Charm and bottom Heavy baryon mass spectrum from Lattice QCD with 2+1 flavors

Heechang Na
with Steven Gottlieb

Indiana University

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Outline

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- Lattices and propagators
- Formalism
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  - Doubly heavy baryons
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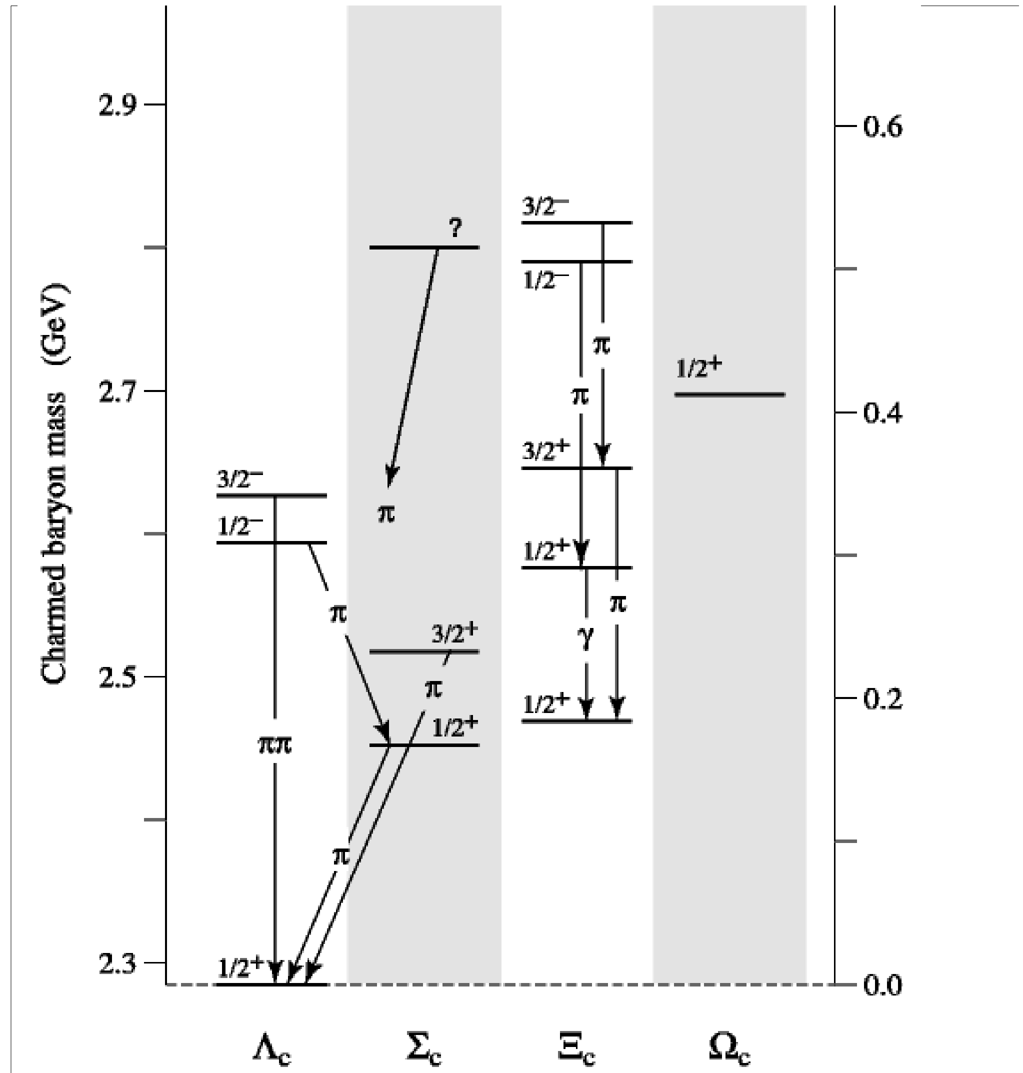
• Introduction

- Singly and doubly charmed heavy baryons
- Singly and doubly bottom heavy baryons :

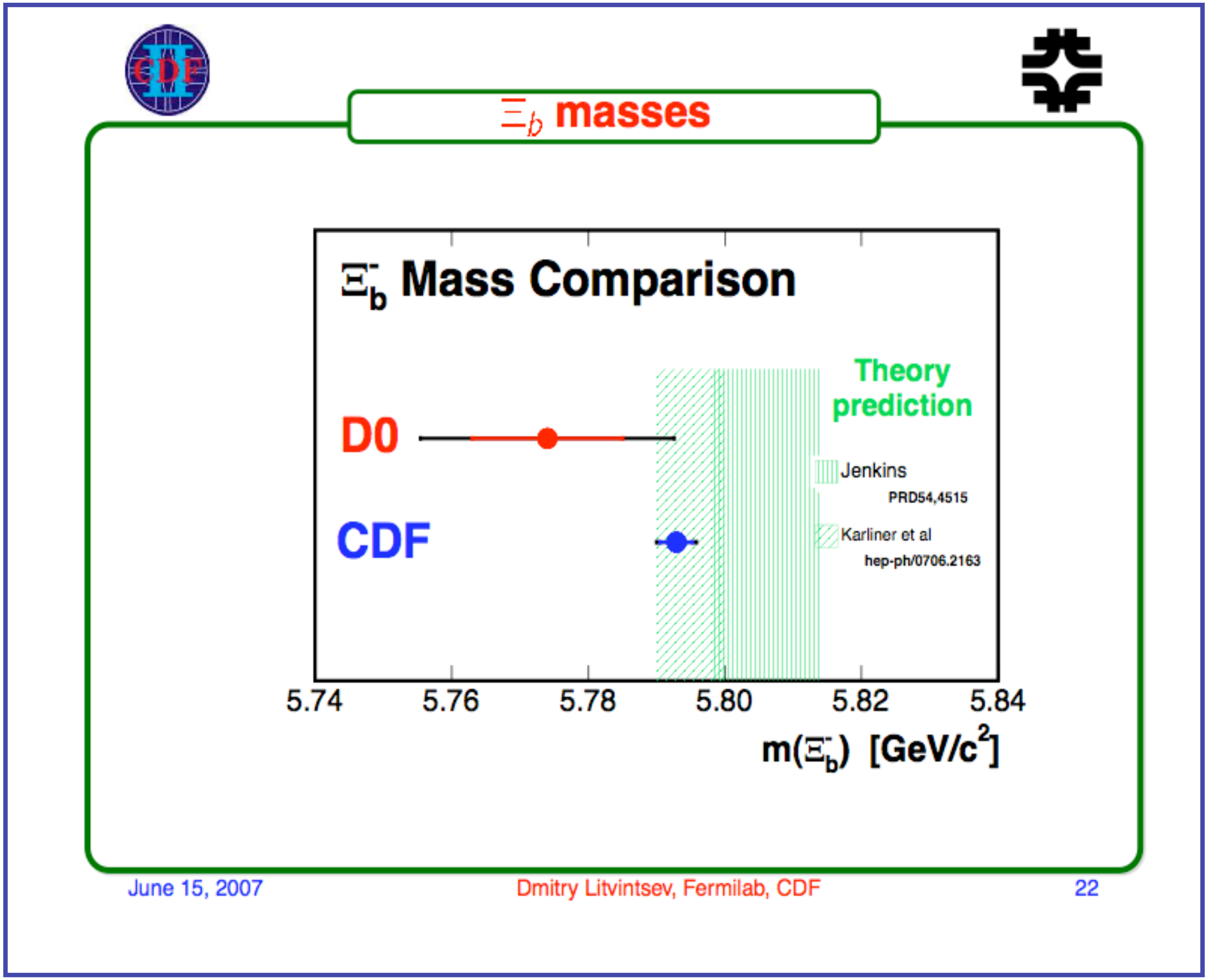
$$\Lambda_H, \Sigma_H, \Sigma^*_H, \Xi_H, \Xi'_H, \Xi^*_H, \Omega_H, \Omega^*_H$$
$$\Xi_{HH}, \Xi^*_{HH}, \Omega_{HH}, \Omega^*_{HH}$$

- Lattice QCD with 2+1 flavors



PDG, J. Phys. G 33, 1 (2006)

- New measurements



• Lattices and Propagators

- MILC coarse lattices
  - $20^3 \times 64$, $a \approx 0.12$ fm
  - 3 ensembles with four different time sources
    - $m_l = 0.007$ $m_s = 0.05$
    - $m_l = 0.01$ $m_s = 0.05$
    - $m_l = 0.02$ $m_s = 0.05$
- Propagators
  - 9 different staggered light valence quarks
    - 0.005 ~ 0.02
  - 3 different staggered strange valence quarks
    - 0.024, 0.03, 0.0415
  - One valence clover heavy quark
    - $k = 0.122$ (Tuned for charm quark)
      - 007 : 545 confs   010 : 591 confs   020 : 459 confs
    - $k = 0.086$ (for bottom)
      - 007 : 554 confs   010 : 590 confs   020 : 452 confs

• Formalism

• Operators (K.C. Bowler et al., PRD 54, 3619 (1996))

$$O_5 = \varepsilon_{abc}(\psi_1^{aT} C\gamma_5 \psi_2^{b})\Psi_H^{c}, \quad O_\mu = \varepsilon_{abc}(\psi_1^{aT} C\gamma_\mu \psi_2^{b})\Psi_H^{c}$$

	Jp	s^π	Content	Baryon
O_5	1/2+	0^+	$\ell\ell h$	Λ_h
			$\ell s h$	Ξ_h
O_μ		1^+	$\ell\ell h$	Σ_h
			$\ell s h$	Ξ'_h
			$s s h$	Ω_h
			$\ell h h$	Ξ_{hh}
			$s h h$	Ω_{hh}
	3/2+		$\ell\ell h$	Σ^*_h
			$\ell s h$	Ξ^*_h
			$s s h$	Ω^*_h
			$\ell h h$	Ξ^*_{hh}
			$s h h$	Ω^*_{hh}

- Two point function with a Staggered light quark and a Wilson heavy quark
  - Conversion between a Naive propagator and a Staggered propagator!

$$G_\Psi(x;y) = \Omega(x) G_\Phi \Omega(y)^+$$
$$G_\Phi = \hat{I}_4 G_\chi(x,y)$$

$$G_\psi(x,y) = \Omega(x)\Omega^+(y)G_\chi(x,y)$$
$$\text{where } \Omega(x) = \prod_\mu (\gamma_\mu)^{x_\mu/a}$$

- Now, we can write the Heavy-Light correlator

$$\sum_x e^{ip\cdot x} < W^+_{\Gamma_{sc}}(x) W_{\Gamma_{sk}}(0) > = \sum_x e^{ip\cdot x} Tr\left[\Gamma_{sc} G_\Psi(0;x)\Gamma^+_{sk} G_H(x;0)\right]$$
$$= \sum_x e^{ip\cdot x} \sum_{c,c'} \left[\mathrm{tr}\{\Gamma_{sc}\Omega^+(x)\Gamma^+_{sk} G_H^{c'c}(x;0)\} G_\chi^{cc'}(0;x)\right]$$
$$\text{where } W_\Gamma = \overline{\Psi}_H(x)\Gamma\Psi(x)$$

- (M. Wingate et al. PRD67, 054505 (2003))

- Two point function for the heavy baryon

$$C_5(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle O_5(\vec{x},t)\overline{O}_5(\vec{0},0)\right\rangle$$
$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[G_1^{aa'T}(x,0)C\gamma_5 G_2^{bb'}(x,0)(C\gamma_5)^+] G_H^{cc'}(x,0)$$

$$C_5(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[\Omega^T(x)C\gamma_5\Omega(x)(C\gamma_5)^+]$$
$$\times G_{1\chi}^{aa'}(x,0) G_{2\chi}^{bb'}(x,0) G_H^{cc'}(x,0)$$

$$\mathrm{tr}[\Omega^T(x)C\gamma_5\Omega(x)(C\gamma_5)^+] = \mathrm{tr}[(-1)^{x_1+x_3}(-1)^{x_1+x_3}]$$
$$= 4$$

Finally,

$$C_5(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} 4\varepsilon_{abc}\varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x,0) G_{2\chi}^{bb'}(x,0) G_H^{cc'}(x,0)$$

This is for O_5, what about O_μ?

-- Surprisingly, C_{ij} is a diagonal matrix for i and j indices

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[\Omega^T(x) C\gamma_i \Omega(x)(C\gamma_j)^+] \times G_{1\chi}^{aa'}(x,0) G_{2\chi}^{bb'}(x,0) G_H^{cc'}(x,0)$$

$$\Omega(x) = \gamma_0^t \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3}$$

$$\mathrm{tr}[\Omega^T(x) C\gamma_i \Omega(x)(C\gamma_j)^+] = 4(-1)^{x_i}\,\delta_{\mathrm{ij}}$$

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} 4(-1)^{x_i}\,\delta_{ij}\varepsilon_{abc}\varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x,0) G_{2\chi}^{bb'}(x,0) G_H^{cc'}(x,0)$$

$$C_{ij}(t) = P_{ij}^{3/2} C_{3/2}(t) + P_{ij}^{1/2} C_{1/2}(t)$$

$$= (\delta_{ij} - \frac{1}{3}\gamma_i\gamma_j) C_{3/2}(t) + \frac{1}{3}\gamma_i\gamma_j C_{1/2}(t)$$

$$C_{3/2}(t) \propto e^{-m_{3/2}t}, \quad C_{1/2}(t) \propto e^{-m_{1/2}t}$$

- Taste mixing?

$$O_\mu = \varepsilon_{abc}(\psi_1^{aT} C\gamma_\mu \psi_2^{\ b})\Psi_H^{\ c} \;\rightarrow\; D_\mu = (\psi_1^{\ T}(x) C\gamma_\mu \psi_2(x))$$

$$\psi^{\alpha'}(x) = \Omega^{\alpha' a}(x)\chi^a(x)$$

$$q^{\alpha\, i,a}(y) = \frac{1}{8}\sum_{\xi}\Omega^{\alpha\, i}(\xi)\chi^a(y+\xi)$$

$$x = y + \xi$$

$$\chi^a(y+\xi) = 2\Omega^{+\, i\alpha}(\xi) q^{\alpha\, i,a}(y)$$

$\psi^{\alpha'}$: Naive quark

χ^a : 4 copies of staggered quark

$q^{\alpha\, i,a}$: Staggered quark in taste basis

$\Omega(x) = \gamma_0^{x_0}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$

a : Copy index

α : Staggered spin index

α' : Naive spin index

i : Taste index

$$\psi^{\alpha'}(x) = \Omega^{\alpha' a}(\xi)\chi^a(y+\xi) = \Omega^{\alpha' a}(\xi) 2\Omega^{+\, i\alpha}(\xi) q^{\alpha\, i,a}(y)$$

- Di-quark operator

$$D_\mu = (\psi_1^T(x) C\gamma_\mu \psi_2(x)) \quad \Rightarrow \quad D_\mu^{conti}(y) = \sum_\xi (\psi_1^T(x) C\gamma_\mu \psi_2(x))$$

$$D_\mu^{conti}(y) = \sum_\xi 2\Omega^{+\,i\alpha}(\xi) q^{\alpha\, i,a}(y) \Omega^{T a\alpha'}(\xi) (C\gamma_\mu)^{\alpha'\beta'} \Omega^{\beta' b}(\xi) 2\Omega^{+\, j\beta}(\xi) q^{\beta\, j,b}(y)$$

$$= \sum_\xi 4\Omega^{+\,i\alpha}(\xi) q^{\alpha\, i,a}(y) (-1)^{\xi_\mu} (C\gamma_\mu)^{ab} \Omega^{+\, j\beta}(\xi) q^{\beta\, j,b}(y)$$

$$\sum_\xi \Omega^{+\,i\alpha}(\xi)(-1)^{\xi_\mu}\Omega^{+\, j\beta}(\xi) = 4(C\gamma_\mu)_{\alpha\beta} \otimes (\gamma_\mu C^{-1})_{ij}$$

$$D_\mu^{conti}(y) = 16 q^{\alpha\, i,a}(y) (C\gamma_\mu)_{\alpha\beta} \otimes (\gamma_\mu C^{-1})_{ij} q^{\beta\, j,b}(y) (C\gamma_\mu)_{ab}$$

$$D_5^{conti}(y) = 16 q^{\alpha\, i,a}(y) (C\gamma_5)_{\alpha\beta} \otimes (\gamma_5 C^{-1})_{ij} q^{\beta\, j,b}(y) (C\gamma_5)_{ab}$$

Overlap with 1^+ and 0^+ spin state with single taste

K. Nagata et al., arXiv:0707.3537

a, b	: Copy index	i, j	: Taste index
α, β	: Staggered spin index	α', β'	: Naive spin index

- Two-point function of the di-quark operator

$$
\begin{aligned}
C_{\mu\nu}^{conti}(y;0) &= < D_{\mu}^{conti}(y)\overline{D}_{\nu}^{conti}(0) > \\
&= 16^2\,\mathrm{Tr}[G_1(y,0)(C\gamma_{\mu}) \otimes (C\gamma_{\mu})^{+} G_2(y,0)(C\gamma_{\nu})^{+} \otimes (C\gamma_{\nu})] \\
&\qquad \times (C\gamma_{\mu})_{\mathrm{ab}} \otimes (C\gamma_{\mu})^{+}_{\mathrm{b'a'}}\delta_{bb'}\delta_{aa'} \\
&= 16^2\,\mathrm{Tr}[G_1(y,0)(C\gamma_{\mu}) \otimes (C\gamma_{\mu})^{+} G_2(y,0)(C\gamma_{\nu})^{+} \otimes (C\gamma_{\nu})] \\
&\qquad \times \mathrm{Tr}[(C\gamma_{\mu})(C\gamma_{\nu})^{+}]
\end{aligned}
$$

$$
\mathrm{Tr}[(C\gamma_{\mu})(C\gamma_{\nu})^{+}] = \begin{cases} 0 & \mu \neq \nu \\ 4 & \mu = \nu \end{cases}
$$

The delta function appears, because the cancellations between copy indices.

- **Data analysis**

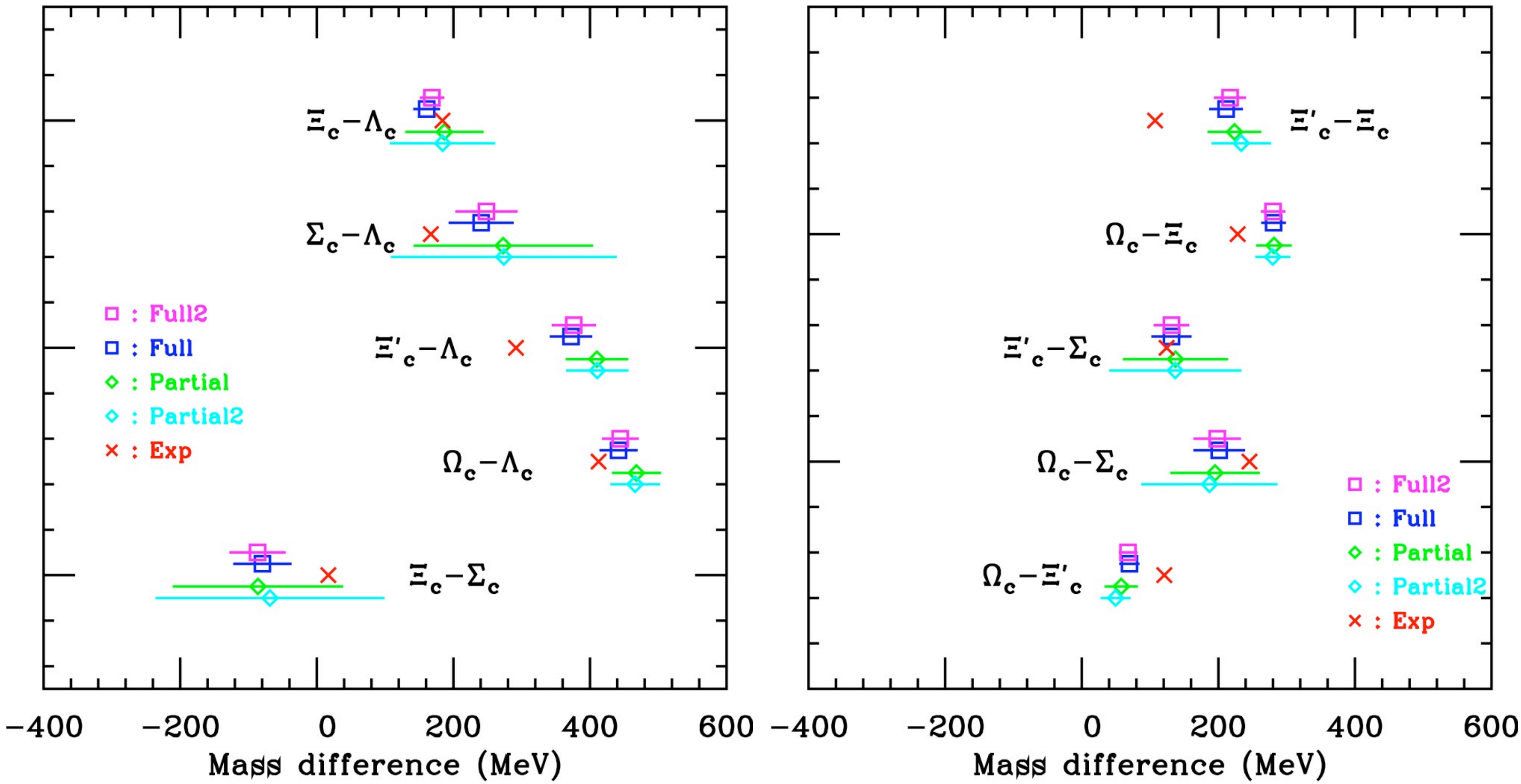
- Fit model function

$$P(t) = Ae^{-mt} + Ae^{-m(T-t)} + (-1)^t \widetilde{A} e^{-\widetilde{m}t} + (-1)^t \widetilde{A} e^{-\widetilde{m}(T-t)}$$
$$+ A^* e^{-m^* t} + A^* e^{-m^*(T-t)} + (-1)^t \widetilde{A}^* e^{-\widetilde{m}^* t} + (-1)^t \widetilde{A}^* e^{-\widetilde{m}^*(T-t)}$$

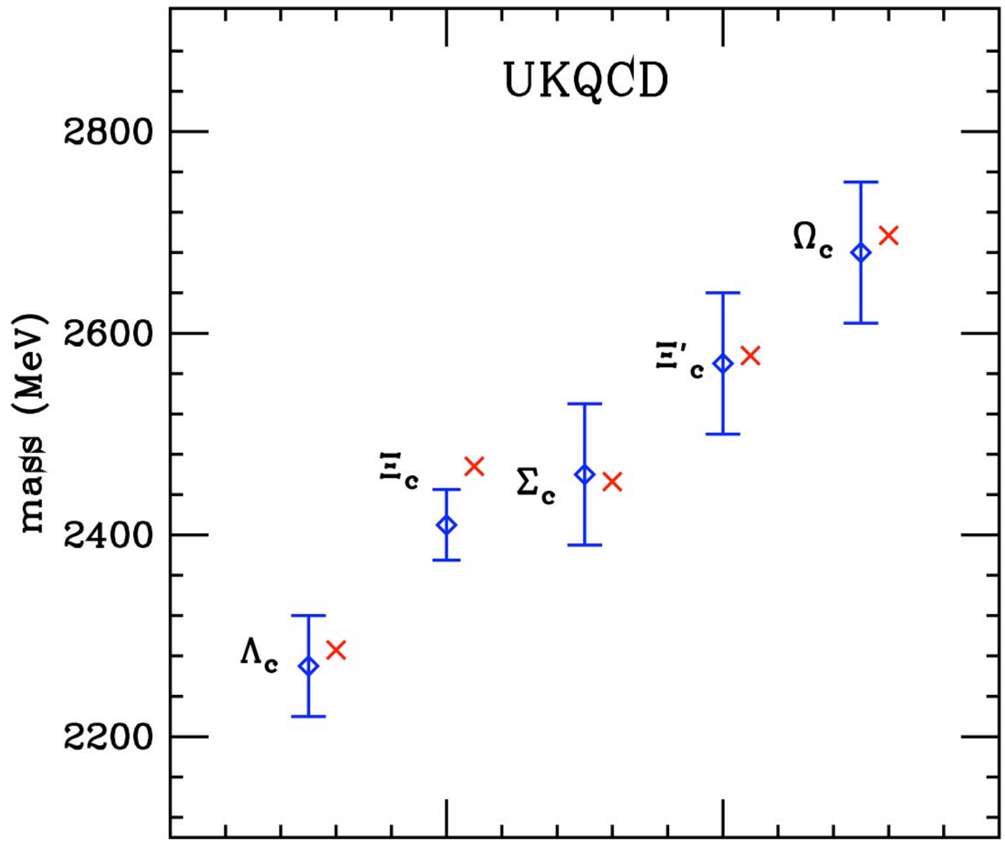
- Correlated least squares fit
- Error estimation
  - 1000 bootstrap samples
- Linear chiral extrapolation

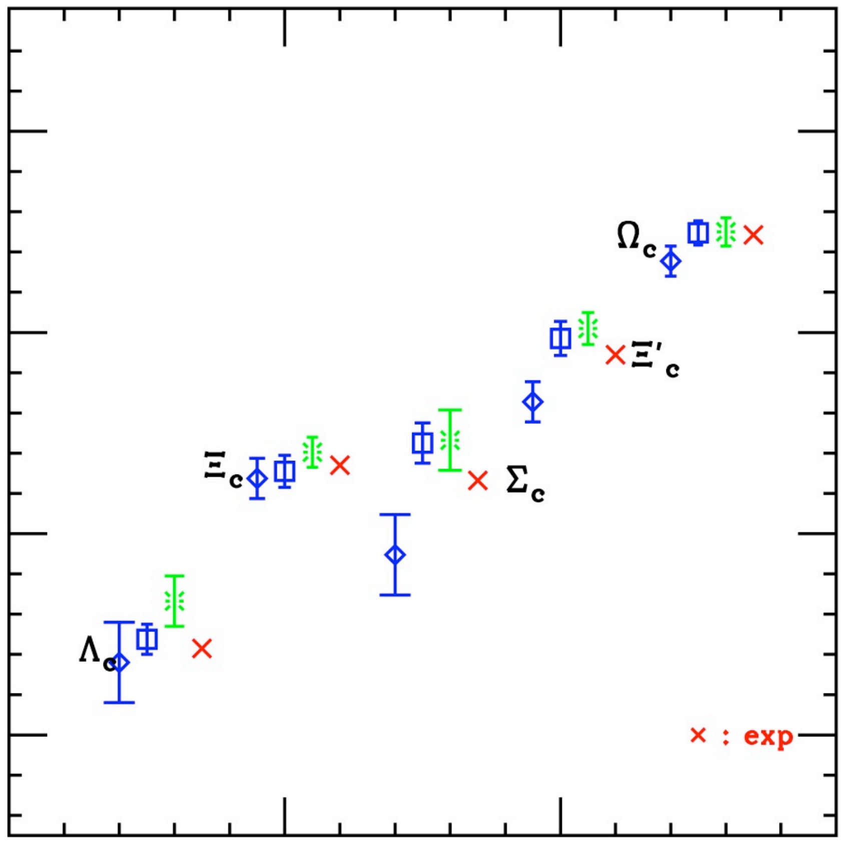
● Results

- $1/2^+$ singly charmed heavy baryons



- $1/2^+$ singly charmed heavy baryons : Other groups (Quenched calculations)





$24^3 \times 48$: 60 confs

($a \approx 0.068$)

K.C. Bowler et al.,

PRD 54,3619 (1996)

$12^3 \times 32$: 720 confs ($a_s \approx 0.22$)

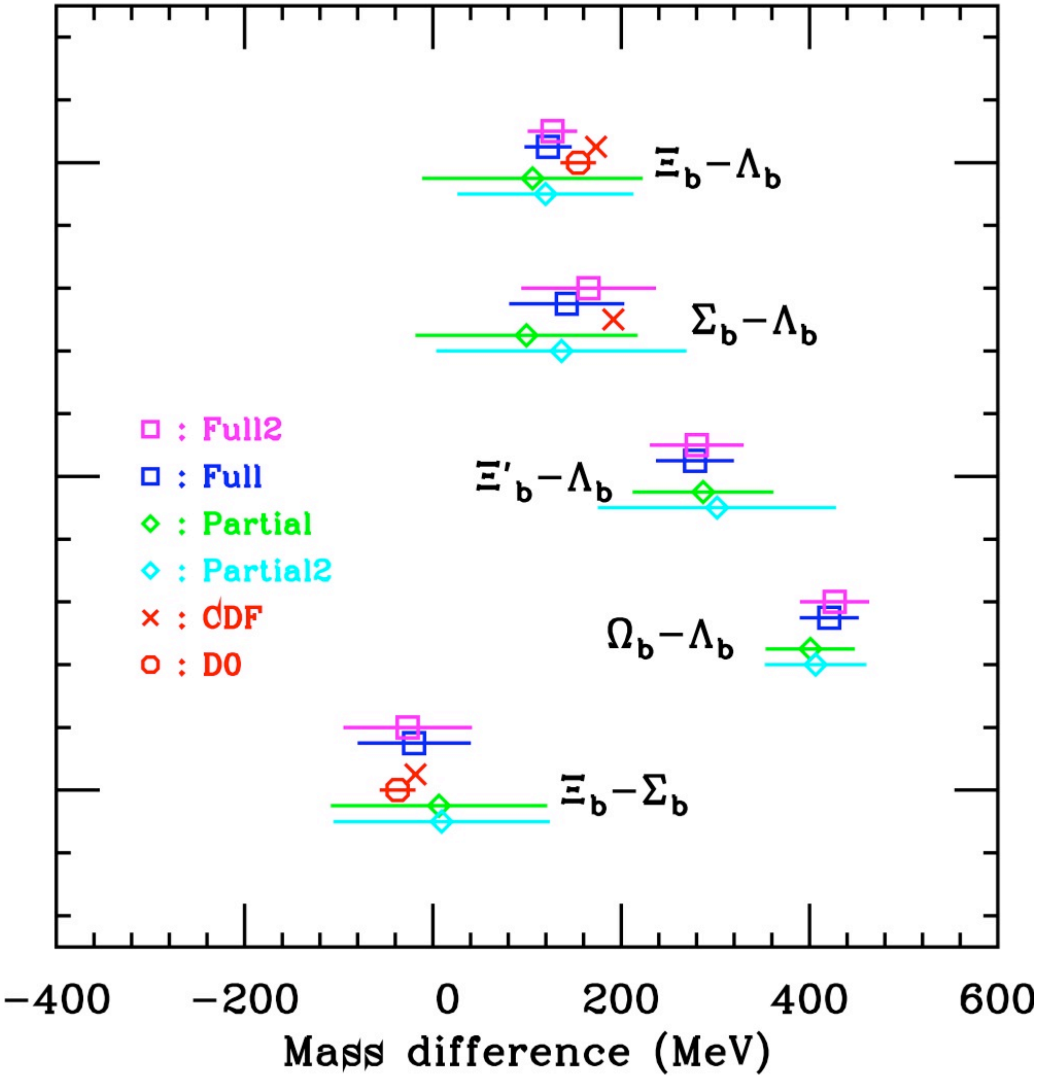
$14^3 \times 38$: 442 confs ($a_s \approx 0.18$)

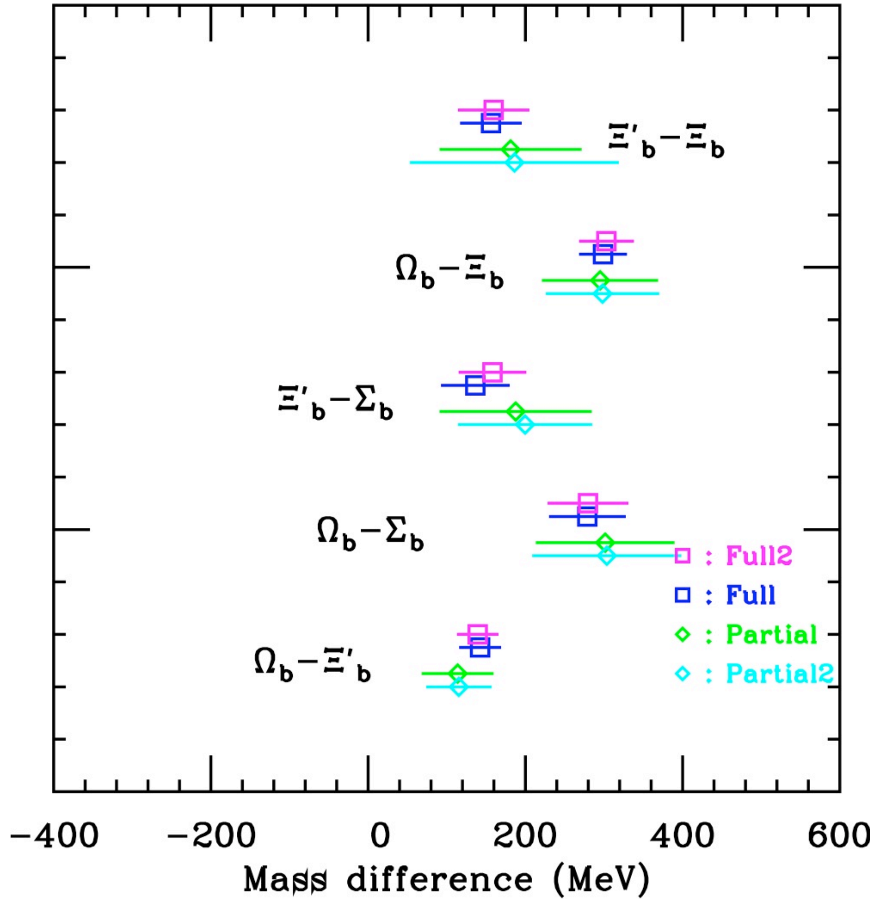
$18^3 \times 46$: 325 confs ($a_s \approx 0.15$)

R. Lewis et al.,

PRD 64,094509 (2001)

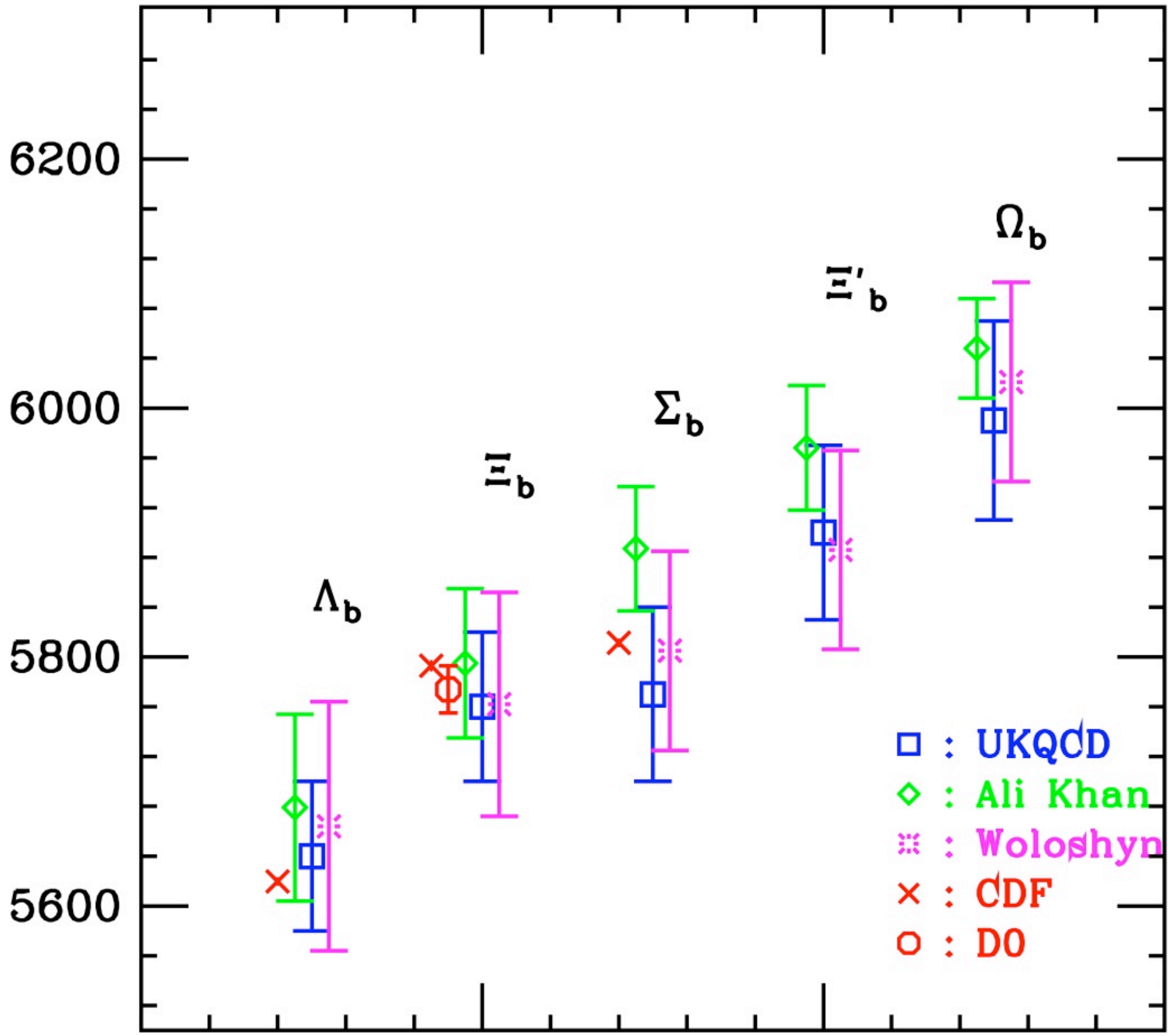
- $1/2^+$ singly bottom heavy baryons





Recent measurements from CDF and D0

- 1/2⁺ singly bottom heavy baryons : Other groups (Quenched calculations)

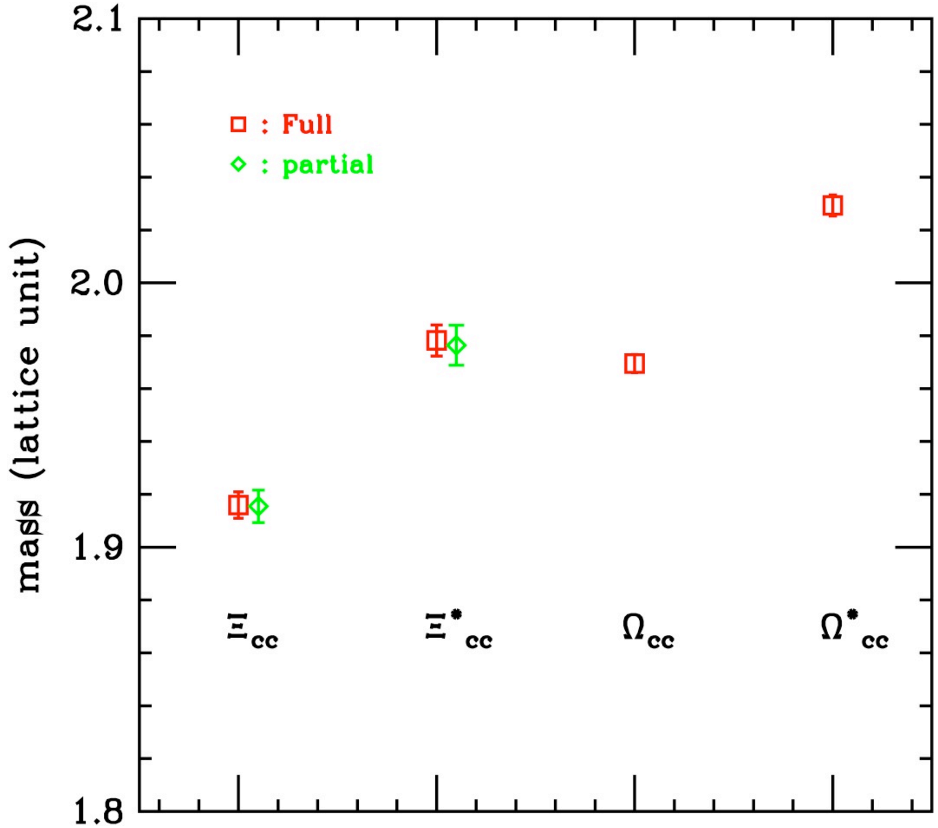


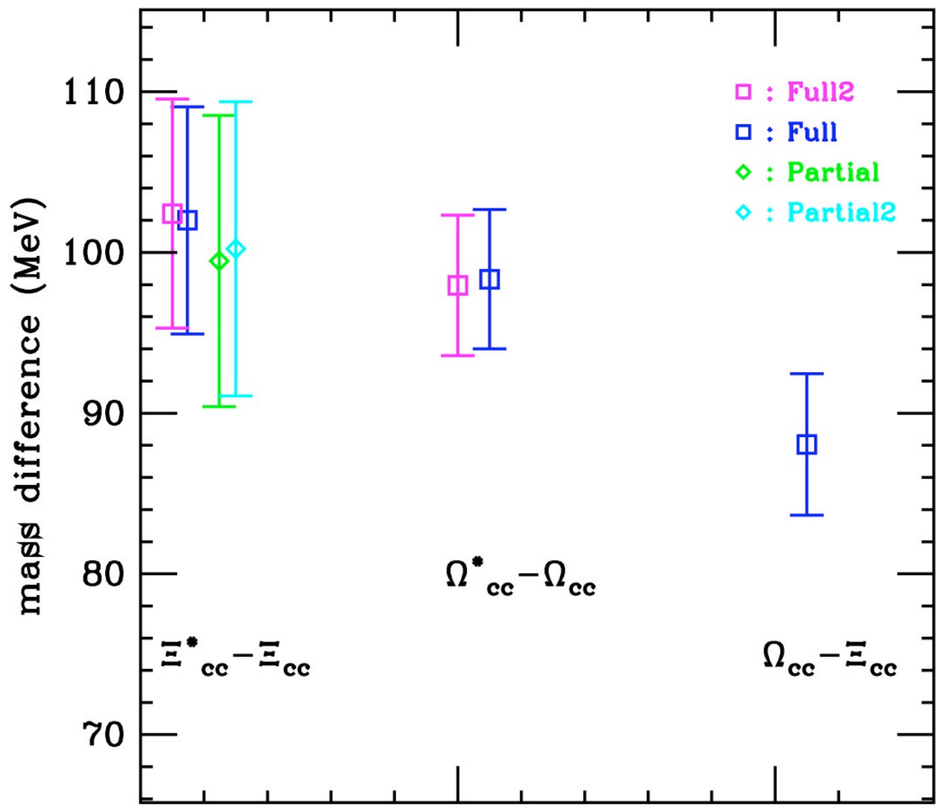
K.C. Bowler et al.,PRD 54,3619 (1996)

A. Ali Khan et al., PRD 62,054505 (2000)

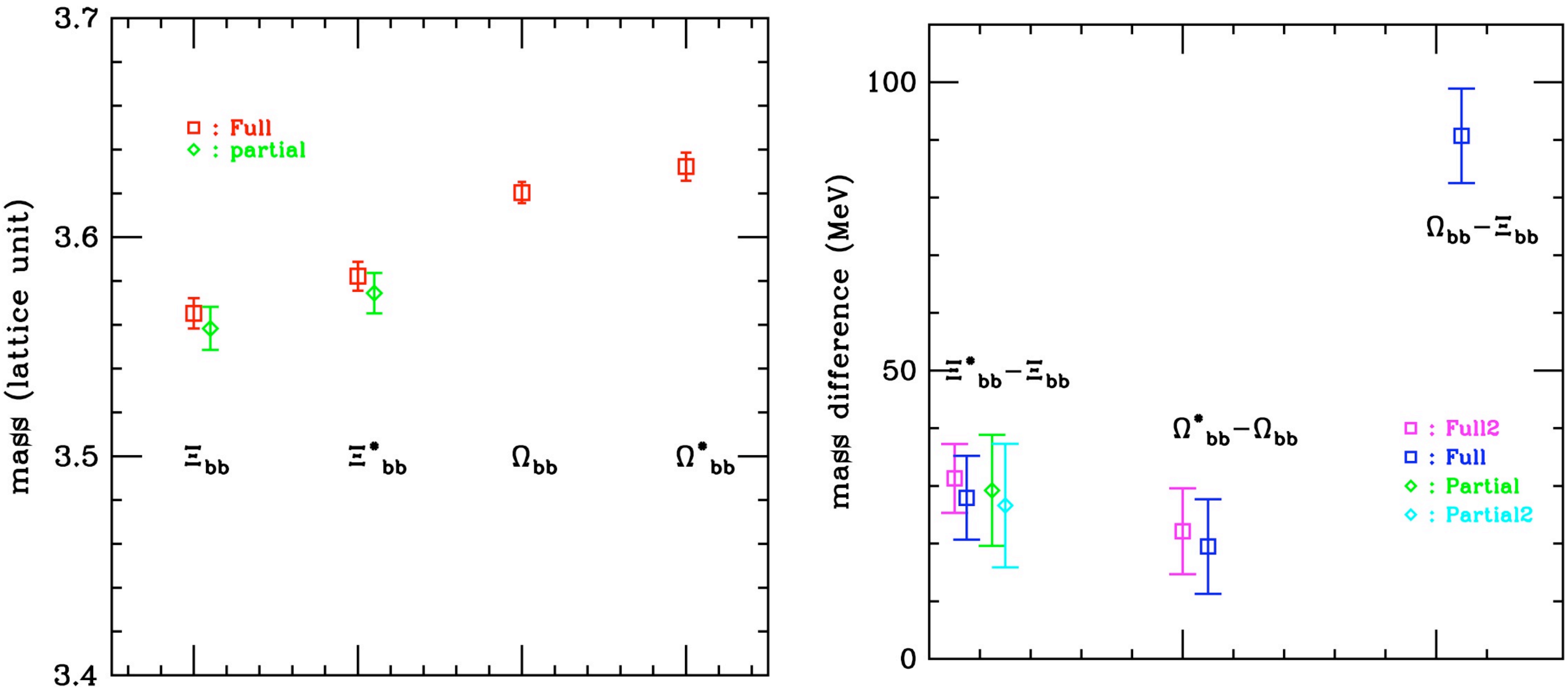
N. Mathur et al., PRD 66,014502 (2002)

- Doubly charmed heavy baryons (Preliminary)





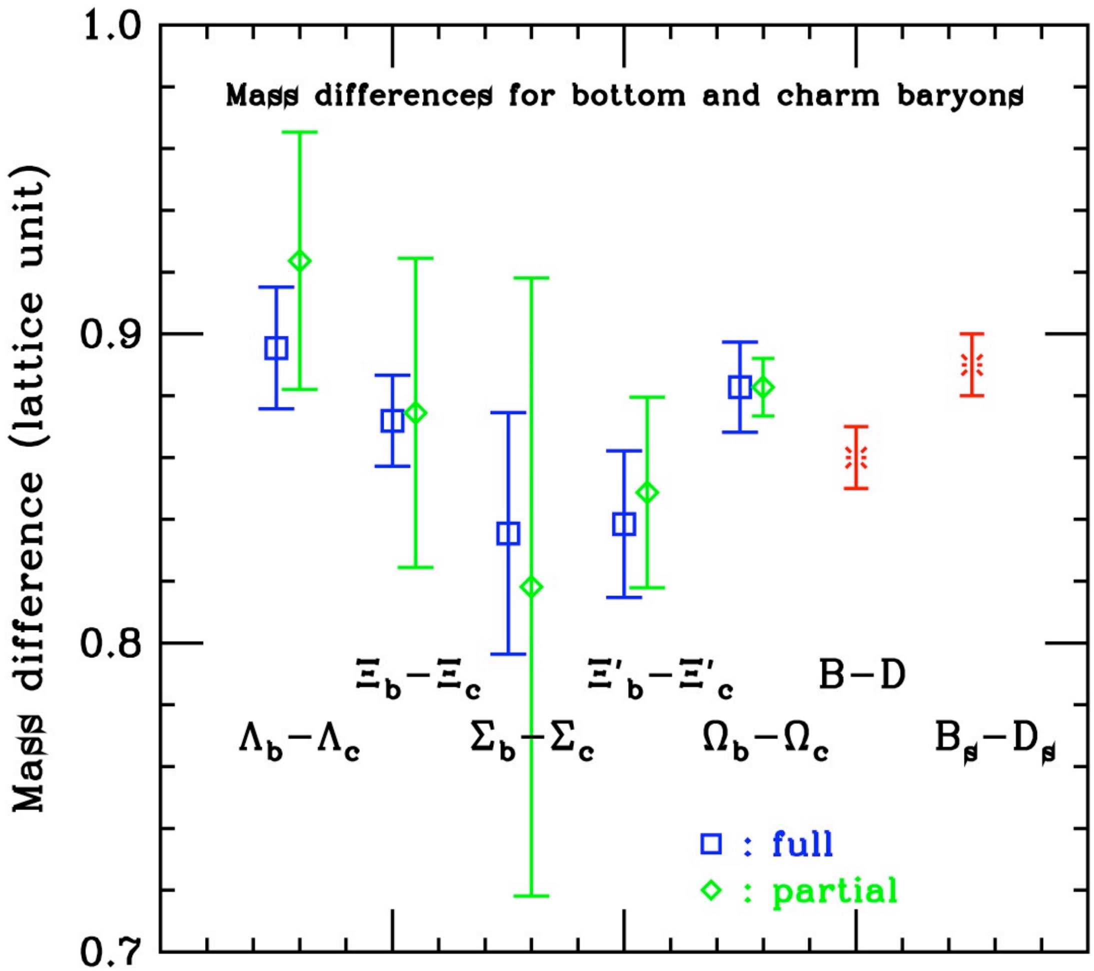
- Doubly bottom heavy baryons (Preliminary)



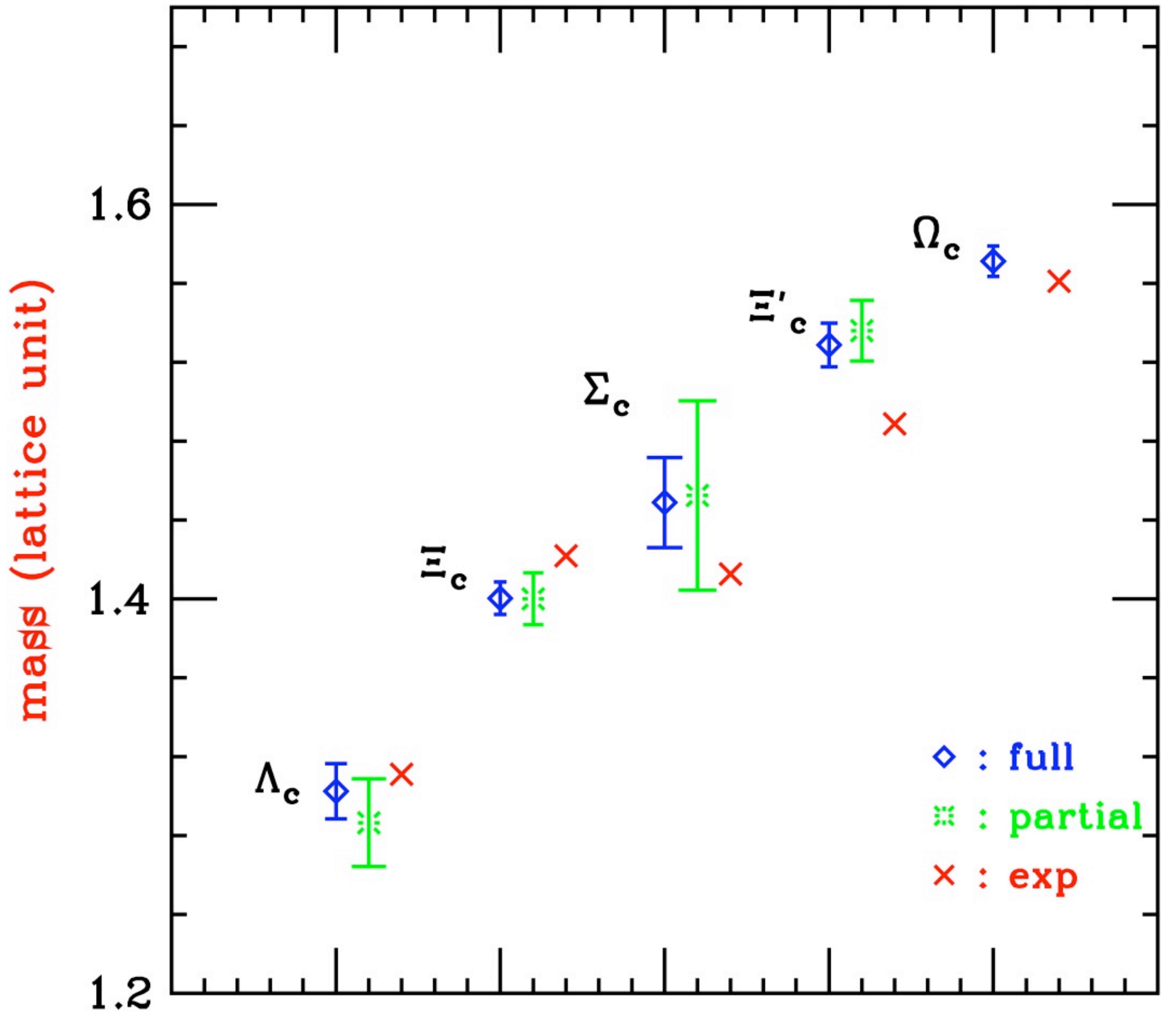
Future study

- Fine lattice
  - $a \approx 0.09$, $m_l = 0.2 m_s$, $m_l = 0.4 m_s$
- Increase statistics
- More about error analysis
- Finite size effect
- Discretization errors
- Excited states $(3/2^+, 1/2^-, 3/2^-)$

- Mass differences between bottom and charm hadrons



- $1/2^+$ singly charmed heavy baryons conti.



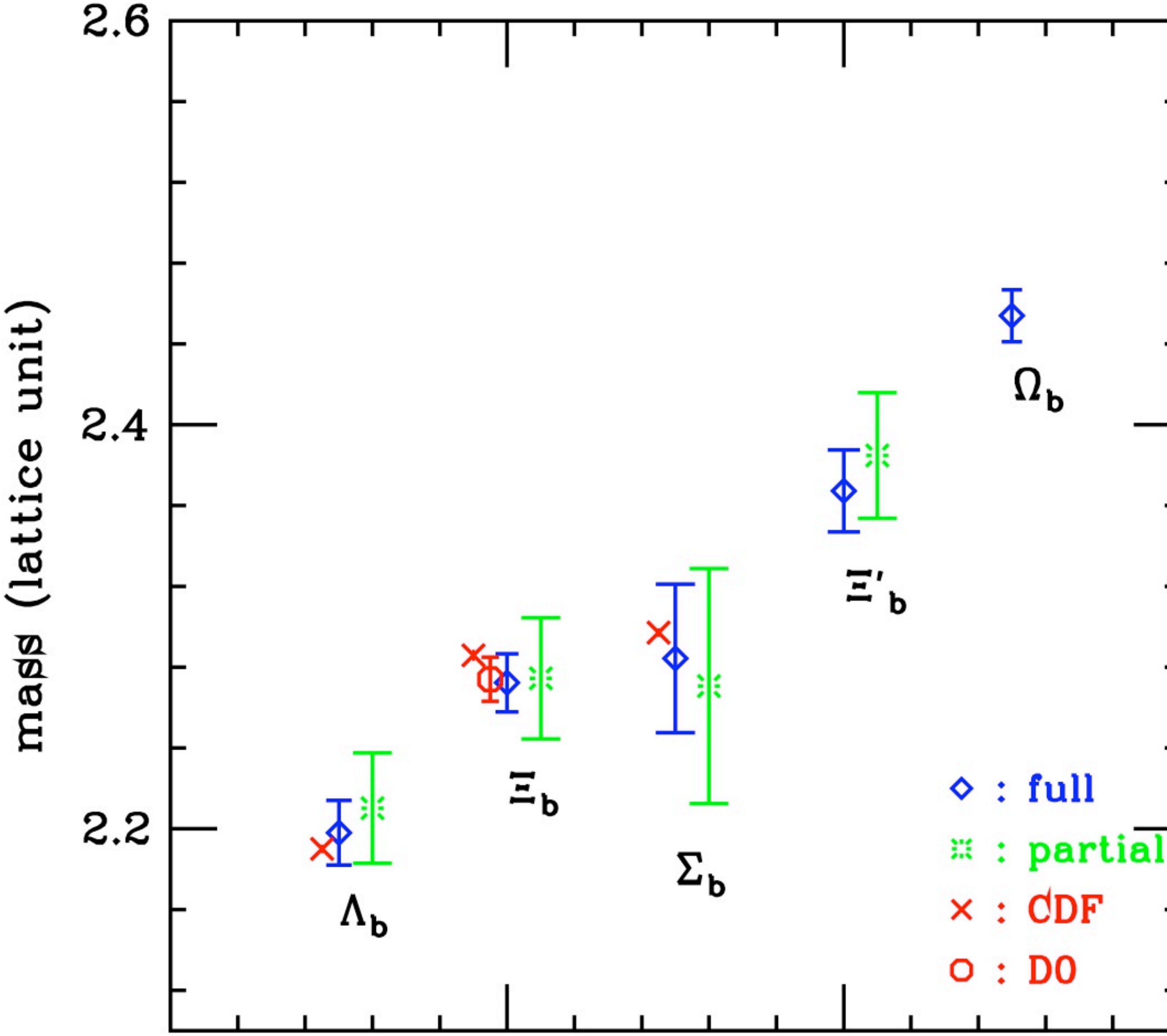
$$M_{phy} = M_{cal} + \Delta$$

$$M_{kin} = \frac{|\vec{p}|^2 - [M_{cal}(\vec{p}) - M_{cal}(0)]^2}{2[M_{cal}(\vec{p}) - M_{cal}(0)]}$$

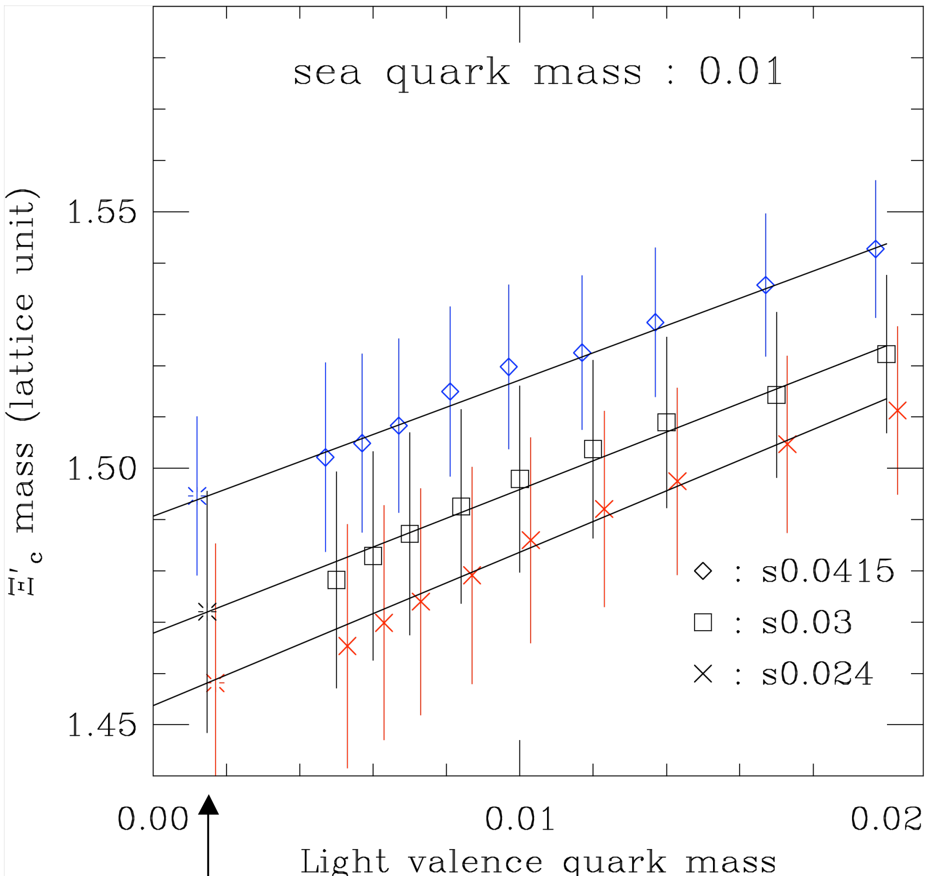
Constant Mass Shift

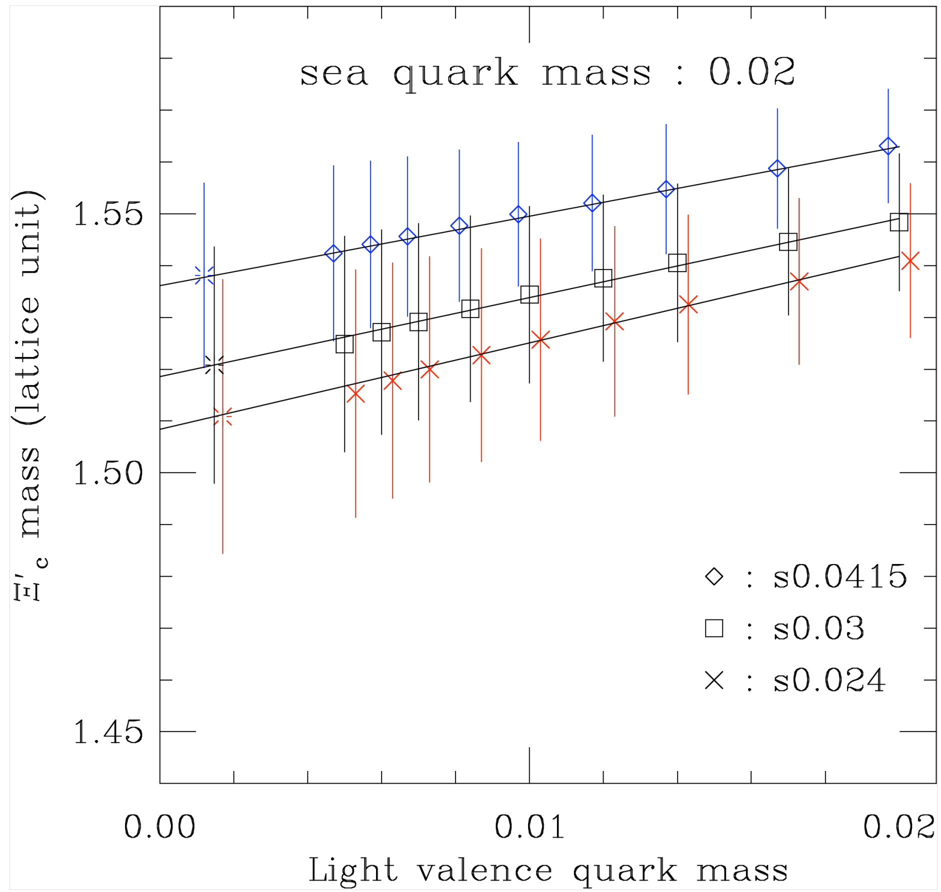
= Average (M_{exp} - M_{cal})

• $1/2^+$ singly bottom heavy baryons conti.



- Extrapolation of light valence quark mass



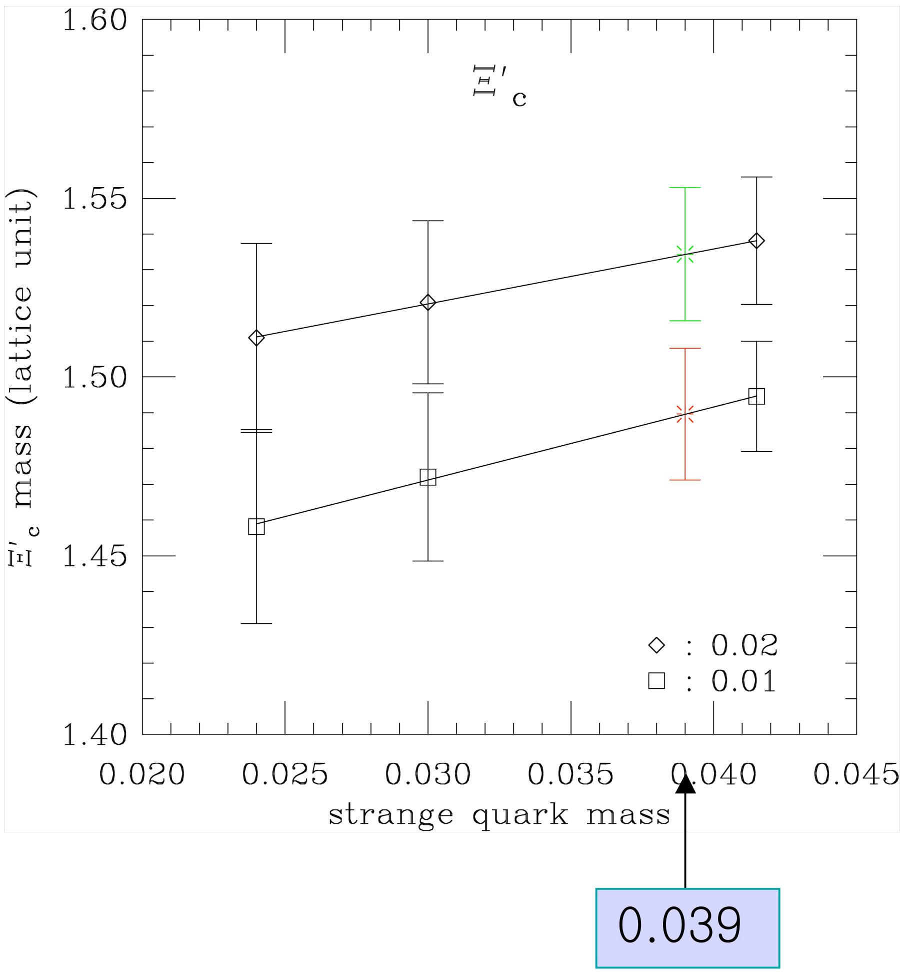


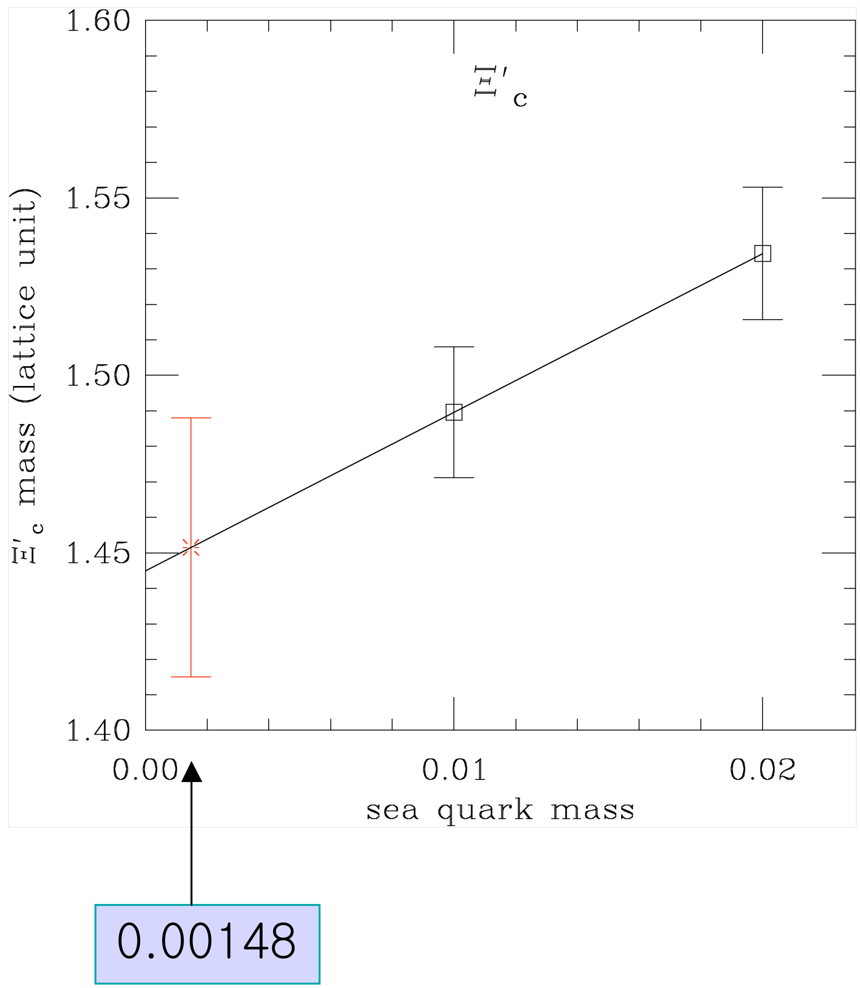
0.00148

Confidence level ~ 40% in averag
e

Real quark masses are quotations from MILC. PRD 70, 114501 (2004)

- Interpolation of Strange quark mass and extrapolation of Light sea quark mass





- Full QCD extrapolation