Transverse Momentum Distributions of Quarks from the Lattice using Extended Gauge Link Operators

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presenting work in collaboration with LHPC and

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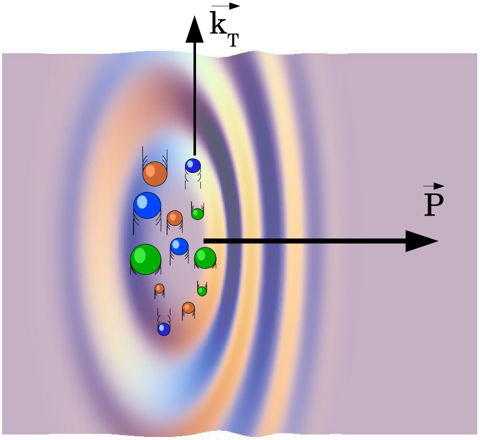
Motivation: Parton Picture



fast nucleon: quarks (and gluons) look like "partons", carrying

- a momentum fraction x of the nucleon momentum P
- an *intrinsic transverse momentum* $\vec{k}_T$

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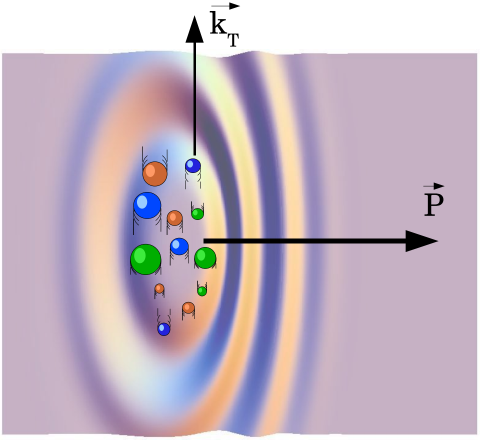

How are the quarks distributed with respect to $\vec{k}_T$?

e.g. $f_1(\vec{k}_T)$?

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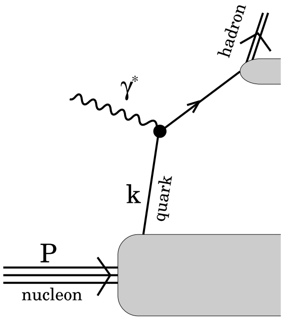
$\vec{k}_T$ dependence and Factorization

example: **S**emi **I**nclusive **D**eep **I**nelastic **S**cattering experiment



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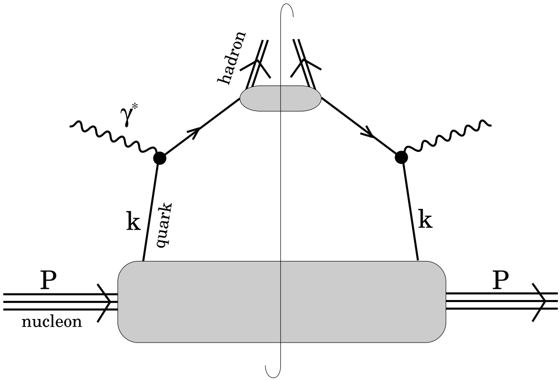
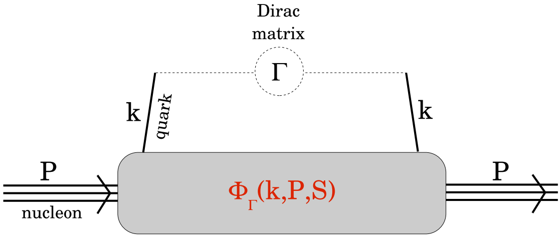


factorization $\Longrightarrow$ hard process + soft blobs (non-perturbative)

→ **T**ransverse **M**omentum dependent **P**arton **D**istribution **F**unctions

[Collins, Soper, Sterman PLB 83, NPB 85]
[Ji, Ma, Yuan PRD (2005)], [Mulders, Tangerman NPB (1996)]

$\vec{k}_T$ dependence and Factorization



non-perturbative correlator ⇒ calculate on the lattice

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \frac{1}{2(2\pi)^4}\int d^4\ell\; e^{ik\cdot\ell}\;\; \langle P,S|\;\; \overline{q}(0)\,\Gamma\,\mathcal{U}\,q(\ell)\;\; |P,S\rangle$$

Gauge Link Operator $\mathcal{U}$

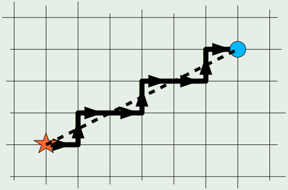
$\langle P,S|\ \overline{q}(0)\,\Gamma\,\mathcal{U}(0\to\ell)\,q(\ell)\ \ |P,S\rangle$ is gauge invariant.

continuum

$$\mathcal{U}(0\to\ell)\ \equiv\ \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

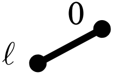
lattice



product of link variables

- factorization in SIDIS :
  path runs to infinity and back

- ***here*** (up to now):
  straight path



$\rightarrow$ probability interpretation of distribution $f(\vec{k}_T)$ [BBHM00]

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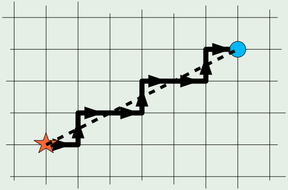

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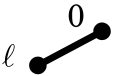
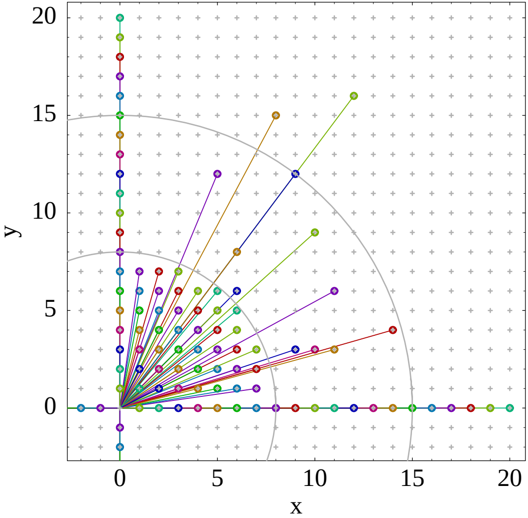


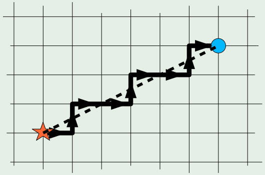
$\to$ probability interpretation of distribution $f(\vec{k}_T)$ [BBHM00]

Evaluated Quark-Quark Separations

quark separations in the x,y-plane :



lattice operator

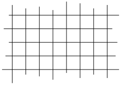


263 paths in total:

- straight paths in x, y, z, $-x$, $-y$, $-z$ direction
- step-like paths in other directions

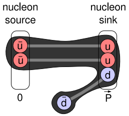
Extracting Nucleon Structure from the Lattice

Ingredients		Output : 3-point correlator $C_{3\mathrm{pt}}$

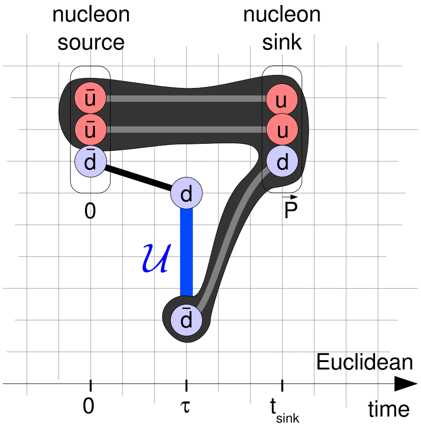


gauge configs.

quark propagators



nucleon sequential propagators



[We calculate isovector quantities $(u - d)$ $\Rightarrow$ no disconnected contributions.]

Parametrization and Ratios

parametrization of nucleon matrix element

$$\langle P,S|\ \overline{q}(0)\,\Gamma\,\mathcal{U}(0\to\ell)\,q(\ell)\ |P,S\rangle\ \equiv \overline{U}(P,S)\,\mathcal{M}_\Gamma(P)\,U(P,S)$$

Transfer matrix formalism ⇒ look at ratios!

ratio

$$R_\Gamma(\tau) \equiv \frac{C_{3\mathrm{pt}}(\tau|\vec{P},t_{\mathrm{sink}})}{C_{2\mathrm{pt}}(t_{\mathrm{sink}},\vec{P})}$$

$$\overset{0\ll\tau\ll t_{\mathrm{sink}}}{\approx} \frac{1}{2E(P)}\,\frac{\mathrm{Tr}\ (\not{P}+m_N)\,\Gamma^{3\mathrm{pt}}\,(\not{P}+m_N)\,\mathcal{M}_\Gamma(P)}{\mathrm{Tr}\ (\not{P}+m_N)\,\Gamma^{2\mathrm{pt}}}$$

⇒ Physics extracted from the plateau region!

Setup for Test Calculations

downloaded components:

gauge configurations — 84 MILC lattices

from NERSC archive [Orginos, Toussaint 1999],

staggered ASQTAD action, 2+1 flavors,
$L^3 \times T = 20^3 \times 64$,
$a \approx 0.124$ fm,

HYP smeared and chopped: only time slices 0..31 are used

propagators and sequential propagators — from LHPC

hybrid action: Domain Wall valence fermions ($L_s = 16$)

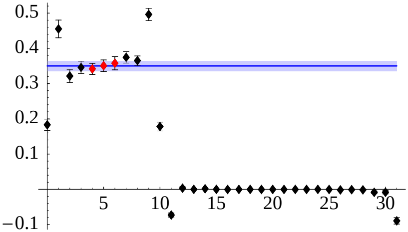
$m_\pi \approx 596$ MeV,
source-sink-separation: 10

2 nucleon momenta available: $\vec{P} = (0, 0, 0)$,

$$\vec{P} = \frac{2\pi}{L}(-1, 0, 0) \triangleq 500 \text{ MeV}/c$$

nucleon spin projection operator $\Gamma^{3\text{pt}} = \frac{1}{2}(\mathbb{1} + \gamma_4)(\mathbb{1} + i\gamma_5\gamma_3)$

Sample Plateaus



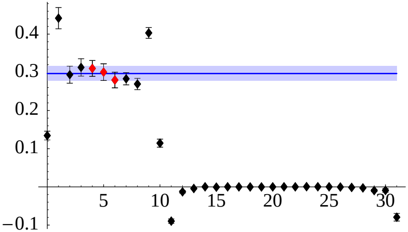
ratio Re $R_\Gamma(\tau)$

$\vec{P} = (0, 0, 0)$

$\Gamma = \gamma^0$

separation $\vec{\ell} = (5, 0, 0)$

$\Rightarrow$ 5 links in x direction

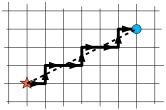


ratio $-$Re $R_\Gamma(\tau)$

$\vec{P} = (0, 0, 0)$

$\Gamma = \gamma^3\gamma^5$

$\vec{\ell} = (6, 3, 0)$, $|\vec{\ell}| = 6.7$



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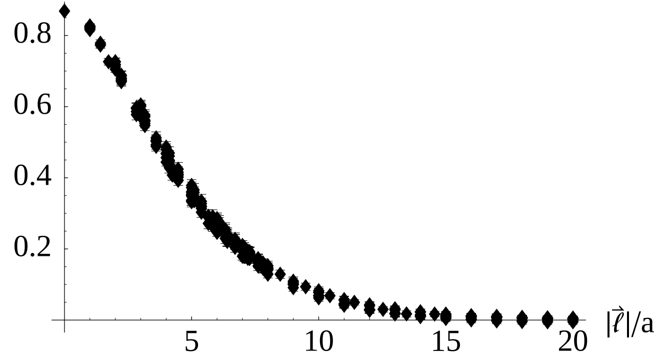
source at $t = 0$ sink at $t = 10$ extraction at $t = 4, 5, 6$

Preliminary Results

survey: $\operatorname{Re} R_\Gamma$ for $\vec{P} = (0,0,0)$, $\Gamma = \gamma^0$

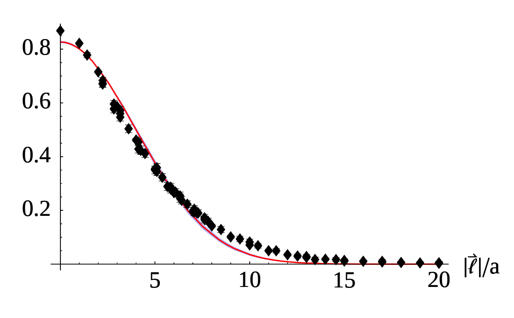


results for all 263 different link paths

$\Rightarrow$ Obviously, only quark separation length $|\vec{\ell}|$ matters (for $\vec{P} = 0$)

Preliminary Results

Re R_Γ for $\vec{P} = (0,0,0)$, $\Gamma = \gamma^0$



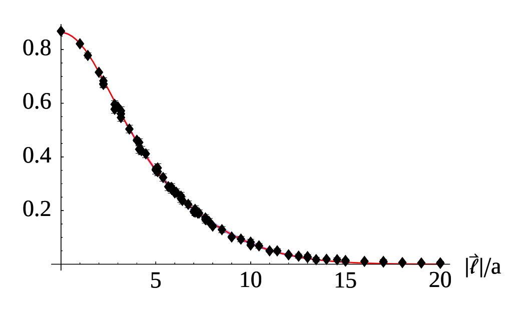
averaged over equivalent link paths in x,y - plane.

fit with Gaussian: $C \exp(-|\vec{\ell}|^2/\sigma^2)$

C	σ	$2/\sigma$
0.826 ± 0.005	$(5.64 \pm 0.12)a = 0.70$ fm	(563 ± 12) MeV/c

Preliminary Results

Re R_Γ for $\vec{P} = (0,0,0)$, $\Gamma = \gamma^0$



averaged over equivalent link paths in x,y - plane.

fit with two Gaussians: $C_1 \exp(-|\vec{\ell}|^2/\sigma_1^2) + C_2 \exp(-|\vec{\ell}|^2/\sigma_2^2)$

C_1	σ_1	$2/\sigma_1$	C_2	σ_2	$2/\sigma_2$
0.49	$7.3a$	(433 ± 15) MeV/c	0.37	$3.4a$	(945 ± 41) MeV/c

Preliminary Results: Fourier Transformed

$R_{\gamma^0} \xrightarrow{\text{Fourier transformation}} f_1^{\text{lat}}(\vec{k}_T)$

$$\int dx \int dk^- \left(\int \frac{d^4\ell}{2(2\pi)^4}\, e^{ik\cdot\ell}\, \langle P,S|\ \overline{q}(0)\,\gamma^+\,\mathcal{U}\,q(\ell)\ |P,S\rangle \right)$$

$$= f_1(\vec{k}_T)$$ see e.g., [Mulders, Tangerman NPB (1996)]

$$= \ ...$$

$$= \int \frac{d^2\vec{l}_T}{(2\pi)^2}\, e^{-i\vec{k}_T\cdot\vec{l}_T}\, R_{\gamma^0}(|\vec{l}_T|, \vec{P}=0)$$

Preliminary Results: Fourier Transformed

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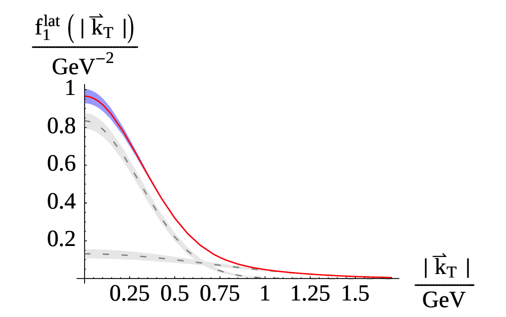


Transverse
Momentum dependent
Parton
Distribution
Function!

$$\sqrt{\langle \vec{k}_T^2 \rangle} = (533{\pm}11)\ \mathrm{MeV}$$

Renormalization (e.g., to $\overline{\mathrm{MS}}$) not done, yet!

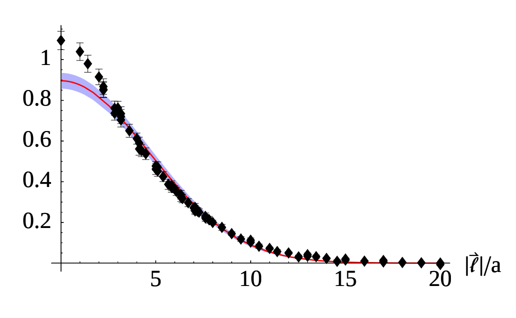
comparison: phenomenology $\longrightarrow$ e.g., [Anselmino et al. PRD (2005)]

usual Ansatz: $f_1(x,\vec{k}_T) \propto f_1(x)\, f_1(\vec{k}_T)$, $\quad f_1(\vec{k}_T) \propto \exp(\,\vec{k}_T^2 \,/\, \langle \vec{k}_T^2 \rangle\,)$,

$\sqrt{\langle \vec{k}_T^2 \rangle} \approx$ 500 MeV describes data.

Preliminary Results

$-\mathrm{Re}\, R_\Gamma$ for $\vec{P} = (0,0,0)$, $\Gamma = \gamma^3\gamma^5$



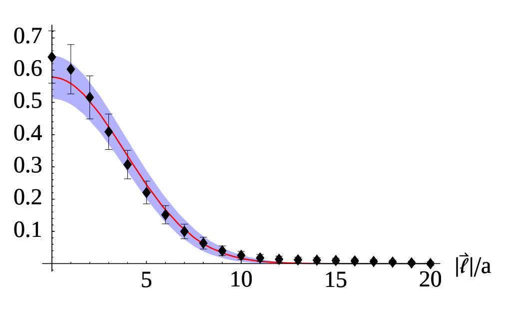
averaged over equivalent link paths in x,y - plane.

fit with Gaussian: $C \exp(-|\vec{\ell}|^2/\sigma^2)$

C	σ	$2/\sigma$
0.90 ± 0.04	$(6.58 \pm 0.12)a = 0.82$ fm	(484 ± 9) MeV/c

Preliminary Results

$\frac{1}{2}\mathrm{Re}\,\{R_\Gamma(\ell) + R_\Gamma(-\ell)\}$ for $\vec{P} = (-1, 0, 0)$, $\Gamma = \gamma^0$



link paths in x - direction.

fit with Gaussian $C\exp(-|\vec{\ell}|^2/\sigma^2)$

C	σ	$2/\sigma$
0.58 ± 0.07	$(5.4 \pm 0.5)a = 0.67$ fm	(666 ± 52) MeV/c

Conclusions & Outlook

first conclusions

- Extraction of non-local correlators
  with good statistics possible.
- Seems like we see approximately Gaussian distribution of quark momenta, as expected.
- Width of the Gaussian is in the expected range.

things to do:

- details of parametrization
- more configurations,
  more Dirac- and link-structures.
- renormalization of the non-local operators
- Can we mimic links "to infinity and back" on the lattice?
  Relation to phenomenological TMPDFs.

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BACKUP

References I

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