Fundamental constants and electroweak phenomenology from the lattice

Lecture V: CKM phenomenology: at loop level

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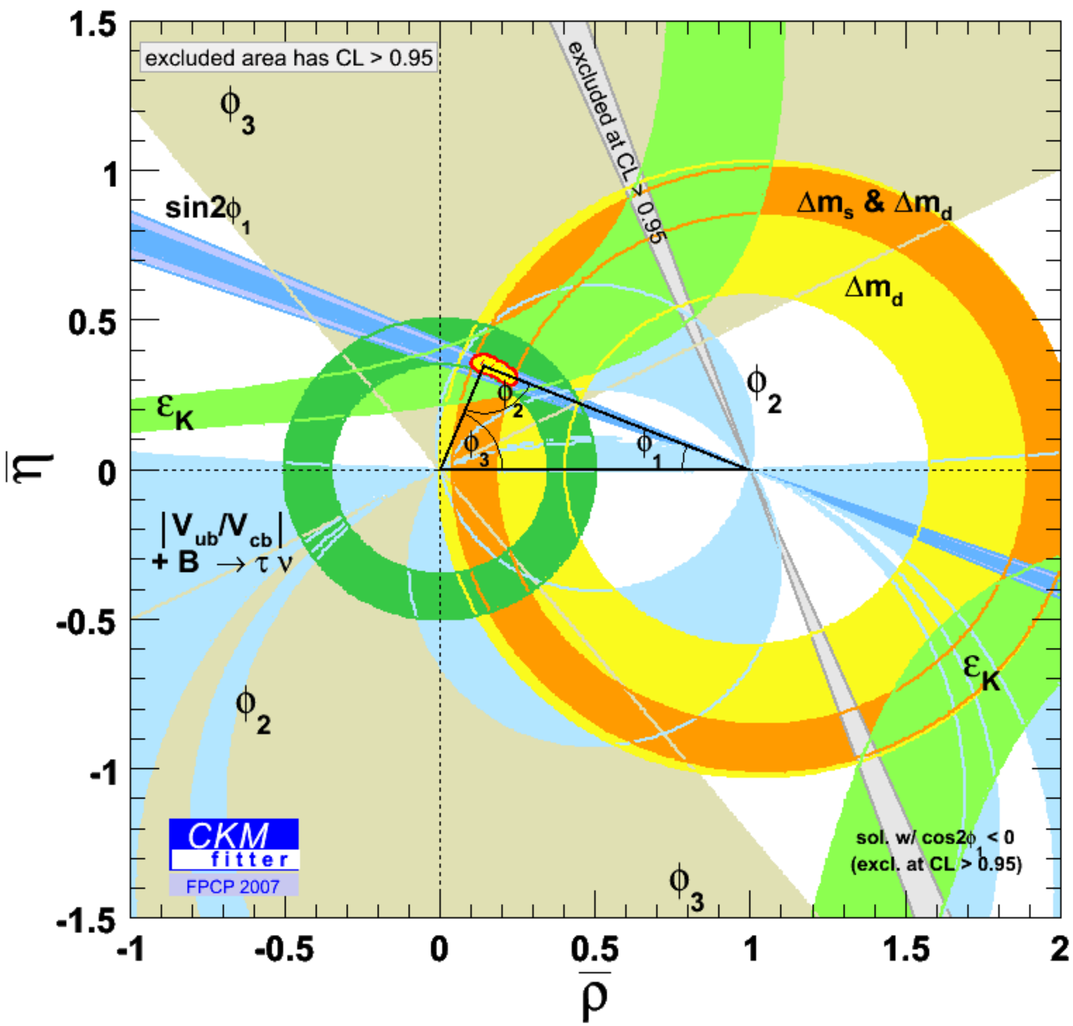
@ INT summer school 2007,
Seattle, August 2007.

CKM Physics



- Our goal:
  - To understand this plot
  - How lattice QCD may contribute to improve it.
  - Tree level decays (mixing angles) discussed yesterday. Today, the loop amplitudes.

V. CKM phenomenology: at loop level

1. Kaon mixing
   - Indirect and direct CP violations
   - Lattice calculation of B_K
   - ε'/ε, the grand challenge for the lattice
2. B meson mixings
   - Lattice calculation, extraction of Vtd, Vts
3. Phenomenology of B meson decays
   - Many interesting decay modes: a few examples
   - Further opportunities for lattice QCD
4. Other applications
   - Muon g-2, neutron electric dipole moment, …

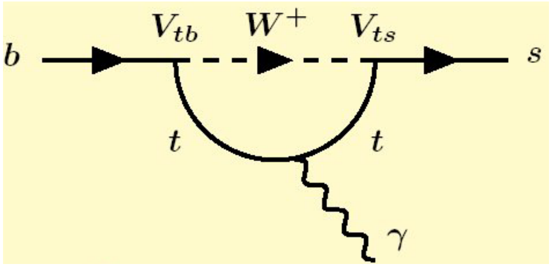
V. CKM phenomenology at loop level
1. Kaon mixing

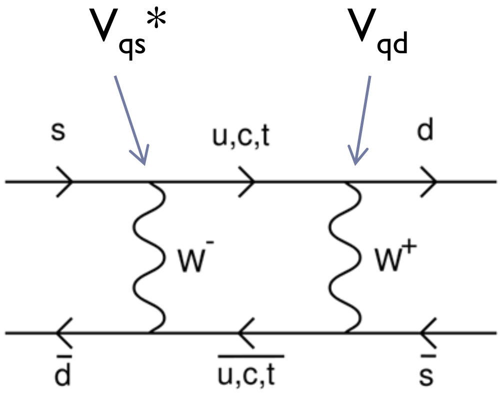
Loop processes

- Loop is not just a small correction, could induce something unusual = Flavor Changing Neutral Current (FCNC)
  - No FCNC at tree level in SM
  - Even at loop level, suppressed by the unitarity, e.g.

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$

thus vanishes when u,c,t are degenerate in mass.

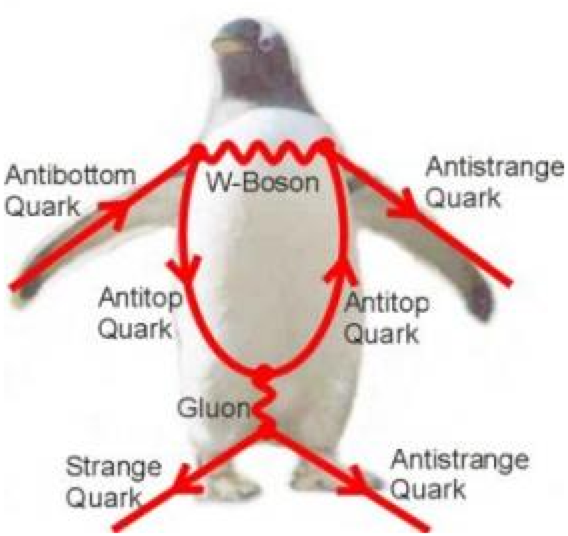




Glashow-Iliopoulos-Maiani (GIM) mechanism (1970)

FCNC processes

- FCNC is suppressed by GIM = a good place to look for New Physics
  - If new physics doesn't have GIM, its effect could be relatively enhanced in FCNC.
- Processes like…
  - Kaon mixing
  - B meson mixing
  - B decays through penguin diagram
  - …
    - contains loop diagram in general.



taken from www.physorg.com

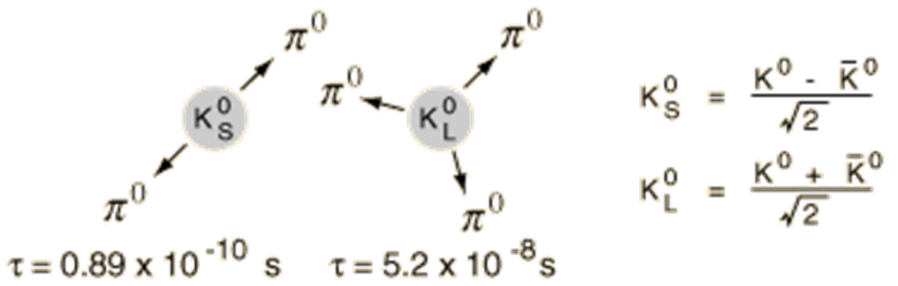
Kaon mixing

- CP violation was first observed in the neutral kaon mixing.
  - Cronin-Fitch (1964)

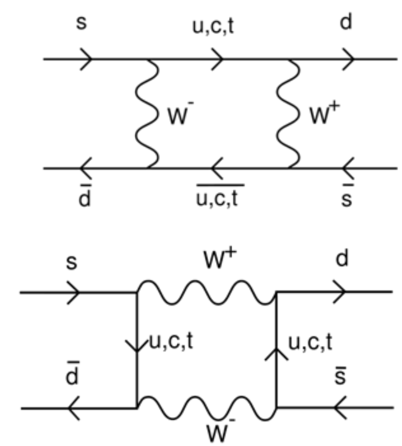
$$K_S \to \pi\pi \quad \text{(CP even)}$$

$$K_L \to \pi\pi\pi \quad \text{(CP odd)}$$

Taken from “Hyper Physics (Georgia State University)
http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html



- Induced by interference between dispersive (real) part and absorptive (imaginary) part of the amplitude.
  - dispersive: u, c, t (thus contains the CP phase)
  - absorptive: u (strong amplitude is imaginary)



Note: CP violation

- In SM, explained by the 3x3 KM matrix. One complex phase remains.

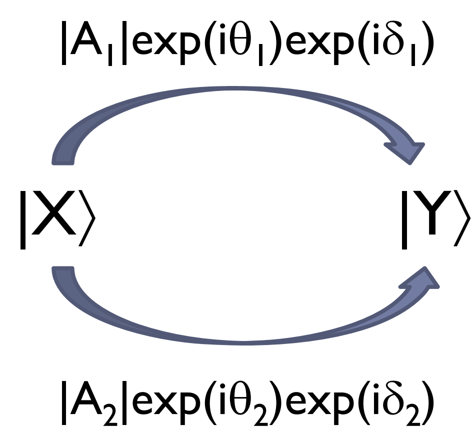
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

  - Must see interference among different amplitudes. Non-CP phase difference $\Delta\delta$ is also necessary.

$$\left|A_{X\to Y}\right|^2 = \left|A_1\right|^2 + \left|A_2\right|^2 + 2\left|A_1 A_2\right|\cos\left(\Delta\theta + \Delta\delta\right),$$

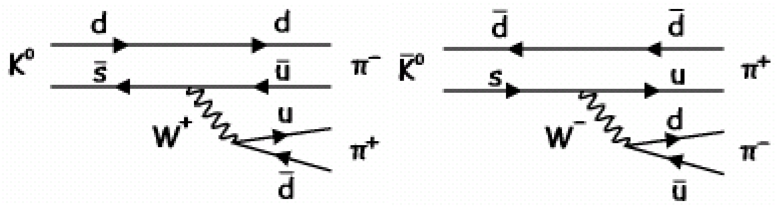
$$\left|A^{CP}_{X\to Y}\right|^2 = \left|A_1\right|^2 + \left|A_2\right|^2 + 2\left|A_1 A_2\right|\cos\left(-\Delta\theta + \Delta\delta\right),$$

$$\left|A_{X\to Y}\right|^2 - \left|A^{CP}_{X\to Y}\right|^2 = -4\left|A_1 A_2\right|\sin\left(\Delta\theta\right)\sin\left(\Delta\delta\right)$$

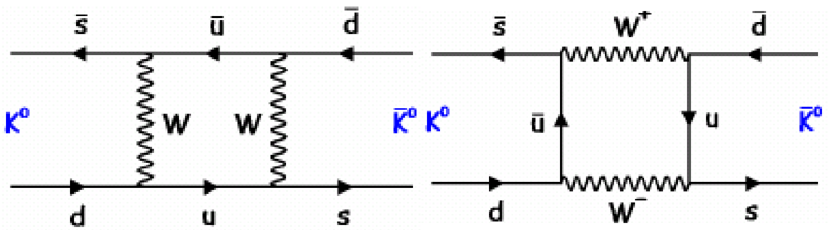


Mixing through…

- Neutral kaon mixing
  - K^0 and K^0bar can decay to the same final state $\pi\pi$, so can mix with each other, in principle.



  - There are also virtual processes to induce the mixing.



  - Can decay to $\pi\pi$ (CP even) or to $\pi\pi\pi$ (CP odd).

$$K_S \cong \frac{1}{\sqrt{2}}\left[\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right] \to \pi\pi \qquad c\tau = 2.7\,\mathrm{cm}$$

$$K_L \cong \frac{1}{\sqrt{2}}\left[\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right] \to \pi\pi\pi \qquad c\tau = 15.5\,\mathrm{m}$$

CPV from mixing

- To be precise, the states are mixture of CP eigenstates.

$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}\left[|K^0_{CP+}\rangle + \bar{\varepsilon}|K^0_{CP-}\rangle\right],$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}\left[|K^0_{CP-}\rangle + \bar{\varepsilon}|K^0_{CP+}\rangle\right].$$

"Im" picks up the CKM phase.

- Characterized by the small parameter ε

$$\bar{\varepsilon} = \frac{i}{2}\frac{\mathrm{Im}\,M_{12} - i\,\mathrm{Im}\,\Gamma_{12}/2}{\mathrm{Re}\,M_{12} - i\,\mathrm{Re}\,\Gamma_{12}/2} = i\frac{\mathrm{Im}\,M_{12} - i\,\mathrm{Im}\,\Gamma_{12}/2}{m_{K_L} - m_{K_S} - i(\Gamma_{K_L} - \Gamma_{K_S})}$$

$$M_{12} - \frac{i}{2}\Gamma_{12} = \langle K^0 | H_W | \bar{K}^0 \rangle$$

width difference

mass difference

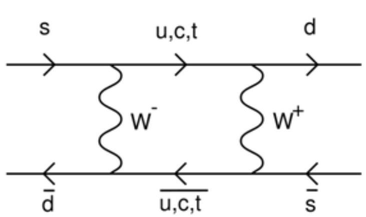
absorptive

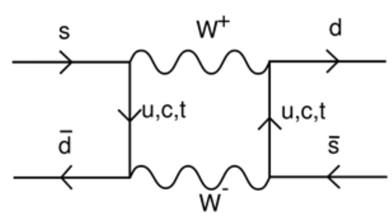
dispersive

Theoretical calculation?

- Re
  - Calculating the mass difference ($\mathrm{Re}M_{12}$) and the width difference ($\mathrm{Re}\Gamma_{12}$) is notoriously difficult, as they involve long distance effects (with π, η, $\pi\pi$, … as intermediate states). Must solve the "$\Delta I=1/2$ rule".
- Im
  - Dominated by short distance physics, as it must go through top quark in the intermediate state.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

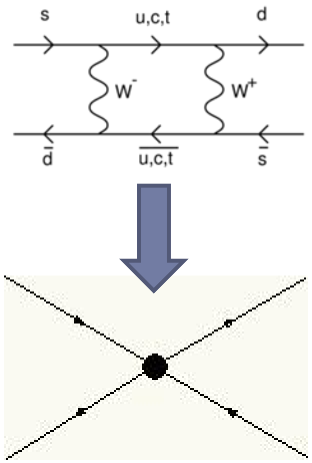




Weak effective Hamiltonian

- Short distance interaction can be represented by an effective operator.

$$O_{LL}=\bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma_\mu(1-\gamma_5)d$$



- The effective Hamiltonian to describe ΔS=2 transition.

$$H_{eff}^{\Delta S=2}=\frac{G_F^2 M_W^2}{16\pi^2}\left[\left(V_{cs}^*V_{cd}\right)^2\eta_1 S_0(x_c)+\left(V_{ts}^*V_{td}\right)^2\eta_2 S_0(x_t)+2\left(V_{cs}^*V_{cd}V_{ts}^*V_{td}\right)\eta_3 S_0(x_c,x_t)\right]O_{LL}$$

  - $S_0(x_c)$, $S_0(xt)$, $S_0(x_c,x_t)$ are Inami-Lim function to describe the box amplitude; a function of $x_i=m_i^2/M_W^2$.
  - "Im" picks up the imaginary part of the KM matrix elements.

B_K

- Problem is reduced to the calculation of a matrix element $\langle \bar{K}^0 | O_{LL} | K^0 \rangle$
  - Often parameterized as
    $$\langle \bar{K}^0 | O_{LL}(\mu) | K^0 \rangle = \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$
  - In the vacuum saturation approximation B_K=1.
    $$B_K(\mu) = \frac{\langle \bar{K}^0 | O_{LL}(\mu) | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K^0 \rangle} \to 1$$
  - Good to take a ratio: bulk of the systematic effects cancels.

Scale μ dependence canceled by the Wilson coefficient of the operator. Scale independent definition:

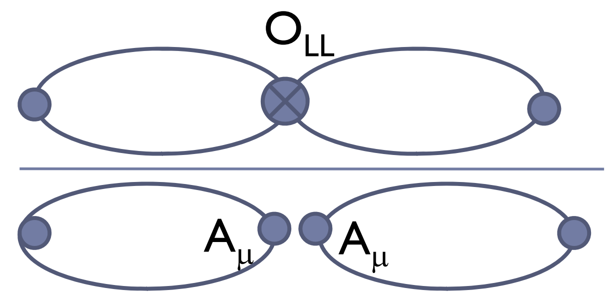
$$\hat{B}_K = B_K(\mu) \left[\alpha_s^{(3)}(\mu) \right]^{-2/9} \left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3 \right]$$

Lattice calculation of B_K

- A matrix element of the local operator O_{LL}.
  - Easy to calculate on the lattice.

$$B_K(\mu) = \frac{\langle \bar{K}^0 | O_{LL}(\mu) | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K^0 \rangle}$$

  - Chiral symmetry is essential to ensure that numerator behaves as $\propto m_K^2$, otherwise the ratio diverges.
  - Use
    - Overlap/domain-wall
    - Staggered (pick the NG pion)
    - Twisted-mass (special care needed)
    - Wilson (not impossible…)



Operator matching of O_{LL} should be done non-perturbatively, especially when the operator mixes with others, like O_{LR}…
Use of the RI/MOM scheme is a popular choice.

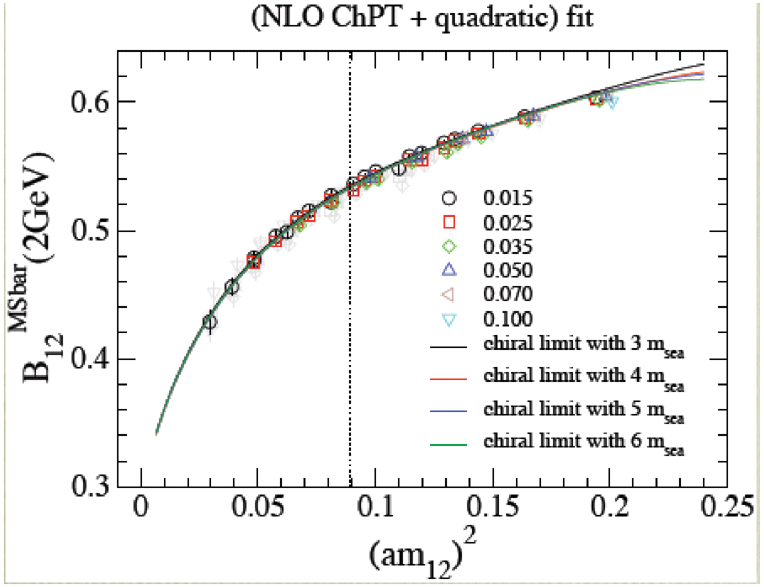
Chiral extrapolation

- For the extraction of B_K, the impact of chiral log is marginal.
  - Only kaon mass appears in the chiral log; interpolation only.
  - Visible in the lattice data.

$$B_P = B_P^{\chi}\left[1 - \frac{6m_P^2}{(4\pi f)^2}\ln\frac{m_P^2}{\mu^2} + bm_P^2 + O(m_P^4)\right]$$

  - Partially quenched ($m_{sea} \neq m_{val}$) formula available (Golterman-Leung, 1998)

JLQCD (2007)
with dynamical overlap
Yamada's talk at Lattice 2007



Some recent results

- RBC with domain-wall
  - Including 2+1 flavors of dynamical quarks.
  - Good control of (unnecessary) operator mixing.

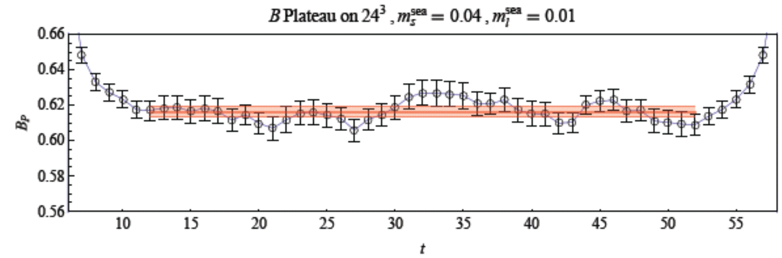
    B_K(2 GeV) = 0.522(10)(15)

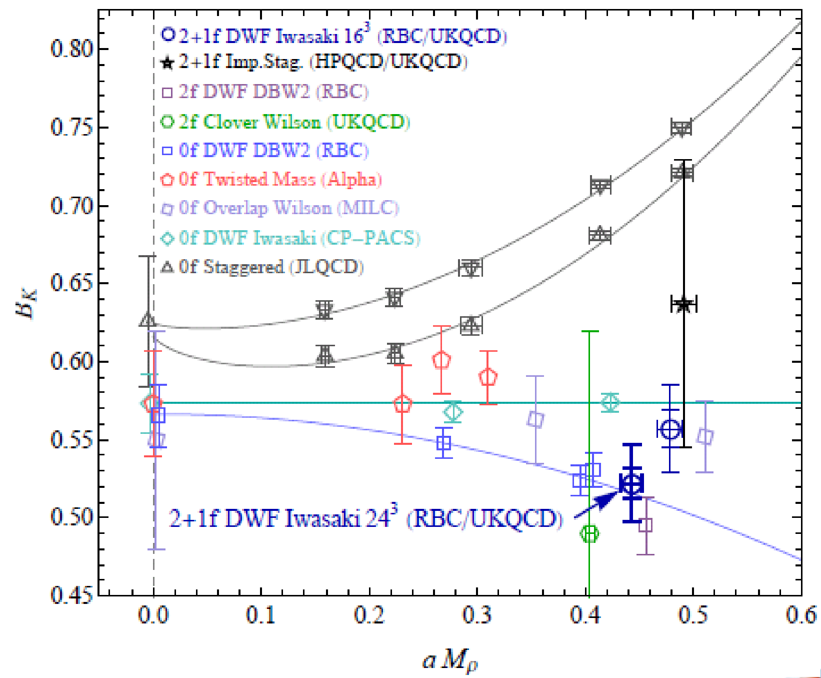
- JLQCD with overlap
  - 2 flavors of dynamical quarks
  - Perfect control of operator mixing

    B_K(2 GeV) = 0.533(7)

Yamada at lat07, error stat only

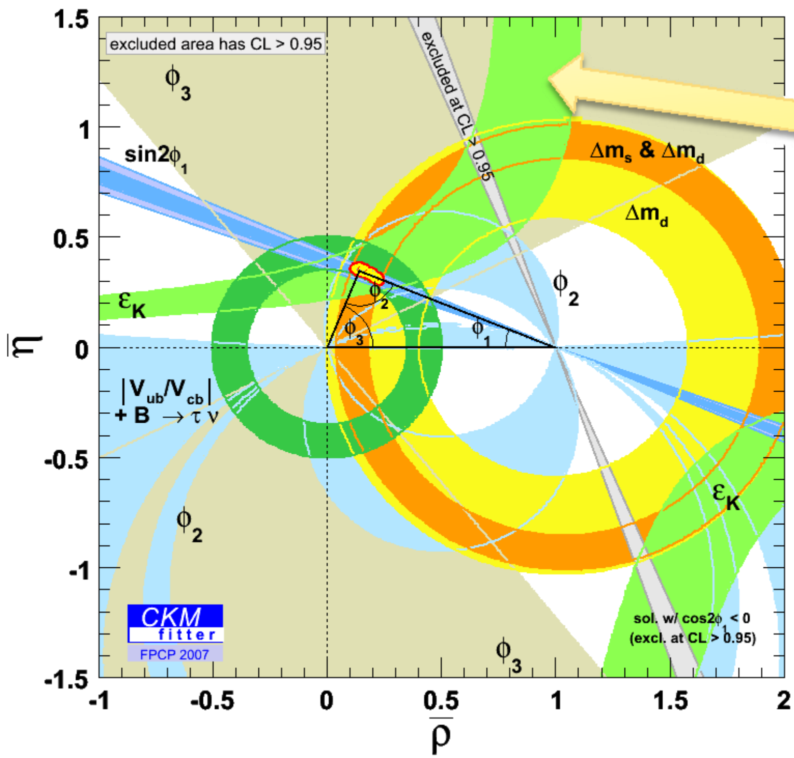
Plots from Cohen at Lettice 2007





ε_K on the unitarity triangle

- Can draw a constraint from ε_K
  - Was the only measurement of CP violation
  - Now, there are quite a few from B facories



12% error was assumed (CKM2006), which covers all recent unquenched calculations.
Too big, already? Eventually, <5% precision will be possible.

Direct CP violation

- Interference among decay amplitudes
  - Between ππ(I=0) and ππ (I=2)

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$$

$$A(\bar{K}^0 \to \pi^+\pi^-) = -\sqrt{\frac{2}{3}} A_0^* e^{i\delta_0} - \sqrt{\frac{1}{3}} A_2^* e^{i\delta_2}$$

$$\varepsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0} \left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} \right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}} \left[|K^0_{CP-}\rangle + \bar{\varepsilon} |K^0_{CP+}\rangle \right].$$

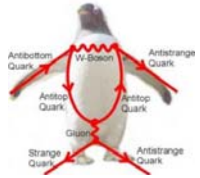
direct → ππ ← indirect

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \cong \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \cong \varepsilon - 2\varepsilon'$$

$$\mathrm{Re}(\varepsilon'/\varepsilon) = \begin{cases} 0.00207(28) & \text{(KTeV 2003)} \\ 0.00147(22) & \text{(NA48 2002)} \end{cases}$$

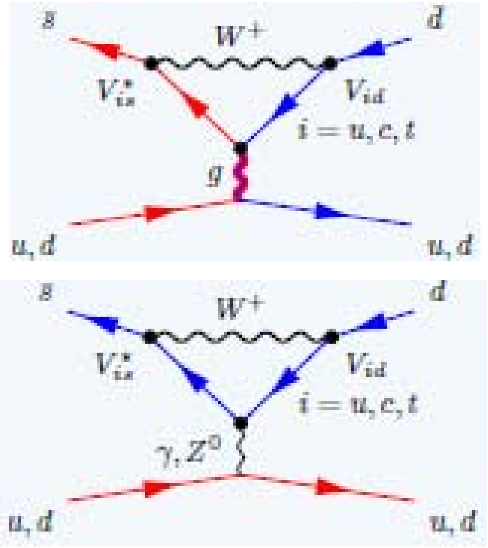
Penguins



- To produce the "Im" parts ($\mathrm{Im}A_0$, $\mathrm{Im}A_2$), loop processes are needed.
  - QCD penguin: Q_3, Q_4, Q_5, Q_6
  - Electro-weak penguin: Q_7, Q_8, Q_9, Q_{10}

$$Q_6 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

$$Q_8 = \frac{3}{2}\left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} e_q \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$



- Calculation of their matrix elements is the grand challenge
  - Operator mixing & power divergence
  - Large cancellation
  - Two pions in the final state

Power divergence

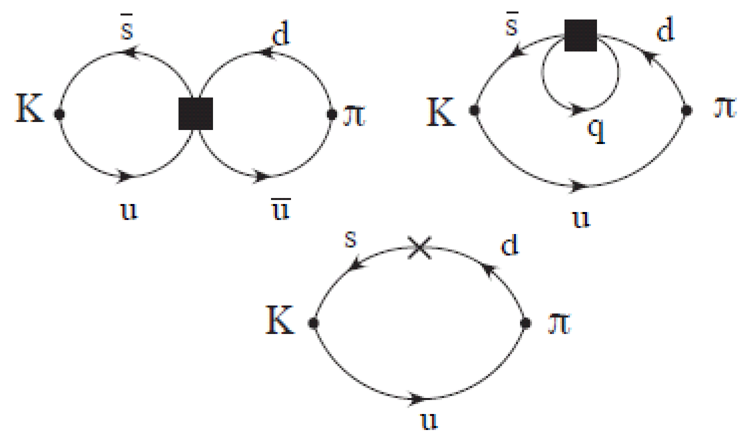
- Q_6 can mix with a lower dimensional operator

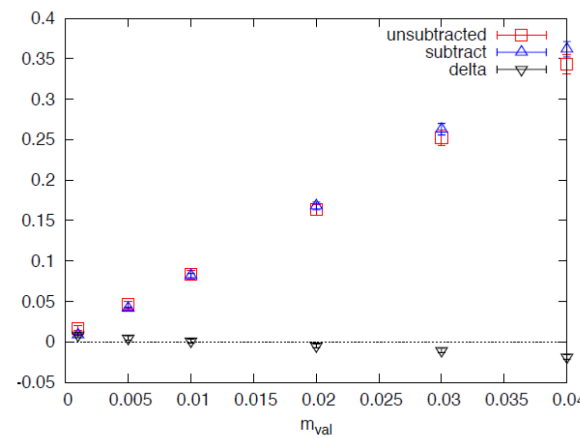
$$Q_6 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

$$\leftrightarrow \frac{1}{a^2}\left[(m_s + m_d)\bar{s}d - (m_s - m_d)\bar{s}\gamma_5 d\right]$$

  - Requires a good chiral symmetry; other operators could also contaminate if chiral symmetry is not exact.
  - In any case, huge cancellation occurs.

RBC 2007
(Mawhinney at Lattice 2007)





Two pions in the final state

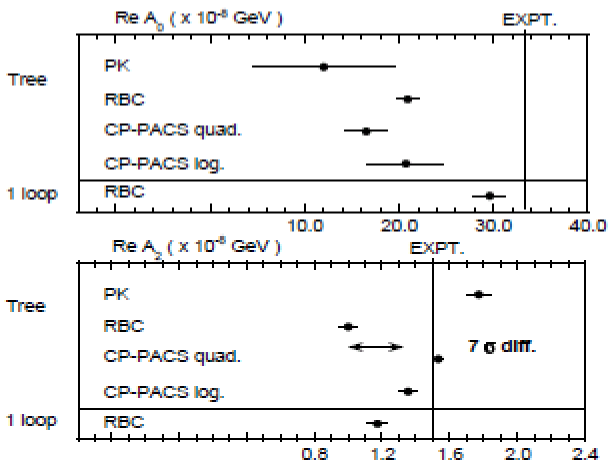
- Maiani-Testa no-go theorem (1990)
  - On the Euclidean lattice, the extraction of ground state relies on the analytic structure of particle pole.

$$C^{(2)}(t) = \int_{-\pi/a}^{+\pi/a} \frac{dq_0}{2\pi} \frac{e^{iq_0 t}}{m^2 + q_0^2 + \mathbf{q}^2} \sim e^{-E(\mathbf{q})t}$$

  - Not simply applied for two-particle state. If fact, the lowest energy state is always the zero momentum state $q_1=q_2=0$, not the state of interest.
  - By tuning the physical volume, an excited state can match the physical state.
  - Relation between finite volume ME and physical amplitude derived (Lellouch-Luscher, 2000); practical application is still very hard.

Use of ChPT

- Relate the $\langle\pi\pi|H_W|K\rangle$ matrix elements to those of $\langle\pi|Q|K\rangle$ and $\langle 0|Q|K\rangle$ using ChPT; no two-body final state appears.
  - LO (Bernard, 1985)
    - Extensive quenched studies by CP-PACS and RBC (2001)



PK	$-38.6 \pm 2.1 \pm 9.1$	Tree
RBC	-3.2 ± 2.2	Tree
	-4.0 ± 2.3	One-loop
CP-PACS	-7.7 ± 2.0	Tree
EXPT	$+20.7 \pm 2.8$	KTeV
	$+15.3 \pm 2.6$	NA48

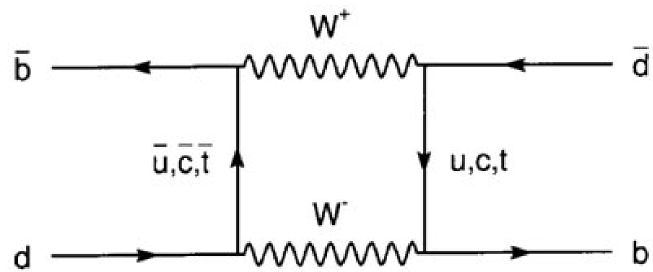
from Ishizuka at Lattice 2002

  - NLO (Laiho-Soni, Lin et al., 2002)
    - Work in progress RBC-UKQCD.

V. CKM phenomenology at loop level
2. B meson mixings

Neutral B meson mixings

- B_d and B_s
- Similar to the kaon mixing. But, different …
  - Dominated by the top loop (Inami-Lim function gives $\sim m_t^2/m_W^2$)
  - Thus, short distance; $\Delta M_{(d,s)}$ can be calculated.



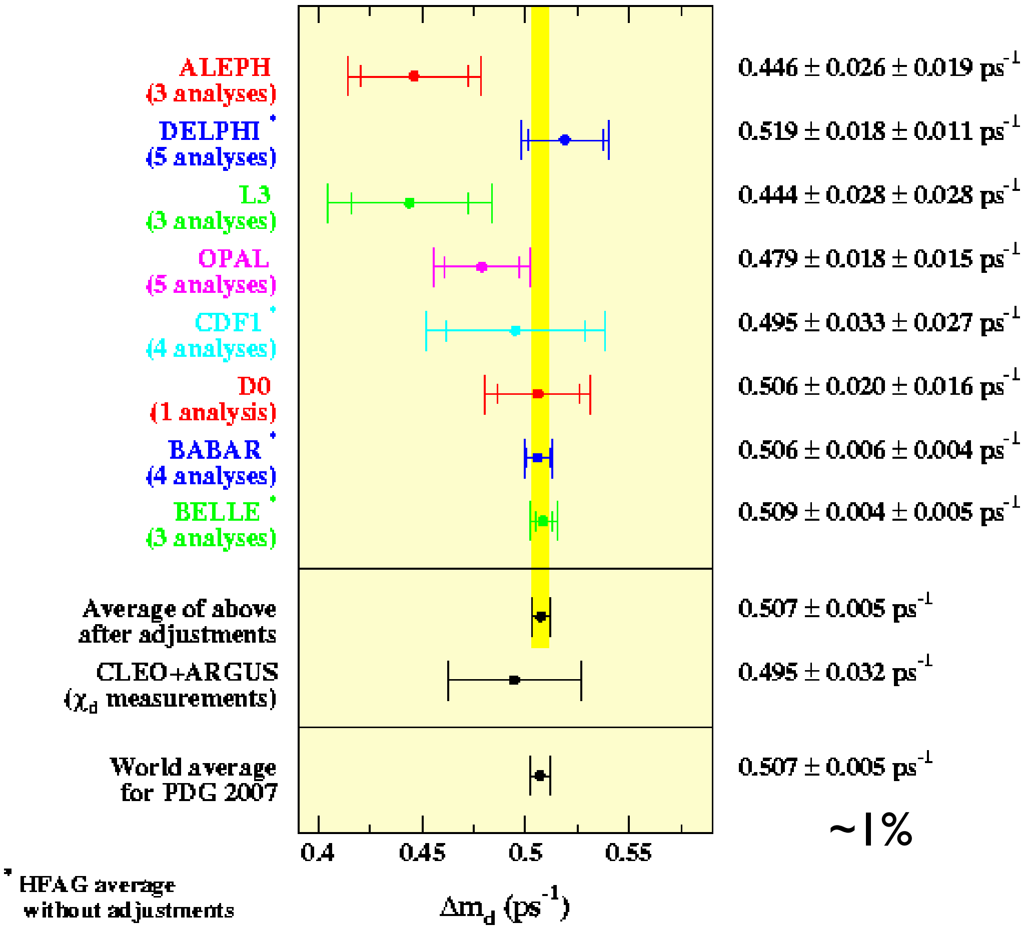
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\Delta M_q = \frac{G_F^2}{6\pi^2}\eta_B m_{B_q}\left(B_{B_q} f_{B_q}^2\right) M_W^2 S_0(x_t)\left|V_{tq}\right|^2$$

$$\left\langle \bar{B}_q^0 \right| (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \left| B_q^0 \right\rangle = \frac{8}{3} B_{B_q}(\mu) f_{B_q}^2 m_{Bq}^2$$

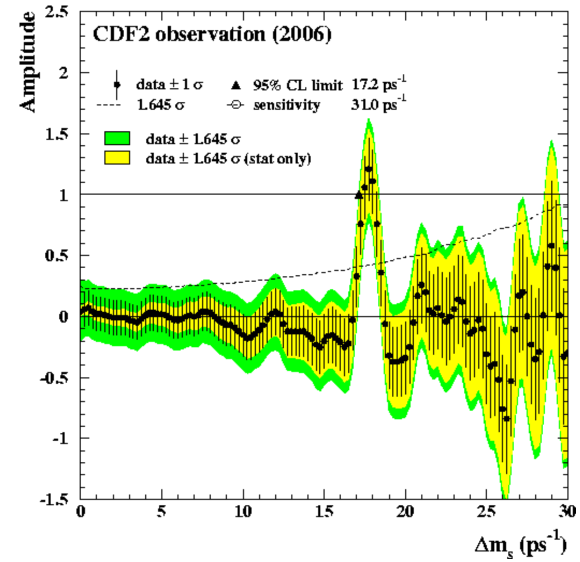
B mixings (experiment)

ΔMd (gives |Vtd|)



~1%

ΔMs (gives |Vts|)



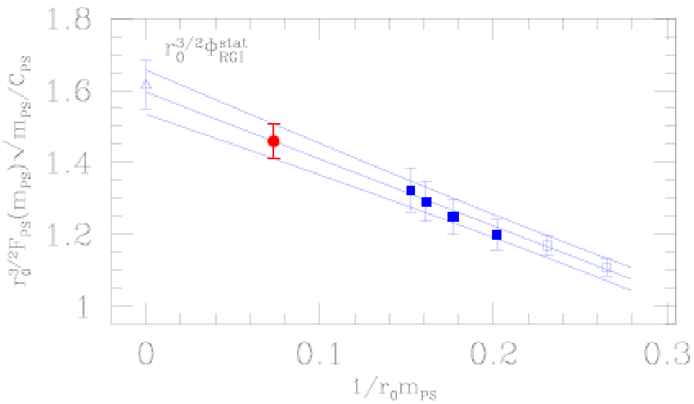
~1% (CDF Run II)

Errors on |Vtd|, |Vts| are now dominated by the lattice calculation.

Lattice calculation

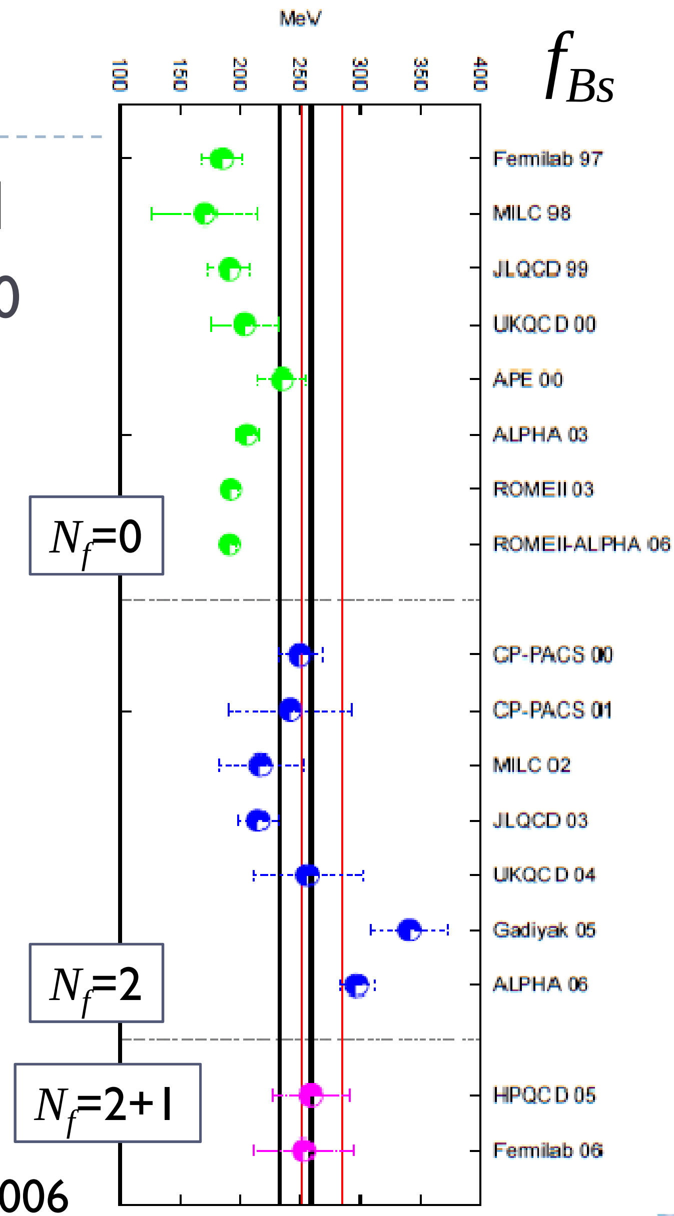
- Similar to f_K and B_K, except that b quark is much heavier.
  - Use a dedicated formulation for the heavy quark.
    - HQET, NRQCD, Fermilab, etc (see Kronfeld's lecture)
  - For f_B, 1/M correction is substantial. Extrapolation from below requires continuum extrapolation first, in order to avoid large $(am_Q)^2$ error.
    - Combining with the static limit (HQET) is helpful.

ALPHA at Lattice 2007
quenched, but NP



Decay constant

- Summary for f_{Bs} from N_f=0 to 2+1
  - The value slightly went up from N_f=0 to 2.
  - Error estimate depends on the group
    - Scale setting, heavy quark action, operator matching, …
  - Renormalization is the key to achieve better than 5%.
    - Mostly perturbative in the past. NPR will be mandatory in the future (how? see Sint's lecture)



Plot from Tantalo at CKM2006

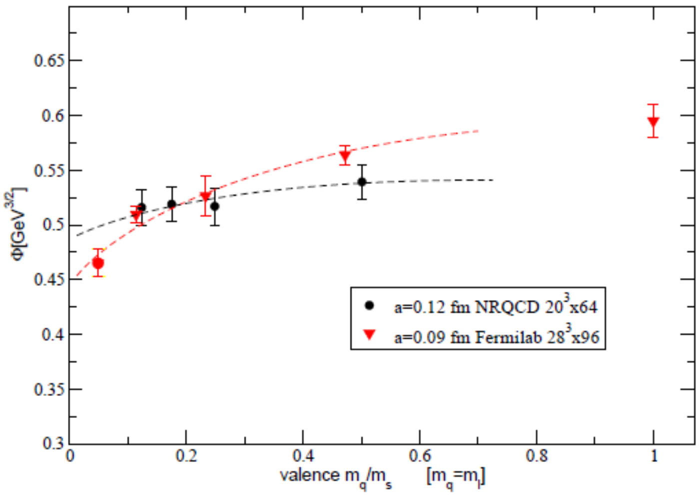
Chiral extrapolation

- Need to get f_{Bd}.
  - Extrapolation with the chiral log effect

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}} = 1 + \frac{1+3g^2}{4(4\pi f)^2}\left(3m_\pi^2 \log\frac{m_\pi^2}{\Lambda} - 2m_K^2 \log\frac{m_K^2}{\Lambda} - m_\eta^2 \log\frac{m_\eta^2}{\Lambda}\right) + \cdots.$$

- Most recent test
  - From HPQCD and MILC; both on the MILC 2+1 lattice
  - Uses SχPT in both cases
  - Matching at one-loop (matters only the overall normalization)

Plot from Della Morte at Lattice 2007

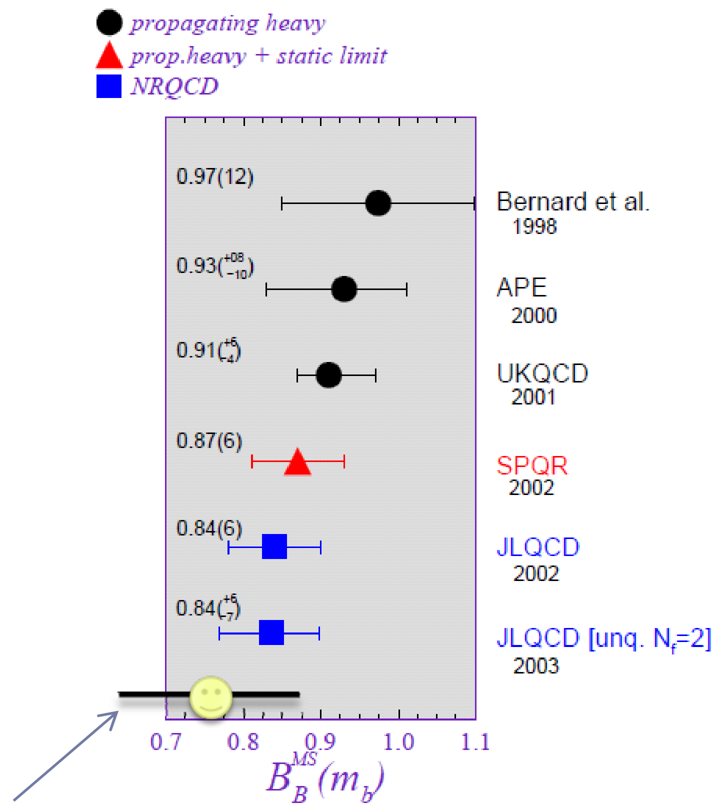


What is going on…?

Bag parameter

- Calculation is similar to B_K, except that b quark is much heavier.
  - Result does not so much depend on heavy quark mass, number of flavors.
  - Matching still perturbative
  - New efforts are emerging
    - Static + domain-wall (RBC/UKQCD)
    - Static + tmQCD with NP renormalization (ETMC)

From Onogi at Lattice 2006



HPQCD (2006)

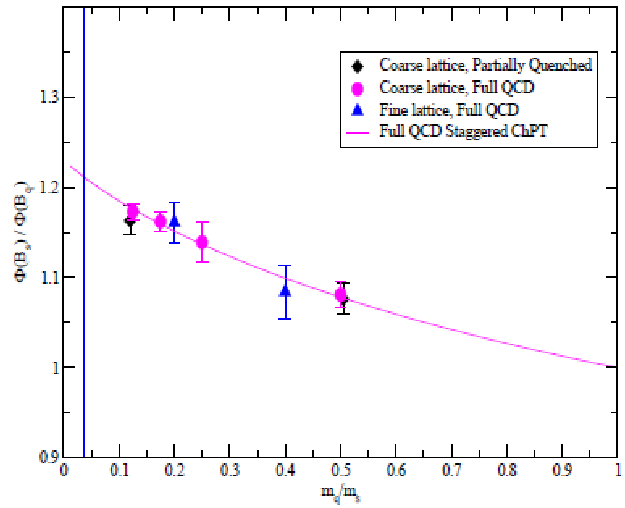
See also a poster by Evans at this school!

$|V_{td}/V_{ts}|$

- Chance to get a better precision for the ratio $\Delta M_d/\Delta M_s$

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_s}{M_d} \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2}$$

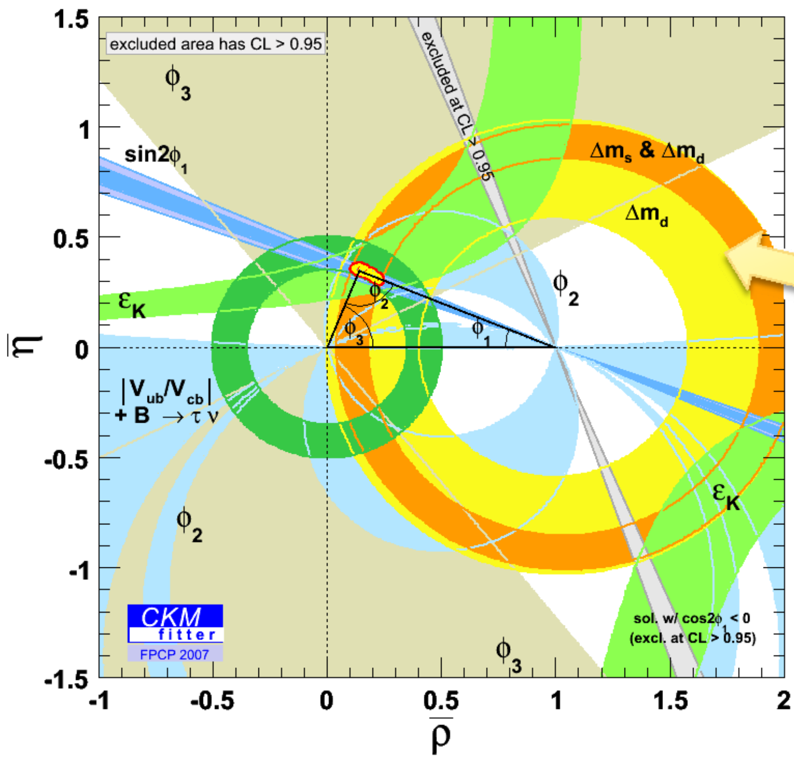
  - Bulk of errors (statistical + systematic) cancels. Only the chiral extrapolation is relevant.
  - Need a further check of the consistency with the NLO ChPT.
    - Note: an additional coupling $B^*B\pi$ appears; may use $D^*D\pi$ as an input



HPQCD (2006)
2+1-flavor calculation
A fit with $S\chi PT$

$|V_{td}/V_{ts}|$ on the unitarity triangle

- Can draw a circle with the center at (1,0)
  - Two circles: one from ΔM_d alone, the other from the ratio $\Delta M_s/\Delta M_d$.



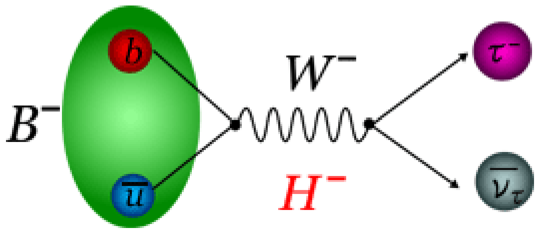
~15% error was assumed for f_B (CKM2006), which covers all recent unquenched calculations.
~5% for the ratio, now constrains better.

Leptonic decay (experiment)

- Info on the decay constant is now available from experiment.

$$B(B^- \to l^- \bar{\nu}) = \frac{G_F^2 m_B m_l^2}{8\pi}\left(1 - \frac{m_l^2}{m_B^2}\right) f_B^2 |V_{ub}|^2 \tau_B$$

$$= \begin{cases} (1.79^{+0.56+0.46}_{-0.49-0.51}) \times 10^{-4} & \text{(Belle)} \\ (1.8^{+1.0+0.3}_{-0.9-0.3}) \times 10^{-4} & \text{(BaBar)} \end{cases}$$



  - Difficult experiment: too small BR for eν and μν due to lepton mass, more than one neutrino for τν.
  - Error is still large, but the deduced value of f_B (=230(50) MeV) is roughly consistent with the lattice calculation.

Super-B expectation

Lum.	$\Delta B(B \to \tau\nu)_{exp}$
414 fb^{-1}	36%
5 ab^{-1}	10%
50 ab^{-1}	3%

V. CKM phenomenology at loop level
3. Phenomenology of B meson decays

B physics is rich

- Not just the mixing mass difference and the semi-leptonic decays.
  - Many FCNCs
  - Many places to find CP violation
  - CKM angles can be measured
  - Some hint of new physics??
- Exp info is rapidly growing. Will continue to be so.
  - LHC-b will start soon.
  - Super-B factory ?

# of pages of the B meson section in PDG

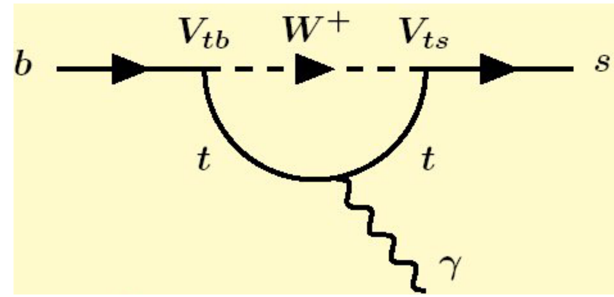
PDG	# pages
PDG 1996	51 pages
PDG 1998	58 pages
PDG 2000	70 pages
PDG 2002	85 pages
PDG 2004	98 pages
PDG 2006	123 pages

with many reviews

FCNCs

- b→sγ and related
  - B →K*γ: the first observed penguin (CLEO 1993)
  - Carries info on $|V_{ts}|^2$
  - b→sl⁺l⁻, other effective operators involved
  - B→ργ; |Vtd/Vts| can be extracted if the form factor ratio is known.
- Lattice calculation?
  - Two-body decay
  - Final states are energetic.

What can we do??



Sciolla at CKM2006

$$\left|\frac{V_{td}}{V_{ts}}\right|^{WA}_{\rho/\omega\gamma} = 0.202^{+0.017}_{-0.016}(\text{exp}) \pm 0.015(\text{th})$$

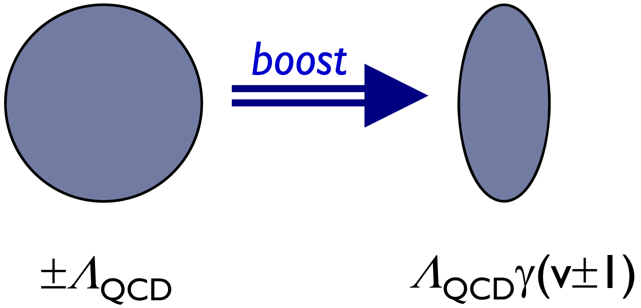
8.2% 7.5%

Moving... NRQCD

- Boosted system may be simulated on the lattice, by constructing an effective theory in the boosted frame
  - HQET with finite velocity; extension to 1/M = Moving NRQCD (SH-Matsufuru, Sloan, Davies-Dougall-Foley-Lepage)

$$\mathcal{L}_v = \bar{h}_v(x) i u \cdot D h_v(x)$$

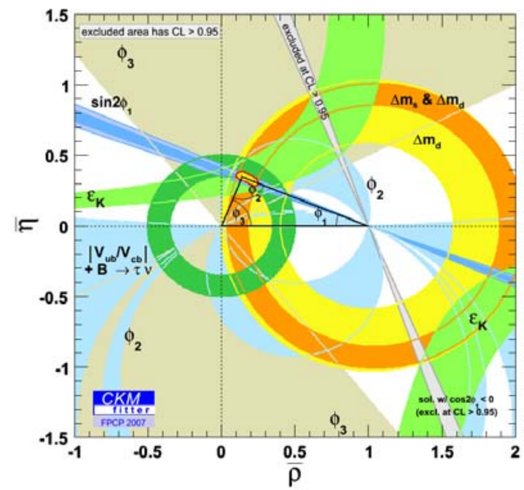
  - Maybe useful for $B \to K^* \gamma$, for instance.
  - Limitation will come from the Lorentz contraction (light quarks + gluons must propagate together)
  - Challenge!
    - Discretization effect enhanced
    - Statistical noise
    - Renormalization

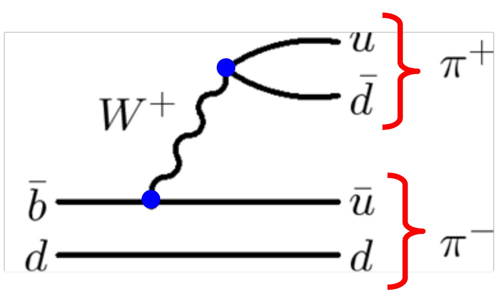


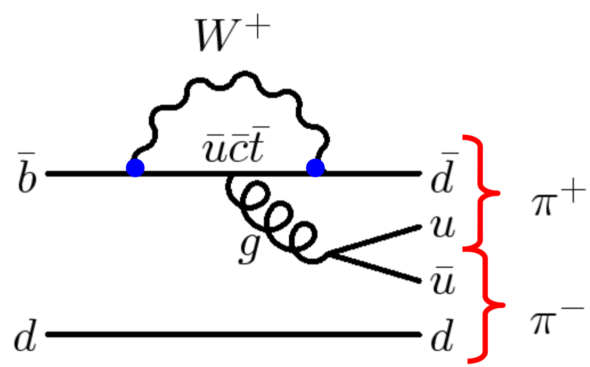
See a poster by Mienel at this school!

CP violation

- Angles are measured
  - $\sin 2\phi_1$: through $B \to J/\psi K_S$
    - the gold-plated mode (no contamination from hadron uncertainty)
  - $\sin 2\phi_2$: through $B \to \pi\pi$, $\rho\pi$, etc.
    - "penguin pollution" cured by the isospin analysis (Gronau-London, 1990; separate different amplitudes using isospin relations)

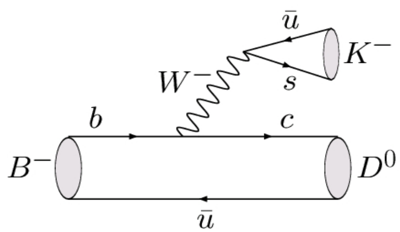


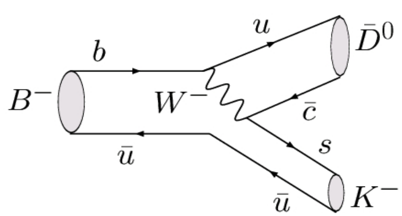




CP violation

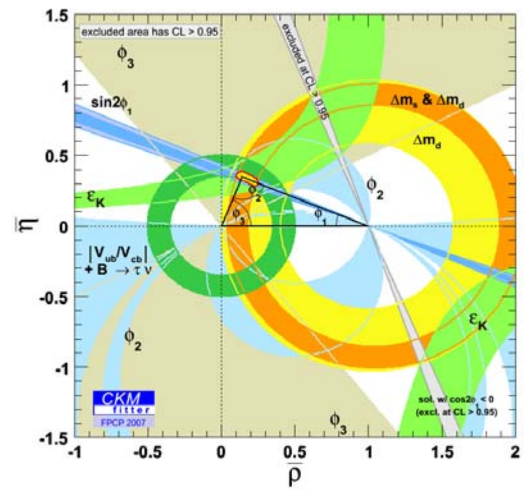
- ϕ_3: through B→DK
  - CP violating angle from tree decays
  - Methods to eliminate unknown strong phase difference (Gronau-London-Wyler (1991), Atwood-Dunietz-Soni (1997), …)





$$A \propto V_{cb}V_{us}^* \propto \lambda^3$$

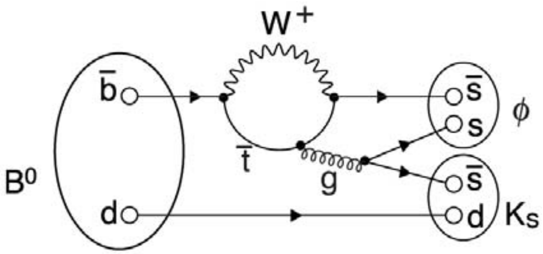
$$A \propto V_{ub}V_{cs}^* \propto \lambda^3\sqrt{\rho^2+\eta^2}\,e^{i\delta_B}e^{-i\gamma}$$

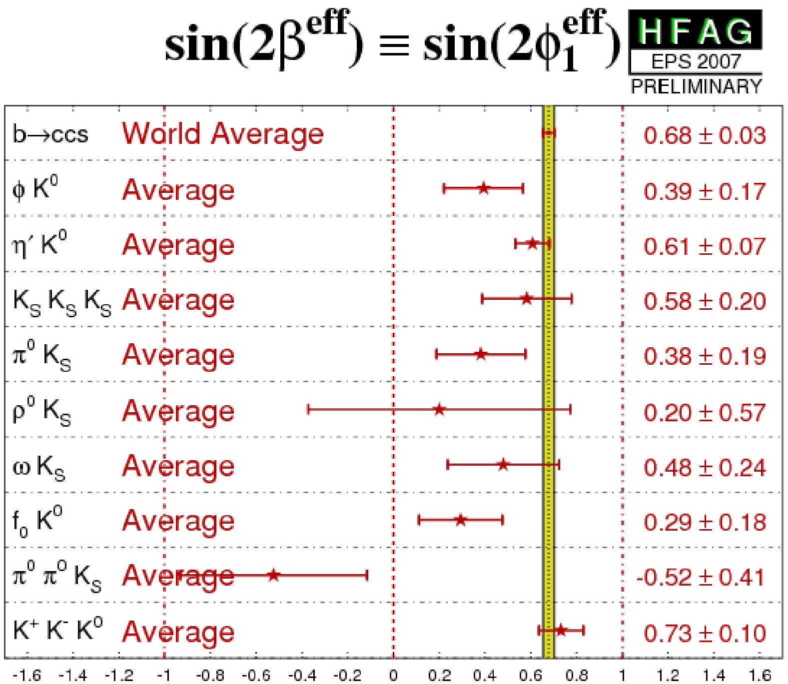


- Lattice calculation?
  - In many cases, useful if the ratio of amplitudes is theoretically calculated.

Sign of new physics?

- Penguin dominated mode
  $b \to s q \bar{q}$





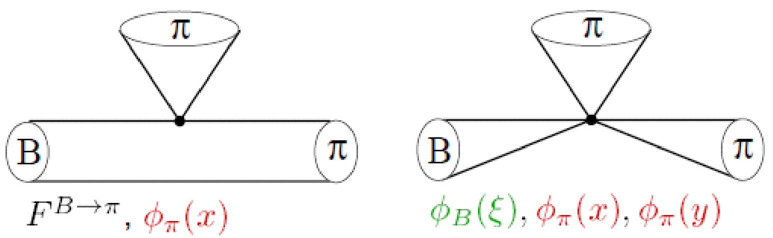
  - Should give $\sin 2\phi_1$
  - Significantly lower than $b \to c\bar{c}s$?

- How does the hadronic uncertainty affect the prediction?

QCD based calculation

- Perturbation theory is most suitable for these energetic decay modes.
  - pQCD for exclusive processes (Brodsky-Lepage, 1980)
  - Application for B decays = QCD factorization (Beneke-Buchalla-Neubert-Sachrajda, 1999)
  - An effective theory = Soft Collinear Effective Theory (SCET) (Bauer-Flemming-Pirjol-Stewart, … 2001~)
- Convolution of
  - Hard part
  - Light-cone distribution amp
  - (+ form factors)

  as in the pQCD calculation of DIS.



Light-cone distribution amplitude

- Defined on the light-cone coordinate

$$\langle \pi^+(q)|\bar{u}_\alpha(z)\,\mathscr{P}(z,-z)\,d_\beta(-z)|0\rangle\big|_{z^2=0} \equiv \frac{if_\pi}{4}(\not{q}\gamma_5)_{\beta\alpha}\int_0^1 du\, e^{i(2u-1)q\cdot z}\phi_\pi(u,\mu)$$

  - Expansion in z gives moments

$$\langle \pi^+(q)|\bar{u}(0)\gamma_5\gamma_{\{\rho}\overleftrightarrow{D}_{\mu\}}d(0)|0\rangle = f_\pi(iq_\mu)(iq_\rho)\int_0^1 du\,(2u-1)\,\phi_\pi(u,\mu)$$

$$\langle \pi^+(q)|\bar{u}\gamma_5\gamma_{\{\rho}\overleftrightarrow{D}_\mu\overleftrightarrow{D}_{\nu\}}d|0\rangle = f_\pi(iq_\rho)(iq_\mu)(iq_\nu)\int_0^1 du\,(2u-1)^2\,\phi_\pi(u,\mu)$$

Lattice calculation is then straight-forward. Almost like a calculation of pion decay constant, with a bit more complicated operator. (For a recent work, see a talk by C. Sachrajda at Lattice 2007; and a poster by Donnellan here!)

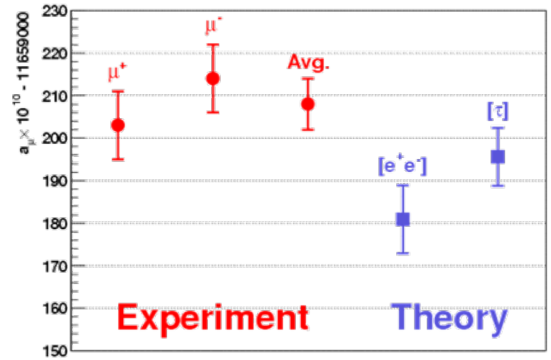
V. CKM phenomenology at loop level
4. Other applications

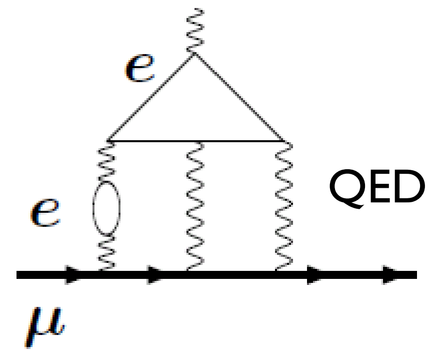
Muon g-2

- Anomalous magnetic moment of muon
  - Experiment is very precise.

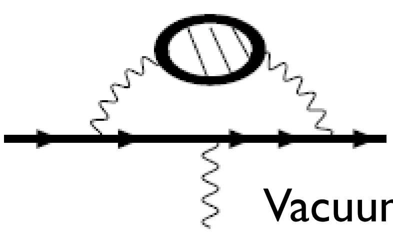
    $$\mu = 1.0011659208(6)\,(e\hbar / 2m_\mu)$$

  - QED correction calculated to α^4 (Kinoshita et al.)
  - Electroweak contribution is small.
  - Hadronic contribution is the major uncertainty



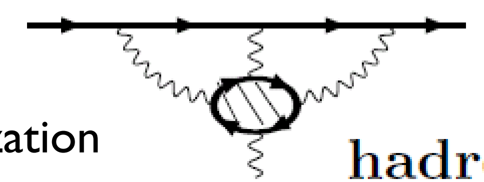


QED α^4



Vacuum polarization

Light-by-light scattering

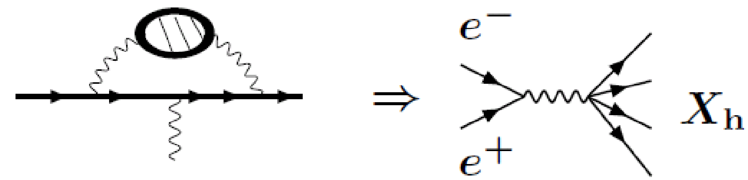


hadrons

Diagrams taken from Melnikov's lecture at SLAC summer institute (2002)

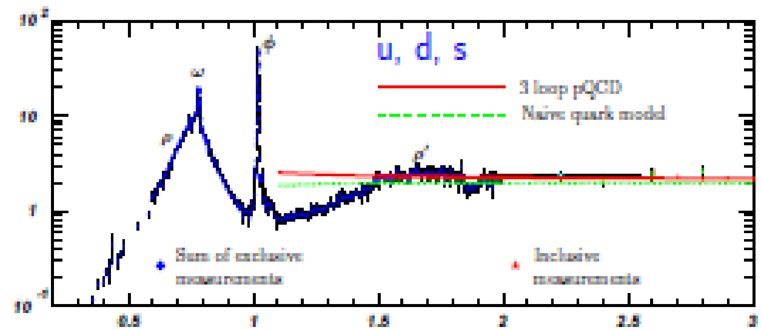
Vacuum polarization

- Usually related to the e+e- cross section using the optical theorem.



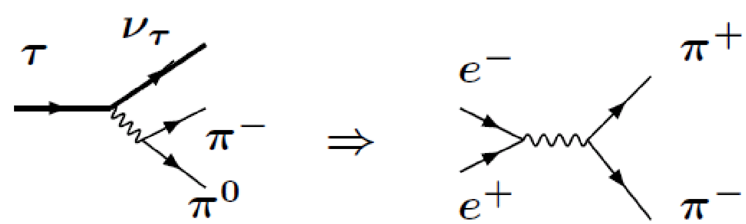
$$a_\mu^{\rm vp} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} {\rm d}s\, K(s)\, \sigma_{\rm h}(s)$$

$$K(s) \sim \frac{m_\mu^2}{s} \text{ for } s \gg m_\mu^2.$$



- Or, τ decay can also be used assuming isospin symmetry. Agreement is not satisfactory.

  (If we believe e+e-, the sign of NP is stronger.)

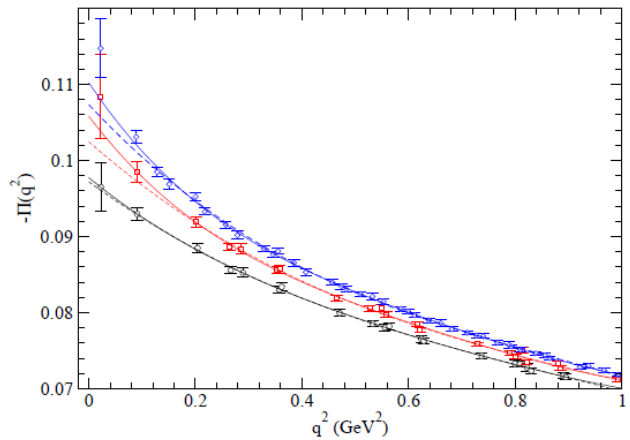


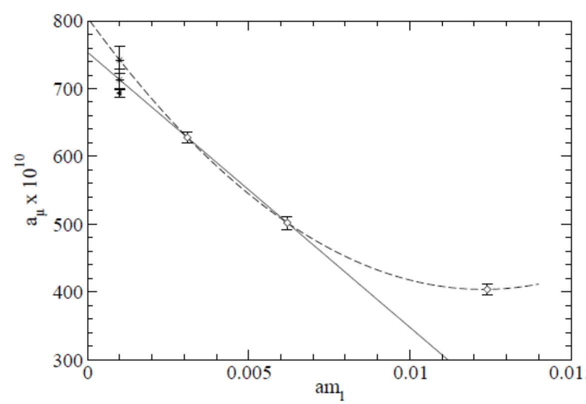
Vacuum polarization on the lattice

- Lattice can provide a direct calculation in the Euclidean region.
  - Data should contain all the equivalent physics as in the e+e- cross section.
  - But, the kinematics is very different at heavier quark masses ($\rho \to \pi\pi$ threshold is not open, etc.)
  - So, the comparison is non-trivial. Chiral extrapolation can be tricky.

Interesting avenue: get the physics info in the Euclidean region and use the optical theorem.

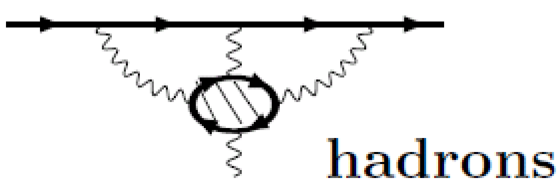
Aubin-Blum (2006)





Light-by-light

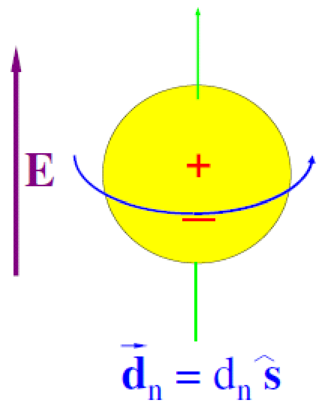
- Much more challenging…, but
  - deserves efforts, because there is no direct exp info available on $\gamma^*\gamma^* \to \gamma^*\gamma^*$.



null

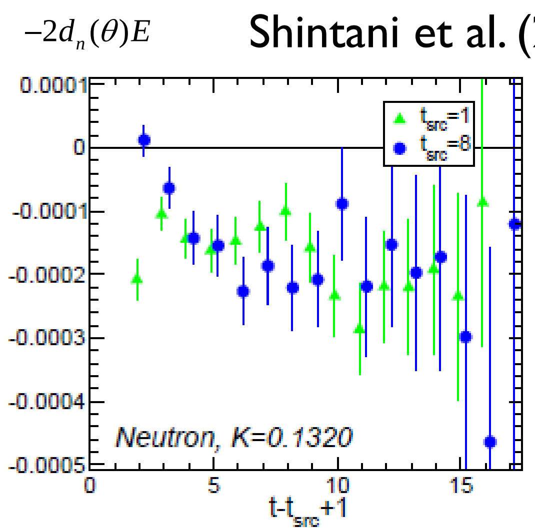
Neutron electric dipole moment

- CP violation not related to flavor
  - Strong CP problem
  - New physics models may induce large NEDM.



  - Need non-perturbative calculation to relate θ (or imaginary quark mass) to d_n.
  - Possible to calculate on the lattice
    - e.g. on a constant electric field
    - Reweighting with exp(i θQ)

Shintani et al. (2006)



And more, …

Final remarks

- Lattice QCD is not a stand-alone business!
  - Interesting physics opportunities often come from outside of QCD… CKM determination and more.
  - There are many other quantities that are waiting for viable non-perturbative tools.

  - Lattice calculation needs help from symmetries and effective theories:
    - Chiral symmetry, heavy quark symmetry
    - χPT, HQET, …

    They are essential to control systematic effects.

Thanks to …

- Organizers
  - to Karl Jansen, Kostas Orginos, Steve Sharpe
  - Especially, to Steve for the exciting night at…

- And, of course, to the excellent audiences!