Lattice QCD with Two Degenerate Dynamical Light Wilson Quarks

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Motivation & Basic Idea

Long term aim:

To probe the chiral regime of lattice QCD with **dynamical** Wilson & Wilson-type fermions

Why Wilson fermions?

Conceptual simplicity, Hadronic operators simply constructed.

Long standing problem:

Because of lack of chiral symmetry, no protection against accidental zero modes.

Recent work (Luescher 2006): Numerical simulations safe from *exceptional configurations* at reasonably large volume

Need to avoid scaling violations & have small enough quark masses

Recent algorithmic developments in Wilson fermion simulations along with increased computational power may enable one to probe the chiral regime

However, presence of sea quarks makes the scale determination non-trivial.

Very accurate data required to make progress

- **Simulation Details:**
  - Standard Wilson gauge and fermion actions (both *unimproved*)
  - $\beta = 5.6$, $16^3 32$, $N_F = 2$ degenerate quarks
  - $\kappa =$ 0.156, 0.1565, 0.15675, 0.157, 0.15725, 0.1575
  - HMC used to generate configurations. Measurements done on 200 trajecs at each values of κ
  - Code based on the MILC structure
- **Gaussian Smearing** of hadronic operators investigated in detail:
  - Both source & sink smearing
  - Optimum smearing parameter found for each operator at each κ
- For accurate determination of the scale, APE smearing of the gauge configurations used & optimum smearing level at each κ used (Optimum smearing levels grow as κ increases)

Data presented still preliminary: Not checked, all error bars not calculated

ON-GOING WORK ...

Pion Propagators

- For π, we measure the following correlation functions of different combinations of the pseudoscalar density ($P = \overline{q}\gamma_5 q$) and the 4^{th} component of the axial vector current ($A_4 = \overline{q}\gamma_4\gamma_5 q$) on lattice:

$$
\begin{aligned}
C_1(t) &= \langle 0 \mid \mathcal{O}^\dagger(t)\mathcal{O}(0) \mid 0\rangle \stackrel{t\to\infty}{\longrightarrow} C^{\mathcal{O}\mathcal{O}}\left[e^{-m_\pi t} + e^{-m_\pi(T-t)}\right] \\
C_2(t) &= \langle 0 \mid \mathcal{O}_1^\dagger(t)\mathcal{O}_2(0) \mid 0\rangle \stackrel{t\to\infty}{\longrightarrow} C^{\mathcal{O}_1\mathcal{O}_2}\left[e^{-m_\pi t} - e^{-m_\pi(T-t)}\right]
\end{aligned}
$$

where, $\mathcal{O}\mathcal{O} \equiv PP$ or AA and $\mathcal{O}_1\mathcal{O}_2 = AP$ or PA

- The coeffi cients are given by,

$$
\begin{aligned}
C^{\mathcal{O}\mathcal{O}} &= \frac{1}{2m_\pi} \mid \langle 0 \mid \mathcal{O}(0) \mid \pi\rangle \mid^2 \\
C^{\mathcal{O}_1\mathcal{O}_2} &= \frac{1}{2m_\pi}\langle 0 \mid \mathcal{O}_1^\dagger(0) \mid \pi\rangle\langle\pi \mid \mathcal{O}_2(0) \mid 0\rangle \ .
\end{aligned}
$$

Pion Decay Constant and PCAC Quark Mass

The pion decay constant F_π and PCAC quark mass m_q are defined via
$\langle 0 \mid A_\mu(0) \mid \pi(q)\rangle = \sqrt{2} F_\pi q_\mu, \quad \partial_\mu A_\mu(x) = 2 m_q P(x)$

- The pion decay constant is calculated using

$$F_\pi \;=\; 2\kappa \sqrt{\frac{C^{AA}}{m_\pi}}, \quad \frac{2\kappa\, C^{AP}}{\sqrt{2 m_\pi C^{PP}}}, \quad \text{or} \quad \frac{2\kappa\, C^{PA}}{\sqrt{2 m_\pi C^{PP}}}$$

- Quark mass defined through PCAC can be calculated from

$$m_q \;=\; \frac{m_\pi}{2} \sqrt{\frac{C^{AA}}{C^{PP}}}, \quad \frac{m_\pi}{2} \frac{C^{AP}}{C^{PP}}, \quad \text{or} \quad \frac{m_\pi}{2} \frac{C^{PA}}{C^{PP}}$$

Rho Mass & Decay Constant

- Two different definitions of ρ deacy constant
  - $\langle O \mid V_\mu(0) \mid \rho \rangle \;=\; \epsilon_\mu \, \frac{m_\rho^2}{f_\rho}$
  - $\langle O \mid V_\mu(0) \mid \rho \rangle \;=\; \epsilon_\mu F_\rho m_\rho$

  where $V_\mu(0) = \overline{q}(0)\gamma_\mu q(0)$ and ϵ_μ is the polarization vector of rho.

  f_ρ is dimensionless and $F_\rho/m_\rho = (f_\rho)^{-1}$

- ρ decay constants and mass calculated from the correlation function

$$
\begin{aligned}
C(t) \quad &= \quad \langle O \mid V_k^\dagger(t) V_k(0) \mid O \rangle \\
&\xrightarrow{t\to\infty} \quad C^{VV}\left[e^{-m_\rho t} \;+\; e^{-m_\rho (T-t)}\right]
\end{aligned}
$$

where, $C^{VV} = \frac{1}{2m_\rho} \mid \langle O \mid V_k(0) \mid \rho \rangle \mid^2$

$$
\Rightarrow 1/f_\rho \;=\; 2\kappa \, \sqrt{\frac{2C^{VV}}{m_\rho^3}} \; .
$$

Smearing of Operators

- For smearing of the π and ρ operators, we have used Gaussian shell model trial wave function with one variational parameter, $\phi(r) \sim \exp\left(-\left(r/r_0\right)^2\right)$
- The correlation function involving local operators $\mathcal{A}$ and $\mathcal{B}$

$$\langle O \mid \mathcal{A}_l^{\mathrm{sink}\,\dagger}(t)\, \mathcal{B}_l^{\mathrm{source}}(0) \mid O\rangle \quad \xrightarrow{t\to\infty} \quad C_{ll}^{\mathcal{AB}}\, e^{-m_H t}$$

where, $C_{ll}^{\mathcal{AB}} = \frac{1}{2m_H}\langle O \mid \mathcal{A}_l^{\mathrm{sink}\,\dagger}(0) \mid H\rangle\langle H \mid \mathcal{B}_l^{\mathrm{source}}(0) \mid O\rangle$

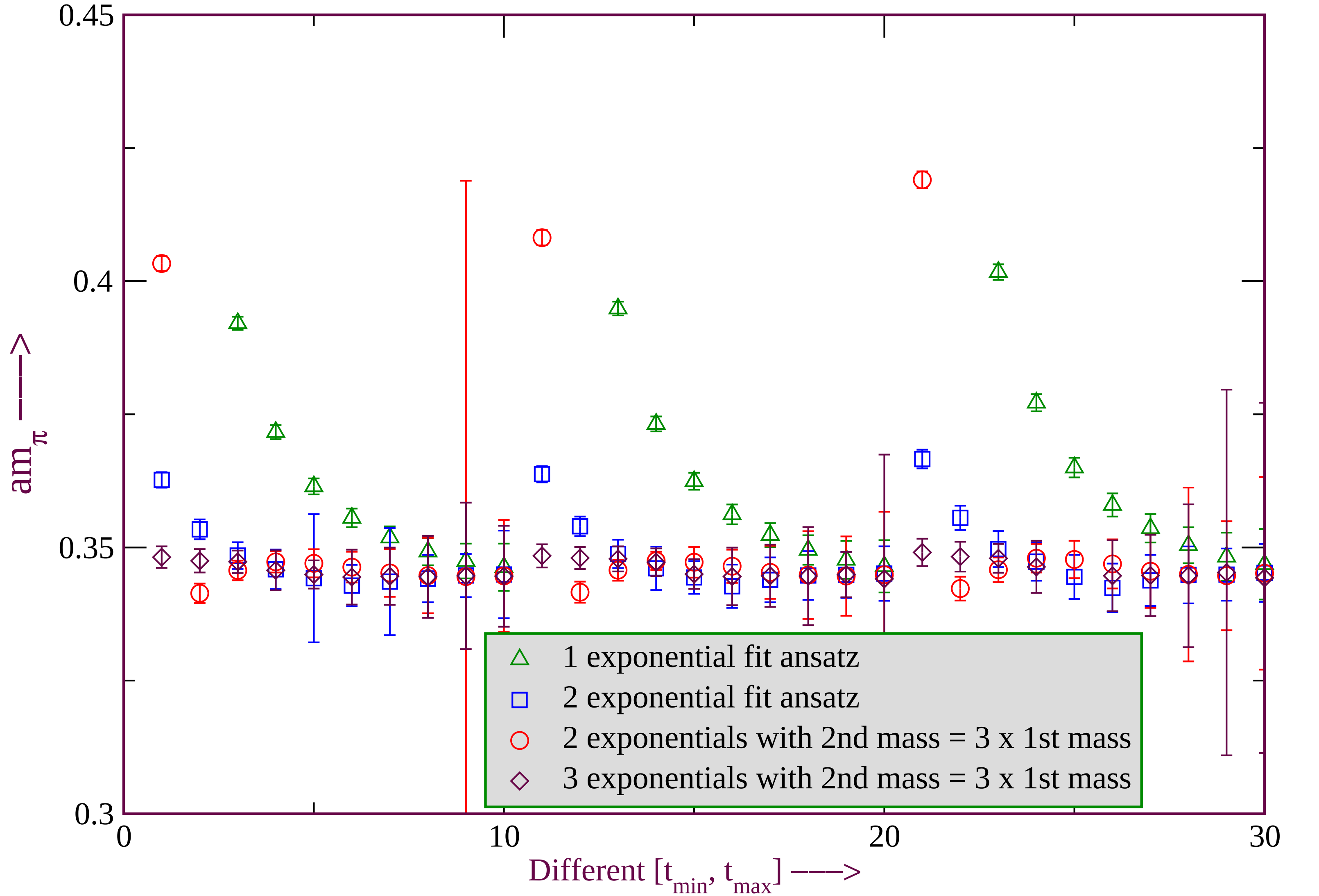
Similarly

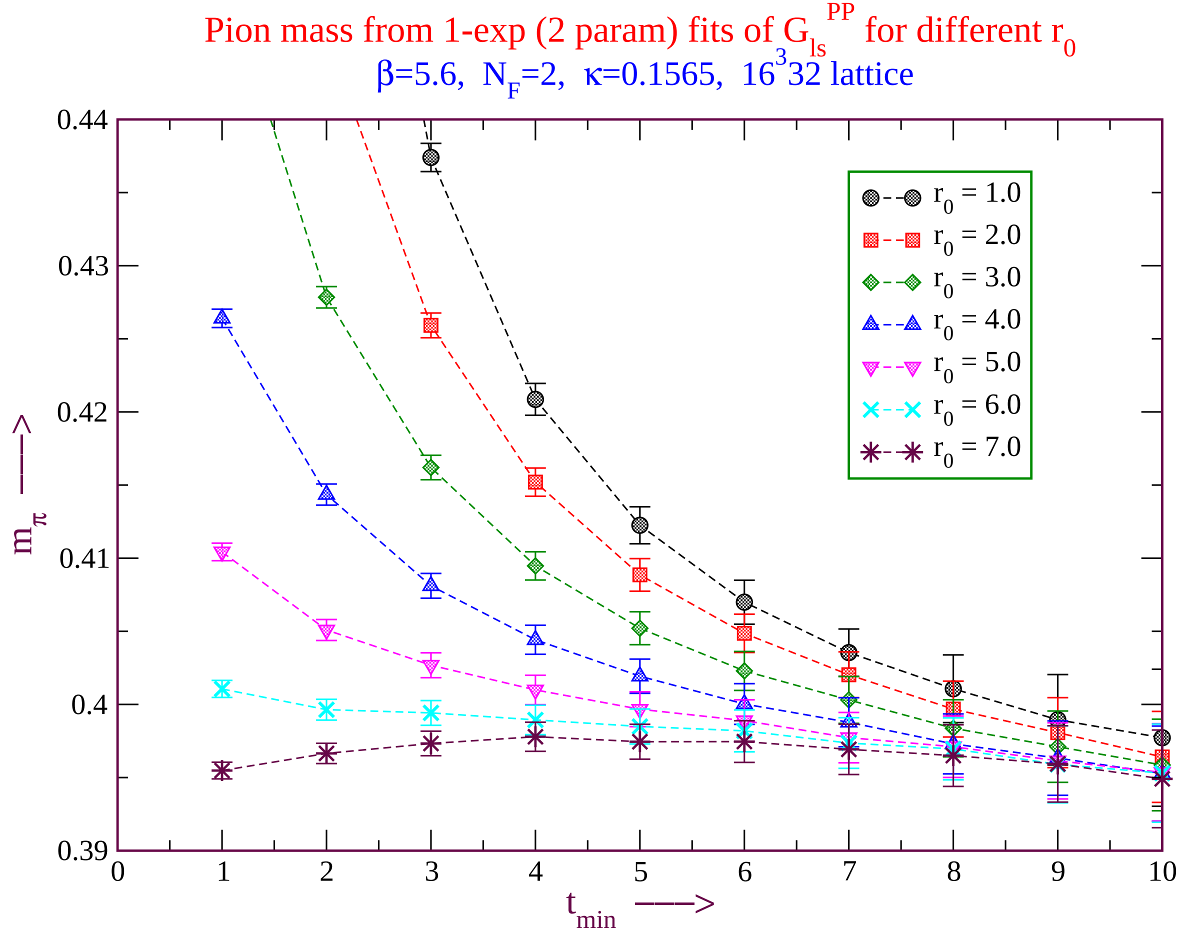
$$\begin{aligned}
C_{ls}^{\mathcal{AB}} &= \frac{1}{2m_H}\langle O \mid \mathcal{A}_l^{\mathrm{sink}\,\dagger}(0) \mid H\rangle\langle H \mid \mathcal{B}_s^{\mathrm{source}}(0) \mid O\rangle \\
C_{sl}^{\mathcal{AB}} &= \frac{1}{2m_H}\langle O \mid \mathcal{A}_s^{\mathrm{sink}\,\dagger}(0) \mid H\rangle\langle H \mid \mathcal{B}_l^{\mathrm{source}}(0) \mid O\rangle \\
C_{ss}^{\mathcal{AB}} &= \frac{1}{2m_H}\langle O \mid \mathcal{A}_s^{\mathrm{sink}\,\dagger}(0) \mid H\rangle\langle H \mid \mathcal{B}_s^{\mathrm{source}}(0) \mid O\rangle\ .
\end{aligned}$$

$\Rightarrow C_{ll} = C_{ls} C_{sl} / C_{ss}\ .$

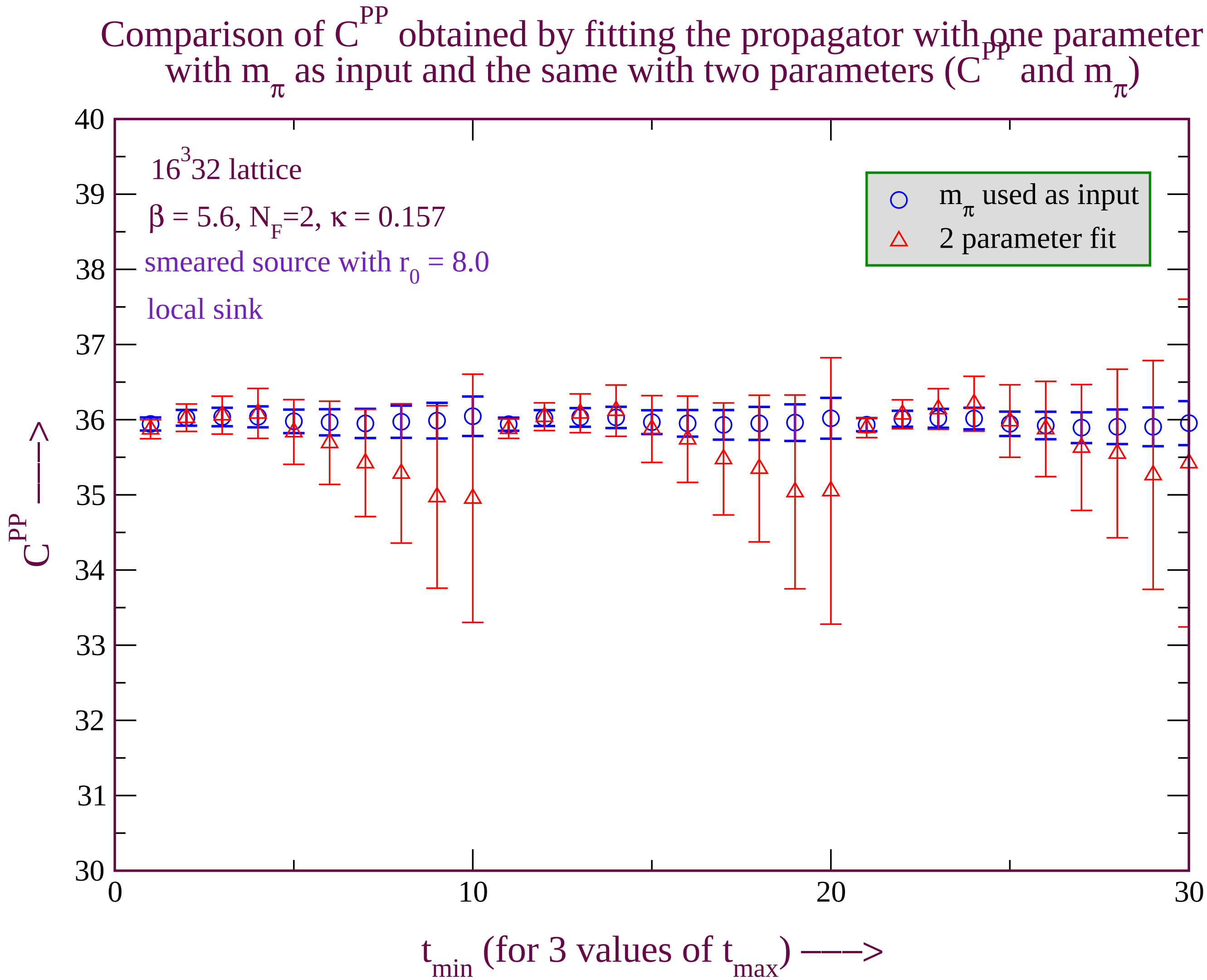
- Before smearing, gauge-fix to Coulomb gauge

Comparison among different values of am_π obtained by fitting the unsmeared propagator with different exponentials and different sets of $[t_{min}, t_{max}]$ at $\beta = 5.6$, $N_F=2$, $\kappa = 0.157$ for $16^3 32$ lattice

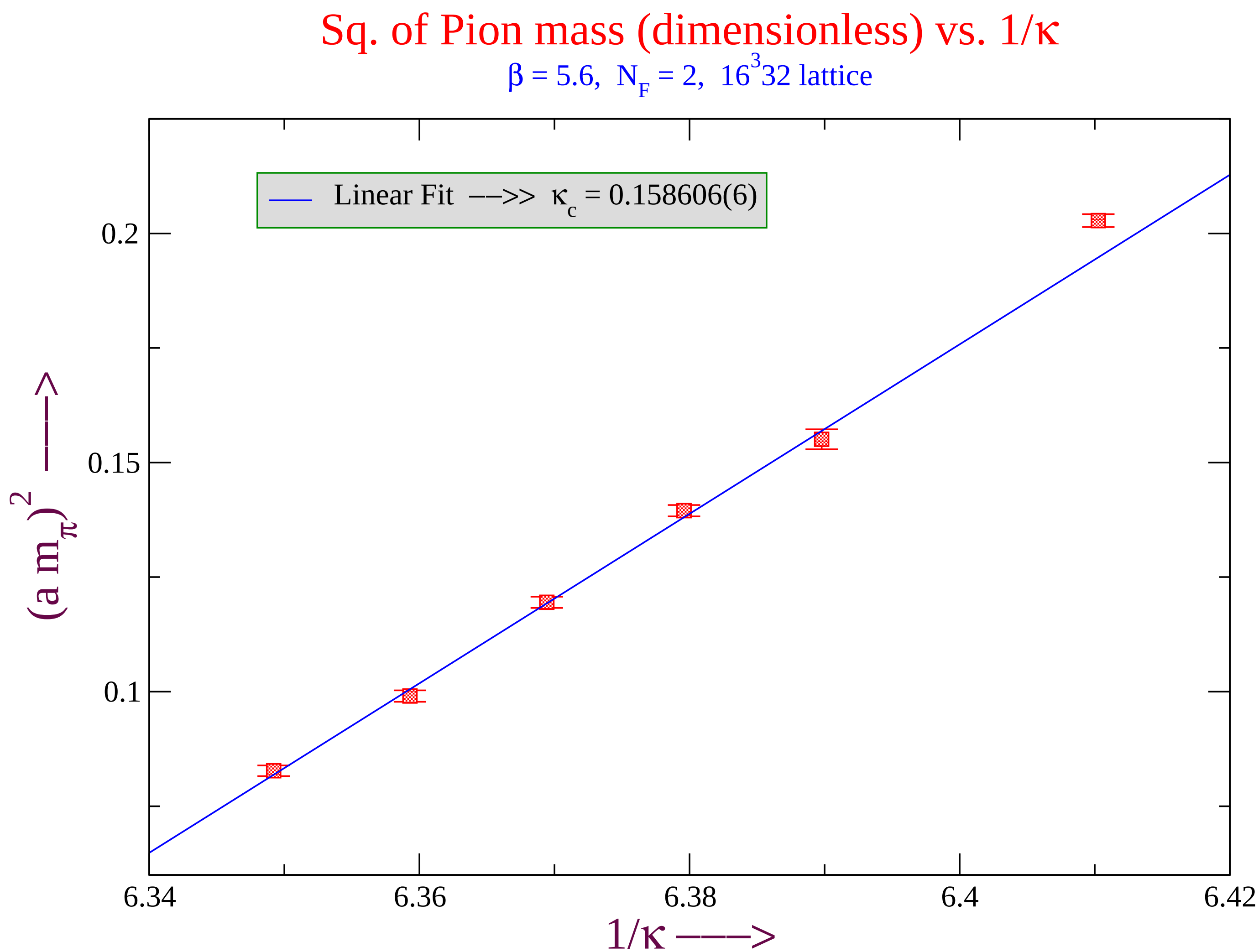




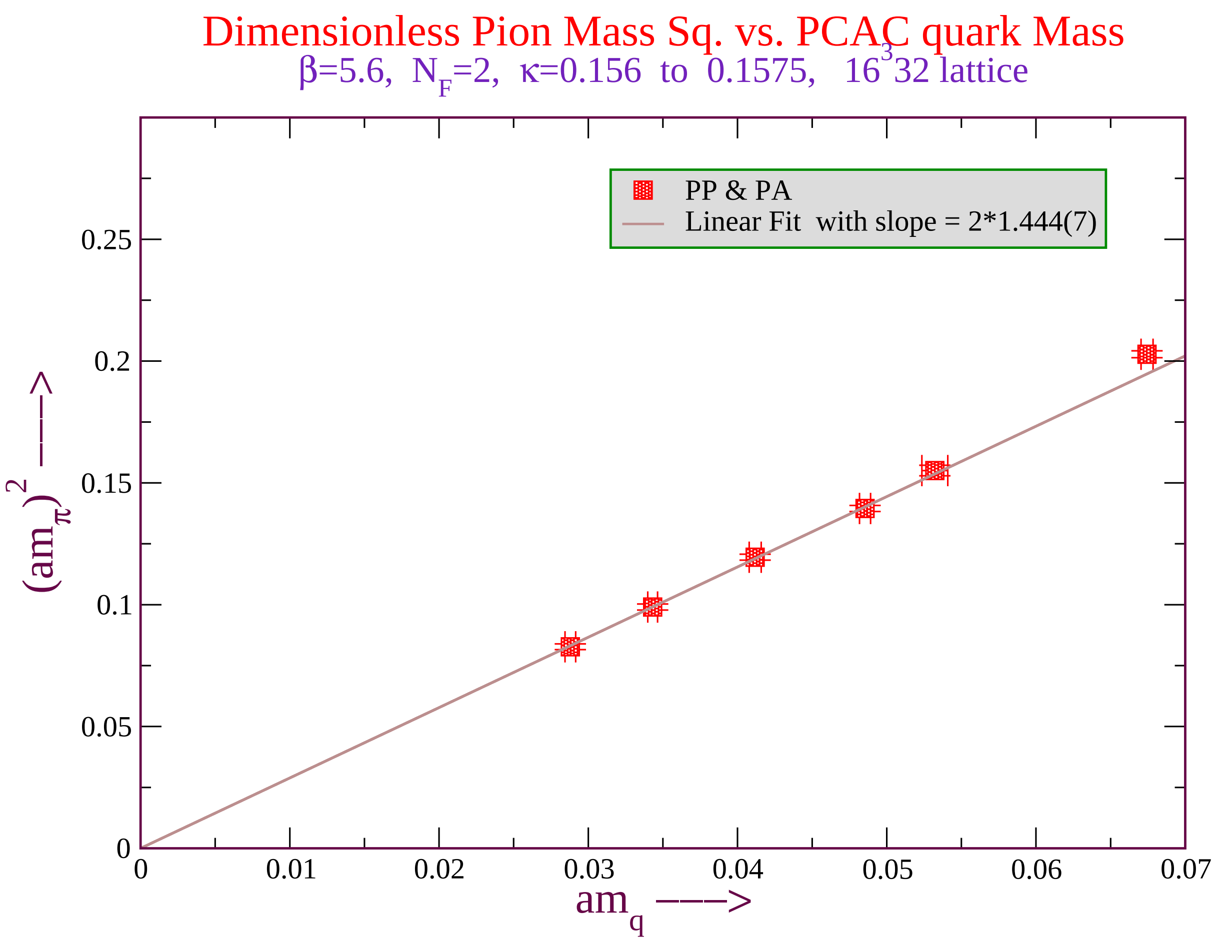
- For each κ, all props (PP, AA, AP, PP, VV etc) & all combination of smearing (ls, sl, ss) were computed with $r_0 = 1, 2, 3, ..., L/2 = 8.0$
- Best r_0 obtained for each type (PP, AA etc) from exponential fi tting. Best r_0 obtained was in general different for different types at a given κ.



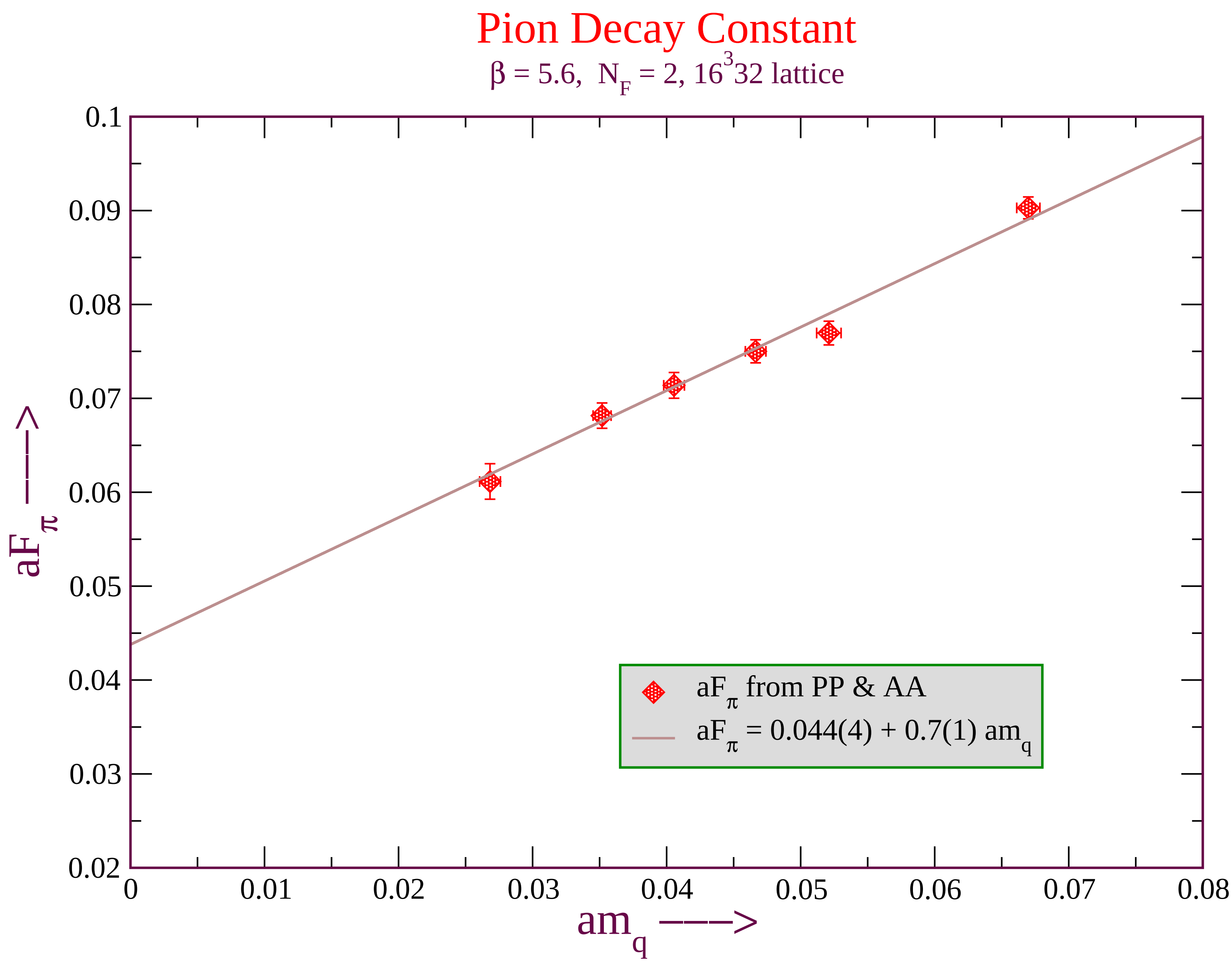
- After the choice of m_π or m_ρ at the optimum r_0 value for a given propagator and a given smearing combination, this value was used as input and all props refi tted with exponential ansatz to get the coeffs.

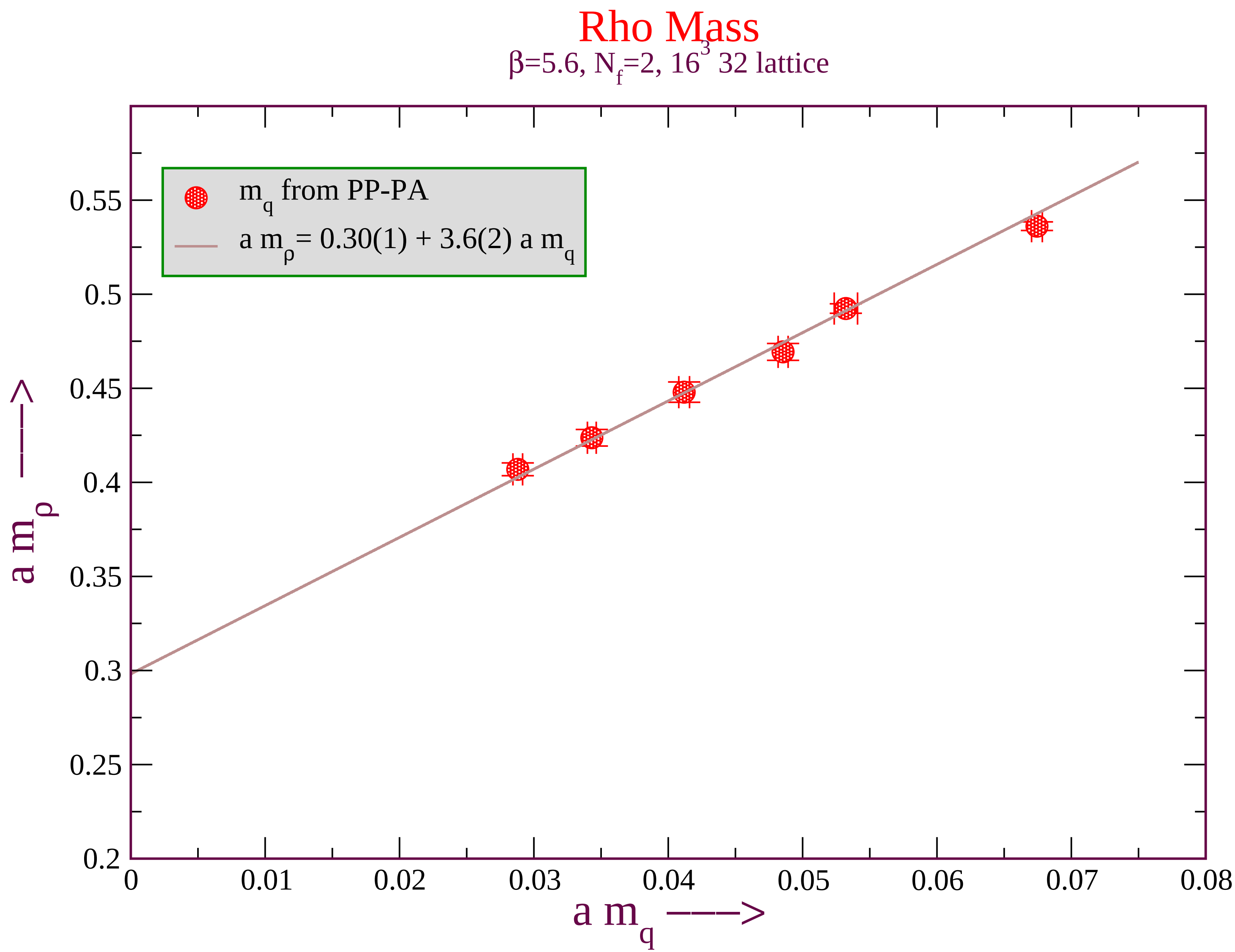


- At the bare or classical level $(1/\kappa - 1/\kappa_c)$ takes care of the additive $\mathcal{O}(a)$ effect, in the quantum theory it does an approximate job as shown above
- $\kappa = 0.156$ is left out of the fit

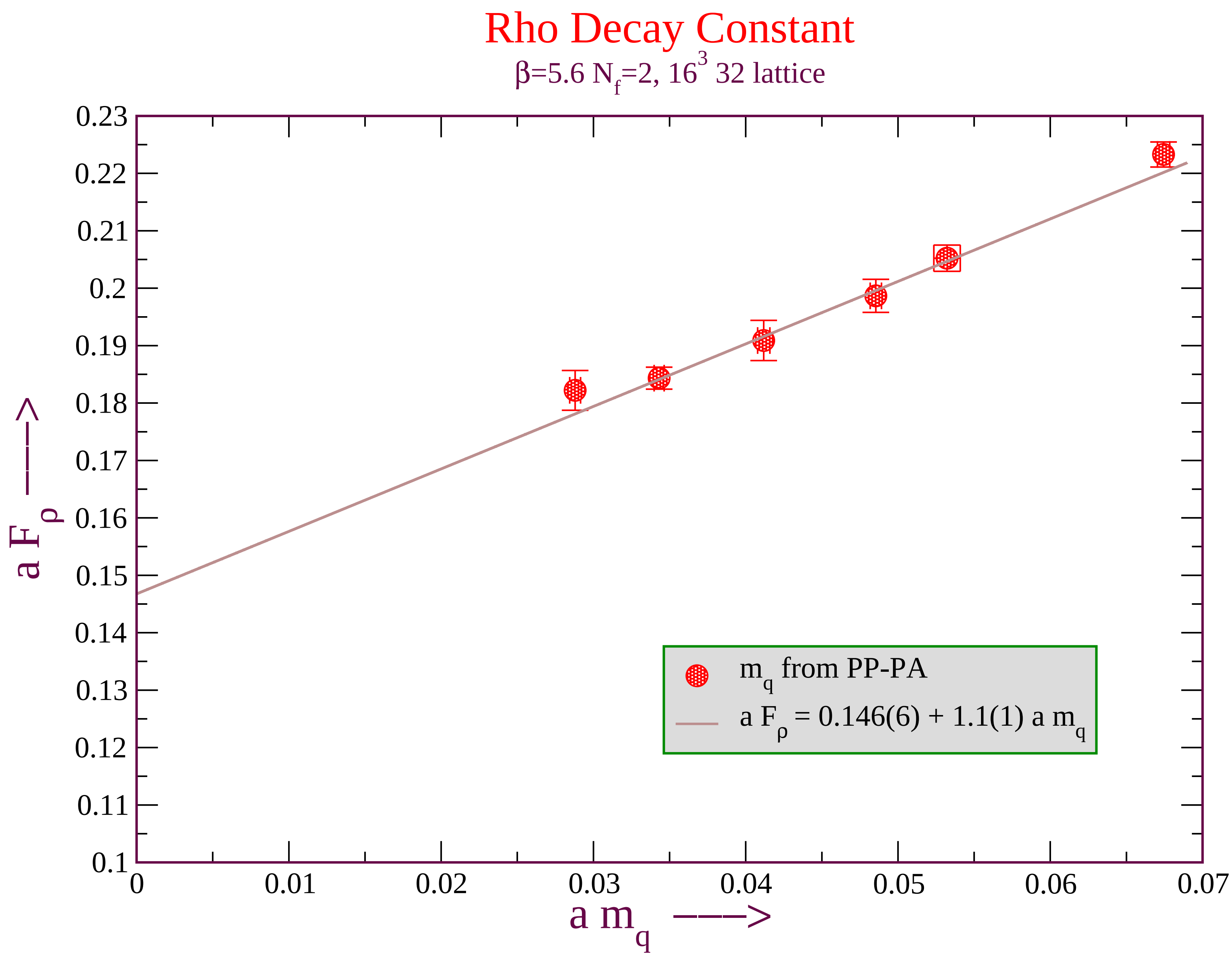


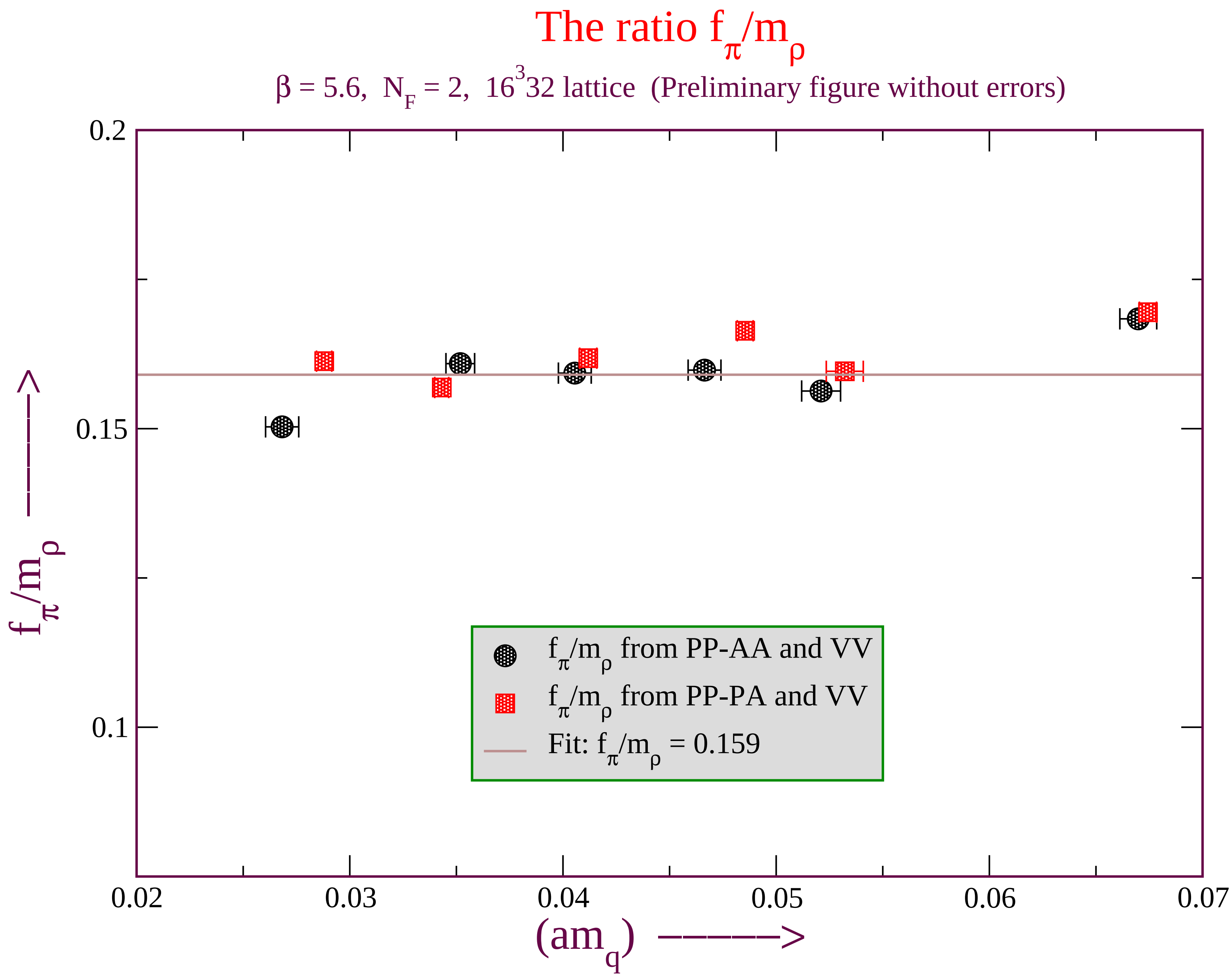
- Better χ^2 than the $m_\pi^2 \ - \ 1/\kappa$ fi t, because of the use of PCAC quark mass, almost a perfect fi t except for the heaviest quark mass


Pion Decay Constant
β = 5.6, N_F = 2, 16^3 32 lattice
aF_π ---->
am_q ---->
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
aF_π from PP & AA
aF_π = 0.044(4) + 0.7(1) am_q



Rho Mass
β=5.6, N_f=2, 16^3 32 lattice
m_q from PP-PA
a m_ρ = 0.30(1) + 3.6(2) a m_q
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
a m_ρ ——>
a m_q ——>


Rho Decay Constant
β=5.6 N_f=2, 16^3 32 lattice
0.23
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
a F_ρ ---->
a m_q ---->
m_q from PP-PA
a F_ρ = 0.146(6) + 1.1(1) a m_q


The ratio fπ/mρ
β = 5.6, NF = 2, 16^3 32 lattice (Preliminary figure without errors)
0.2
0.15
0.1
0.02
0.03
0.04
0.05
0.06
0.07
fπ/mρ ------->
(amq) ------->
fπ/mρ from PP-AA and VV
fπ/mρ from PP-PA and VV
Fit: fπ/mρ = 0.159

Calculation of lattice spacing

- Lattice spacing calculated by Sommer's parameter method using scale defined through force between heavy quark-antiquark pair (quarkonia)

$$R^2 \frac{\partial V(R)}{\partial R} |_{R=R_0} = 1.65$$

- Simplest *ansatz* for a confining potential is the Cornell potential

$$V(R) = V_0 + \sigma R - \frac{\alpha}{R}, \qquad \sigma : \text{string tension}$$

Cornell potential yields $R_0 \sim 0.49$ fermi $\Rightarrow a = 0.49 \times \sqrt{\frac{\sigma}{1.65-\alpha}}$ fermi.

- $SU(3)$ gauge links are smeared for this purpose. We have used APE smearing

$$\begin{aligned} U_i(x) \to U_i'(x) &= \alpha U_i(x) + \sum_{\text{staples}} \tilde{U}_i(x) \\ \text{where, } \tilde{U}_i(x) &= U_j(x+i) U_i^\dagger(x+j) U_j^\dagger(x) \end{aligned}$$

followed by projection back to $SU(3)$

Scale determination

- R_0/a changes with κ (effect of changing the sea quark mass)

κ	0.156	0.1565	0.15675	0.157	0.15725	0.1575
R_0/a	4.940(25)	5.120(27)	5.355(29)	5.426(30)	5.581(32)	5.711(13)

- In a mass-independent scheme, for a given β (independent of κ) the scale a is usually set by going to the chiral limit. R_0 is then interpreted to change with sea quark mass. We are yet to set the this scale.
- Then all the previous plots of am_π^2, aF_π, am_ρ, aF_ρ with am_q stay.
- Chiral extrapolation is perfectly linear in $(am_\pi)^2 - am_q^{LAT}$.
- Need to multiply by appropriate renormalization constants to get to $\overline{MS}(\mu)$, only changes slope

- If a fixed $R_0 = 0.49 fm$ is taken, the scale obviously varies with κ or sea quark mass

κ	$\sigma(GeV^2)$	$a(fm)$	$a^{-1}(GeV)$
0.156	0.215	$0.0992(5)$	$1.989(9)$
0.1565	0.216	$0.0957(5)$	$2.061(10)$
0.15675	0.213	$0.0915(5)$	$2.157(11)$
0.157	0.213	$0.0903(5)$	$2.186(13)$
0.15725	0.213	$0.0878(5)$	$2.248(12)$
0.1575	0.213	$0.0858(2)$	$2.301(5)$

If we accept the change of scale

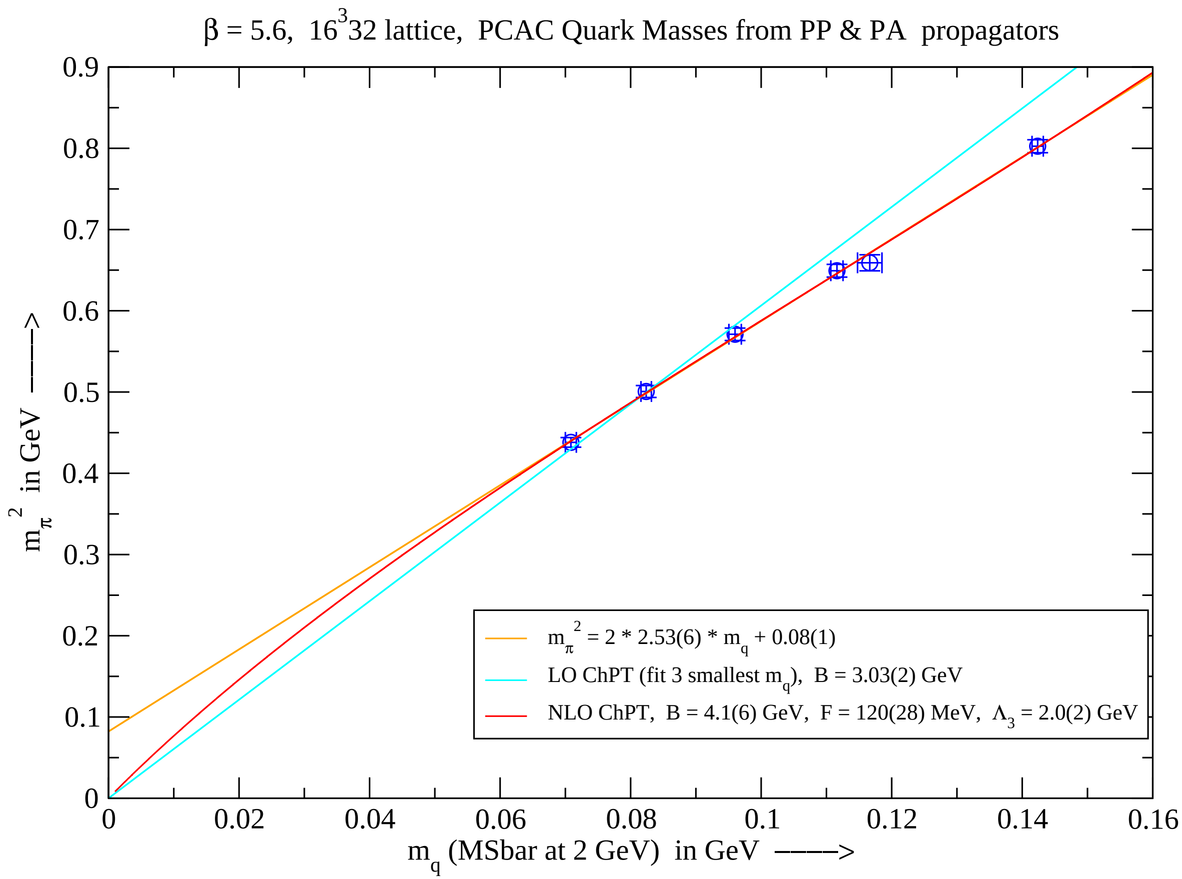
- Need dimensionful m_q's to renormalize to $\overline{MS}(\mu)$

$$m^{\overline{MS}}(\mu) = \frac{Z_A}{Z_P(a\mu)} m^{LAT}(a)$$

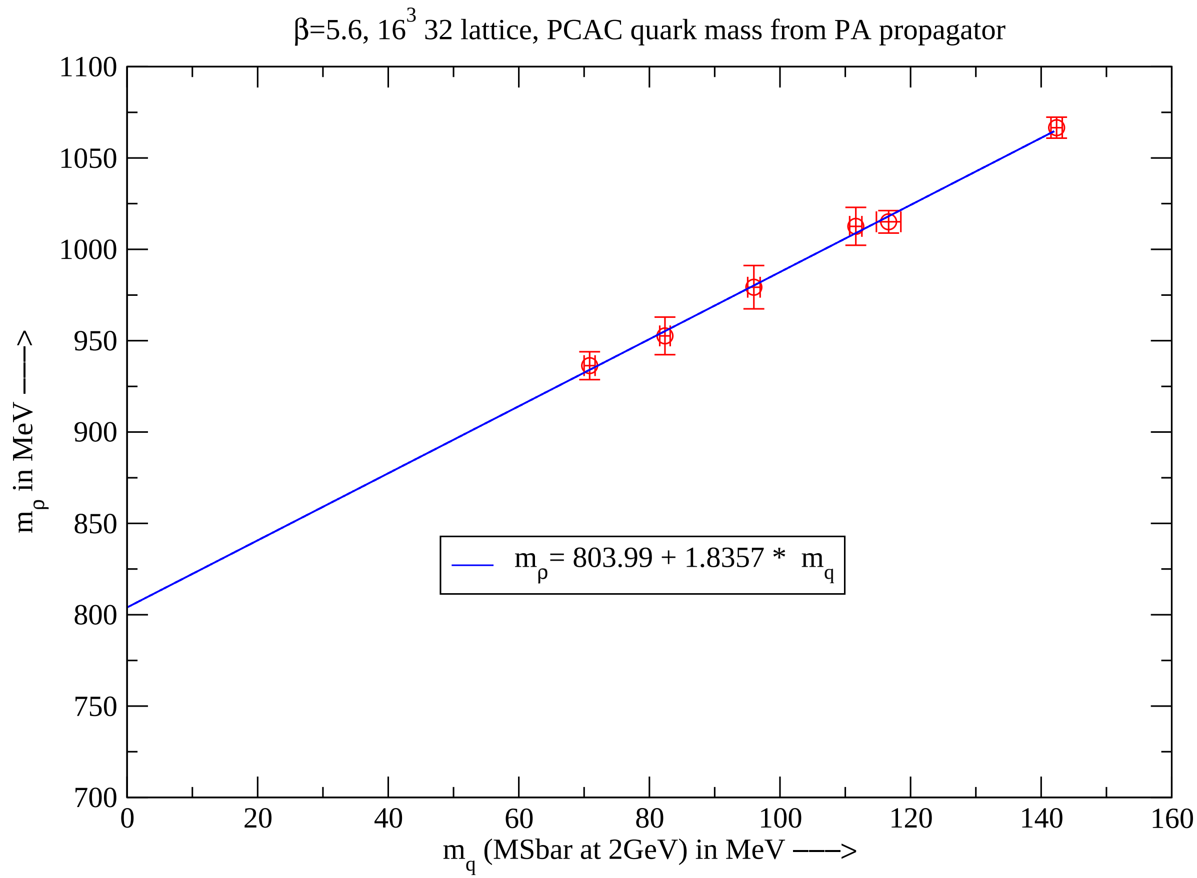
- Let's try the perturbative 1-loop formulas:

$$\begin{aligned} Z_A &= 1 - 15.797 \frac{g^2}{12\pi^2} \\ Z_P &= 1 - 22.596 \frac{g^2}{12\pi^2} + [\ln(a^2\mu^2)] \frac{g^2}{4\pi^2} \end{aligned}$$

- Use continuum $\chi \; P \; T$



- LO χPT barely fits the lowest 3 quark masses
- NLO χPT fits all our data almost perfectly including the heaviest quark mass at $\kappa = 0.156$, $\Rightarrow m_q^{\overline{MS}(2GeV)} \sim 3.0 MeV$ at the physical point
- A linear fit which does NOT go through origin also describes the data



- After all that, linearity still intact
- Leads to $m_\rho \sim 809 MeV$ at $m_q^{\overline{MS}(2GeV)} \sim 3 MeV$

Outlook

- Thorough investigation of *gaussian smearing* $\Rightarrow$ Quite accurate masses & coeffs
- Important to use m_q^{PCAC} for our analysis. Discretization errors in other quantities seem small for the lattice scales achieved ($\geq 2GeV$).
- Our $m_\pi L > 4.5$ and the physical volumes vary from $(1.59fm)^3$ to $(1.37fm)^3$. Hopefully the fi nite size effects are under control.
- Need smaller pion and quark masses $\Rightarrow$ Larger volumes. Plan in near future.
- Partially quenched runs complete. Need to analyse.