Z(3)-symmetric effective theory of hot QCD

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Introduction

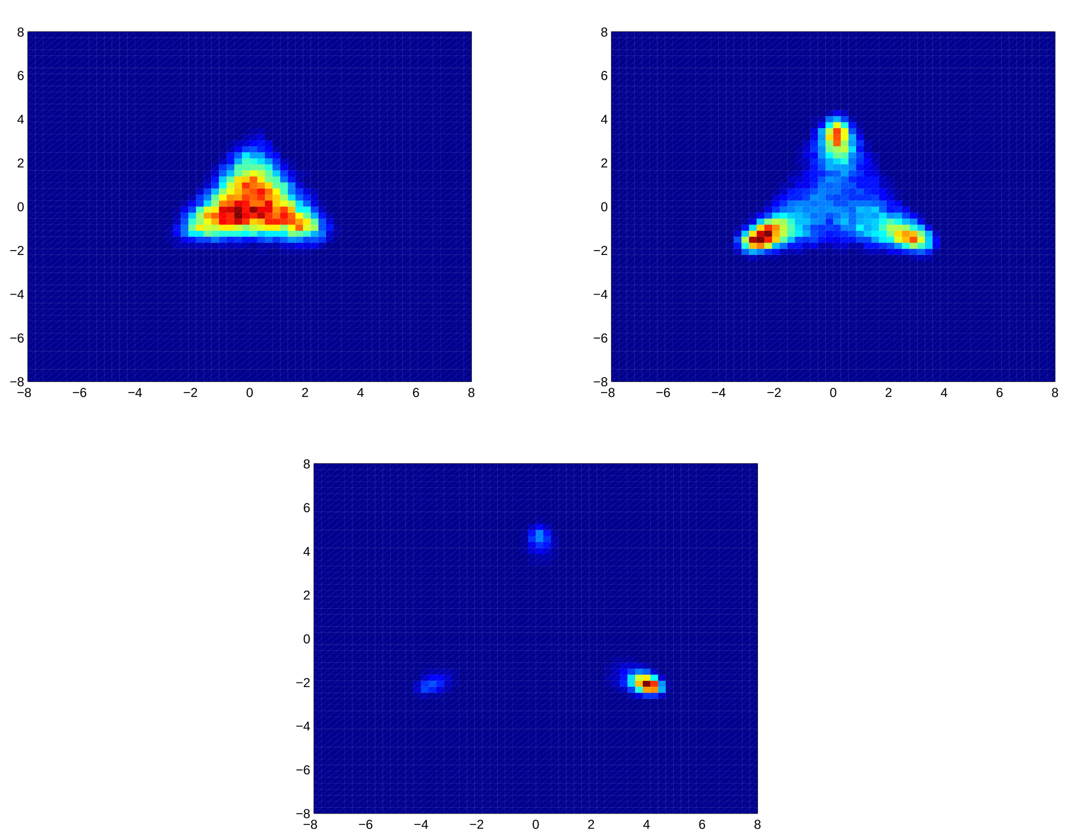
At high temperature, QCD matter undergoes a deconfinement transition, where ordinary hadronic matter transforms into strongly interacting quark-gluon plasma. In the absence of quarks, $N_f = 0$, the transition is a symmetry-breaking first order transition, where the order parameter is the thermal Wilson line. The non-zero expectation value of the Wilson line signals the breaking of the Z(3) center symmetry of quarkless QCD at high temperatures.

The transition has been studied extensively using lattice simulations, but becomes computationally exceedingly expensive at high temperatures $T \sim 5T_c$. At high T, the complementary approach has been to construct perturbatively effective theories, such as EQCD, using the method of dimensional reduction. In the dimensional reduction procedure, however, one expands the temporal gauge fields around one of the Z(3) vacua and thus explicitly violates the center symmetry and the models fail for T below $5T_c$.

As a unification of these strategies, a 3D effective theory of hot QCD respecting the Z(3) symmetry has been constructed in [2]. At high temperatures, the effective theory is matched to EQCD and still preserves the center symmetry. The effective theory is further connected to full QCD by matching the domain wall profile separating two different Z(3) minima.

The new theory relies on the scale separation between the inverse correlation length and the lowest non-zero Matsubara mode, which is still modest at T_c. Thus, one hopes that the range of validity of this theory would extend down to T_c.

In order to perform lattice simulations the theory must be formulated on a lattice and the lattice theory needs to be matched to the continuum theory. The effective theory is super-renormalizable, and thus the connection between the continuum $\overline{\text{MS}}$ and lattice regulated theories can be obtained exactly to the desired order in the lattice spacing a via two-loop lattice perturbation theory.



Theory

Continuum:

$$S = \int \mathrm{d}^{3-2\epsilon}x \Big\{ \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}\left(D_i Z^\dagger D_i Z\right) + c_1 \mathrm{Tr}[Z^\dagger Z] + 2c_2 \mathrm{Re}(\mathrm{Det}[Z]) + c_3 \mathrm{Tr}[(Z^\dagger Z)^2] + d_1 \mathrm{Tr}[M^\dagger M] + 2d_2 \mathrm{Re}(\mathrm{Tr}[M^3]) + d_3 \mathrm{Tr}[(M^\dagger M)^2] \Big\},$$

$$F_{ij} = \partial_i A_j - \partial_j A_i + i g_3 [A_i, A_j]$$
$$D_i = \partial_i - i g_3 [A_i, \quad]$$

Z **is a** 3×3 **complex matrix** and $M = Z - \frac{1}{3}\mathrm{Tr}[Z]\mathbb{1}$ is the traceless part of Z. Scale dependence of the mass terms:

$$c_1(\bar\mu) = \frac{1}{16\pi^2}\left[64 c_3 g_3^2 + \frac{88}{9}c_3^2\right]\log\left(\frac{\Lambda}{\bar\mu}\right)$$
$$d_1(\bar\mu) = \frac{1}{16\pi^2}\left[\frac{280}{9}c_3^2 - 64 d_3 g_3^2 + \frac{92}{3}\left(2d_3c_3 + d_3^2\right) + \frac{9}{2}g_3^4\right]\log\left(\frac{\Lambda}{\bar\mu}\right).$$

Lattice: $S = S_W + S_Z$, where S_W is the Wilson action and

$$S_Z = 2\left(\frac{2N_c}{\beta}\right)\sum_{x,i}\mathrm{ReTr}\left[\hat Z^\dagger \hat Z - \hat Z^\dagger(x) U_i(x) \hat Z(x+\hat\imath) U_i^\dagger(x)\right] + \left(\frac{2N_c}{\beta}\right)^3 \sum_x \left(\hat c_1 \mathrm{Tr}[\hat Z^\dagger \hat Z] + 2\hat c_2 \mathrm{ReDet}\hat Z + \hat c_3 \mathrm{Tr}[(\hat Z^\dagger \hat Z)^2]\right) + \left(\frac{2N_c}{\beta}\right)^3 \sum_x \left(\hat d_1 \mathrm{Tr}[\hat M^\dagger \hat M] + 2\hat d_2 \mathrm{ReTr}\hat M^3 + \hat d_3 \mathrm{Tr}[(\hat M^\dagger \hat M)^2]\right),$$

$$\boxed{\beta = \frac{6}{g_3^2 a}}$$

with, via **Two-Loop Lattice Perturbation Theory Calculation**[1],

$$\hat c_1 = \frac{c_1}{g_3^4} - \frac{1}{4\pi}6.3518228\hat c_3\beta - \frac{1}{16\pi^2}\left[\left(64\hat c_3 + \frac{88}{9}\hat c_3^2\right)(\log\beta + 0.08849) + 37.0863\hat c_3\right] + \mathcal{O}(\beta^{-1})$$
$$\hat d_1 = \frac{d_1}{g_3^4} - \frac{\beta}{4\pi}\left(3.17591 + 5.64606\hat d_3\right) - \frac{1}{16\pi^2}\Big\{41.780852 + 37.0863\hat d_3 - \left(\frac{280}{9}\hat c_3^2 - 64\hat d_3 + \frac{184}{3}\hat d_3\hat c_3 + \frac{92}{3}\hat d_3^2 + \frac{9}{2}\right)[\log\beta + 0.08849]\Big\} + \mathcal{O}(\beta^{-1}).$$

Other terms are matched to order $\mathcal{O}(a^0)$ on tree-level by scaling with g_3:

$$\begin{aligned} Z &= g_3\hat Z & M &= g_3\hat M \\ c_2 &= g_3^3\hat c_2 & d_2 &= g_3^3\hat d_2 \\ c_3 &= g_3^2\hat c_3 & d_3 &= g_3^2\hat d_3 \end{aligned}$$

Also various condensates have been renormalized in order to convert their expectation values to $\overline{\text{MS}}$.

Phase diagram of the Theory at $c_i = 0$

Two distinct phases separated by a first order transition:

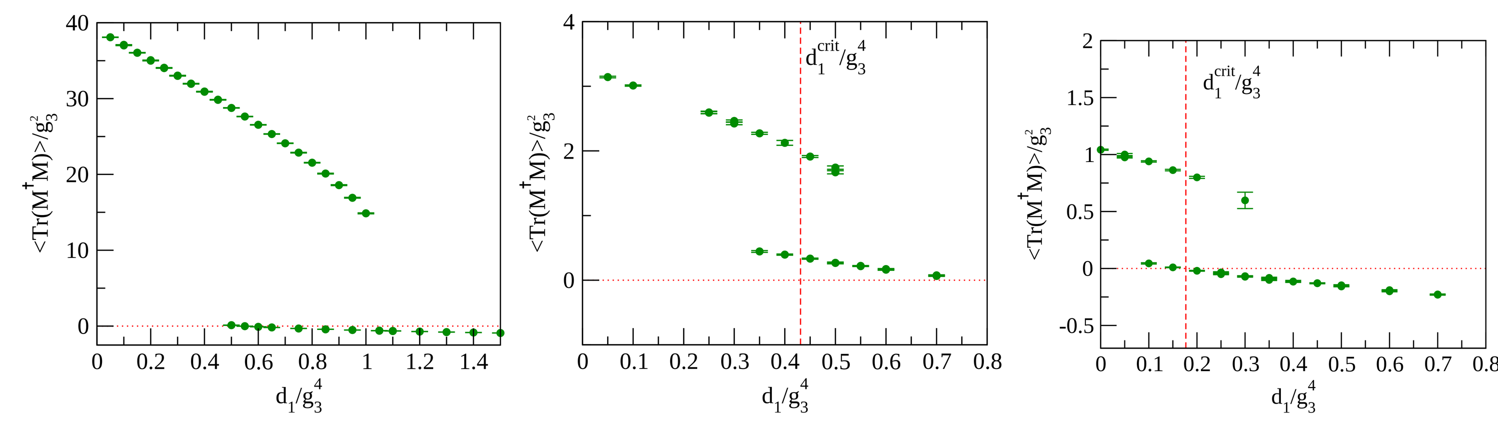


FIGURE 1: Discontinuity in the quadratic condensate in continuum regularization $\langle \mathrm{Tr}M^\dagger M\rangle_{\overline{\text{MS}}}$ for $d_3 = 0.1, 1, 3$. The phase transition gets weaker as the coupling d_3 grows. The metastable regions shrink and the discontinuity diminishes

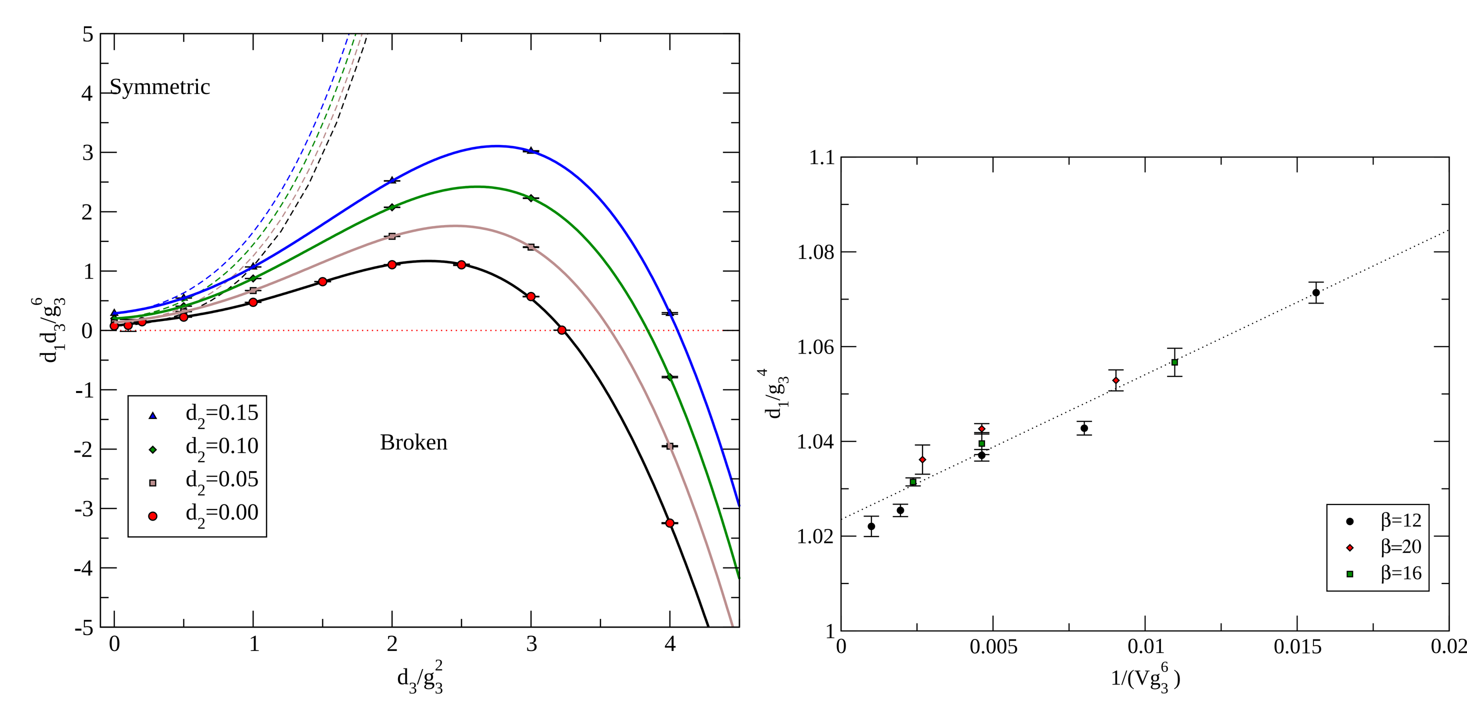


FIGURE 2: (LEFT) The phase diagram as a function of d_1, d_2 and d_3. First order critical line separates two phases. Dashed lines are one-loop perturbative predictions. The symmetric phase refers to the phase where $\langle \mathrm{Tr}M^\dagger M\rangle$ is smaller. (RIGHT) Volume dependence of the pseudo-critical point with $d_3 = 2$ and $d_2 = 0.1$. The pseudo-critical point was determined by requiring equal probability weight for $\mathrm{Tr}M^\dagger M$ in both phases.

Conclusions

The exact relations between the lattice and continuum $\overline{\text{MS}}$ regulated formulations of the Z(3)-symmetric 3D effective theory of hot QCD have been calculated. The Lagrangians and the operators up to cubic ones have been matched to $\mathcal{O}(a^0)$. These results make the non-perturbative lattice study of the theory possible.

References

[1] A. Kurkela, arXiv:0704.1416 [hep-lat], to be published in Phys. Rev. D

[2] A. Vuorinen and L. G. Yaffe, Phys. Rev. D **74** (2006) 025011 [arXiv:hep-ph/0604100].