Heavy Quark Masses in 3-flavor Lattice QCD

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Fermilab Lattice and MILC Collaborations

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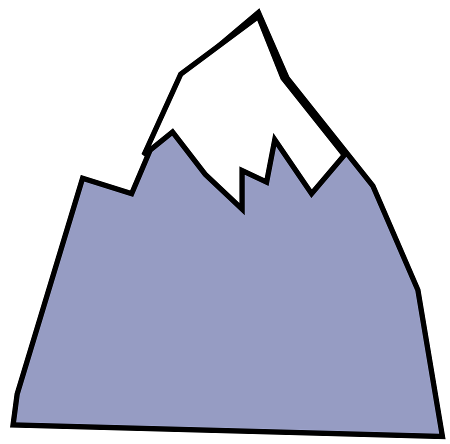
The Annotated Edition

INT Lattice Summer School 2007



Related Lectures:

Shoji Hashimoto: Lectures I & II: pert theory scales & schemes

Stefan Sint: Symanzik, other m_Q methods

Andreas Kronfeld: Heavy Quarks

Introduction

$m_b \qquad m_c$

Goal:

calculate masses of the bottom and charm quarks

Do this by:

combining non-perturbative calculations of heavy-light meson masses M

with (lattice) perturbation theory for the quark pole masses m

$$\overline{M} = \frac{1}{4}\left(M_{\text{pseudoscalar}} + 3M_{\text{vector}}\right)$$

Pole Mass(es)

a distorted energy-momentum relation,

$$E^2(\mathbf{p}) = m_1^2 + \frac{m_1}{m_2}\mathbf{p}^2 + \ldots$$

leads to two pole masses which we refer to as the

"rest" mass

$$m_1 = E(\mathbf{0})$$

"kinetic" mass

$$m_2 = \left(\frac{\partial^2 E}{\partial p_1^2}\right)^{-1}_{\mathbf{p}=\mathbf{0}}$$

and similarly for Mesons.

A.X.El-Khadra, A.S. Kronfeld, P. B. Mackenzie, Phys. Rev. D 55, 3933 (1997), hep-lat/9604004

Rest Mass Method

S. Hashimoto Lec 2 Sec 2 slide 21

Calculate the quark mass via the binding energy

$$\overline{M}_1 - m_1 = \overline{M}_{\text{exp}} - m_{\text{pole}}$$

Uncertainties are from:

- light quark and gluon discretization
- heavy quark discretization
- truncation of the perturbation theory for m_1

A. S. Kronfeld, Phys. Rev. D **62**, 0104505 (2000), hep-lat/0002008.

Kinetic Mass Method

$$m_{\text{pole}} = a\, m_2 \frac{\overline{M}_{\text{exp}}}{a\overline{M}_2}$$

$\overline{M}_2$ is obtained by fitting to the dispersion relation.

Same uncertainties as rest-mass method but expected to be less sensitive to heavy-quark tuning.

Statistical error is larger.

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Statistical error is larger.

In the continuum,
both methods must yield the same result

Non-perturbative Elements

Ensembles & Mesons

MILC lattices:

2+1 flavors of sea quarks (asqtad, staggered)

$m_{u,d} \approx 0.1$ to $0.3, 0.4 m_s$

improved gluons

three lattice spacings ~ 0.09, 0.12, 0.15 fm

bottom and charm quarks: Fermilab method

S. Sint Lec 1 Sec 2 slides 9-12 - Symanzik Eff Th method(s)
A. Kronfeld

valence light quark: staggered

Meson masses

$\overline{M}_1$ Rest mass, p =0

Fit two-point correlators and spin average

$\overline{M}_2$ Kinetic mass, mesons with momentum

Fit correlators at each momentum and spin average

Then fit to the (lattice) dispersion relation

physical strange mass

To avoid needing a chiral extrapolation, we work with the $\overline{B}_s$ and $\overline{D}_s$ meson masses.

work with the physical strange quark mass because experimental input is needed

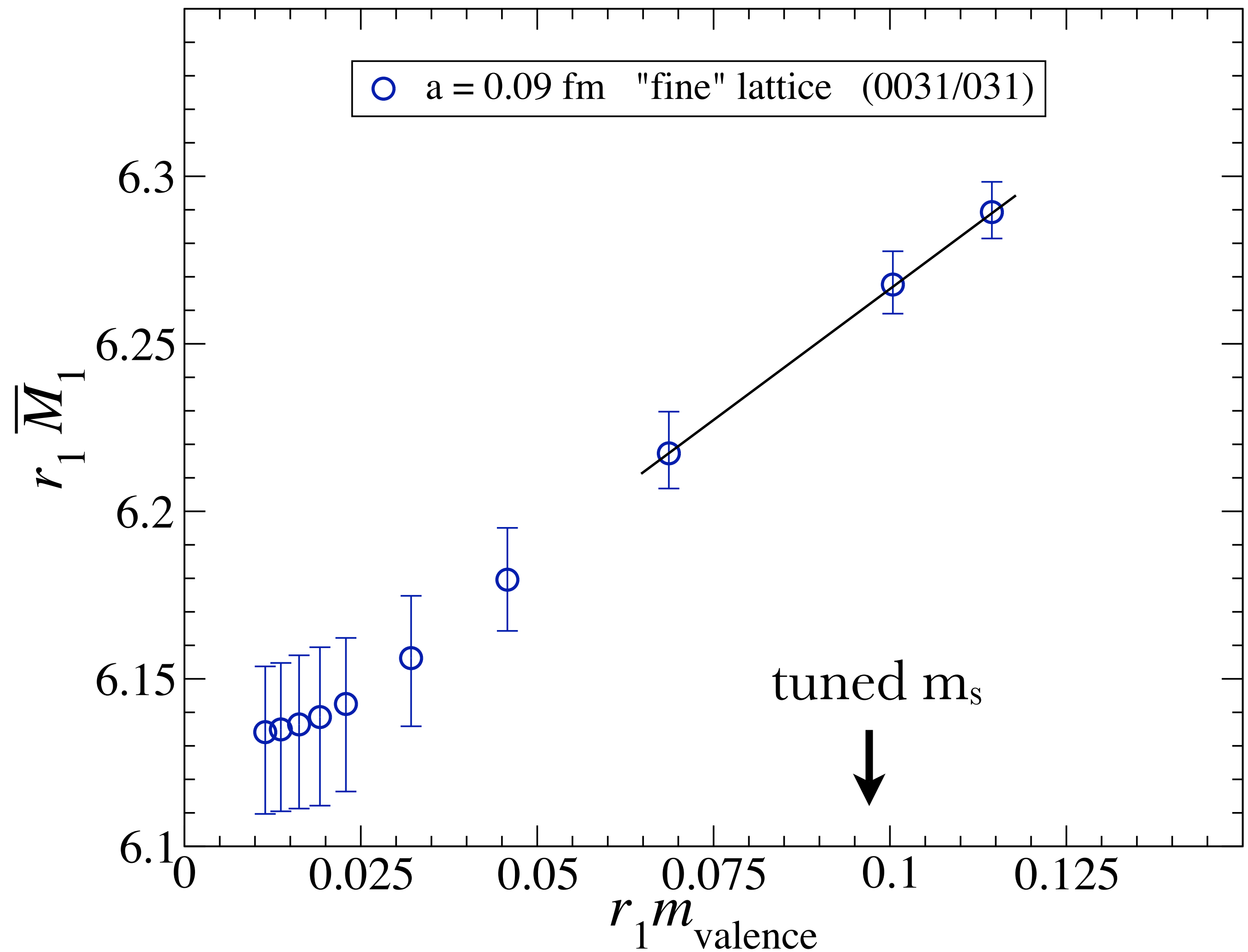
Monte Carlo data is not at the physical strange quark mass

$\Rightarrow$ linearly interpolate to m_s

Interpolating to m_s

interpolate to the physical strange mass

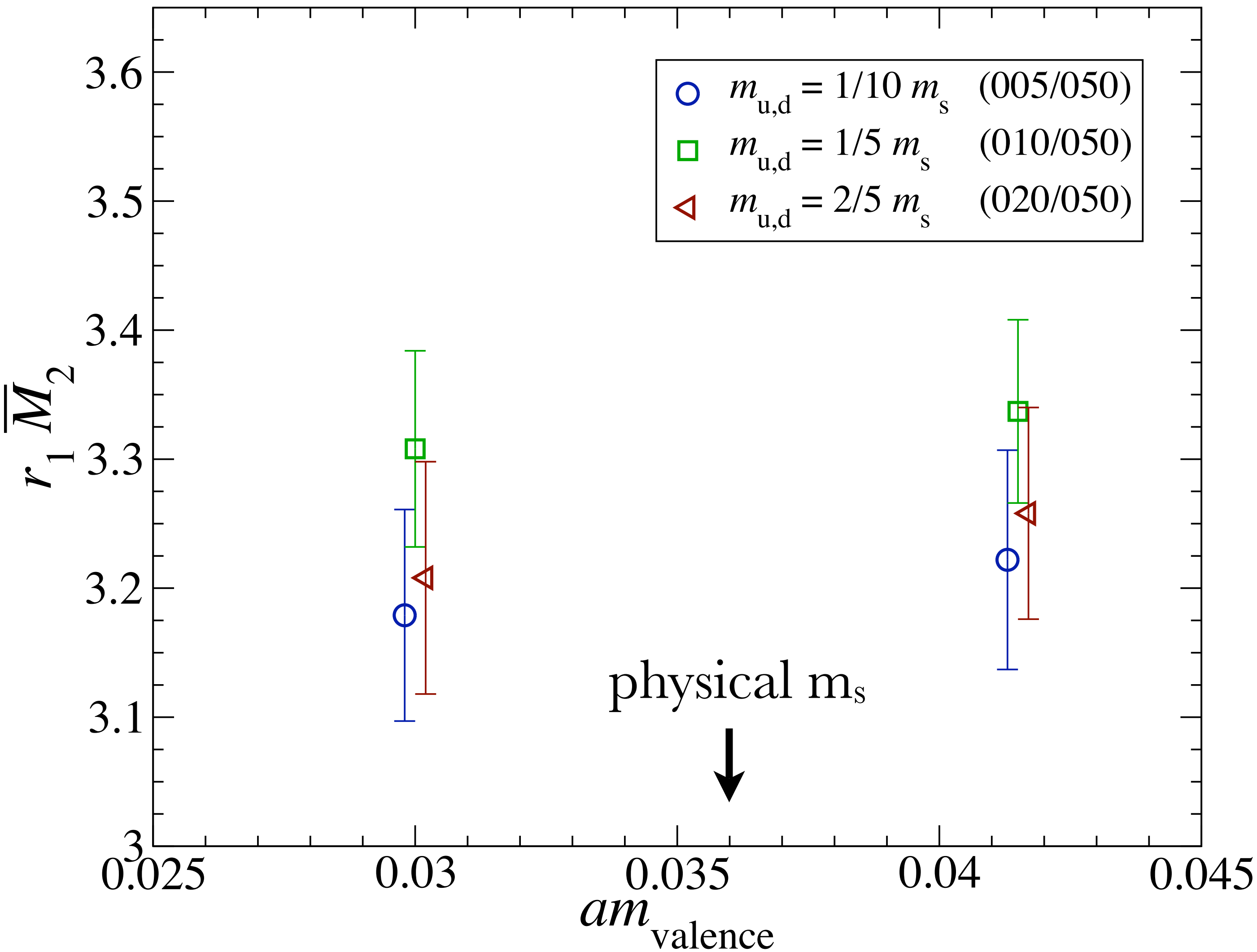
$$\Rightarrow \quad \overline{D}_s, \quad \overline{B}_s$$



Sea Quark Effects

"$\overline{D}_s$" meson kinetic mass, a = 0.12 fm lattice

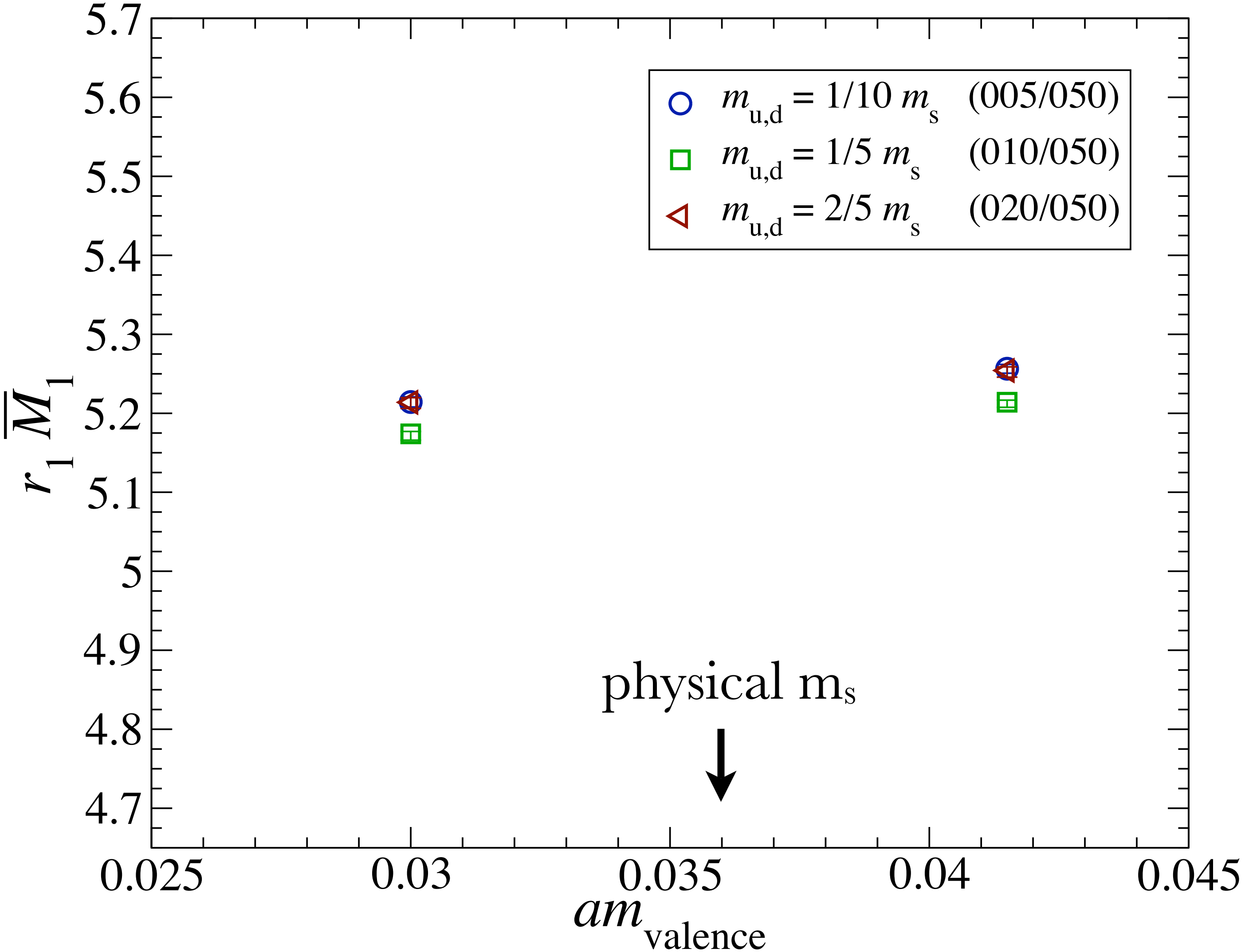
errors are statistical



Sea Quark Effects

"$\overline{B}_s$" meson rest mass, a = 0.12 fm lattice

errors are statistical



Perturbation Theory

Lattice Perturbation Theory

We use one-loop results to obtain the (lattice) pole masses m_1, m_2.

Formulae:

B. P. G. Mertens, A. S. Kronfeld and A. X. El-Khadra,
Phys. Rev. D **58**, 034505 (1998) hep-lat/9712024.

Automated perturbation theory using an improved gluon action:

M. Nobes, H. Trottier,
PoS LAT2005:209,2006 hep-lat/0509128; private communication.
A. El-Khadra private communication

Need: mass scheme and scale
coupling scheme and scale

α_s Schemes & Scales

We use V-scheme for $\alpha_s(q^*)$. ***S. Hashimoto Lec 1 slides 32 - 25***

$\alpha_s(q^*)$ is obtained as described in Q. Mason et al.

Q. Mason et al. [HPQCD Collaboration], Phys. Rev. Lett. **95**, 052002 (2005) hep-lat/0503005 ***S. Hashimoto Lec 1 slides 39 - 42***

G.P. Lepage (private communication).

T. van Ritbergen et al., Phys. Lett. B 400, 379 (1997) hep-ph/9701390

The scale, q*, is set via BLM, HLM prescription.

S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983);
G. P. Lepage and P. B. Mackenzie, Phys. Rev. D 48, 2250 (1993), hep-lat/9209022
K. Hornbostel, G. P. Lepage, C. Morningstar, Phys. Rev. D 67, 034023 (2003), hep-ph/0208224

S. Hashimoto Lec 1 slide 36

q^* “BLM”

q^* = the typical momentum of the gluon in the loop

$$\alpha(q^*) = \int d^4q\, f(q)\, \alpha(q)$$

$$= \int d^4q\, f(q)\, \alpha(q^*) \left\{ 1 - \alpha(q^*) \frac{\beta_0}{4\pi} \ln \frac{q^2}{q^{*2}} + \ldots \right\}$$

“BLM”: Choose q^* such that the $\alpha^2(q^*)\, \beta_0$ term is zero.

$$\ln q^{*2} = \frac{\int d^4q\, f(q) \ln(q^2)}{\int d^4q\, f(q)}$$

q^* "HLM"

"HLM", is in the spirit of LM, BLM.

Designed for cases where the 1-loop coefficient is zero or anomalously small.

Use higher order information (log moments).
Zero the $\alpha^3(q^*)\,\beta_0^2$ term.

Mass Scheme

S. Hashimoto Lec 2 slide 10

Renormalon ambiguity in the pole mass

$\Rightarrow$ need a short-distance mass

In particular, use a "threshold mass."

a short distance mass designed to run at low mass scales

Of the several threshold masses available, we've chosen to use the

potential subtracted mass

M. Beneke, Phys. Lett. B **434**, 115 (1998) hep-ph/9804241

Potential Subtracted Mass

- based on the static quark potential
- introduces a separation scale, μ_f.

$$m_{\rm PS}(\mu_f) = m_{\rm pole} - \frac{C_F\,\mu_f}{\pi}\alpha(q^*) + O(\alpha^2)$$

$$\Lambda_{\rm QCD} \;<\; \mu_f \;<\; m_{\rm quark}$$

bottom:

$$\mu_f = 2.0 \text{ GeV}$$

charm:

$$\mu_f = 1.0 \text{ GeV}$$

methods for higher orders

Setting the mass-scale is approached in two ways.

Method one ("q^*-method"):

Set $\mu_f = 1.0$ GeV for charm

$\mu_f = 2.0$ GeV for bottom

and use $\alpha_s(q^*)$ for the 1-loop coupling

Method two ("zero and run"):

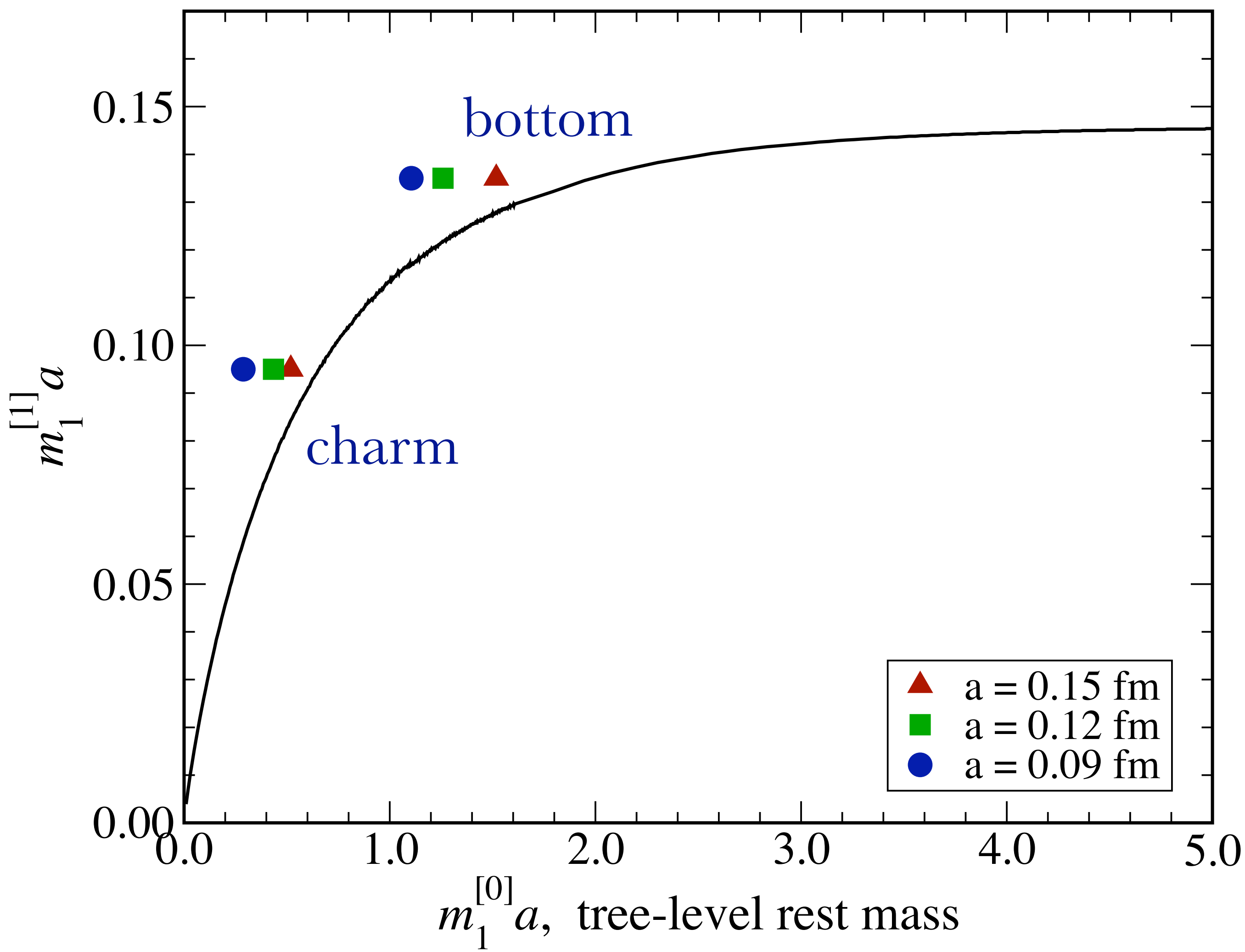
Set μ_f such that the one-loop term is zero

Run to the conventional value, 1.0 or 2.0 GeV, using the two-loop solution to the renormalization group equations.

and a bit about the lattice perturbation theory....

How do the corrections behave as the lattice spacing decreases?

$$am_1 = am_1^{[0]} + g^2 am_1^{[1]} + g^4 am_1^{[2]} + \ldots$$



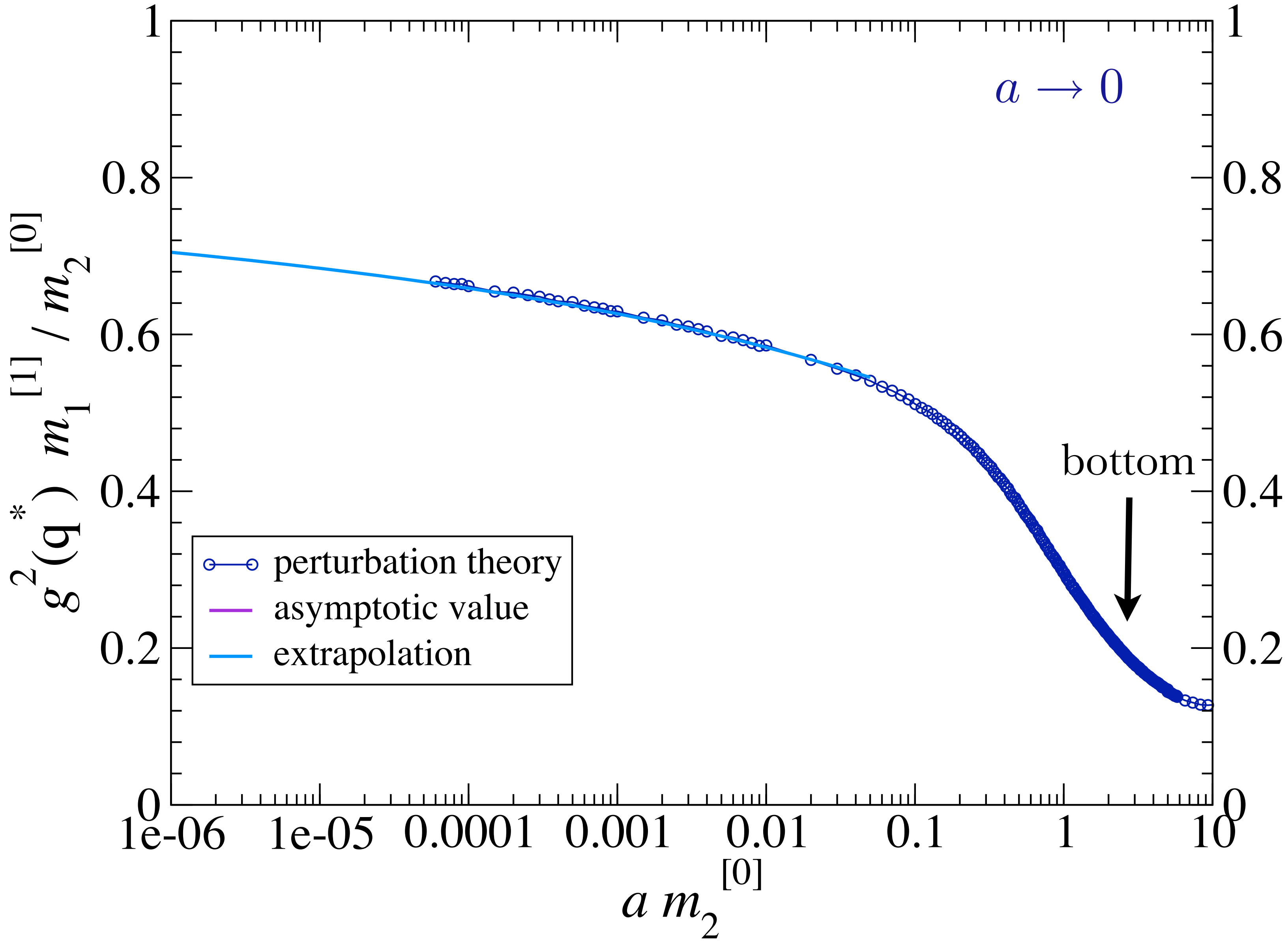
How do the corrections behave as the lattice spacing decreases?

$$am_1 = am_1^{[0]} + g^2 am_1^{[1]} + g^4 am_1^{[2]} + \ldots$$

$$m_1 = m_1^{[0]} \Big\{ 1 + \alpha(q^* a) \Big[C + \gamma_0 \ln(am_1^{[0]}) \Big]$$

$$+ \alpha^2(q^* a) \Big[C' + \gamma_1 \ln(am_1^{[0]}) + \gamma_0 \ln^2(am_1^{[0]}) \Big] \Big\}$$

keep the kinetic mass fixed while taking the lattice spacing to zero:



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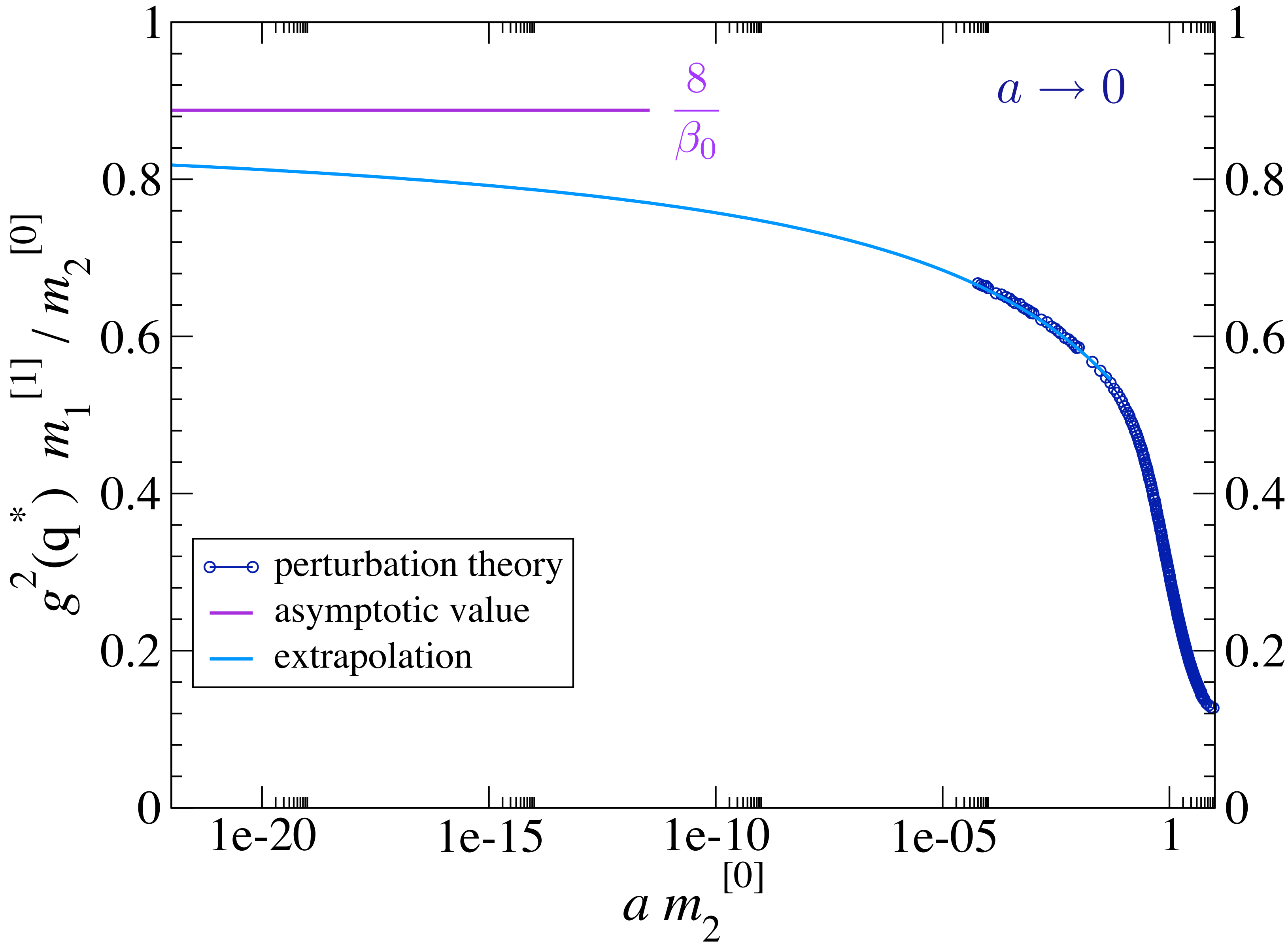

Preliminary Results

New for 2007

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added a smaller lattice spacing:
a = 0.09 fm “fine” lattice

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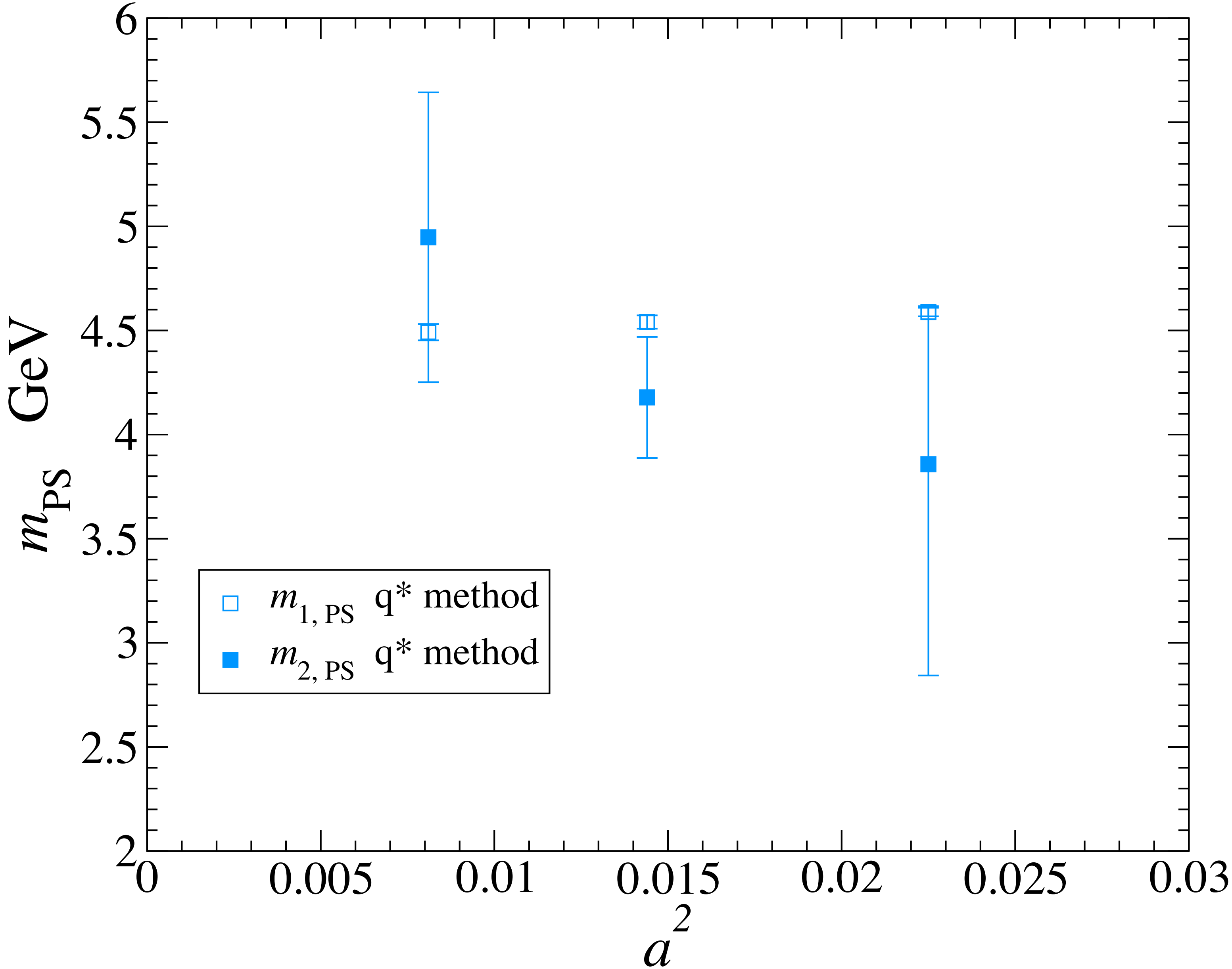
“zero and run” method of mass-scale setting
to test higher order effects

Bottom m_b $\mu_f = 2.0$ GeV



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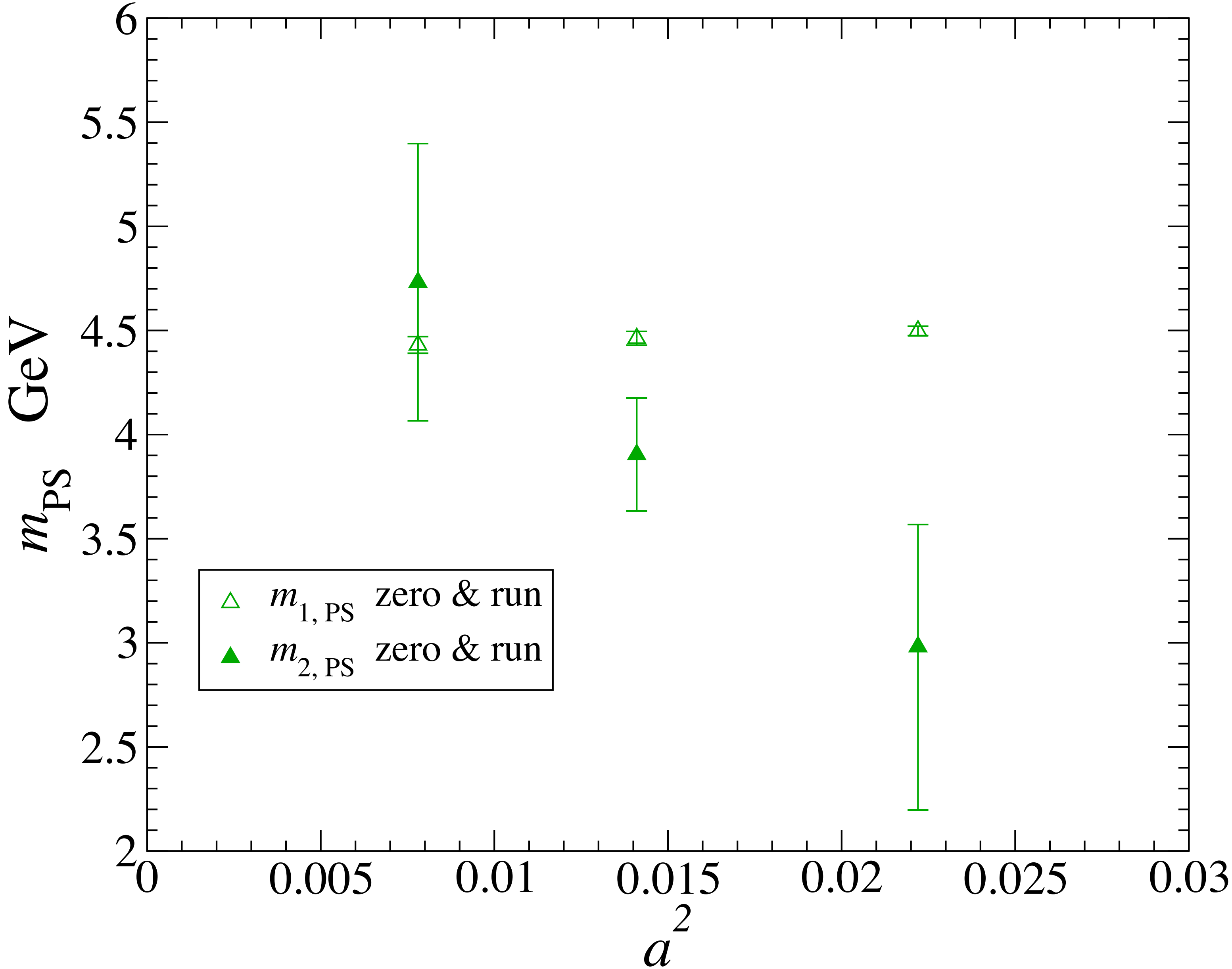

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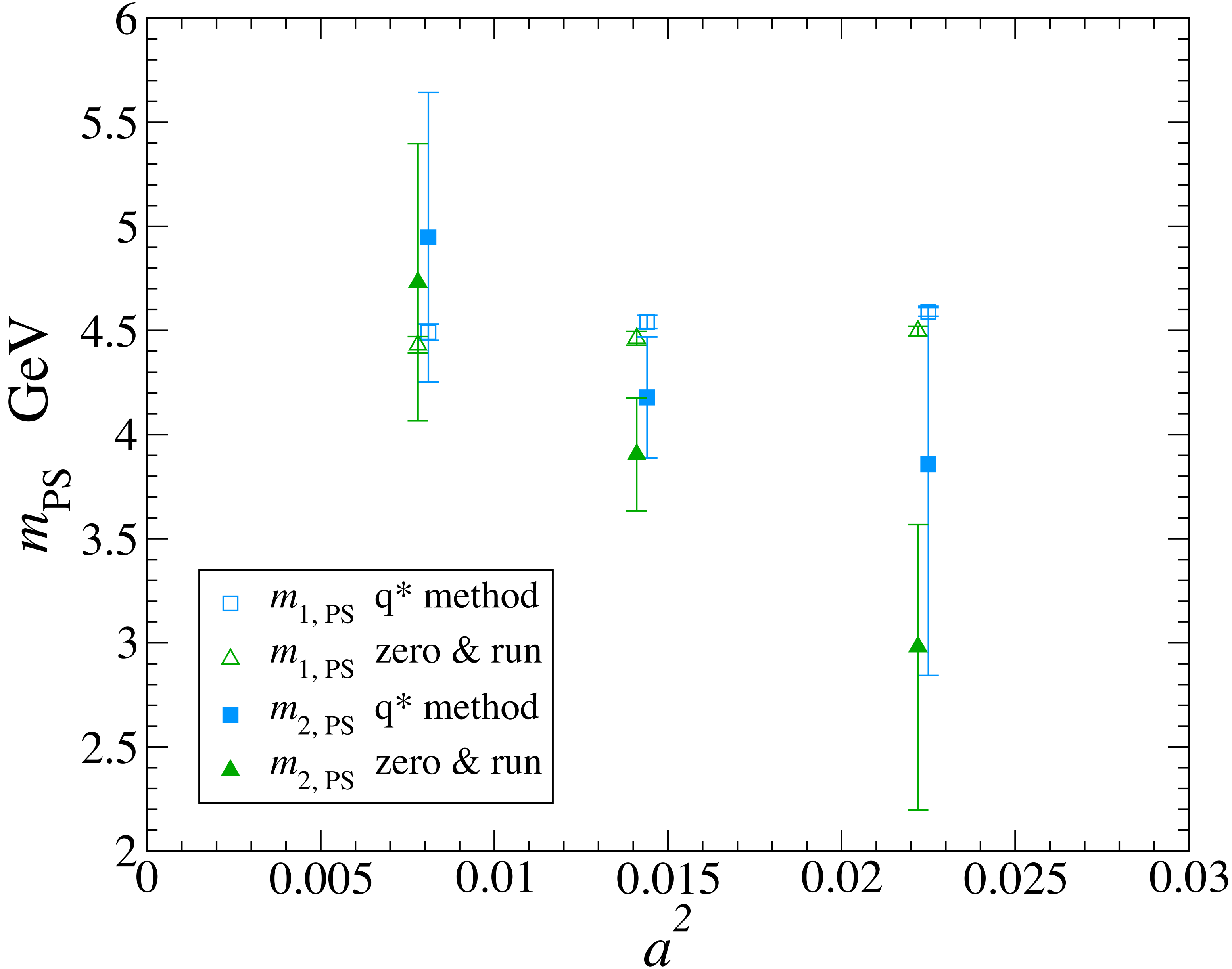

Heavy Quark Masses

Bottom

Uncertainty	% error
statistical	0.1
lattice spacing determination	0.4
heavy-quark tuning	0.4
sea quark effects	0.7
strange mass interpolation	0.4
perturbation theory truncation	3 to 5
heavy quark discretization	0.1 to 0.6
Total	3 to 5

Preliminary

potential subtracted mass

$$m_{b,PS}(\mu_f) = 4.46(22)\ \text{GeV}$$

$$\mu_f = 2.0\ \text{GeV}$$

S. Hashimoto Lec 2 Sec 2 - continuum

S. Hashimoto Lec 2 Sec 3 - lattice

S. Hashimoto Lec 2 Sec 4 - heavy quarks

Summary

Three lattice spacings: a = 0.09, 0.12, 0.15 fm

$m_{u,d} = 0.1$ to $0.4\, m_s$

Multiple methods to assess discretization effects and higher order perturbative contributions

Working on bottom:

$$m_{b,PS}(\mu_f) = 4.46(22)\ \text{GeV}$$

$$\mu_f = 2.0\ \text{GeV}$$

Also, working on charm :-)

M. Hildred Blewett Scholarship (APS)

for women who have interrupted their physics careers for family reasons

Hildred Blewett

accelerator theorist

as a student,
worked with Bethe at Cornell
later at Argonne, Fermilab,
CERN

died 2004
created this scholarship