Heavy-light form factors on the lattice

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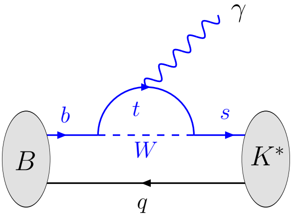
INT Summer School on "Lattice QCD and its applications"
Seattle 2007

Outline

Motivation

B $\longrightarrow$ K*(892)γ decay

- Flavour Changing Neutral Current (FCNC): Loop - suppressed in Standard Model
- Exp. error $\approx$ 5%:



$$B(B^0 \to K^{*0}\gamma) = \begin{cases} (3.92 \pm 0.20 \pm 0.24) \times 10^{-5} & \text{BaBar '04} \\ (4.01 \pm 0.21 \pm 0.17) \times 10^{-5} & \text{Belle '04} \end{cases}$$

$\Rightarrow$ Need precise (nonperturbative) QCD matrix elements to constrain BSM physics.

Lattice results for relativistic b-quarks

Problem in lattice calculations: $am_b > 1$

$\Rightarrow$ Extrapolation in heavy quark mass $m_h \to m_b$ *[Becirevic, Lubicz, Mescia (2007)]*.

Tensor form factor at $q^2 = 0$

$$\langle K^*(p';\varepsilon)|i\bar{s}\sigma^{\mu\nu}b|B(p)\rangle = \varepsilon^*_\alpha \epsilon^{\alpha\mu\nu\beta}\left(p_\beta + p'_\beta\right) T^{B\to K^*}(q^2=0)$$
$$\text{with} \quad q = p - p'$$

Their result:

$$T^{B\to K^*}(q^2=0;\mu=m_b) = 0.24 \pm 0.03(ext.)^{+0.04}_{-0.01}(sys.)$$

$\Rightarrow$ $>10\%$ error from quark mass extrapolation.

Effective theories for heavy quarks

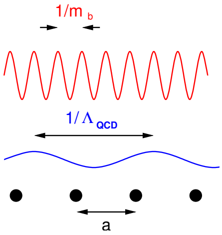
Alternative solution to $am_b > 1$:

$\Downarrow$

Integrate out high energy modes

$\Downarrow$

Effective Lagrangian
expansion in $1/m_b$



$$\mathcal{L}_{\text{eff}} = Q^\dagger iD_0 Q - Q^\dagger \frac{\mathbf{D}^2}{2m_b} Q - C(\mu) g\, Q^\dagger \frac{\sigma^{\mu\nu} G_{\mu\nu}}{2m_b} Q + \mathcal{O}(1/m_b^2)$$

$Q(x) \sim \int d^4p\; Q(p) e^{ipx}$ with $p \sim \Lambda_{\text{QCD}}$

$\Rightarrow$ expect discretisation errors $\sim a\Lambda_{\text{QCD}} \ll 1 < am_b$

NRQCD on the lattice

Lattice action *[Lepage et al. (1992)]*

$$S_{\mathrm{eff}}^{(\mathrm{lat})} = \sum_{\mathbf{x},t} \Big[Q_t^\dagger Q_t - Q_t^\dagger \left(1 - \frac{a\delta H}{2}\right)_t \left(1 - \frac{\tilde{H}_0}{2n}\right)_t^n \times (U_4^\dagger)_{t-a} \left(1 - \frac{a\tilde{H}_0}{2n}\right)_{t-a}^n \left(1 - \frac{a\delta H}{2}\right)_{t-a} Q_{t-a} \Big]$$

$$H_0 = -\frac{\Delta^{(2)}}{2m_b}, \qquad \tilde{H}_0 = H_0 - \frac{a}{4n}H_0^2, \qquad \delta H = -C(a)g\frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_b}$$

- Stability parameter $n = 1, 2, \ldots$
- Lattice derivative $\Delta^{(2)}$, chromomagnetic field $\mathbf{B}_k = -\frac{1}{2}\epsilon_{ijk}G_{ij}$
- Can be systematically improved to include $\mathcal{O}(1/m_b^2, a^2, \ldots)$

Current matching I

Hamiltonian for $b \to s$ transition:

- Full theory in the continuum

$$\mathcal{H}_{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_n C_n(\mu) \mathcal{Q}_n$$

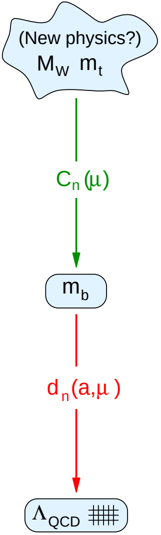
Wilson coefficients $C_n(\mu)$ known at NLO

- Effective Lattice theory

$$\mathcal{H}_{b\to s}^{(\mathrm{lat})} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_n d_n(a,\mu) C_n(\mu) \mathcal{Q}_n^{(\mathrm{lat})}$$

Matching coefficients $d_n(a,\mu)$ determined by

$$\langle s\gamma|\mathcal{H}_{b\to s}|b\rangle \stackrel{!}{=} \langle s\gamma|\mathcal{H}_{b\to s}^{(\mathrm{lat})}|b\rangle_{\mathrm{lat}}$$



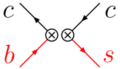
Current matching II - full theory

$b \to s$ **current in full theory**, e.g. *[Greub, Hurth, Wyler (1996)]*

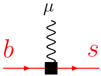
$$\mathcal{H}_{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \Big(C_2(\mu)\mathcal{Q}_2 + C_7(\mu)\mathcal{Q}_7 + C_8(\mu)\mathcal{Q}_8 + \dots \Big)$$

Operators:

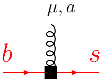
$$\mathcal{Q}_2 = (\overline{c}_L \gamma^\mu b_L)(\overline{s}_L \gamma_\mu c_L)$$



$$\mathcal{Q}_7 = \frac{e}{16\pi^2} m_b (\overline{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$



$$\mathcal{Q}_8 = \frac{g}{16\pi^2} m_b (\overline{s}_L T^a \sigma^{\mu\nu} b_R) G^a_{\mu\nu}$$



Current matching III - effective theory

$b \to s$ **current in the effective lattice theory** (at LO in $1/m_b$)

$$\mathcal{H}^{(\mathrm{lat})}_{b\to s} = -\frac{4G_F}{\sqrt{2}} V^*_{ts} V_{tb} \Big(d_2 C_2(\mu) \mathcal{Q}^{(\mathrm{lat})}_2 + d_7 C_7(\mu) \mathcal{Q}^{(\mathrm{lat})}_7 + d_8 C_8(\mu) \mathcal{Q}^{(\mathrm{lat})}_8 + \mathcal{O}(a, 1/m_b) \Big)$$

with lattice operators

$$\mathcal{Q}^{(\mathrm{lat})}_{2,7,8} = \mathcal{Q}_{2,7,8}\Big|_{b\to Q}$$

Q: heavy quark field on the lattice

Perturbative matching

Expansion in $\alpha_s = \alpha_s(m_b) \ll 1$

$$d_n = d_n^{(0)} + \alpha_s d_n^{(1)} + \dots$$

- **Tree level**: $d_7^{(0)} = 1,\ d_{2,8}^{(0)} = 0$
- $\mathcal{O}(\alpha_s)$: **Mixing matrices**

$$\langle s\gamma|\mathcal{Q}_i|b\rangle = \left(\delta_{ij} + \alpha_s Z_{ij}^{(1)} + \dots\right) \langle s\gamma|\mathcal{Q}_j|b\rangle_{\text{tree}}$$

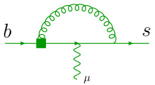
Matching coefficient

$$C_7(\mu) d_7^{(1)} = C_2(\mu)\delta Z_{27}^{(1)} + C_7(\mu)\delta Z_{77}^{(1)} + C_8(\mu)\delta Z_{87}^{(1)}$$

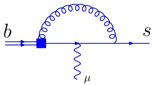
$$\delta Z_{ij}(\mu, a) = Z_{ij}^{(1)}\Big|_{\text{full},\overline{MS},\mu} - Z_{ij}^{(1)}\Big|_{\text{lat},a}$$

Mixing matrix evaluation

$\delta Z_{87}^{(1)}(\mu, a) =$



$\Big|_{\text{full},\overline{MS},\mu} -$



$\Big|_{\text{lat},a} + \dots$

- Full theory: Calculation of $Z_{i7}^{(1)}, i = 2, 7, 8$ completed *[Greub, Hurth, Wyler (1996)]*.
  UV & IR regularised in dim. Reg.
- Lattice calculation:
  UV regularised by lattice, IR by gluon mass λ

! **IR regulator has to be the same**
$\Rightarrow$ Repeat continuum calculation with gluon mass

Lattice integrals

Feynman rules on the lattice very complicated

$\Rightarrow$ **Numerical integration**:
Adaptive Monte-Carlo algorithm VEGAS *[Lepage (1978)]*

Integrand peaks in some regions of phase space
$\Rightarrow$ separate IR - divergence first:

$$\int \frac{d^4k}{(2\pi)^4} I^{(\mathrm{lat})} = \underbrace{\int \frac{d^4k}{(2\pi)^4} I^{(\mathrm{sub})}}_{\substack{\text{analytical} \\ \text{IR divergent}}} + \underbrace{\int \frac{d^4k}{(2\pi)^4} \left(I^{(\mathrm{lat})} - I^{(\mathrm{sub})} \right)}_{\substack{\text{numerical} \\ \text{IR finite}}}$$

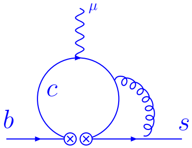
with suitable subtraction function

$$I^{(\mathrm{sub})}(k) \approx I^{(\mathrm{lat})}(k) \qquad \text{for} \quad k \to 0$$

To do I

Next steps

1. Repeat calculation in full theory with finite gluon mass λ (Includes 2-loop diagrams at $\mathcal{O}(\alpha_s)$)
2. Subtract lattice calculation
   - Improved gluonic action
   - ASQTAD (HISQ?) light quarks



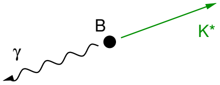
Done:

- matching calculation for the (partially conserved!) axial-vector current $\overline{b}\gamma_\mu\gamma_5 u$ in the static limit
- compared to results in *[Dalgic, Shigemitsu, Wingate (2004)]*

To do II

Further ahead

1. Include $1/m_b$ corrections, improve to $\mathcal{O}(a)$
2. Momentum of K^* as large as $\approx m_b/2$, discretisation errors for light quarks in final state significant



$\Rightarrow$ Work in moving frame $\rightarrow$ mNRQCD
[Dougall, Foley, Davies, Lepage (2005,2006)],
see poster by Stefan Meinel at this school.

3. Finally: Compute matrix elements using $\mathcal{H}^{(\mathrm{lat})}_{b\rightarrow s}$

Conclusion

- **Heavy quarks on the lattice**:
  Effective theory to avoid high energy modes at m_b which cannot be represented on the lattice.
- **Matching coefficients for heavy-light currents**:
  Perturbative matching.
  - Outlined calculation for $b \longrightarrow s$ current.
  - Numerical evaluation of lattice diagrams using VEGAS
  - IR divergences have to be understood
- **Systematic improvement** possible
  $\mathcal{O}(1/m_b)$ corrections, mNRQCD

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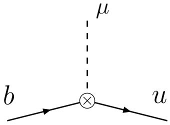
Travel grant from The University of Edinburgh Campaign
Enlightenment in the 21st century

Matching of axial $b \to u$ current

Continuum calculation:

Heavy quark momentum: $p = m_b v + p_{res}$



$$J_\mu^{(1)} = \overline{u}\gamma_5\gamma_\mu b \qquad J_\mu^{(2)} = \overline{u}\gamma_5 v_\mu b$$

Matrix elements

$$\begin{aligned}\langle u(p')|J_\mu^{(1)}|b(p)\rangle &= \left[1 + \frac{\alpha_s}{3\pi}\left(\frac{3}{2}\log m_b^2/\lambda^2 - \frac{11}{4}\right)\right]\langle u|J_\mu^{(1)}|b\rangle_{\text{tree}} \\ &\quad + \frac{2\alpha_s}{3\pi}\langle u|J_\mu^{(2)}|b\rangle_{\text{tree}}\end{aligned}$$

- **NO scale dependence** as current is partially conserved.
- **Logarithmic IR divergence**, gluon mass λ

Lattice calculation

Matrix elements on the lattice

$$\langle u|J_0^{(1,2)}|b\rangle_{\text{lat}} = \left[1+\alpha_s\left(\xi_{1PI}^{(\text{lat})}+\frac{1}{2}(R_q^{(\text{lat})}+R_h^{(\text{lat})})\right)\right]\langle u|J_0^{(1,2)}|b\rangle_{\text{tree}}$$

(Split off UV/IR divergent terms)

$$= \left[1+\alpha_s\left(\overline{\xi}_{1PI}^{(\text{reg})}+\frac{1}{2}(\overline{R}_q^{(\text{reg})}+\overline{R}_h^{(\text{reg})})\right.\right.$$
$$\left.\left.-\frac{1}{2\pi}\left(\log a^2+\gamma_E-\log 2\right)-\frac{1}{2\pi}\log\lambda^2\right)\right]\langle u|J_0^{(1,2)}|b\rangle_{\text{tree}}$$

- $\overline{\xi}_{1PI}^{(\text{reg})}$, $\overline{R}_q^{(\text{reg})}$ and $\overline{R}_h^{(\text{reg})}$: IR and UV finite
- **IR divergence:** same as in full theory → cancel!
- **UV divergence:** → $\log a^2 m_b^2$ terms in matching coefficients

Matching coefficient

Subtract to get one loop matching coefficient of $\overline{u}\gamma_5\gamma_\mu b$:

$$d^{(1)} = \frac{1}{2\pi}\log a^2 m_b^2 + \frac{1}{2\pi}\left(-\frac{11}{6} + \gamma_E - \log 2\right) - \left(\overline{\xi}_{1PI}^{(\text{reg})} + \frac{1}{2}(\overline{R}_q^{(\text{reg})} + \overline{R}_h^{(\text{reg})})\right)$$

Numerical results

	My data	*[Dalgic et al. ('04)]* [1]	*[Morningstar ('93)]* [1,2]
$\overline{R}_h^{(\text{reg})}$	0.3933(5)	0.31(2)	0.404(2)
$\overline{R}_q^{(\text{reg})}$	—	−0.924(3)	—
$\overline{\xi}_{1PI}^{(\text{reg})}$	0.4797(2)	0.49(1)	—

[1] Extrapolated to $m_b \to \infty$. [2] Gluon action different at $\mathcal{O}(a^4)$

Zero point energy

$aE_0^{(lat)}$ for finite m_b and $m_b \to \infty$