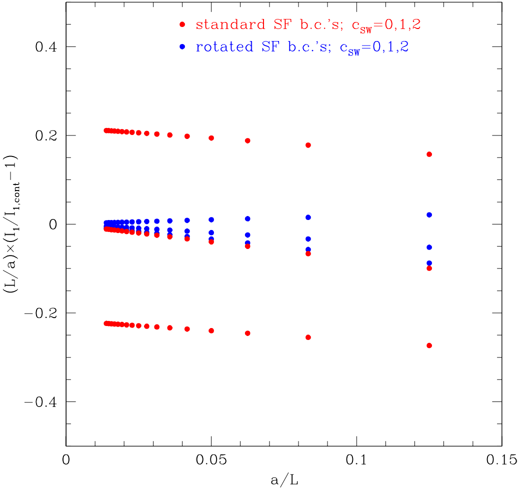
In the continuum limit this should be equal to

$$I_1' = a^3 \sum_{\mathbf{x}} \langle \overline{\chi}'(x) P_+ \chi'(y) \rangle_{(0,0,P_+)}$$

Setting $x_0 = T/4$, $y_0 = T/2$, $\theta = 0.5$, $T = L$, and with a color electric background field, we expect:

- I_1 reaches the continuum limit $\propto (a/L)^2$
- I_1' reaches the same continuum limit $\propto (c_{\rm sw}^{(0)} - 1)(a/L)$

A tree-level check



The SF coupling to one-loop order

In perturbation the SF coupling can be related to the $\overline{\mathrm{MS}}$-coupling

$$\bar{g}^2(L) = g^2_{\overline{\mathrm{MS}}}(\mu) + k_1(\mu L) g^4_{\overline{\mathrm{MS}}} + O(g^6)$$

here: consider fermionic contribution $\propto N_\mathrm{f}$ [Sommer, S. '95]

$$k_1 = k_{1,0} + N_\mathrm{f} k_{1,1}, \qquad k_{1,1} = -0.039863(2)/(4\pi)$$

in practice one computes for a sequence of lattices
$f(L/a) \sim r_0 + (a/L)\left[r_1 + s_1 \ln(a/L)\right] + O(a^2)$

- the correct continuum limit $r_0 = k_{1,1}$ is reproduced
- r_1 is to be cancelled by boundary O(a) counterterm $\propto c_\mathrm{t}\,\mathrm{tr}\left\{F_{0k}F_{0k}\right\}$
- observation: s_1 vanishes independently of c_sw; (in standard SF $s_1 \propto (c_\mathrm{sw}^{(0)} - 1)$)

Conclusions and Outlook

- Successful implementation of chirally rotated SF boundary conditions for Wilson quarks; compatibility with automatic O(a) improvement has been checked in perturbative examples
- A finite dimension 3 counterterm needs to be fixed by parity
- Achievement:
  O(a) improvement in the bulk of massless standard or partially improved Wilson quarks
  $\Rightarrow$ Z-factors can be O(a) improved by tuning a couple of boundary O(a) counterterms;
- Expect improved control over continuum extrapolation of SSF's with Wilson-type quarks (will benefit anybody using the continuum RG evolution to connect to RGI quantities)
- The chirally rotated SF Wilson-Dirac operator can be used in the Neuberger relation $\Rightarrow$ the overlap operator inherits the b.c.'s $\Rightarrow$ easy implementation of the SF for overlap and Domain wall quarks (so far: even number of flavours)