Glueballs and Scalar Mesons in Lattice QCD

Chris Richards†(a), Alan Irving(a), Eric Gregory(a), Craig McNeile(b)

UKQCD collaboration

†cmr@liv.ac.uk
(a) Department of Mathematical Sciences, University of Liverpool
(b) Department of Physics and Astronomy, University of Glasgow

Introduction

Members of the Lattice QCD group at the University of Liverpool are actively engaged in the study of the flavour-singlet sector [1] of quantum chromodynamics (QCD). The group has a strong track record in this field of Lattice QCD, with publications spanning nearly 25 years.

The group is currently performing a high statistics investigation of scalar ($J^{PC} = 0^{++}$) singlet states, including the f_0 mesons and their mixings with so called 'glueball' states.

Staggered Fermions

We have chosen to generate the configurations for this study using the Asqtad staggered fermion action. Asqtad is well known to be a fast action (due to the fewer degrees of freedom) which makes it feasible for us to build large, light dynamical ensembles. This is necessary when considering 0^{++} glueballs as they are rather heavy so require high statistics to increase the signal to noise ratio.

The structure of the staggered fermion action differs significantly from all Wilson type actions. Rather than working with spinors directly we define Kogut-Susskind fermion fields $\chi(x)$ and $\overline{\chi}(x)$ by performing the following spin-diagonalisation transformations:

$$\psi(x) = \Omega(x)\chi(x), \quad \overline{\psi}(x) = \overline{\chi}(x)\Omega^\dagger(x) \qquad \text{with} \qquad \Omega(x) = \prod_{\mu=1}^{4} (\gamma_\mu)^{x_\mu} .$$

The resulting action

$$S_{\text{stag}}(x) = \sum_{x,y} \overline{\chi}(x) \left(m\delta_{xy} + \frac{1}{2}\sum_\mu \alpha_\mu(x) \left(U_\mu(x)\delta_{x,y-\hat{\mu}} - U_\mu^\dagger(x-\hat{\mu})\delta_{x,y+\hat{\mu}} \right) \right) \chi(y)$$

carries no spin indices — the Dirac matrices are hidden in the Kogut-Susskind phases $\alpha_\mu(x)$. The action is diagonal in spinor space, and the four components of χ are identical — i.e. the action S_{stag} describes 4 flavours (which we refer to as tastes) of fermion which are degenerate in the free field case.

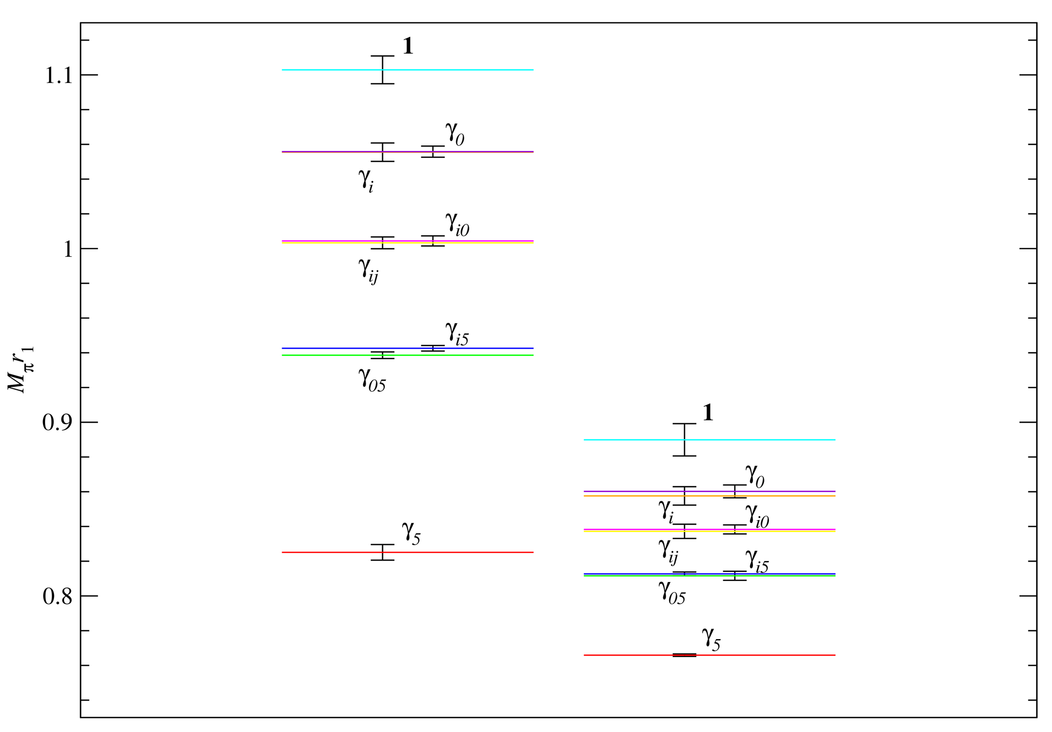
The Fourth Root Trick

In order to represent one physical flavour we reduce this fourfold degeneracy by taking the fourth-root of the fermion determinant. It is this 'fourth-root trick' (FRT) which attracts so much criticism — the FRT renders the action non-local for $a \neq 0$. However it is expected that rooted staggered fermions still have the correct continuum limit (see [3]) and the rooted theory is described well by rooted $\mathrm{S}\chi\mathrm{PT}$.

Taste Symmetry Violations

The fourfold degeneracy above exists only in the free-field case, interactions with the gauge fields break the taste symmetry. These taste breaking interactions lead to a more complicated spectrum — for example, when considering the pseudoscalar spectrum, rather than having a single pion we obtain 16 pions grouped into 8 multiplets, each with a different mass. We work in the $spin \otimes taste$ basis in order to distinguish between the different tastes.

These taste breaking effects are an $\mathcal{O}(a^2)$ effect for the Asqtad action and hence vanish in the continuum limit as the figure below indicates. Due to the remnant $U(1)$ chiral symmetry only one of the 16 pions become massless in the chiral limit, the $\gamma_5 \otimes \gamma_5$ pion.



Taste symmetry violations for the pions on MILC asqtad coarse ($am_{l/s} = 0.02/0.05$) and fine ($am_{l/s} = 0.0124/0.031$) lattices (from [2])

The Asqtad action includes terms in the action which attempt to reduce the taste symmetry violations by suppressing the coupling of quarks to gluons with high momentum — the single link in the action above is replaced by a fattened link which includes paths with up to 7 links.

Glueballs

Since $SU(3)$ is a non-Abelian group the field strength tensor includes a product of the gluon fields due to the commutator of 2 gluon fields being non-zero. This leads to self interactions of the gluons, in particular three and four gluon vertices. This allows us to consider colourless bound states of the gluons themselves, known as glueballs.

Glueball Operators

In order to extract the physics of glueballs from gauge configurations we need to choose operators which have the maximum overlap with the glueball wavefunction. In order to obtain the correct J^{PC} quantum numbers we sum over different orientations of plaquettes, and in order to achieve the desired definite momentum $\mathbf{p}$ we sum over all sites of the lattice with a weighting factor $e^{i\,\mathbf{p}\cdot\mathbf{x}}$.

Since the 0^{++} glueball corresponds to the A_1^{++} irrep of the cubic subgroup $\mathcal{O}_h$ we only need choose plaquettes which transform under the same irrep, and as such the following operator is sufficient to represent $|\mathbf{p}| = 0$ glueballs

$$\mathcal{O}_i^{A_1^{++}}(t) = \mathrm{Tr} \sum_{\mathbf{x}} \left(\Box_{12}^i(\mathbf{x},t) + \Box_{23}^i(\mathbf{x},t) + \Box_{31}^i(\mathbf{x},t) \right) .$$

Glueballs are quite large objects (effective radius ~ 0.3 fm) and so operators formed from elementary plaquettes do not reveal much of the large distance physics. In order to enhance the large distance physics we both APE smear and Teper block the gauge field. Both are iterative procedures which makes them computationally attractive.

$$U_\mu^1(n) = U_\mu^0(n)U_\mu^0(n+l_0\hat{\mu}) + \sum_{\substack{\pm\nu\neq\mu \\ \nu\neq 4}} U_\nu^0(n)U_\mu^0(n+l_0\hat{\nu})U_\mu^0(n+l_0\hat{\nu}+l_0\hat{\mu})U_\mu^{0\dagger}(n+2l_0\hat{\mu}) .$$

Singlet Mesons on the Lattice

We study mesons on the lattice by creating a state using an operator $\mathcal{O}$ with the desired properties (e.g. J^{PC} quantum number) at time t, and letting it propagate on the lattice until time $t+\tau$, when we annihilate it by creating its antiparticle. A (non-staggered) mesonic operator is thus written

$$\mathcal{O}(\mathbf{x},t) = \overline{\psi}_i(\mathbf{x},t)\Gamma\psi_j(\mathbf{x},t)$$

where Γ is some product of the Euclidean gamma matrices.

The correlation function for this general mesonic operator is written

$$C(\tau) = \langle \overline{\psi}_j(\mathbf{x}',t+\tau)\overline{\Gamma}\psi_i(\mathbf{x}',t+\tau)\overline{\psi}_i(\mathbf{x},t)\Gamma\psi_j(\mathbf{x},t)\rangle \quad .$$

In the case of the flavour-singlet mesons we consider mesons which are made from a quark and antiquark of the same flavour, e.g. $\bar{u}u$. For flavour-singlets there are four non-zero contributions to the correlation function from the Wick contraction

$$\langle \overline{\psi}_{i,\mathbf{x}'}\overline{\Gamma}\psi_{i,\mathbf{x}'}\overline{\psi}_{i,\mathbf{x}}\Gamma\psi_{i,\mathbf{x}}\rangle \quad .$$

These contractions result in *connected* (lower contractions) and *disconnected* (upper contractions) contributions to the correlation function. The disconnected correlators are formed from correlations of quark fields at the same site, and the connected ones from those at different sites.

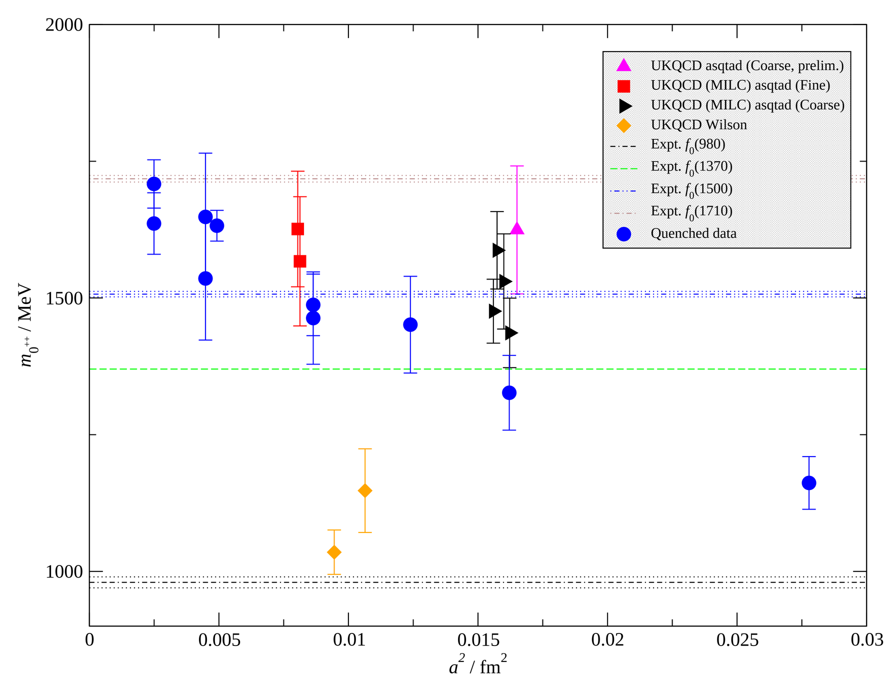
Glueball and Singlet Scalar Meson Mixing

In pure gauge QCD (i.e. the quenched approximation) the glueball is a perfectly well defined concept since the glueball operators G do not mix with the fermionic $\overline{\psi}\Gamma\psi$ operators. However, in unquenched or *dynamical* QCD the sea quarks, which were previously neglected, will cause the glueball and flavour-singlet fermionic 0^{++} operators to couple to the same set of states [4].

For this reason we call the 'glueball operators' glueball interpolating operators since we can design operators as well as possible to couple more strongly to glueball components than fermionic components, but there will always be mixing between them.

Scalar singlets — the f_0 meson and glueballs

Members of the lattice QCD group at Liverpool have made determinations of the masses of scalar singlet states as well. The $J^{PC} = 0^{++}$ sector includes the f_0 meson. The 0^{++} glueball will mix with the 0^{++} $\overline{q}q$ operators. From the lattice we would expect to get the masses of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. A summary of our results is below.



References

[1] E. B. Gregory, A. C. Irving, C. C. McNeile, S. Miller and Z. Sroczynski, PoS **LAT2005** (2006) 027 [arXiv:hep-lat/0510066].

[2] C. Aubin *et al.*, Phys. Rev. D **70** (2004) 094505 [arXiv:hep-lat/0402030].

[3] S. R. Sharpe, PoS **LAT2006** (2006) 022 [arXiv:hep-lat/0610094].

[4] A. Hart, C. McNeile, C. Michael and J. Pickavance [UKQCD Collaboration], Phys. Rev. D **74** (2006) 114504 [arXiv:hep-lat/0608026].