Meson spectroscopy with derivative quark sources

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Introduction

Lattice QCD has the possibility of providing a model independent \textit{ab initio} calculation of the QCD mass spectrum. Extracting signals from excited states has however proven to be a formidable task. A previous study of the BGR-Collaboration \cite{1} employed the variational technique \cite{2, 3} to study excited mesons on the lattice using standard meson interpolators with Gaussian-smearred sources and sinks. For an accurate description of excited states it is crucial to use a basis with good overlap with those states. We therefore construct additional meson-interpolators with derivative sources obtained by a covariant derivative acting on the Jacobi smeared sources and explore their effect in the variational approach. For our analysis we used 90 (100 for pseudovectors) quenched gauge configurations with those states. We therefore construct additional meson-interpolators with derivative sources by taking the mass difference between the state \(i\) and the closest lying state. For an accurate description, the interpolators should be linearly independent, as orthogonal as possible and possess a strong overlap with physical states.

Smeared sources and sinks

The first step for all sources is Jacobi smearing \cite{4, 5} of point sources \(s_j\):

\[
\sigma_j^{(n)}(\vec{x}, 0) = \delta(\vec{x}, 0) s_j \delta_{\text{n/w}},
\]

\[
e^{\alpha n} = \sum_n H^n,
\]

\[
H(\vec{x}, \vec{y}) = \sum_{j=1}^N \left[ U_j(\vec{x}, 0) | \delta(\vec{x} + j \vec{y}) + U_j(\vec{x} - j \vec{y}) - \delta(\vec{x} - j \vec{y}) \right].
\]

In \cite{1} two different parameter values for \(n\) and \(\alpha\) giving rise to wide and narrow sources \((S_n, S_w)\), have been used. In addition, we now construct covariant derivatives which act upon a wide smeared source to form our derivative quark sources \(W'_j\):

\[
P'_j(\vec{x}, \vec{y}) = U_j(\vec{x}, 0) | \delta(\vec{x} + j \vec{y}) + U_j(\vec{x} - j \vec{y}) - \delta(\vec{x} - j \vec{y}) ,
\]

\[
W'_j = P'_j S'_w.
\]

With these sources, meson interpolators of definite quantum numbers are constructed.

Interpolators used

Table 1 shows the interpolators used for different meson channels. In some cases, an (anti-)symmetrization of the interpolators is necessary to obtain the correct behavior under charge conjugation. Therefore, interpolators of the type \(u \Gamma_{d_j} \bar{d}\) should be read as \(u \Gamma_{d_j} \bar{d}\). We restrict ourselves to light, isovector \((I = 1)\) mesons with degenerate quark masses \(m_u = m_d\).

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>old interpolators</th>
<th>new interpolators</th>
<th>( # )</th>
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<tbody>
<tr>
<td>pseudoscalar</td>
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<td>10</td>
</tr>
<tr>
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<td>(u \gamma_\mu \gamma_5 \bar{d})</td>
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<tr>
<td>vector</td>
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Table 1: Meson interpolators; \(n/w\) denote narrow and wide Jacobi smearing and \(d_j\) stands for a derivative source in the \(j\)th direction. The last column shows the total number of different interpolators.

Results

To get an idea about the properties of the correlators, we first plot the normalized diagonal entries of the correlation matrices for the pseudoscalar and vector mesons. In Fig. 1 the simple interpolators have been colored blue, while the new correlators from derivative sources appear in red. As can be seen from the behavior at small times, the new correlators have stronger contributions from excited states.

Fig. 1: Diagonal correlators for the pseudoscalar \(\text{(left)}\) and vector \(\text{(middle)}\) mesons. For the pseudoscalar mesons, overlap with excited states has however proven to be a formidable task. A previous study of the BGR-Collaboration \cite{1} employed the variational technique \cite{2, 3} to study excited mesons on the lattice using standard meson interpolators with Gaussian-smearred sources and sinks. For an accurate description of excited states it is crucial to use a basis with good overlap with those states. We therefore construct additional meson-interpolators with derivative sources obtained by a covariant derivative acting on the Jacobi smeared sources and explore their effect in the variational approach. For our analysis we used 90 (100 for pseudovectors) quenched gauge configurations with those states. We therefore construct additional meson-interpolators with derivative sources by taking the mass difference between the state \(i\) and the closest lying state. For an accurate description, the interpolators should be linearly independent, as orthogonal as possible and possess a strong overlap with physical states.

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Conclusions and upcoming investigations

It has been demonstrated that interpolators constructed with derivative quark sources lead to an enhanced signal for a variety of different meson channels. For pseudoscalar and vector mesons, the correlators constructed from such interpolators display a significantly better overlap with excited states. For the scalar and pseudovector mesons, overlap with the ground states can be improved. We are currently extending our calculations to dynamical configurations obtained in BGRs dynamical CI project. Furthermore, we investigate the construction of baryon interpolators from derivative quark sources.

References