New Phases of Finite Temperature Gauge Theory from an Extended Action

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Abstract

We study the behavior of the order parameter, the phase diagram, and the thermodynamics of exotic phases of finite temperature gauge theory. Lattice simulations were performed in SU(3) and SU(4) with an adjoint Polyakov loop term added to the standard Wilson action. In $SU(3)$, the pattern of $Z(3)$ symmetry breaking in the new phase is distinct from the pattern observed in the deconfined phase. In $SU(4)$, the $Z(4)$ symmetry is spontaneously broken down to $Z(2)$, representing a partially-confined phase. The existence of the new phases is confirmed in analytical calculations of the free energy based on high-temperature perturbation theory.

Background

The QCD phase transition of SU(N) ($N \geq 3$) gauge theories in $3+1$ dimensions is characterized by a low-temperature confined phase, where $Z(N)$ symmetry is unbroken and quarks and gluons are bound, and a high-temperature deconfined phase where $Z(N)$ symmetry is spontaneously broken (Svetitsky and Yaffe, Nucl. Phys. B210, 423 (1982)), this is also known as the quark-gluon plasma (QGP) phase. Simulations indicate that the transition between the confined and deconfined phases has the following key properties:

- The transition is first order for all $N \geq 3$.
- The global $Z(N)$ symmetry appears to always break completely in the deconfined phase, with no residual unbroken subgroup.

One way to explore this transition and the phase structure surrounding it is to extend the Euclidean action of the pure $SU(N)$ gauge theory with a simple $Z(N)$ invariant term, the adjoint Polyakov loop:

$$-\int d^3x\, h_A Tr_A P(\vec{x}) = -T\int_0^\beta dt \int d^3x\, h_A Tr_A P(\vec{x})$$

Here $P(\vec{x})$ is the Polyakov loop at the spatial location $\vec{x}$, given by the path ordered exponential of the temporal component of the gauge field, $P(\vec{x}) = e^{i\beta A_0(\vec{x})}$. This addition leads to the appearance of new phases with interesting properties for a small range of $h_A < 0$.

What would motivate us to extend the action in this way? Well, we expected that it would provide a way to examine the possibility of confinement restoration at high temperatures. This is important because currently we know of know way to perturbatively study the confined phase. Extending the action to provide this ability in a region of high temperature where perturbation theory is applicable should prove quite useful. To show confinement restoration and to better understand the phase structure we look at how the effective potential is minimized:

$$V_{eff} = \sum v_R Tr_R P - T h_A Tr_A P$$

- Since $Tr_A P = |Tr_F P|^2 - 1$, positive h_A favors maximization of $Tr_A P$, this implies $|Tr_F P| > 0$. This breaking of $Z(N)$ symmetry suggests the deconfined phase.
- negative h_A favors minimization of $Tr_A P$, this implies $Tr_F P = 0$, which defines the confined phase

It was therefore reasonable to expect that a sufficiently negative h_A might lead to a restoration of confinement at temperatures above the deconfinement temperature.

$SU(3)$ Simulation Results

Well, sometimes things aren't as they seem. As expected, increasing positive h_A does decrease the deconfinement temperature. And, when exploring negative h_A we did indeed find confinement to be restored, however, we also found that the phase structure was even richer than just a deconfined and a confined phase. We found unexpected new phases in both $SU(3)$ and $SU(4)$.

- In $SU(3)$, the new phase breaks $Z(3)$ symmetry in an unfamiliar way, characterized by $Proj\,\langle Tr_F P\rangle < 0$, where the projection is taken onto the nearest $Z(3)$-axis.
- In $SU(4)$, global $Z(4)$ symmetry is spontaneously broken to $Z(2)$. The residual $Z(2)$ symmetry ensures that in the fundamental representation $\langle Tr_F P\rangle = 0$, but that $\langle Tr_R P\rangle \neq 0$ for representations that transform trivially under $Z(2)$.

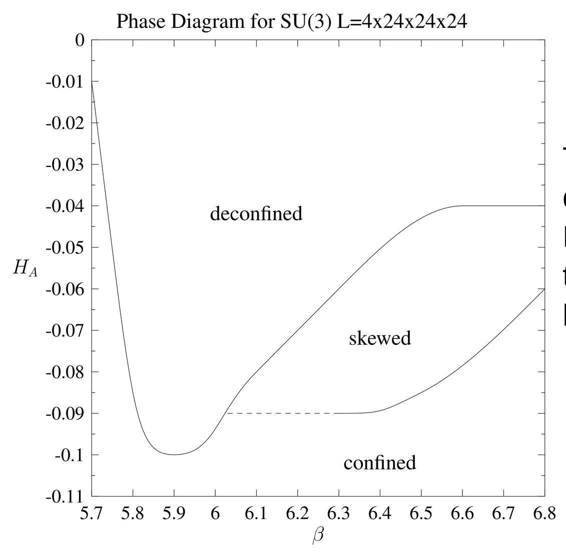
This phase structure was revealed to us by simulations performed in $SU(3)$ and $SU(4)$. We added the $Tr_A P$ term onto the standard lattice action to get:

$$S = S_W + \sum_{\vec{x}} H_A Tr_A P(\vec{x})$$

where S_W is the Wilson action, the sum is over all spatial sites, and we have a simple relationship, $H_A = h_A a^3$, between our variable lattice parameter H_A and the parameter used in our analytical calculations h_A (in addition to an unknown renormalization factor).

The phase diagram from $SU(3)$ simulations

Here we look at the phase structure in the β - h_A parameter space of $SU(3)$. In the exploration of negative h_A we discovered 3 distinct phases:

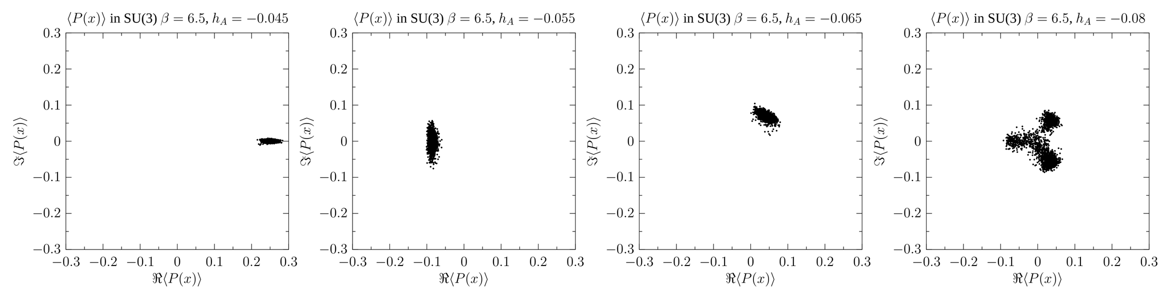


- deconfined: $Proj\,\langle Tr_F P\rangle > 0$
- confined: $Proj\,\langle Tr_F P\rangle = 0$
- skewed: $Proj\,\langle Tr_F P\rangle < 0$

The locations of the phase transitions were determined from the peaks of the adjoint Polyakov loop susceptibility, checked against the histograms of the fundamental Polyakov loop.

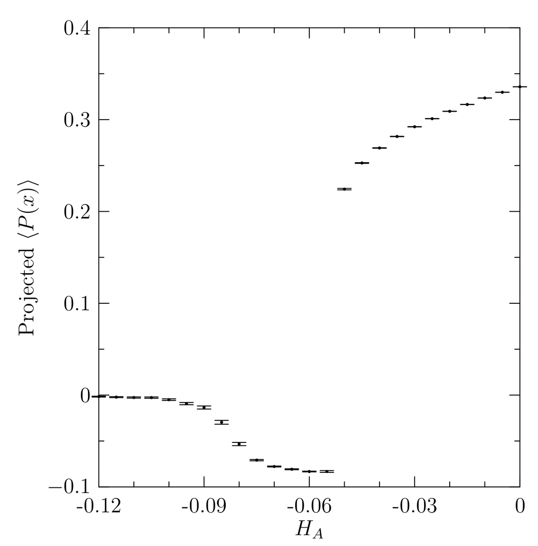
Indicators of phase transition in $SU(3)$

The histograms in $SU(3)$ show the three phases clearly. As we see in the phase diagram, going down in h_A and for fixed β, we first encounter the deconfined phase, then the skewed phase, then the confined phase. Tunneling observed in the skewed phase indicates that the transition to the confined phase is weak.

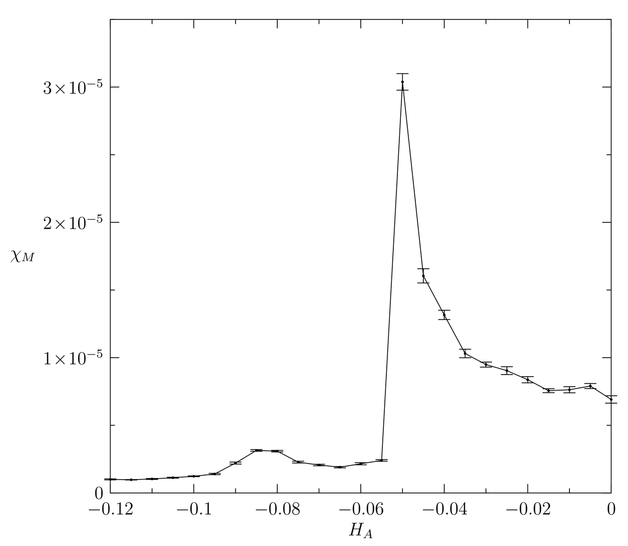


deconfined | **skewed** | **skewed** | **skewed, tunneling**

There are a few more ways to see the transitions clearly. One way is to look at the Polyakov loop projected onto the nearest $Z(3)$ axis, another is to look at the adjoint Polyakov loop susceptibility.



Projected $Tr_F P$



Adjoint susceptibility χ_M

Both graphs show that the transition between the deconfined and skewed phases is clearly first-order from the obvious discontinuity in the order parameters. The transition between the skewed phase and the confined phase is likely to be first order as well since this model is associated with the universality class of the 3d Potts model, but this is not obvious from the much smaller changes in the order parameters.

$SU(3)$ Theory

We now want to confirm our lattice results with some analytical calculations of the thermodynamics. The effective potential we use is adapted from the one-loop free energy density first evaluated by Gross, Pisarski, and Yaffe (Rev. Mod. Phys. 53, 43 (1981)), to get:

$$V_{eff} = -2\frac{1}{2}Tr_A \int \frac{d^3k}{(2\pi)^3}\sum_n \ln[(\omega_n - A_0)^2 + k^2] - h_A T\, Tr_A P$$

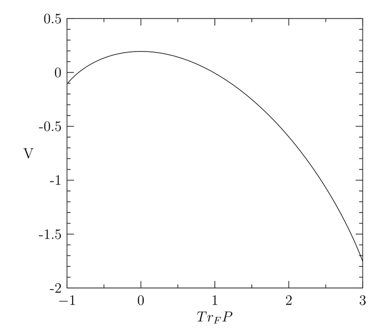
where the sum is over Matsubara frequencies $\omega_n = 2\pi n T$. It is useful to convert this into a function of the eigenvalues of the Polyakov loop:

$$V_{eff} = -2T^4 \sum_{j,k=1}^{N}\left(1 - \frac{1}{N}\delta_{jk}\right)\left[\frac{\pi^2}{90} - \frac{1}{48\pi^2}|\Delta\theta_{jk}|^2(2\pi - |\Delta\theta_{jk}|)^2\right]$$

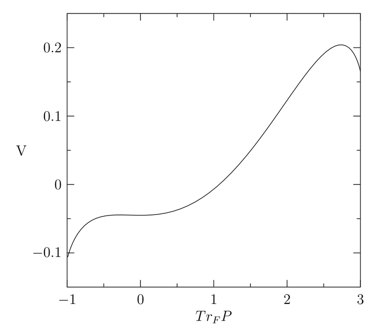
$$- h_A T\left(\left|\sum_{j=1}^{N} e^{i\theta_j}\right|^2 - 1\right)$$

where the angles θ_j are the eigenvalues of βA_0.

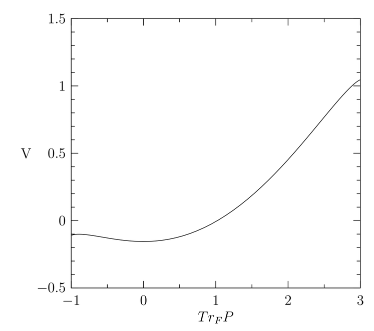
We would like to know if the effective potential shows all 3 phases. In $SU(3)$, it is sufficient to consider V_{eff} for the Polyakov loop projected onto the nearest $Z(3)$ axis. $P = diag[1, exp(i\phi), exp(-i\phi)]$.



deconfined: $(Tr_F P)_{crit} = 3$



skewed: $(Tr_F P)_{crit} = -1$



confined: $(Tr_F P)_{crit} = 0$

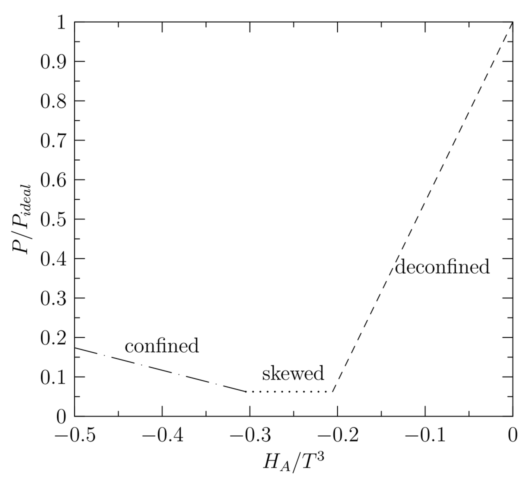
Since we know the values of ϕ for which V_{eff} is minimized for all 3 phases, we can set V_{eff} in two phases equal to find the location of the phase transitions in terms of the dimensionless quantity h_A/T^3.

- deconfined-skewed phase transition: $h_A/T^3 = -\pi^2/48 \simeq -0.206$
- skewed-confined phase transition: $h_A/T^3 = -5\pi^2/162 \simeq -0.305$

The ratio of these values is similar to that from simulations.

Comparison of $SU(3)$ Theory to simulation

We can also compare values for the pressure from the effective potential to the pressure determined from simulations.



Theoretical prediction for pressure normalized to black body pressure as a function of h_A.

In simulations pressure is calculated along a path of constant β,

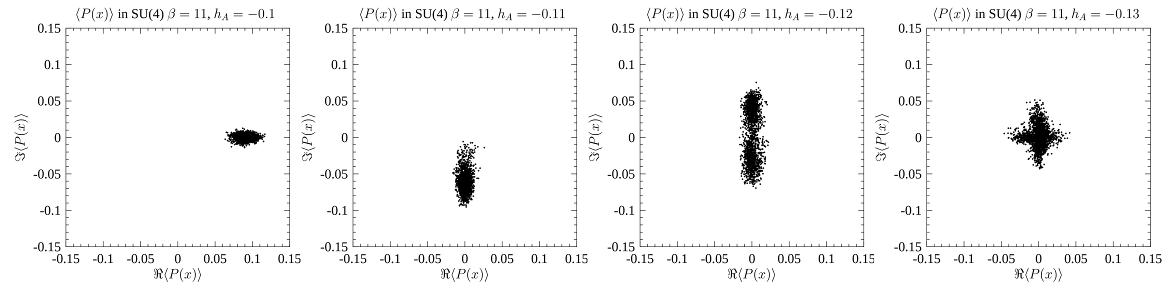
$$\frac{p_2}{T^4} - \frac{p_1}{T^4} = N_t^3 \int_1^2 dH_A \langle Tr_A P\rangle$$

Comparing ΔP across the deconfined and skewed phases:

- Theory:
  Deconf: $\Delta p/T^4 = \pi^2/6 \simeq 1.64$
  Skewed: $\Delta p/T^4 = 0$
- Simulations:
  Deconf: $\Delta P = 1.64 \pm 0.03$
  Skewed: $\Delta P = -0.18 \pm 0.07$

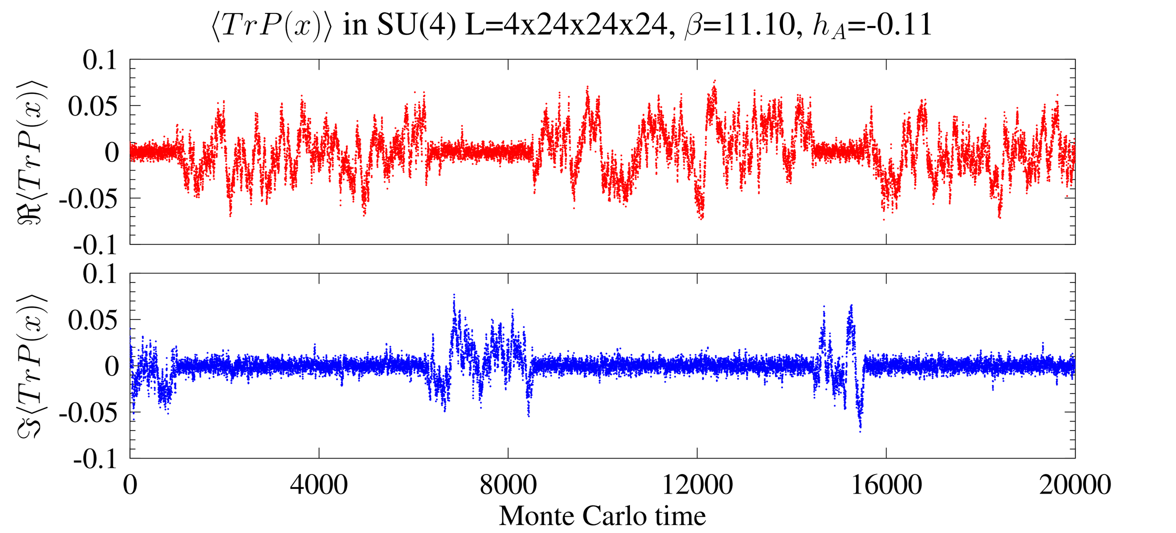
$SU(4)$ Simulation: histograms of $\langle Tr_F P\rangle$

Now let's take a look at the phases of $SU(4)$. Again decreasing negative H_A and keeping β fixed we find the new phase in approximately the same region. We first encounter the deconfined phase, then the new partially confined phase. Tunneling is observed as we continue decreasing H_A in the partially confined phase, the fluctuations gradually reduce in size, but we are uncertain if there is a transition into the confined phase.



deconfined | **deconfined** | **partially confined** | **partially confined, tunneling**

Let's take a look at the new phase in $SU(4)$. As in the case of $SU(3)$, the new phase is found in the region $h_A < 0$. But, in this, partially confined phase, global $Z(4)$ symmetry is spontaneously broken to $Z(2)$. This break down becomes clear when looking at the time history of variations of the real and imaginary parts of the Polyakov loop during a long run in which tunneling is observed.



Real and imaginary parts of $SU(4)$ Polyakov loop versus Monte Carlo time

$SU(4)$ Theory

For $SU(4)$ theory we have used again the one-loop effective potential to examine the possible occurrence of four different phases in $SU(4)$:

- the confined phase, which has full $Z(4)$ symmetry
- the deconfined phase
- a partially-confined, $Z(2)$-invariant phase
- a skewed phase similar to that of $SU(3)$

$--\rangle$ However, only the deconfined phase and the $Z(2)$ phase are predicted by our simple theoretical model. A more complicated model with additional terms should be able to locate the confined phase.

Let's compare the phase structure by predicted by the one-loop effective potential with our simulation results in $SU(4)$.

- V_{eff} predicts a first-order transition between the deconfined and $Z(2)$-invariant phases at $h_A/T^3 = -\pi^2/48 \simeq -0.205617$. This is in the same region as in simulations.
- The theoretical value of $\Delta\left(p/T^4\right)$ across the deconfined phase is $\pi^2/3 \simeq 3.289$.
- The change in pressure ΔP we obtained from simulations was 2.21 ± 0.07

Conclusions

- We have considerable evidence, from lattice simulation and from theory, for the existence of new phases of finite temperature gauge theories in $SU(3)$ and $SU(4)$ when a $Z(N)$-invariant, adjoint Polyakov loop term is added to the gauge action.
- In $SU(3)$, confinement is restored at high temperatures
- In $SU(3)$, the skewed phase was found, but its interpretation is unclear...
- In $SU(4)$, we found a partially-confined phase where $Z(4)$ is spontaneously broken to $Z(2)$.
- In the general case of $SU(N)$, there is good reason to expect a very rich phase structure may exist. For example, in $SU(6)$, we can consider partial breaking of $Z(6)$ to either $Z(2)$ or $Z(3)$.