

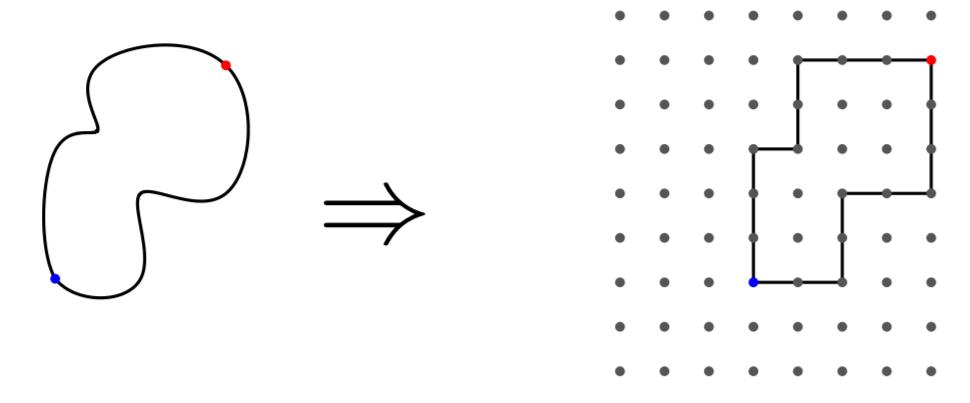
Huey-Wen Lin\* and Kostas Orginos  
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

\*presenter

## Lattice QCD

- First-principles calculation of nonperturbative physics in QCD
- Lattice QCD is a discrete version of continuum QCD theory
- Use Monte Carlo integration to calculate physical observables directly from path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int [dA] [d\bar{\psi}] [d\psi] O(\bar{\psi}, \psi, A) e^{i \int d^4x \mathcal{L}^{\text{QCD}}(\bar{\psi}, \psi, A)}$$



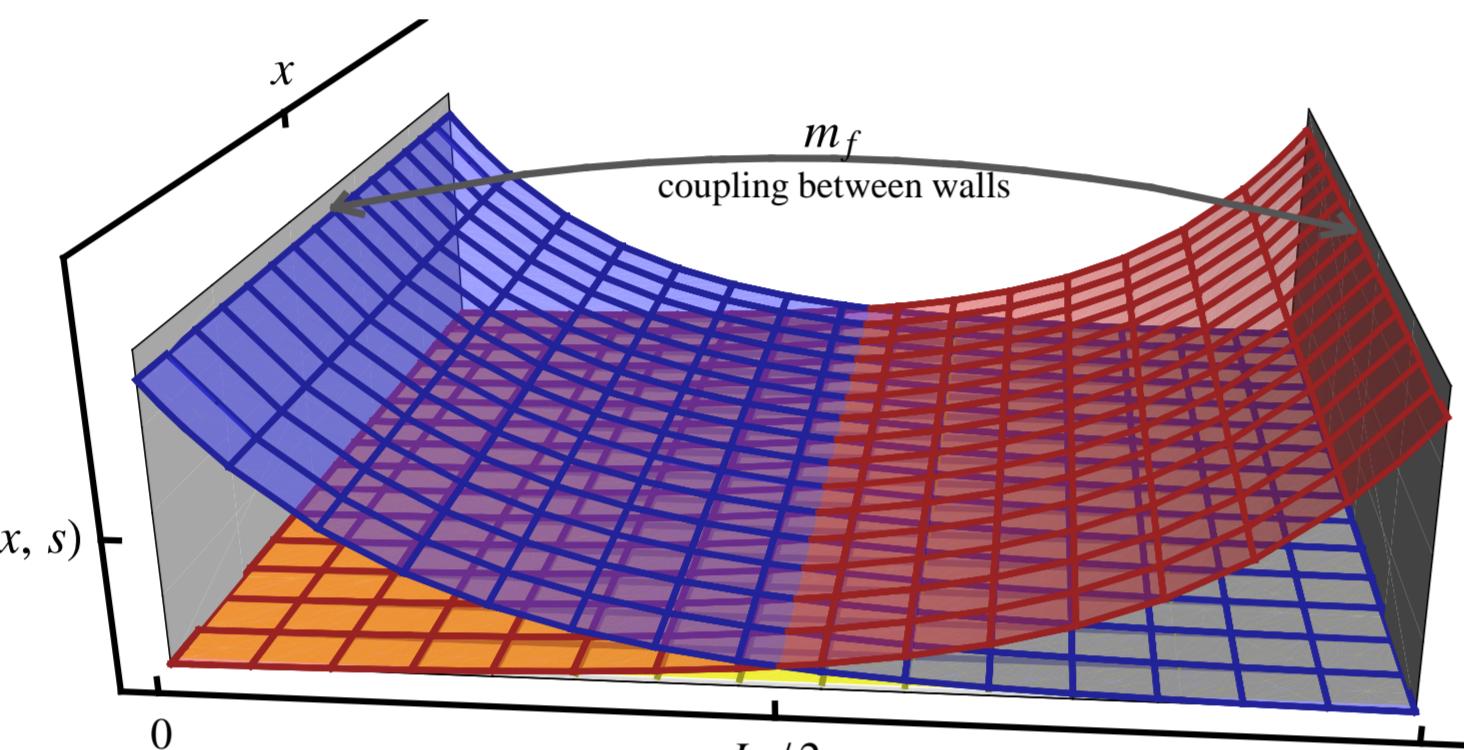
for quantity of interest

• Take  $V \rightarrow \infty$  and  $a \rightarrow 0$  to continuum limit

## Gauge action

- One-loop Symanzik  $O(a^2)$ -improved action

$$S_g = \frac{\beta}{3} \text{ReTr} \left( c_0 \left( 1 - \begin{array}{c} \square \\ \square \end{array} \right) + c_1 \left( 1 - \begin{array}{c} \square \\ \square \\ \square \end{array} \right) + c_2 \left( 1 - \begin{array}{c} \square \\ \square \\ \square \\ \square \end{array} \right) \right)$$



## Fermion action

- Domain-wall fermion formulation

$$D_{x,s;x',s'} = \delta_{x,x'} D_{s,s'}^\perp + \delta_{s,s'} D_{x,x'}^\parallel$$

$$D_{s,s'}^\perp = \frac{1}{2} \left[ (1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2 \delta_{s,s'} \right] - \frac{m_f}{2} \left[ (1 - \gamma_5) \delta_{s,L_s-1} \delta_{0,s'} + (1 + \gamma_5) \delta_{s,0} \delta_{L_s-1,s'} \right],$$

$$D_{x,x'}^\parallel = \frac{1}{2} \sum_{\mu=1}^4 \left[ (1 - \gamma_\mu) U_\mu(x) \delta_{x+\mu,x'} + (1 + \gamma_\mu) U_\mu^\dagger(x') \delta_{x-\mu,x'} \right] + (M_5 - 4) \delta_{x,x'}$$

– Controllable chiral symmetry breaking with  $L_s$   
 ⇒ No complicated operator mixing  
 – Automatic  $O(a)$  off-shell improvement  
 ⇒ Easy to implement RI/MOM NPR  
 ⇒ No extra  $O(a)$  off-shell improvement on the action nor the operators  
 – Expensive in computational resources

- (Improved) Staggered fermions (asqtad): no tree-level  $a^2$  effect

– Relatively cheap for dynamical fermions (good)

– Mixing among parities and tastes

– Baryonic operators a nightmare — not suitable

- Mixed action: Match the sea Goldstone pion mass to the DWF pion

## Numerical Parameters/Ensembles

- MILC 2+1-flavor staggered lattices [1] (lattice spacing  $a \approx 0.125$  fm with volume fixed at 2.6 fm)
- Mixed action: staggered sea (cheap) with DWF fermion (chiral symmetry) with  $L_s = 16$ ,  $M_5 = 1.7$  [2]
- Ensemble parameters:

Label	$m_\pi$ (MeV)	$m_K$ (MeV)	$\Sigma$ conf.	$\Xi$ conf.
m010	358(2)	605(2)	600	600
m020	503(2)	653(2)	420	436
m030	599(1)	688(2)	561	561
m040	689(2)	730(2)	306	319

- HYP (hypercubic)-smeared gauge and Gaussian-smeared fermion fields

• Source-sink separation fixed at 10

• Unitary sea quark mass only

• Interpolating field

$$J_\alpha(\vec{p}, t) = \sum_{\vec{x}, a, b, c} e^{i \vec{p} \cdot \vec{x}} \epsilon^{abc} [u_a^T(y_1, t) C \gamma_5 d_b(y_2, t)] u_{c,\alpha}(y_3, t) \phi(y_1 - x) \phi(y_2 - x) \phi(y_3 - x)$$

- Two-point correlation function

$$C_{2\text{pt}}(\vec{p}, t) = \sum_{\alpha, \beta} \left( \frac{1 + \gamma_4}{2} \right)_{\alpha\beta} \langle J_\beta(\vec{p}, t) \bar{J}_\alpha(\vec{p}, 0) \rangle$$

- Three-point correlation function

$$C_{3\text{pt}}^\Gamma(\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta(\vec{p}, t) \mathcal{O}(\tau) \bar{J}_\alpha(\vec{p}, 0) \rangle$$

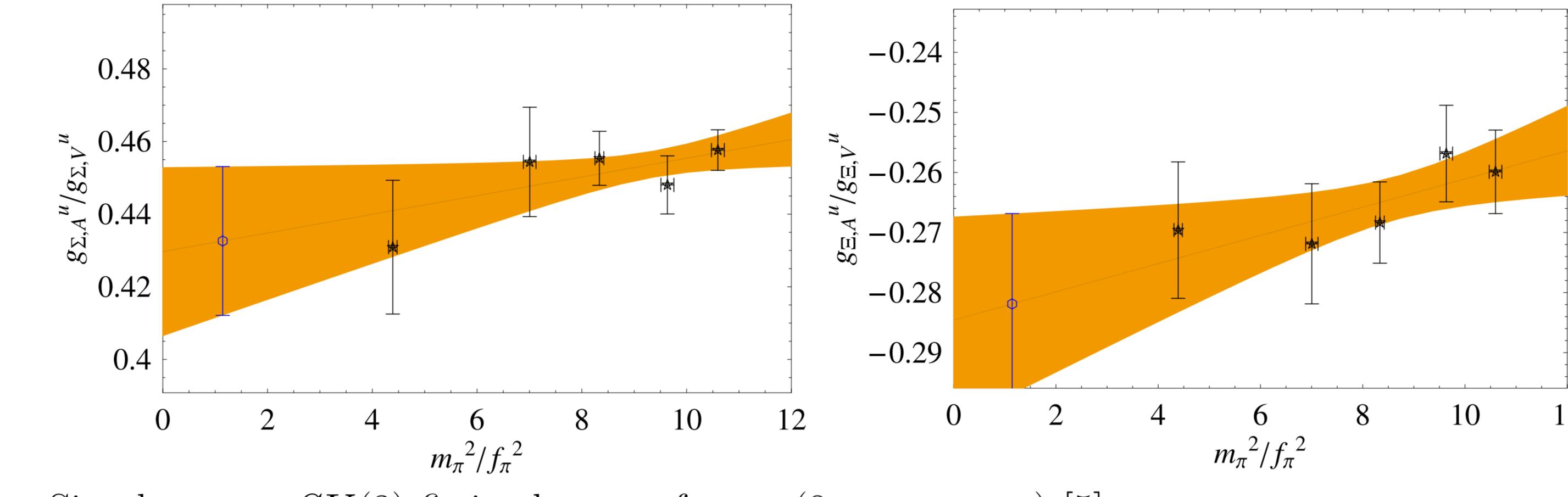
## Axial Coupling Constants

- Important applications such as in hyperon scattering and non-leptonic decays
- Only existing predictions are from chiral perturbation theory and large- $N_c$  calculations [3]:

$$0.18 < -g_{\Xi\Xi} < 0.36$$

$$0.30 < g_{\Sigma\Sigma} < 0.55$$

- Poor fit into the formulation of  $SU(3)$  chPT [4] even with large- $N_c$  constraints
- Naive linear extrapolation against  $(m_\pi/f_\pi)^2$ .  
We find  $g_{\Sigma\Sigma} = 0.441(14)$  and  $g_{\Xi\Xi} = -0.277(11)$ .



- Simultaneous  $SU(3)$  fit in the near future (8 parameters) [5]

## Hyperon Form Factors

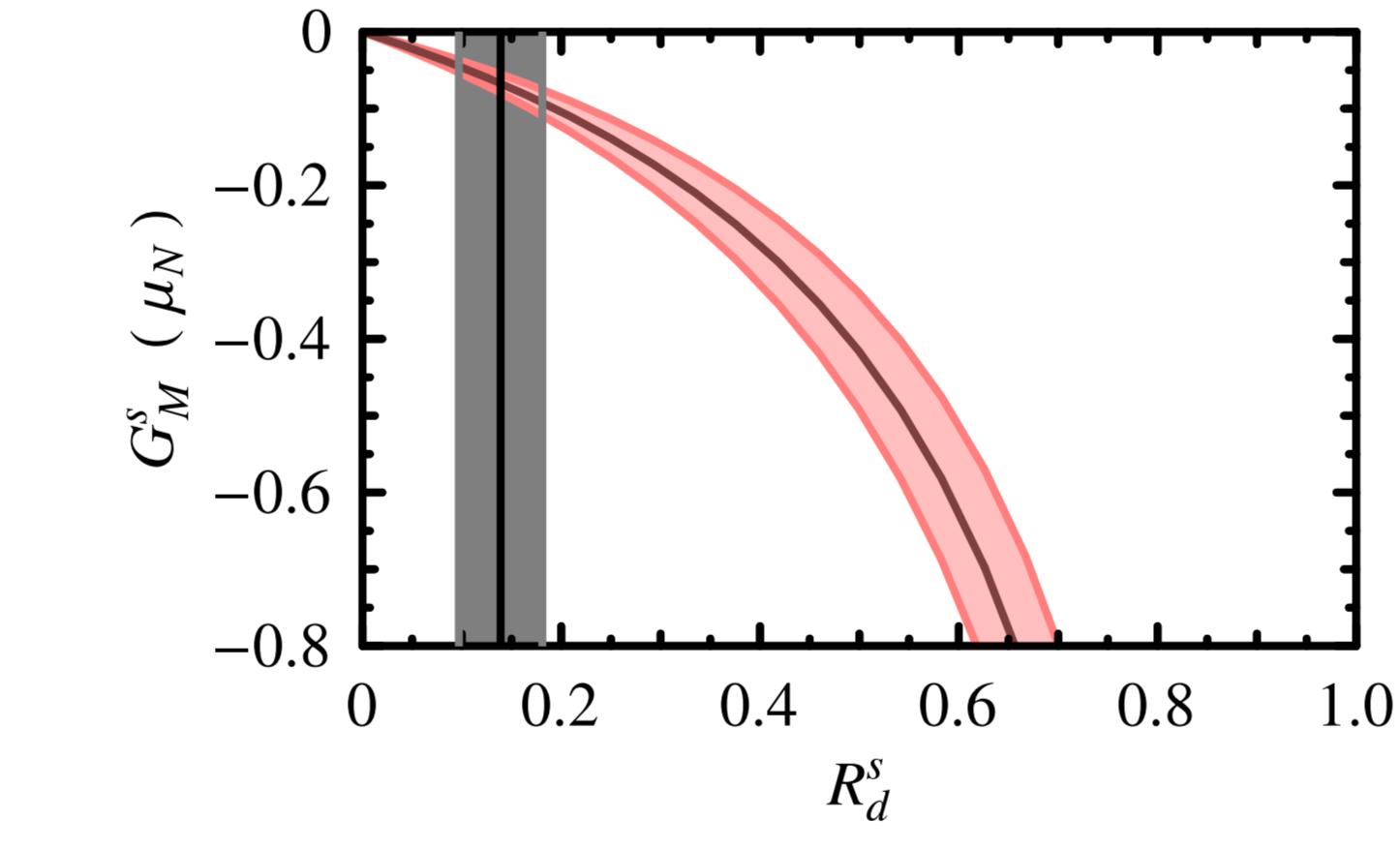
- A proxy for studying the strangeness of the nucleon
- Many experiments are devoted to understanding strange quark contributions to the electromagnetic form factors of the nucleon: HAPPEX and G0 at JLab, SAMPLE at MIT-BATES, and A4 at Mainz.

## Strange Magnetic Moment $G_M^s$

- Quenched approximation in lattice QCD gives values of  $G_M^s$  ranging from  $-0.28(10)$  to  $+0.05(6)$
- Adelaide-JLab Collaboration used an indirect approach with the help of charge symmetry and chiral perturbation theory to correct for quenching effects, obtaining  $-0.046(19)$
- Charge symmetry gives

$$G_M^s = \frac{R_d^s}{1 - R_d^s} \left[ \mu^p + 2\mu^n - \frac{u^N}{u^{\Xi}} (\mu^{\Xi^0} - \mu^{\Sigma^-}) \right]$$

- $\frac{u^N}{u^{\Xi}}$ : Linear ansatz chiral extrapolation to the physical pion mass (right)



- (left) The pink band is the constraint for the proton strangeness magnetic moment from our data, and the grey band indicates the  $R_d^s$  given by Adelaide-JLab Collaboration

## Strange Electric Moment $G_E^s$

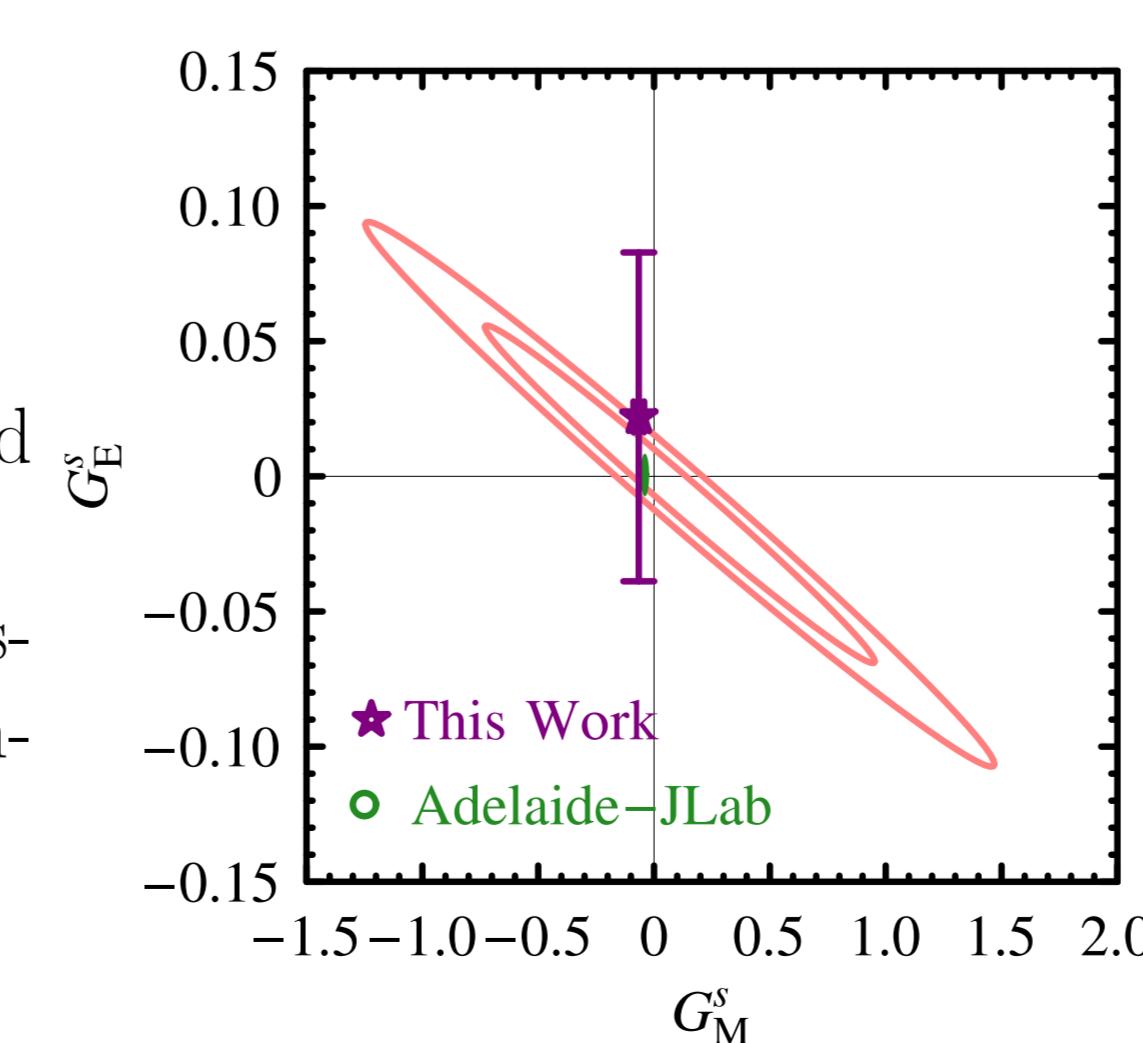
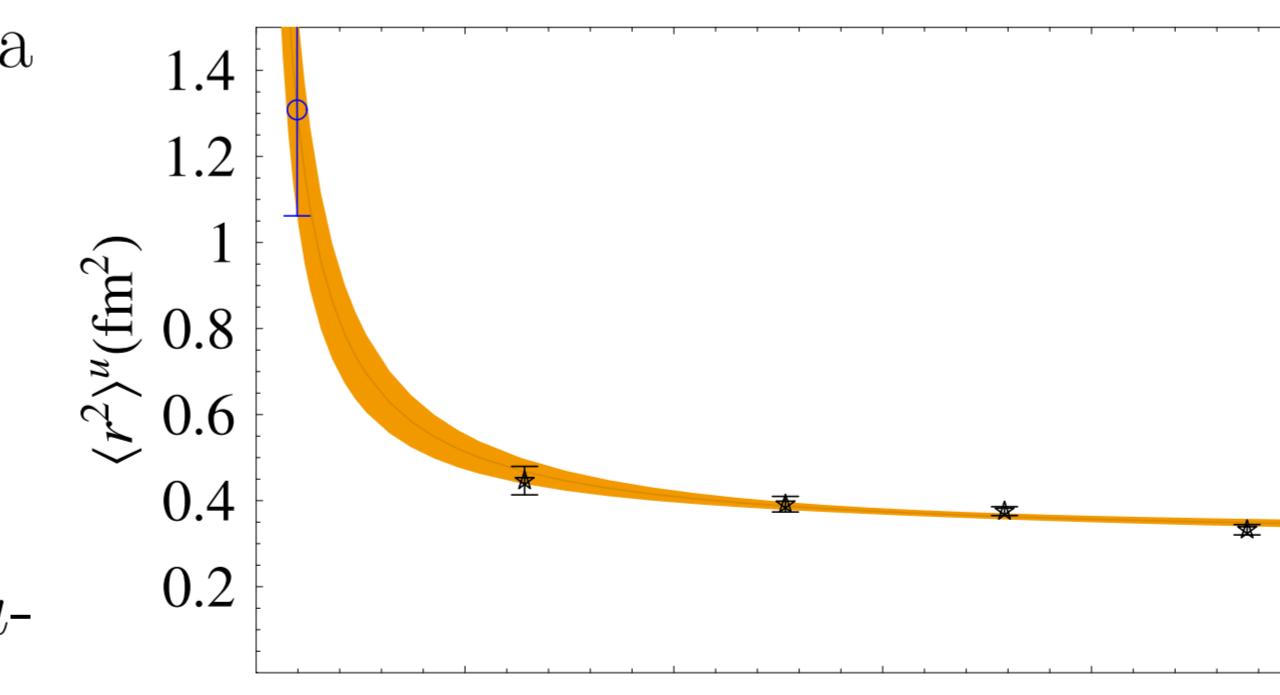
- Similarly, charge symmetry connects us to a strange quantity through  $\langle r^2 \rangle^u$ :

$$\langle r^2 \rangle^s = \frac{r_d^s}{1 - r_d^s} [2 \langle r^2 \rangle^p + \langle r^2 \rangle^n - \langle r^2 \rangle^u].$$

- Chiral extrapolation for proton  $u$ -component charge radius shown on the right

- $G_E^s(Q^2 = 0.1 \text{ GeV}) = 0.022(61)$ , if taking  $r_d^s = 0.16(4)$  from chPT [6]

- $G_M^s - G_E^s$  plane:  
Pink: experimental result  
Purple: our preliminary result from mixed action  
Green: Adelaide-JLab Collaboration [6] using a quenched simulation but with quenching corrections



## $V_{us}$ from Hyperon Semi-leptonic Decays

- $V_{us}$  can be obtained from the semileptonic decay ( $B_1 \rightarrow B_2 \ell \nu$ ) width via

$$\Gamma = G_F^2 |V_{us}|^2 \frac{\Delta m^5}{60\pi^3} (1 + \delta_{\text{rad}}) \times \left[ \left( 1 - \frac{3}{2} \beta \right) (|f_1|^2 + |g_1|^2) + \frac{6}{7} \beta^2 (|f_1|^2 + 2|g_1|^2 + \text{Re}(f_1 f_2^*) + \frac{2}{3} |f_2|^2) + \delta_{q^2} \right]$$

with  $\Delta m = m_{B_1} - m_{B_2}$ ,  $\beta = \Delta m/m_{B_1}$ , the radiative corrections  $\delta_{\text{rad}}$ , and  $\delta_{q^2}(f_1, g_1)$  taking into account the transfer-momentum dependence of  $f_1$  and  $g_1$  [7]

- In this work, only  $\Sigma^- \rightarrow n$  is analyzed. ( $\Xi^0 \rightarrow \Sigma$  underway)
- Ademollo-Gatto (AG) theorem [8]:  $SU(3)$ -breaking corrections start at second order

$$f_1(0) = f_1(0)^{SU(3)} + O(\lambda^2)$$

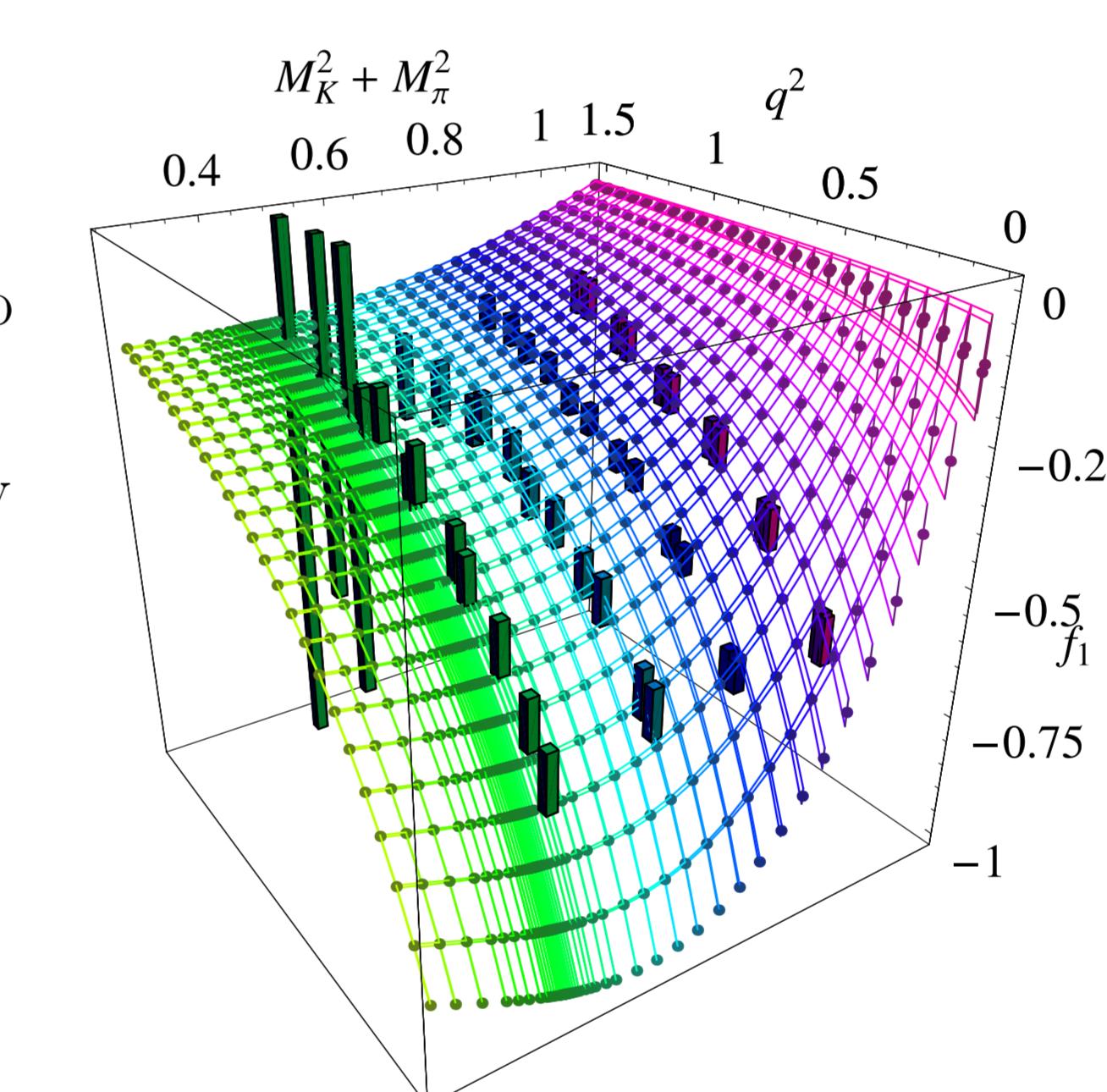
- Dipole form fit to the  $q^2 = 0$  point at each pion ensemble
- Construct a ratio

$$R(m_K, m_\pi) = \frac{f^{SU(3)} - |f'(0)|}{a^4 (m_K^2 - m_\pi^2)^2},$$

and linearly extrapolate it to the physical point

$$R(m_K, m_\pi) = c_0 + c_1 a^2 (m_K^2 + m_\pi^2).$$

- Very weak constraint to the fit from the last point; hence, this is not an ideal solution for our data.



$$\bullet \text{Improvement: a single simultaneous fit (left)}$$

$$f_1(q^2) = \frac{1 + (M_K^2 - M_\pi^2)^2 (A_1 + A_2 (M_K^2 + M_\pi^2))}{(1 - \frac{q^2}{M_0 + M_1 (M_K^2 + M_\pi^2)})^2}$$

$$\bullet \text{Our preliminary result: } f_1 = -0.88(15).$$

## Conclusion and Outlook

- Preliminary dynamical results for hyperon three-point data analysis

– First lattice hyperon axial coupling constants:

$$g_{\Sigma\Sigma} = 0.441(14) \text{ and } g_{\Xi\Xi} = -0.277(11)$$

– Strangeness in the proton:

$$G_M^s = -0.066(12)_{\text{stat}}(23)_{\text{syst}}, G_E^s(Q^2 = 0.1 \text{ GeV}) = 0.022(61)$$

– First dynamical hyperon semileptonic decays:

$$f_1 = -0.88(15)$$

- More statistics needed since the pion mass is low

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