Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\, J_H^{+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\bar{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

Physical processes take place in real time

$$D^{>}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})\, J_H\ (0,0)\right\rangle,$$

$$D^{<}(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})\, J_H\ (t,\vec{x})\right\rangle$$

$$\frac{D^{>}(\omega)-D^{<}(\omega)}{2\pi}=\frac{1}{\pi}\operatorname{Im} D_R(\omega)=\sigma(\omega)$$

$$G(\tau,T)=D^{>}(-i\tau)$$

$$G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T)))}{\sinh(\omega/(2T))}$$

if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ ⇒ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\dots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0$, $\tau_{max}=1/T$

and in the case of resonances, e.g.

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\sigma(\omega)/\omega^2$$

Spectral functions at T>0 and physical observables

Heavy meson spectral functions:

$$J_H = \overline{\psi}\,\Gamma_H\,\psi$$

→ quarkonia properties at finite temperature
heavy quark diffusion in QGP: D

Light vector meson spectral functions:

$$J_\mu = \overline{\psi}\,\gamma_\mu\,\psi$$

$$\lim_{\omega \to 0} \sigma_{ii} = \zeta\omega$$

→ thermal dilepton production rate functions :

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\sigma_{\mu\mu}(\omega,p,T)$$

thermal photon production rate :

$$p\frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

electric conductivity: ζ

Energy momentum tensor :

$$T_{\mu\nu}(x)$$

→ shear η and bulk viscosity ξ

crucial input into hydrodynamic models of heavy ion collisions

Reconstruction of the spectral functions : MEM

$$G(\tau,T) = \int_0^\infty d\omega \sigma(\omega,T) \cdot K(\omega,T)$$

$\mathcal{O}(10)$ data and $\mathcal{O}(100)$ degrees of freedom to reconstruct

Bayesian techniques: find $\sigma(\omega,T)$ which maximizes

$$P[\sigma|DH] \sim P[D|\sigma H]P[\sigma|H]$$ (Bayes' theorem)

data

Prior knowledge

prior probability

H : $\sigma(\omega,T) > 0$ → Maximum Entropy Method (MEM): $P[\sigma|H] = e^{\alpha S}$

Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH] = P[\sigma|D\alpha m] = \exp(-\frac{1}{2}\chi^2 + \alpha S)$$

Likelihood function

Shannon-Janes entropy:

$$S = \int_0^\infty d\omega[\sigma(\omega) - m(\omega) - \sigma(\omega)\ln\frac{\sigma(\omega)}{m(\omega)}]$$

$m(\omega)$ - default model $m(\omega \gg \Lambda_{QCD}) = m_0\omega^2$ -perturbation theory

Procedure for calculating the spectral functions

How to find numerically a global maximum in the parameters space of O(100) dimenisons for fixed α ?

$$\sigma(\omega) = m(\omega) \exp\left[\sum_{i=1}^{N} s_i u_i(\omega)\right], \quad N \leq N_\tau/2$$

Find the basis $u_i(\omega)$ through

SVD of $K = U\Sigma V, \; u_i(\omega_j) = U_{ji}$ Bryan, Europ. Biophys. J. 18 (1990) 165

or use $u_i(\omega) = K(\omega, \tau_i)$, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

⇩

maximization of $P[\sigma|D\alpha m]$ reduces to minimization of

$$U = \frac{\alpha}{2} \sum_{i,j=1}^{N} s_i C_{ij} s_j + \sum_{l=0}^{N_\omega} \sigma(\omega_l) \Delta\omega - \sum_{i=1}^{N} \bar{G}(\tau_i) s_i$$

covariance matrix (C_{ij}); ensemble average ($\bar{G}$)

which can be done using Levenberg-Marquardt algorithm ⇨ $\hat{\sigma}_\alpha$

Procedure for calculating the spectral functions (cont'd)

How to deal with the α-dependence of the result ?

$$\sigma(\omega) = \int d\alpha \ \hat{\sigma}_\alpha(\omega) \ P[\alpha|Dm]$$

For good data $P[\sigma|D\alpha m]$ is sharply peak around $\sigma(\omega) = \hat{\sigma}_\alpha(\omega)$ and using Bayes' theorem

$$\begin{aligned} P[\alpha|Dm] &\sim \int [d\sigma] P[D|\sigma\alpha m] P[\sigma|\alpha m] P[\alpha|m] \\ &\sim P[\alpha|m] \int [d\sigma] \exp\left[-\frac{1}{2}\chi^2 + \alpha S\right] \\ &\sim P[\alpha|m] \exp\left[\frac{1}{2}\sum_k \frac{\alpha}{\alpha+\lambda_k} + \alpha S(\hat{\sigma}_\alpha) - \frac{1}{2}\chi^2(\hat{\sigma}_\alpha)\right] \end{aligned}$$

λ_k are the eigenvalues of $\Lambda_{ll'} = \frac{1}{2}\sqrt{\sigma_l}\frac{\partial \chi^2}{\partial\sigma_l \partial\sigma_{l'}}\sqrt{\sigma_{l'}}|_{\sigma=\hat{\sigma}_\alpha}$ and common choices for $P[\alpha|m]$ are $P[\alpha|m] = const$ and $P[\alpha|m] = 1/\alpha$.

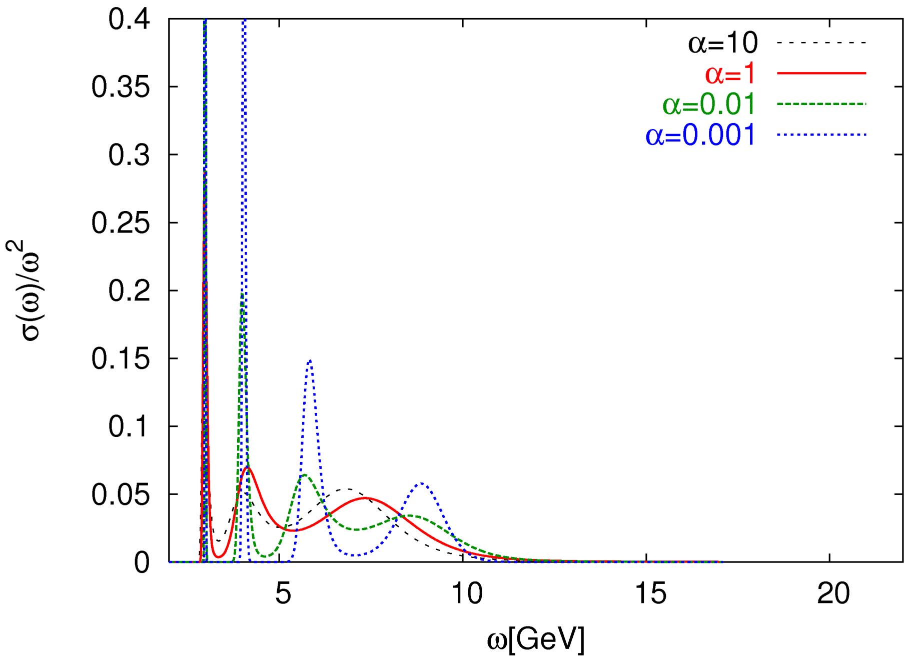
In practice $P[\alpha|Dm]$ is peaked at some α_{max}.

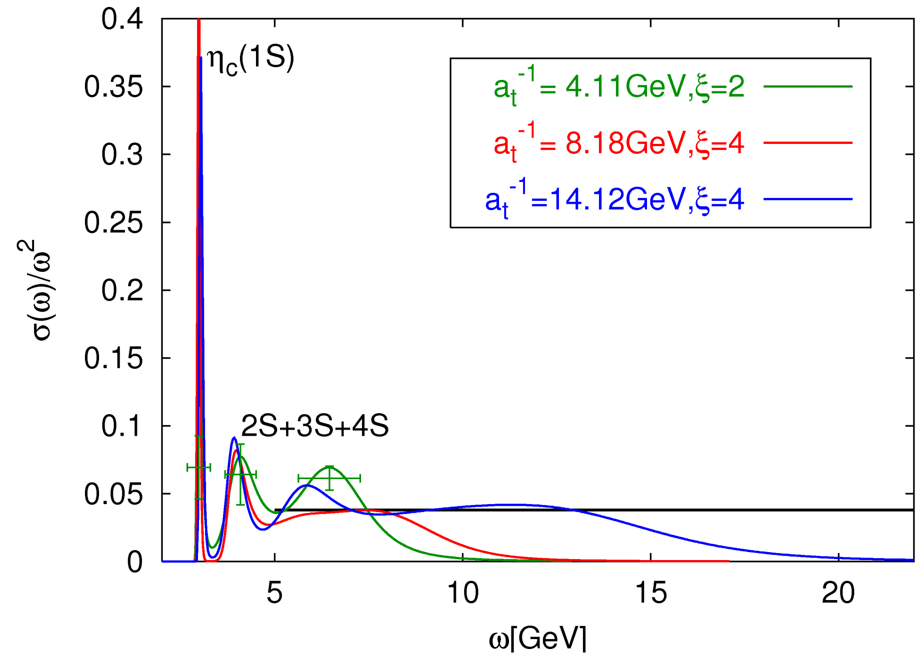
Charmonium spectral functions at T=0

Anisotropic lattices: $16^3 \times 64, \xi = 2$ $16^3 \times 96, \xi = 4$, $24^3 \times 160, \xi = 4$
$L_s = 1.35 - 1.54$fm, #configs=500-930;
Wilson gauge action and Fermilab heavy quark action

Pseudo-scalar (PS) $\rightarrow$ S-states

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

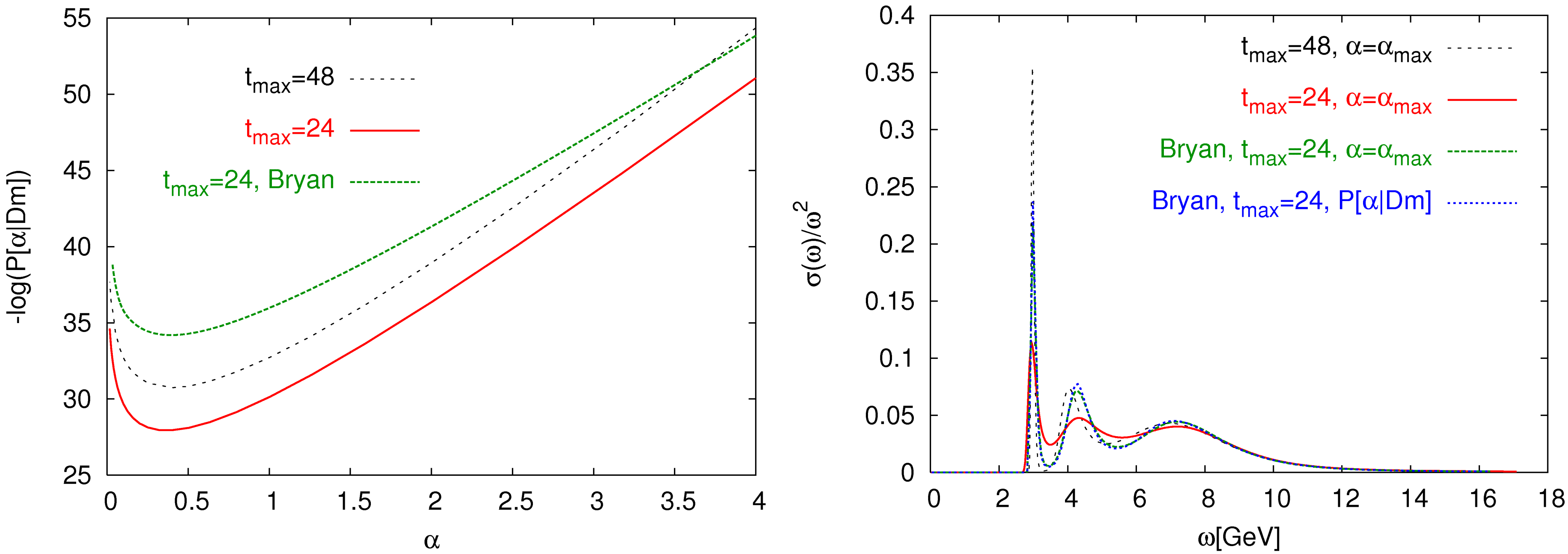




For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;
good agreement with 2-exponential fit for peak position and amplitude

Charmonium spectral functions at T=0 (cont'd)

PS, $16^3 \times 96$, $a_t^{-1} = 8.18$ GeV, $\xi = 4$



$P[\alpha|Dm]$ has a well defined maximum at some α_{max}

Bryan algorithm and the new algorithm give similar results, but the use of the former is limited $\tau_{max} = 24$.

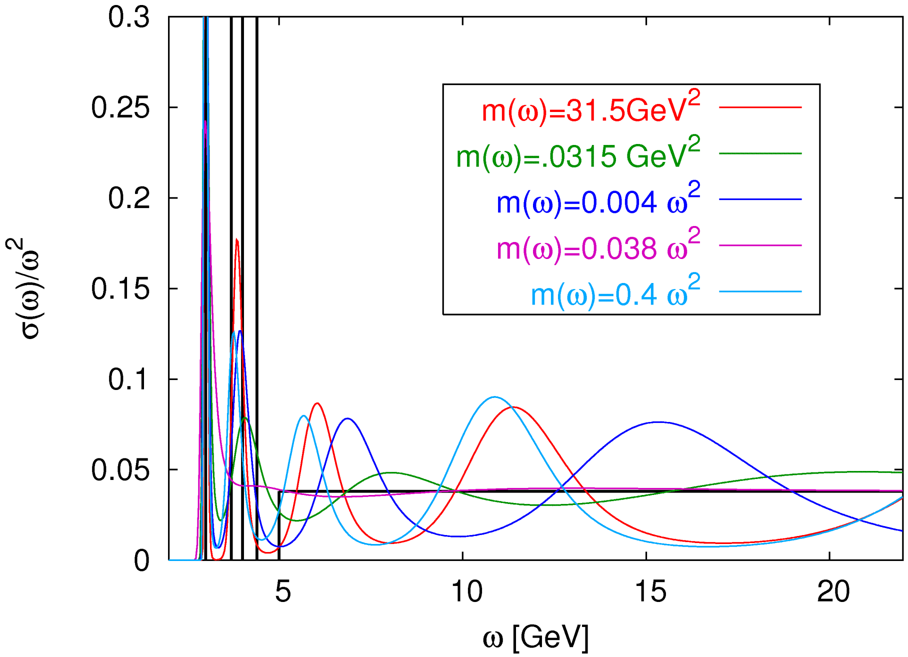
Charmonium spectral functions at T=0 (cont'd)

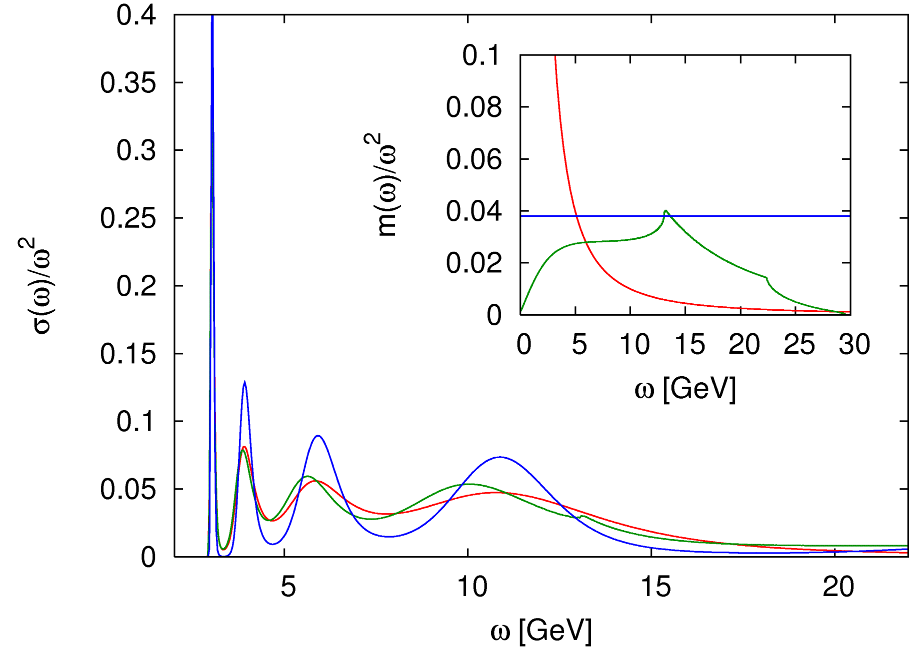
What states can be resolved and what is the dependence on the default model ?

Reconstruction of an input spectral function :

Lattice data in PS channel for:

$a_t^{-1} = 14.12\text{GeV},\ N_t = 160$





Ground states is well resolved, no default model dependence;
Excited states are not resolved individually, moderate dependence on the default model;
Strong default model dependence in the continuum region, $\omega > 5$ GeV

Comparison of MEM and 2-exponential fit

Meson masses , $\beta = 6.5,\ \xi = 4, 24^3 \times 160$:

	MEM	2-exp fit
$m_{ps}(n=1)$	0.2147(7)	0.2154(2)
$m_{ps}(n=2)$	0.281(8)	0.285(2)
$m_{sc}(n=1)$	0.255(5)	0.251(1)[5]

Amplitudes, $A = \int_{peak} d\omega\sigma(\omega)$, $\beta = 6.5,\ \xi = 4, 24^3 \times 160$:

	MEM	2-exp fit
$A_{ps}(n=1)$	0.042(2)	0.0432(5)
$A_{ps}(n=2)$	0.107(13)	0.117(7)
$A_{sc}(n=1)$	0.028(4)	0.021(1)[8]

MEM works at T=0

Charmonia correlators at T>0

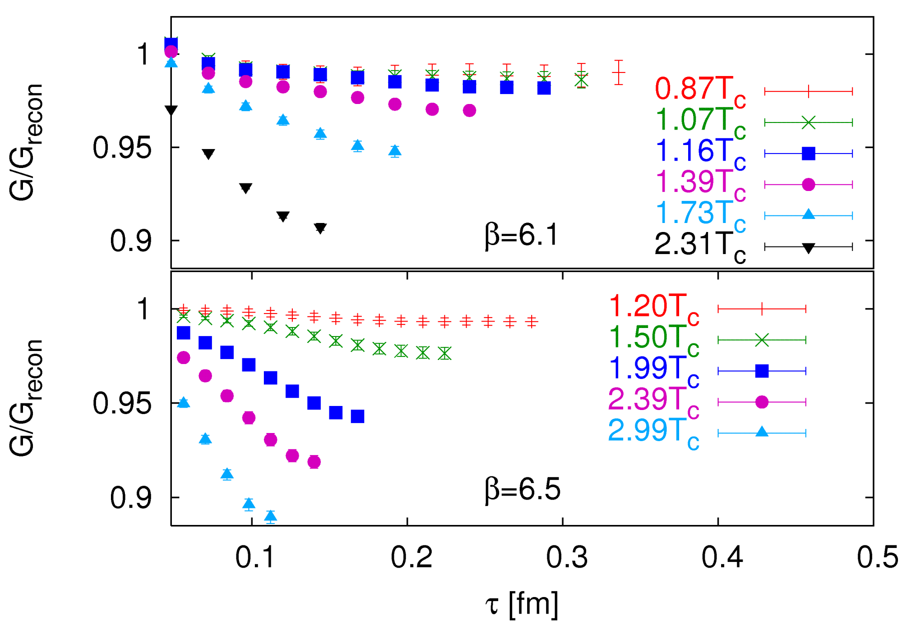
temperature dependene of $G(\tau,T)$

$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

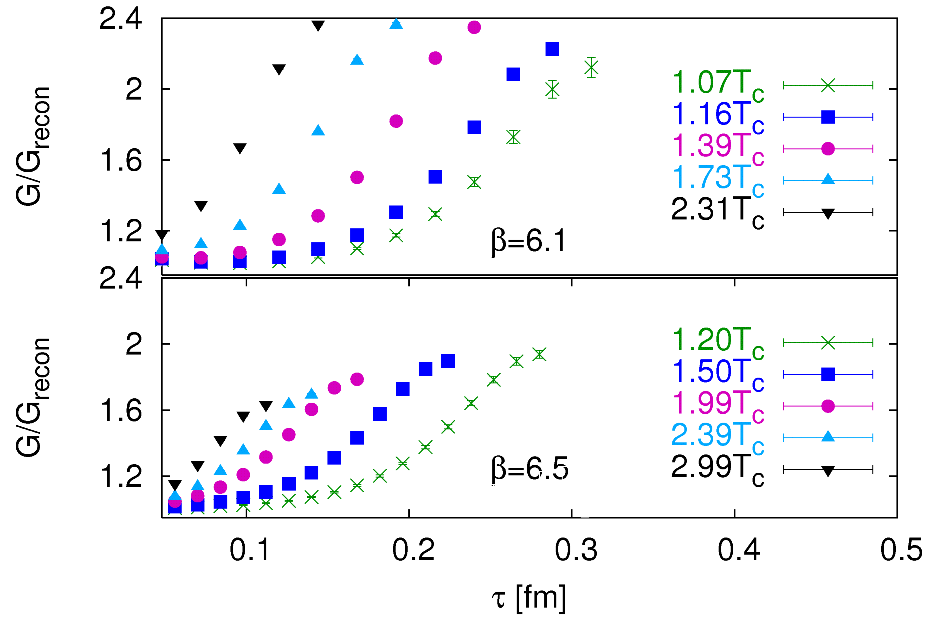
If there is no T-dependence in the spectral function, $G(\tau,T)/G_{recon}(\tau,T)=1$

$$G_{recon}(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

PS, $\Gamma_H=\gamma_5$



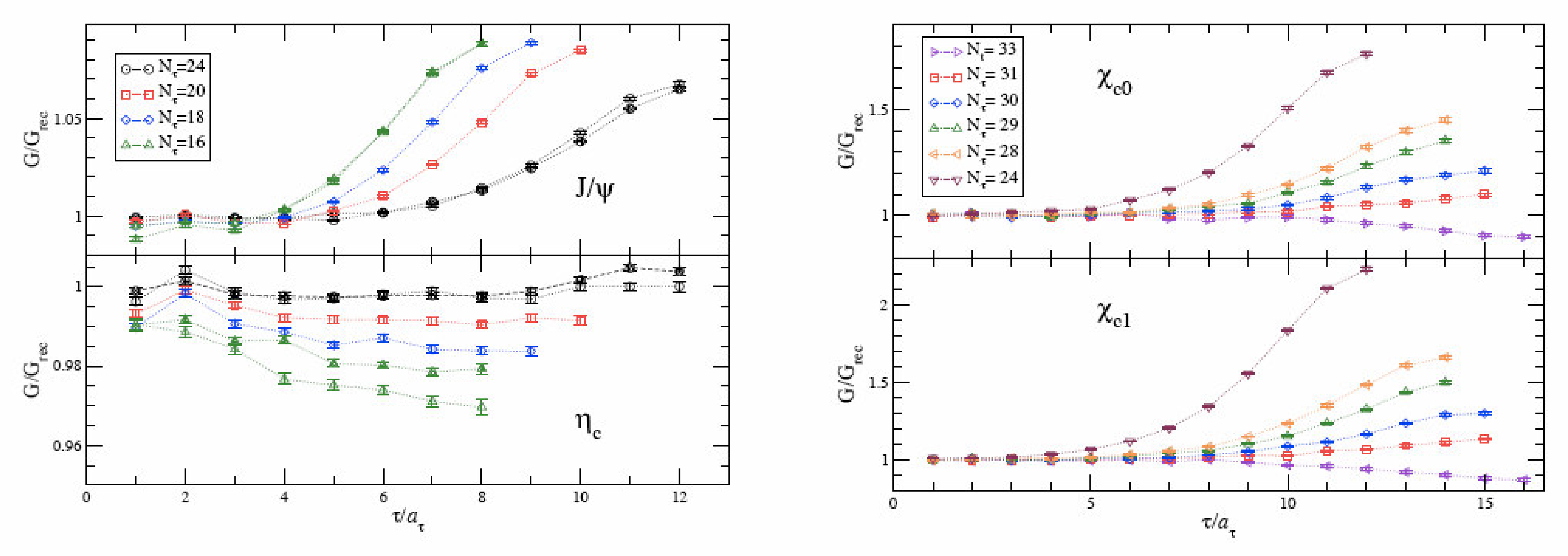
SC, $\Gamma_H=1$



in agreement with previous calculations: Datta et al, PRD 69 (04) 094507

Calculation in full QCD :

Aarts,Allton,Oktay,Peardon,Skullerud, arXiV:0705.2198 [hep-lat]



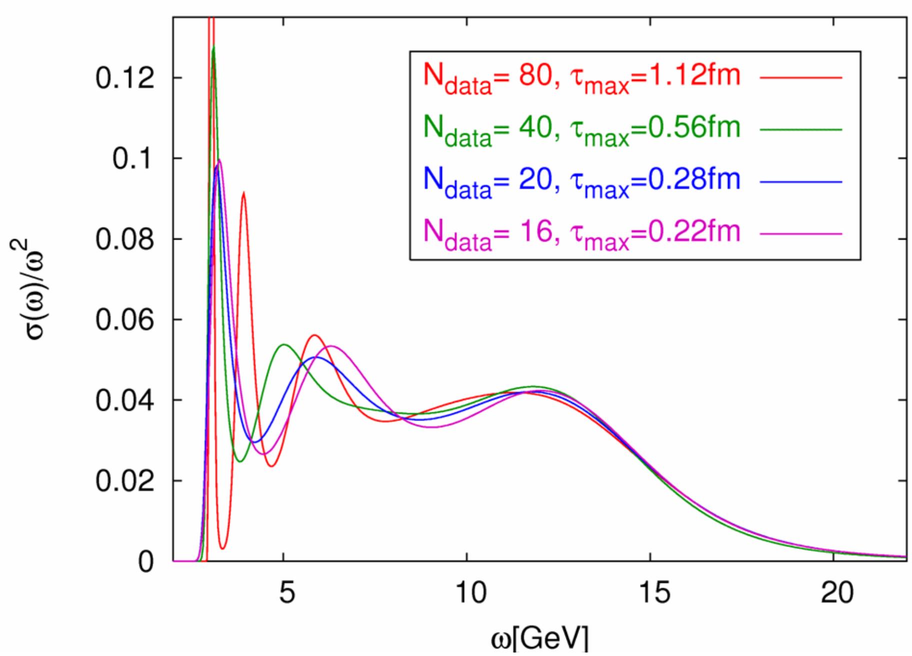
results are qualitatively the same as in quenched QCD !

Charmonia spectral functions at T>0

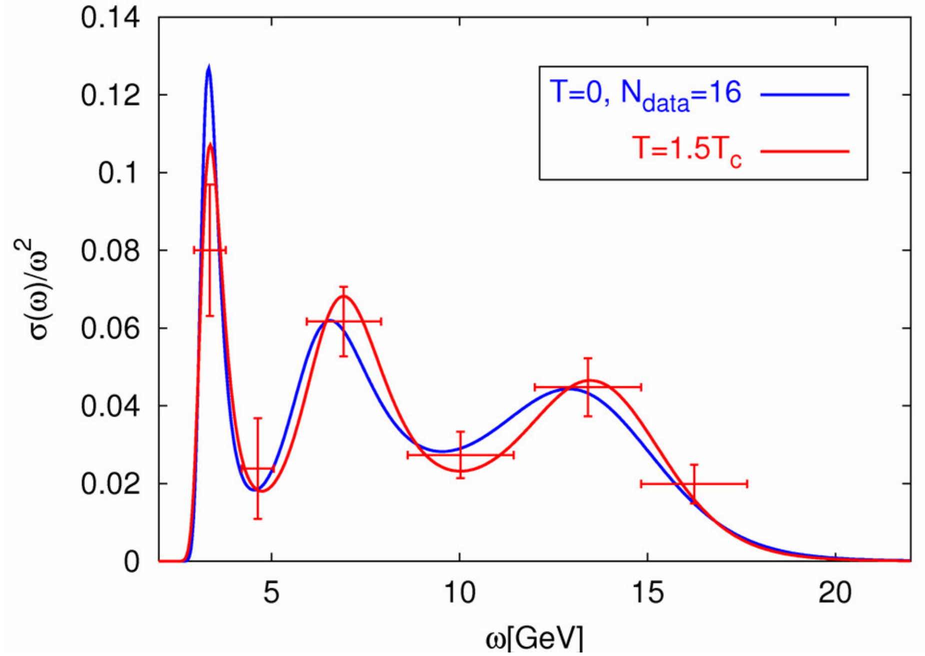
$$T = 1/(N_t a) \quad \leftrightarrow \quad \tau_{max} = 1/(2T)$$

$$\text{PS},\ 24^3 \times N_t,\ a_t^{-1} = 14.12\ \text{GeV},\ \xi = 4$$

$T = 0, \quad N_t = 160$



$T = 1.5T_c, \quad N_t = 32$



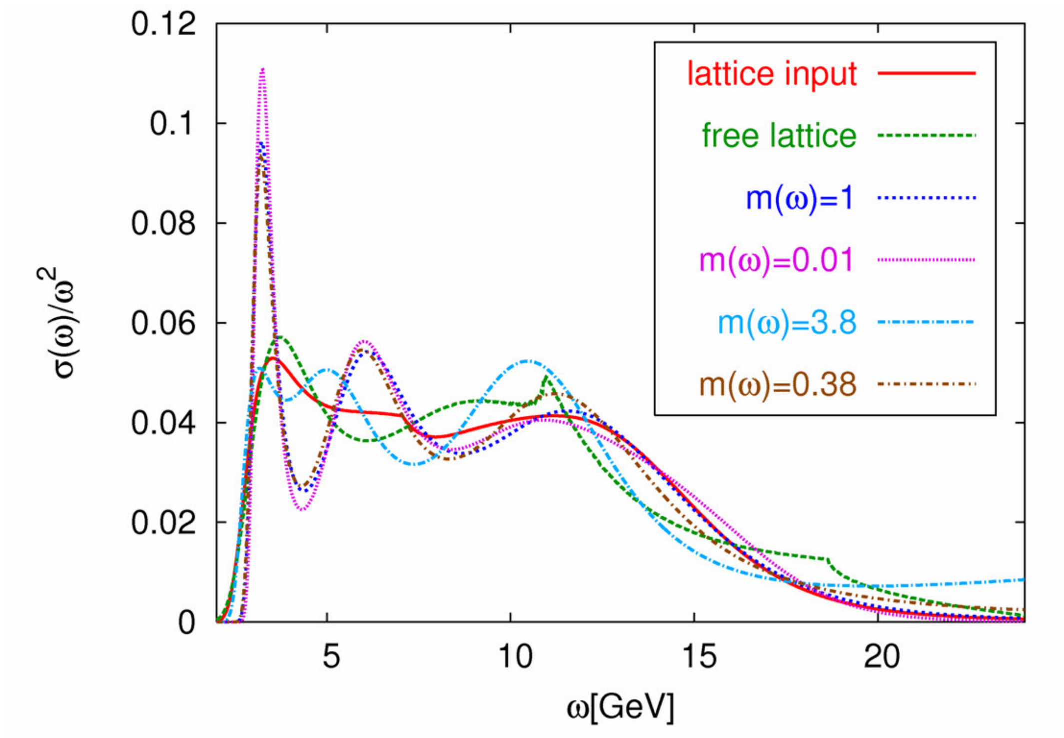
ground state peak is shifted, excited states are not resolved when $\tau_{max}, \quad N_{data}$ become small

no temperature dependence in the PS spectral functions within errors

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

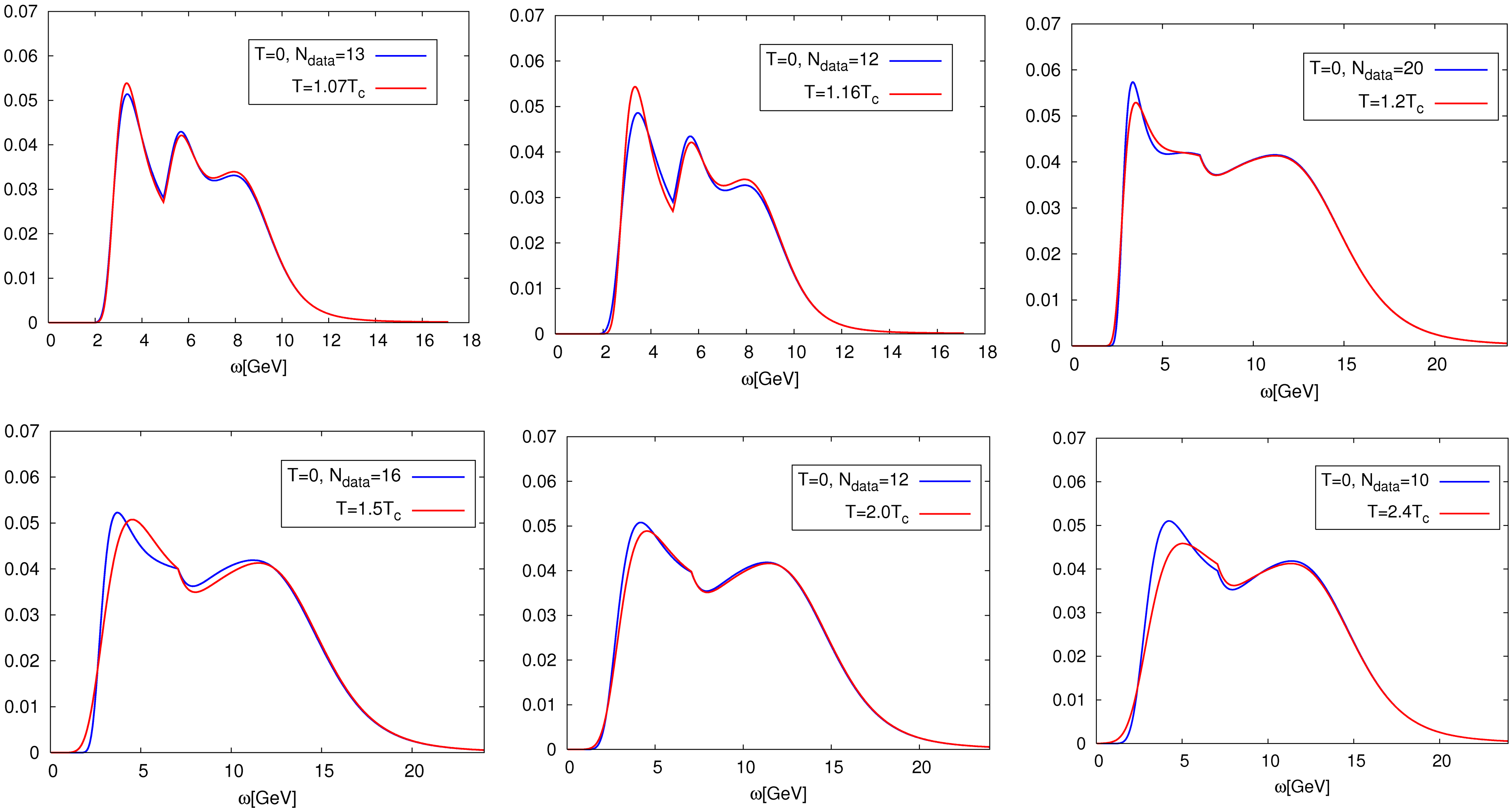
Charmonia spectral functions at T>0 (cont'd)

PS, $24^3 \times 40$, $a_t^{-1} = 14.12$ GeV, $\xi = 4, T = 1.2T_c$



there is a strong dependence on the default model $m(\omega)$ at finite temperature

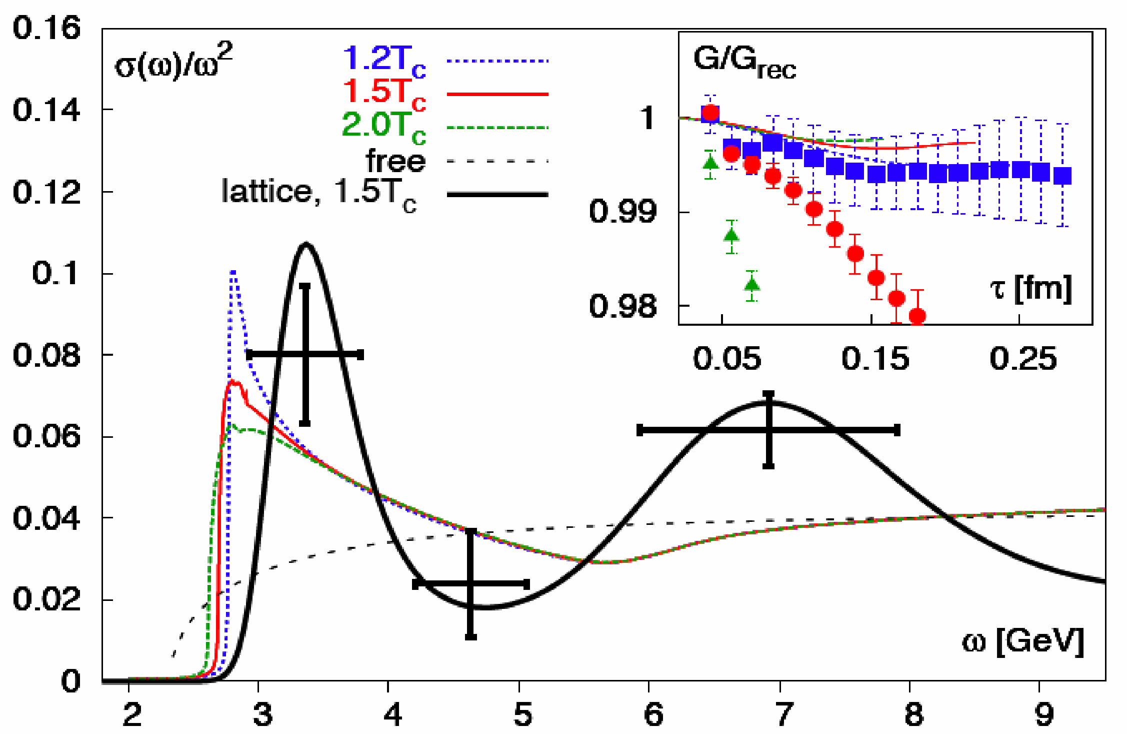
Using default model from the high energy part of the T=0 spectral functions :
resonances appears as small structures on top of the continuum,
almost no T-dependence in the PS spectral functions till $T \simeq 2.4T_c$



Can 1S charmonia state survive deconfinement ?

What aboutcolor screening ?

$$\left[-\frac{1}{m}\nabla^2 + V(\vec{r}) + E\right]G^{NR}(\vec{r},\vec{r}',E) = \delta^3(\vec{r}-\vec{r}')$$



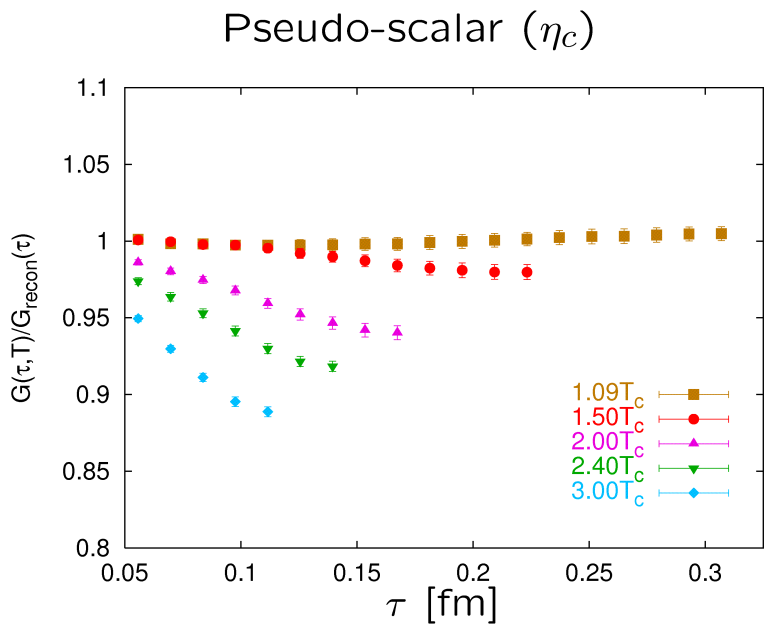
$$\sigma(E) = \frac{2N_c}{\pi}\operatorname{Im}G^{NR}(\vec{r},\vec{r}',E)_{\vec{r}=\vec{r}'=0}$$

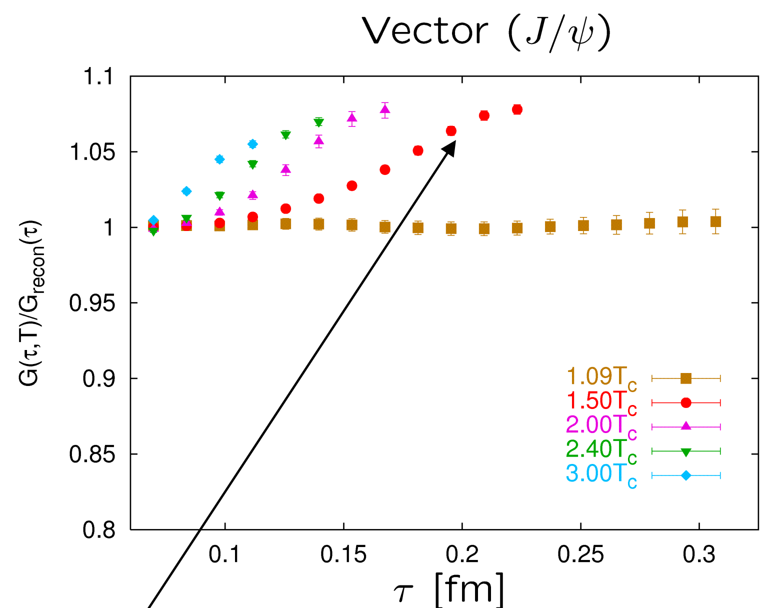
$$\sigma(E) = \frac{2N_c}{\pi}\frac{1}{m^2}\vec{\nabla}\cdot\vec{\nabla}'\operatorname{Im}G^{NR}(\vec{r},\vec{r}',E)_{\vec{r}=\vec{r}'=0}$$

Mócsy, P. P.,
arXiV:0705.2559 [hep-ph]

- resonance-like structures disappear already by 1.2Tc
- strong threshold enhancement
- contradicts previous claims ? No !

Vector correlator and heavy quark diffusion





P.P., Petrov, Velytsky, Teaney, hep-lat/0510021

Vector current is conserved $\longrightarrow$ fluctuations of charm number

$$\sigma_V^{ii}(\omega) = F_{J/\psi}^2(T)\delta(\omega^2 - m_{J/\psi}^2(T)) + \frac{1}{4\pi^2}\omega^2\sqrt{1 - \frac{4m_D^2(T)}{\omega^2}} \quad + \chi_s(T) v_{therm}^2 \omega \delta(\omega)$$

$$\frac{1}{3}\chi_s(T) v_{ther}^2 \cdot \omega \cdot \frac{1}{\pi} \cdot \frac{\eta}{\omega^2 + \eta^2}$$

Interactions

Effective Langevin theory

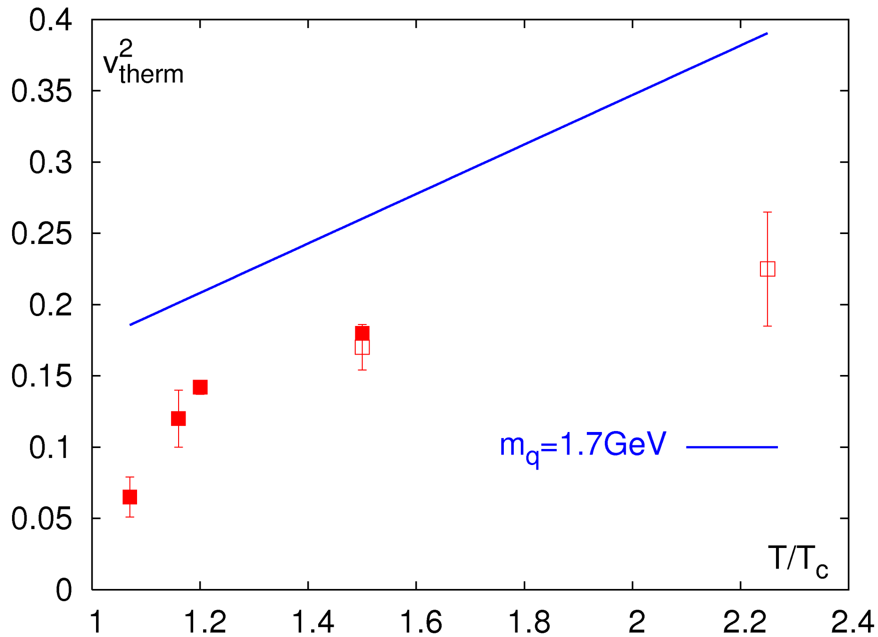
Free streaming :
Collision less
Boltzmann equation

$$\eta = \frac{T}{m_q}\frac{1}{D} \qquad \partial_t N_c + D\nabla^2 N_c = 0 \qquad D > 1/(2\pi T)$$

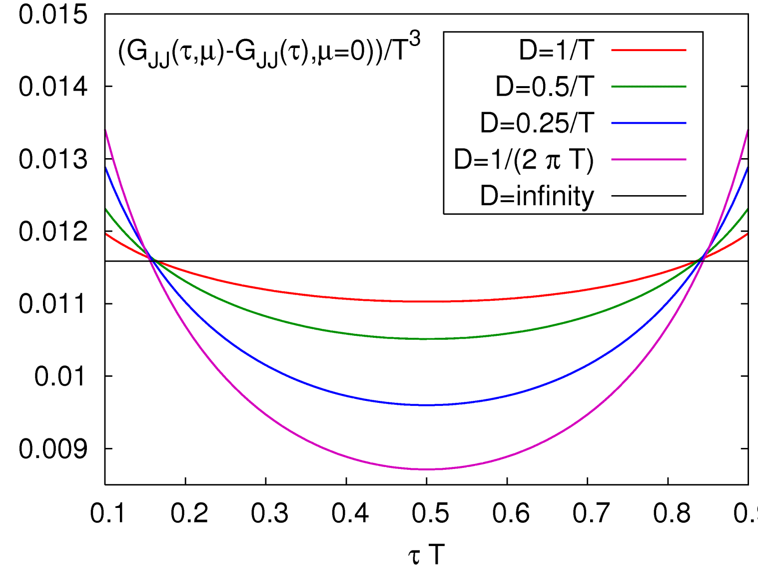
$$G_{ii}(\omega) = G_{ii}^{\text{low}}(\omega) + G_{ii}^{\text{high}}(\omega)$$

$$G_{ii}^{\text{low}}(\tau) \simeq \chi_s(T) v_{therm}^2 \qquad G_{00}^{\text{low}}(\tau) \simeq -\chi_s(T)$$

non-inetacring quarks: $v_{therm}^2 = T/m_q$

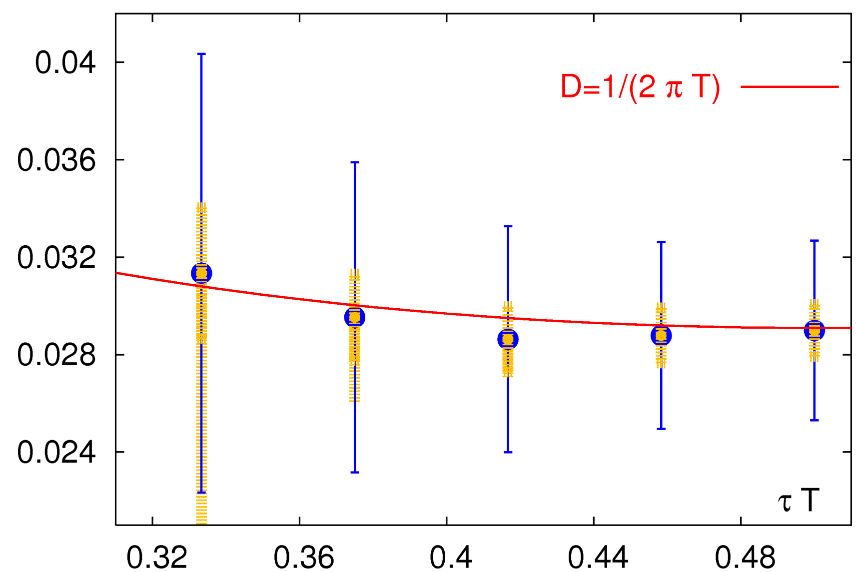


heavy quark thermal velocity is significantly smaller than expected



P.P., Teaney PRD73 (06) 014508

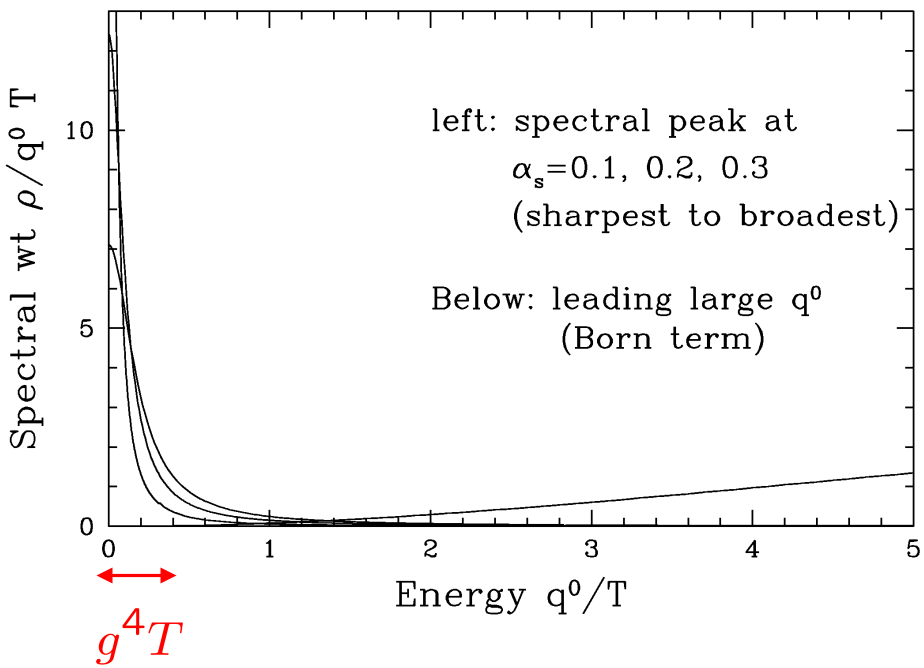
$64^3 \times 24, \ a^{-1} = 9.72GeV, \#conf = 60$



Strongly coupled or weakly coupled QGP ?

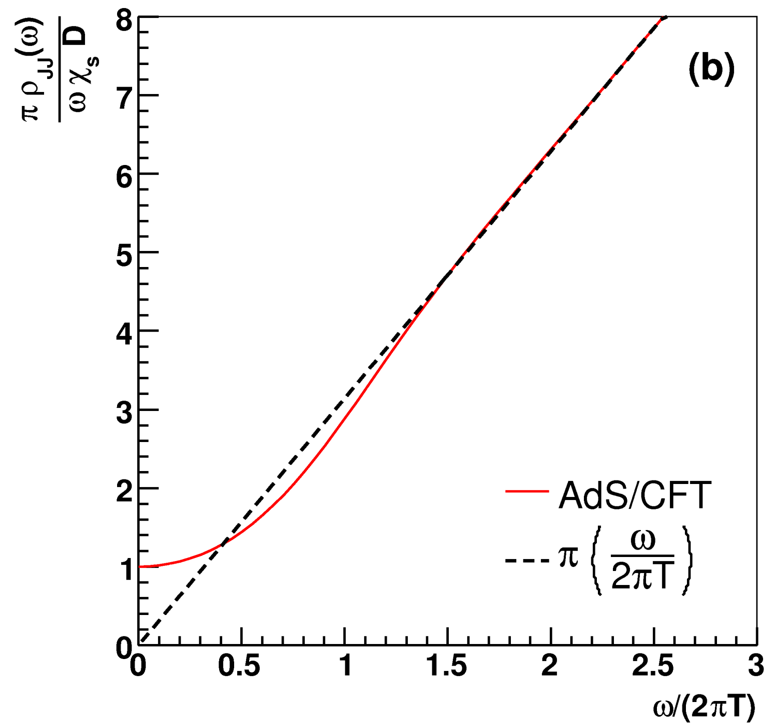
Weak coupling caculation of the vector current spectral function in QCD

Moore, Robert, hep-ph/0607172



vector current correlator in N=4 SUSY at strong coupling

Teaney, PRD74 (06) 045025



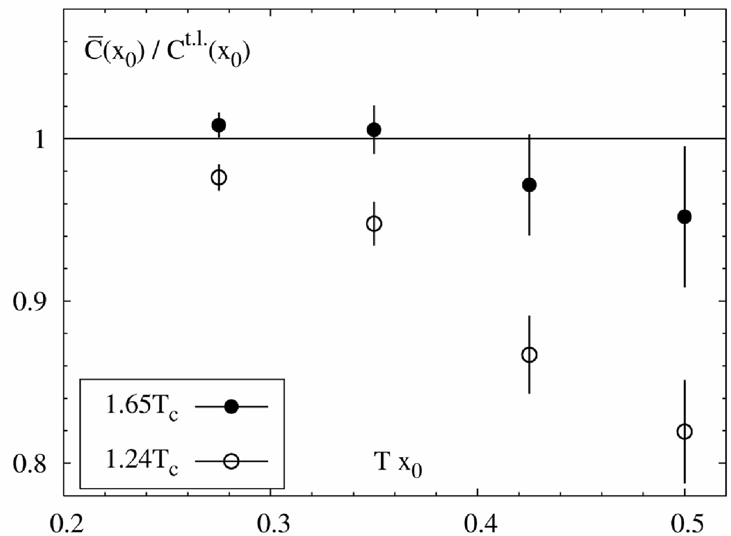
can lattice decide ?

Correlation function of energy momentum tensor

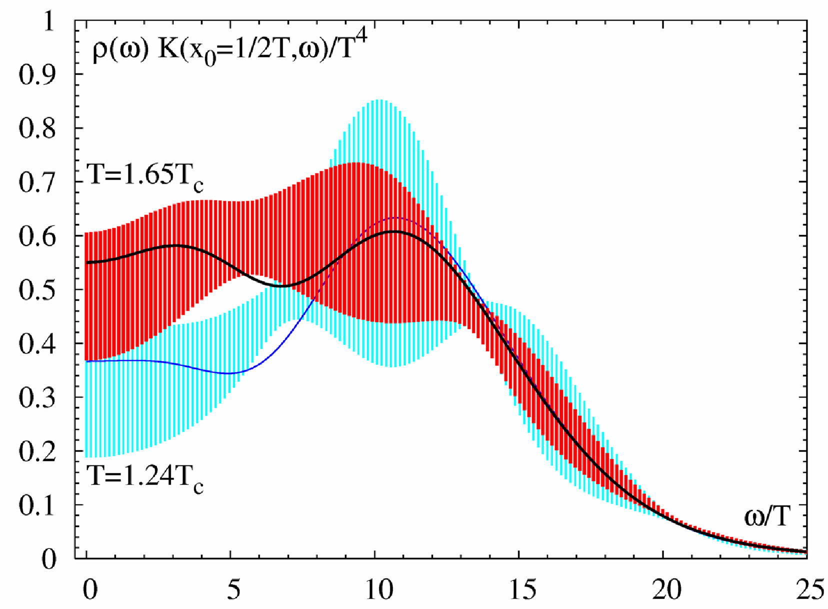
Correlation function is very noisy (see Nakamura, Sakai, PRL 94 (05) 072305)
⇒2-level algorithm
⇒(exponential noise reduction)

H. Meyer, arXiv:0704.1801[hep-lat]

$\eta/s = 0.134(33)$



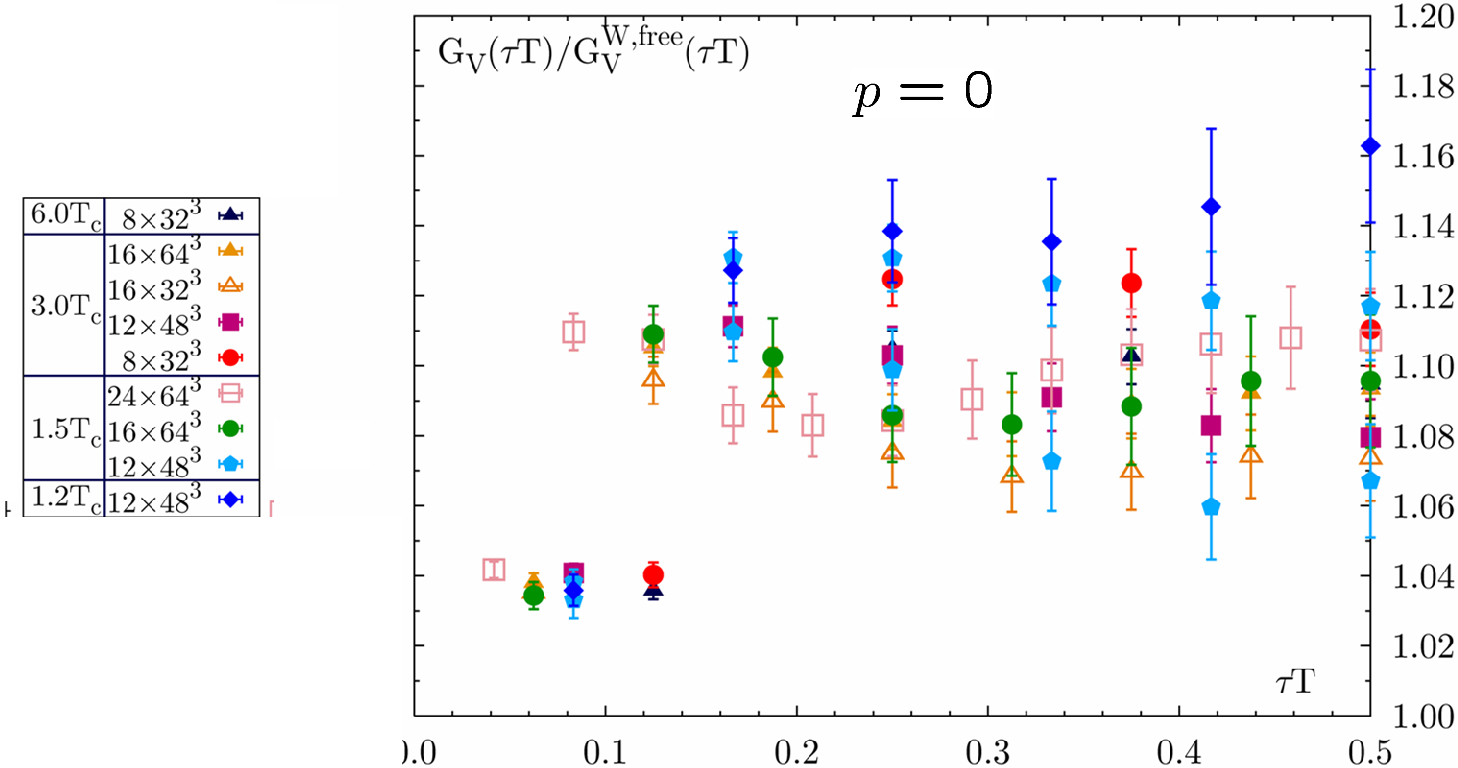
Note that the correlator at $1.65T_c$ is compatible with the free correlator corresponding to infinitely narrow transport peak !



Correlator of the vector currents

non-perturbative clover action on Wilson gauge action

Karsch et al., PLB 530 (02) 147



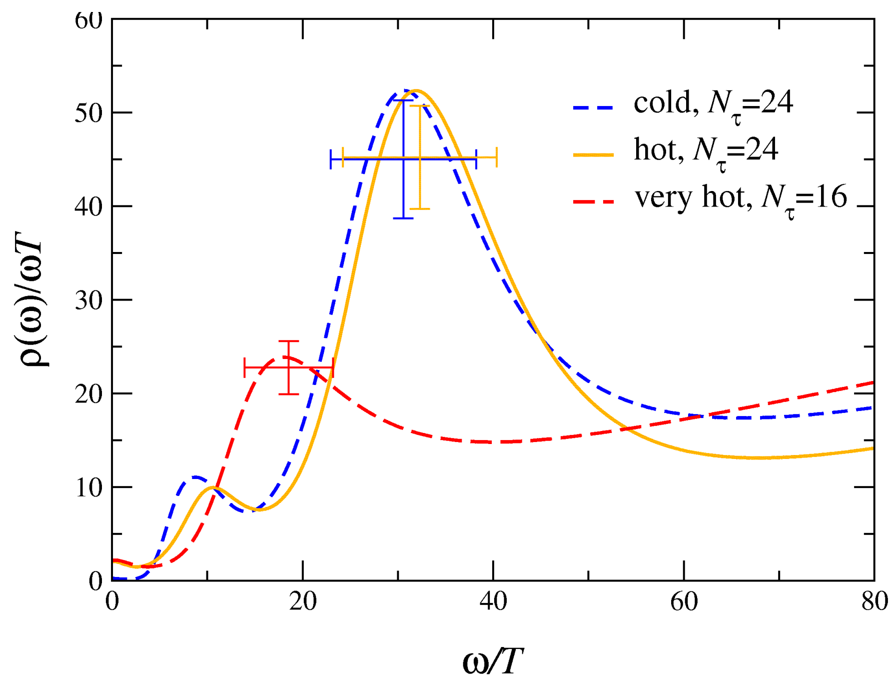
Lattice spacing dependence is small !

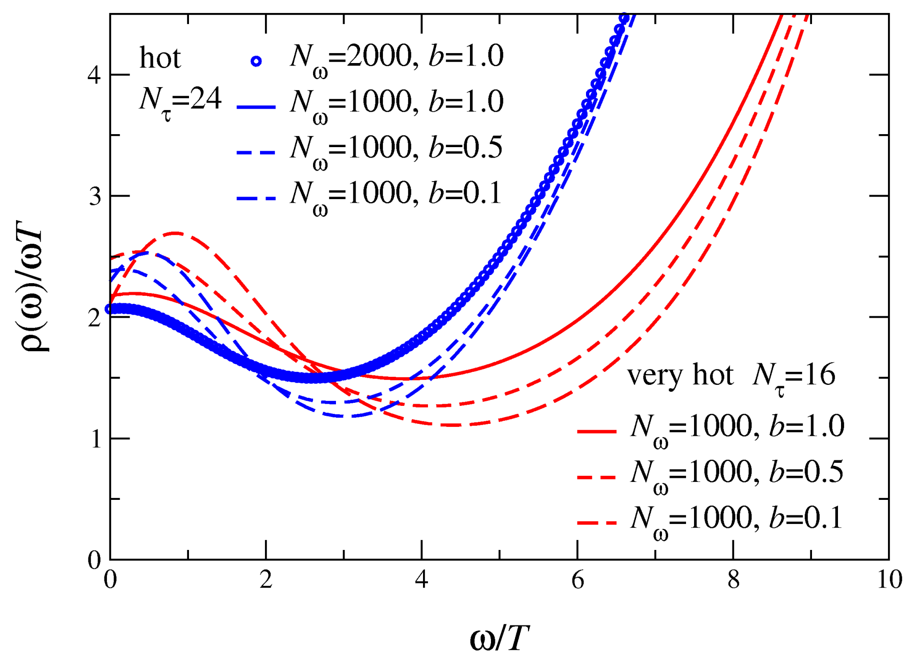
$$G(\tau = \frac{1}{2T}, p, T) = \int_0^\infty d\omega \frac{\sigma(\omega, p)}{\sinh(\omega/(2T))}$$

→ constraints on the spectral functions at small energies

Aarts et al, PRL 99 (06) 022002, std. staggered on Wilson gauge action

ζ/T=0.4 (1)





$N_\tau = 24$, hot : $1.5T_c$

$N_\tau = 16$, veryhot : $2.25T_c$

Is the plasma strongly coupled also at $2.25T_c$?