The SF running coupling with four flavours of staggered quarks

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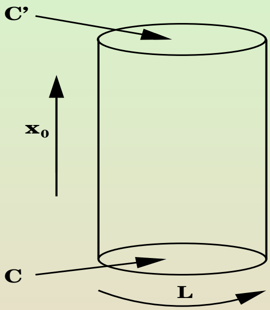
Outline

Schrödinger functional

- Schrödinger functional:
  $\mathcal{Z}[C,C'] = \int \mathcal{D}[A,\psi,\bar\psi] e^{-S[A,\psi,\bar\psi]}$
- Boundary conditions:

$$\begin{cases} A_k(x)\big|_{x_0=0} = C_k \quad A_k(x)\big|_{x_0=T} = C'_k \\ (1+\gamma_0)\psi\big|_{x_0=0} = 0 = (1-\gamma_0)\psi\big|_{x_0=T} \\ \bar\psi(1-\gamma_0)\big|_{x_0=0} = 0 = \bar\psi(1+\gamma_0)\big|_{x_0=T} \end{cases}$$

  C, C' abelian, spatially constant (Narayanan et al. '92).
- Effective action: $\Gamma[B] = -\ln \mathcal{Z}[C,C']$.
- B is the Background field induced by the b.c.'s.
- Consider a one parameter family, $B(\eta)$, $m_q = 0$, $C, C' \sim L^{-1}$, $T = L$.
- L is the only scale of the problem.



Running coupling

- For small couplings g_0, the effective action has an asymptotic expansion,

$$\Gamma[B] \overset{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \dots$$

- Definition of the running coupling (η is a dimensionless parameter):

$$\frac{1}{\bar{g}^2(L)} = \frac{\Gamma_0'}{\Gamma'} \qquad \Gamma' = \partial_\eta \Gamma[B(\eta)]\big|_{\eta=0}.$$

- Previous work: **Pure Yang Mills** (Lüscher et. al. '93) and **QCD with 2 flavours** (ALPHA collaboration '05). The running of the coupling has been computed using finite size scaling methods.
- Here, we try to deal with **QCD with 4 flavours**. Staggered fermions seem to be a natural choice.

Staggered fermions and continuum limit

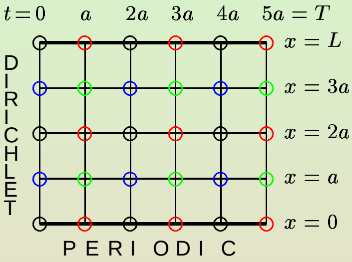


Figure: 2^d fermionic degrees of freedom on a two dimensional lattice

- Technical problem with staggered fermions (Miyazaki and Kikukawa '94 & Heller '97): T/a odd and L/a even.
- Modified conventions: take the continuum limit at $T' = L$. $T' = T + sa$ is the extent of the dual lattice ($s = \pm 1$).
- This modifies the $\mathrm{O}(a)$ effects in the pure gauge theory even at tree level.

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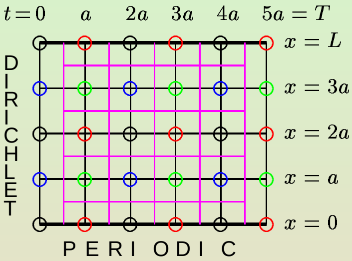

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Tree level calculation

Classical Yang-Mills action

- $S[U] = \frac{1}{g_0^2} \sum_p w(p)$ tr $\{1 - U(p)\}$ $\quad p \equiv$ oriented plaquette.
- Fields at the boundaries: $\begin{array}{l} C_k = i/L \; \mathrm{diag}(\phi_1, \phi_2, \phi_3) \\ C'_k = i/L \; \mathrm{diag}(\phi'_1, \phi'_2, \phi'_3) \end{array}$
- $\mathrm{O}(a)$ Counterterms :

$$a \int_{x_0=0,T} d^3\mathbf{x} \; \mathrm{tr}\,\{F_{0k}F_{0k}\}, \qquad a \int_{x_0=0,T} d^3\mathbf{x} \; \mathrm{tr}\,\{F_{kl}F_{kl}\}.$$

- $c_t(g_0) = c_t^{(0)} + c_t^{(1)} g_0^2 + \dots$
- Taking the continuum limit at fixed $T'/L = 1 \Rightarrow c_t^{(0)} \neq 1$.

Equations of motion

- Covariant divergence of the plaquette (Narayanan et al. '92), modified by the weight factor,
  $d_w^* P(x,\mu) =$
  $\sum_{\nu=0}^{3} \left\{ w[P_{\mu\nu}(x)] P_{\mu\nu}(x) - w[P_{\mu\nu}(x-\hat{\nu})] U_\nu^\dagger(x-\hat{\nu}) P_{\mu\nu}(x-\hat{\nu}) U_\nu(x-\hat{\nu}) \right\}$
- Equations of motion:
  $d_w^* P(x,\mu) - d_w^* P^\dagger(x,\mu) - \frac{1}{3}\mathrm{tr}\left\{ d_w^* P(x,\mu) - d_w^* P^\dagger(x,\mu) \right\} = 0.$
- We assume the solution to be $V_\mu(x) = \exp(aB_\mu(x_0))$ spatially constant and diagonal in the colour structure.
- Up to gauge equivalence,
  $$B_k(x_0) = \begin{cases} \left(x_0 - \frac{T}{2}\right) f + \frac{C_k + C_k'}{2} & x_0 \in [a, T-a] \\ C_k & x_0 = 0 \\ C_k' & x_0 = T \end{cases}$$
  $B_0(x_0) = 0.$
  f had to be computed numerically.
- A cooling program has been written to confirm that B is indeed the minimum action configuration.

Choice of $c_t^{(0)}$

- $c_t^{(0)}$ is to be chosen in such a way that makes the $\mathrm{O}(a)$ effects vanish. If $T' = T + sa$ with, $s = \pm 1$,
  $S_{latt} = S_{cont}\left\{1 + \frac{a}{L}\left[-2 + s + \frac{4c_t^{(0)} - 2}{c_t^{(0)}}\right] + \mathrm{O}(a^2)\right\}.$
- We thus conclude that the appropriate value for $c_t^{(0)}$ is,

$$c_t^{(0)} = \frac{2}{2+s}.$$

- With that choice $S_{latt} = S_{cont} + \mathrm{O}(a^4)$.

First order correction of the running coupling

- $\Gamma_0[B] = g_0^2 S[B]; \quad \Gamma_1[B] = \frac{1}{2} \ln \det \Delta_1 - \ln \det \Delta_0.$
- $\bar{g}^2(L) = g_0^2 + m_1(L/a) g_0^4 + \ldots; \quad m_1(L/a) = -\Gamma_1'/\Gamma_0'.$
- Compute: $\partial_\eta \ln \det \Delta_j / \Gamma_0'$ with $j = 0, 1.$
- $m_1(L/a) \overset{L\to\infty}{\sim} \sum_{n=0}^{\infty} (a/L)^n (B_n + A_n \ln(L/a)).$
- Results for $m_1(L/a)$ confirmed by independent calculation performed by S.Takeda and U.Wolff,

s	A_0	B_0	A_1	B_1
-1	$22/(4\pi)^2$	$0.368283(1)$	0	$-0.2318(3)$
0	$22/(4\pi)^2$	$0.3682817(7)$	0	$-0.1779(3)$
1	$22/(4\pi)^2$	$0.3682818(7)$	0	$0.1232(4)$

Table: Coefficients of asymptotic expansion (in collab. with S.Takeda & U.Wolff)

Determination of $c_t^{(1)}$

- $S_{latt} = S_{latt}\big|_{c_t=c_t^{(0)}} + g_0^2 \frac{\partial S_{latt}}{\partial c_t}\big|_{c_t=c_t^{(0)}} c_t^{(1)}$.
- $\bar{g}^2(L) = g_0^2 + \left(m_1(L/a) - c_t^{(1)} \Gamma_0' / \partial_{c_t} \Gamma_0' \big|_{c_t=c_t^{(0)}} \right) g_0^4 + \mathrm{O}(g_0^6)$.
- $c_t^{(1)} \Gamma_0' / \partial_{c_t} \Gamma_0' \big|_{c_t=c_t^{(0)}} = \mathrm{O}(a/L)$. It can thus remove the contribution of B_1.
- We arrive at,

$$c_t^{(1)} = \frac{B_1}{2} \left(c_t^{(0)} \right)^2 .$$

Fermion action

- Fermionic part of the action (Heller '97),
  $S_{SF} =$
  $\sum_{\vec{x}} \sum_{x_0}^{T-1} \frac{1}{2}\eta_\mu(x)\bar{\chi}(x) \left[\lambda_\mu U_\mu(x)\chi(x+\mu) - \lambda_\mu^\dagger U_\mu^\dagger(x-\mu)\chi(x-\mu)\right] +$
  $S_B^{(0)} + S_B^{(T)}$
- $c_t^{(1)}$ has a contribution coming from the fermionic part of the action.
- We have preliminary results for the contribution to $m_1(L/a)$ from the fermionic action.
- Universal factors of the asymptotic expansion (A_0, B_0) coincide with the results obtained by Heller '97.
- However, a more detailed analysis is required.

Future and summary

- We want to calculate the SF running coupling for QCD with 4 flavours.
- Staggered fermions are a natural choice. They oblige lattices to have $T \neq L$.Continumm limit is taken at fixed $T' = L$.
- This special feature modifies the coefficient of the $\mathrm{O}(a)$ improvement counterterms.
- Coefficient for the countertems: $c_t^{(0)} = \frac{2}{2+s}$ and $c_t^{(1)} = \frac{B_1}{2}\left(c_t^{(0)}\right)^2$.
- At the moment, pure Yang-Mills simulations are being performed, in collaboration with U.Wolff and S.Takeda.
- Currently we are performing a more detailed analysis of the fermionic action, determining the coefficients of the contributing $\mathrm{O}(a)$ boundary counterterms.
- Once the set up of the theory is well established, numerical simulations with fermions will be carried out.