Finite size scaling of left current correlators for non degenerate masses

Fabio Bernardoni[1] and Pilar Hernandez[1]

[1]IFIC (CSIC-Universitat de València), València (Spain)

Abstract: We study the volume dependence of the left-current correlator with non-degenerate quark masses to NLO in the chiral expansion. We consider three possible regimes: all quark masses are in the e-regime, all are in the p-regime and a mixed regime where the lightest quark masses satisfy $m_v \Sigma V \leq 1$ while the heavier $m_s \Sigma V \gg 1$. These results can be used to match LQCD and the Chiral Effective Theory in a box in which the Compton wavelength of the lightest pions is of the order of the box size. We consider both the full and Partially Quenched results.

1 Introduction:

ChPT is an EFT of QCD at low energies. The expansion parameters are p and m_q. It describes the interactions among the lightest pseudoscalar octet (pions).

ASSUMPTIONS:

1-flavor $SU(3)_R \times SU(3)_L$ is spontaneously broken to $SU(3)_V$
2-pions are the would be Goldstone bosons associated to the axial generators

PROPERTIES:

1-the coupling constants are not predicted by chiral symmetry
2-can predict finite volume effects provided $L \gg 4\pi F$
3-coupling constants are not modified by finite volume with Periodic Boundary Conditions

The coupling constants can be determined from experiment or by matching with correlation functions simulated on the lattice. However simulations are still perfformed at relatively large quark masses, where ChPT is less accurate. We calculate the left correlator in the NON degenerate mass case, in the full and Partially Quenched theory. This can be useful to match the next lattice results with the chiral effective theory.

2 Conventions:

We start from the QCD Lagrangian:

$$L_E = \sum_{r=1}^{N_v} \overline{\psi}_r (\gamma_\mu D_\mu + m_v)\psi_r + \sum_{r=N_v+1}^{N_v+N_s} \overline{\psi}_r (\gamma_\mu D_\mu + m_s)\psi_r$$

So the LO effective Lagrangian is:

$$L^{LO}_{ChPT} = \frac{F^2}{4} Tr\left[\partial_\mu U \partial_\mu U^+\right] - \frac{\Sigma}{2} Tr\left[\mathcal{M} U + U^+ \mathcal{M}\right] \quad \mathcal{M} = e^{\frac{i\theta}{N}} \begin{pmatrix} m_v & 0 \\ 0 & m_s \end{pmatrix}$$

The squared pion masses are linear in the quark ones
We just considered sources belonging to $SU(N_v)$.

Going to higher orders new operators are allowed by the symmetry and consequently new constants enter into the game.

NB F=pseudoscalar decay constant, Σ=quark condensate (both in the chiral limit), θ=vacuum angle

3 p-regime ($LM\pi \gg 1$)

The only changes with respect to infinite volume concern the propagator, integrals are replaced by sums.

Power counting: $\quad m_v \approx m_s \approx p^2 \quad p = \frac{2\pi n}{L}$

The left charge correlator is:

$$C_v(x_0) = \frac{\tilde{F}^2 \tilde{M}^2_{vv}}{2} \frac{\cosh[\tilde{M}_{vv}(T/2 - |x_0|)]}{2\tilde{M}_{vv} \sinh[\tilde{M}_{vv} T/2]} + VE \quad M^2_{ab} = \frac{2\Sigma}{F^2}(m_a + m_b)$$

-the time dependence is like at LO, apart from Volume Effects

-NLO corrections are encoded in the renormalization of F and M, plus Volume Effects, which are exponentially suppressed in LM_π

- $\tilde{M}^2_{vv}$ and $\tilde{F}$ depend on NLO coupling constants

4 ε-regime ($LM\pi \leq 1$)

One adopts the following power counting:

$$m_v \approx m_s \approx \varepsilon^4 \quad p \approx 1/L \approx \varepsilon$$

-since the zero mode contribution to propagator $M_\pi^{-2} V^{-1}$ is O(1), loops with zero modes circulating are not suppressed
-we need to factorize zero modes and integrate them Non Perturbatively

$$U = U_0 \exp\left(\frac{2i\xi}{F}\right) \quad U_0 \in SU(N) \quad \int dx\, \xi = 0$$

$$\mathcal{Z} \cong \int_{SU(N)} dU_0 e^{\frac{\Sigma V}{2} Tr\left[\mathcal{M} U + hc\right]} \int d\xi e^{\frac{1}{2}\int dx\, \mathrm{Tr}[\partial_\mu \xi \partial_\mu \xi]}$$

-gluon configurations with non trivial topology ν are suppressed at very small volumes (ν is canonically conjugate to θ). This is an advantage: quantities evaluated at different ν are independent, while in p regime they are quite the same for low ν
-to fix ν extend the domain of NP integration to U(N): $\quad \mathcal{Z}_\nu = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-i\nu\theta} \mathcal{Z}(\theta)$

-advantage: even at NLO $C(x_0)$ only depend on θ, Σ, F
-the factorization leads to a non trivial jacobian at NLO with respect to the Haar measure of U(N)

5 mixed-regime (full theory)

Defined by power counting: $p \approx 1/L \approx \varepsilon \quad m_v \approx \varepsilon^4 \quad m_s \approx \varepsilon^2$

Pions associated to $SU(N_v)$ have masses $O(\varepsilon^4)$:

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix} \exp\left(\frac{2i\xi}{F}\right) \quad U_0 \in \mathrm{SU}(N_v) \quad \int dx\, \mathrm{Tr}[T^a \xi] = 0 \quad T^a \in su(N_v)$$

We calculated the Jacobian associated with this factorization.

We calculated the left charge correlator at NLO. The result looks like the ε-regime one for N_v degenerate quarks apart from:

-a renormalization of F (a decoupling effect: depends on m_s)

-exp. suppressed VE (the sea quarks are in the p-regime)

$$C_v(x_0) = \frac{\tilde{F}^2}{2T}\left[1 + N_v f(L) + \frac{2T^2 \Sigma m_v}{F^2} h_1\left(\frac{x_0}{T}\right)\sigma_v(M)\right] + N_s g(LM_{ss}, \mathrm{L})$$

-σ_v comes from the NP integration:

$$2N_v \sigma_v(\mathcal{M}) = \int_{U(N_v)} dU_0\, \mathrm{Tr}[U_0 + \mathrm{h.c.}](\det U_0)^\nu \exp\left(\frac{\Sigma V}{2} \mathrm{Tr}[\mathcal{M} U_0 + \mathrm{h.c.}]\right)$$

-$\mathcal{Z}_\nu$ is known in terms of Bessel functions, correlation functions are obtained by putting appropriate sources and taking derivatives with respect to them

6 Partially Quenched ChPT (N_v quarks quenched)

Quenching a quark means discarding its internal loops. One can achieve this by adding N_v quarks of the wrong statistics (SUSY formalism):

$$\mathcal{Z}^{PQ} = \int [dA_\mu] \frac{\det(D_\rho \gamma_\rho + m_v + J)^{N_v}}{\det(D_\rho \gamma_\rho + m_v)^{N_v}} \det(D_\rho \gamma_\rho + m_s + J)^{N_s} e^{-S_g[A_\mu]}$$

At perturbative level the translation in the chiral effective theory consists of extending SU(N) to the supergroup $SU(N_v+N_s|N_v)$.

Otherwise just take the $N_v \to 0$ limit after the functional derivatives are taken (Replica Method):

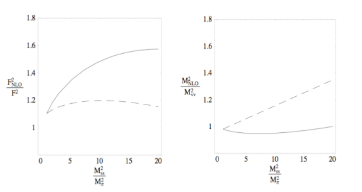
$$\mathcal{Z}_J = \int [dA_\mu] \det(D_\rho \gamma_\rho + m_v + J)^{N_v} \det(D_\rho \gamma_\rho + m_s)^{N_s} e^{-S_g[A_\mu]}$$

$$\langle \dots \rangle = \lim_{N_v \to 0} \frac{\delta^{\dots}}{\delta \dots} \mathcal{Z}_J$$

The NP integrals met have to be performed over the SUSY group $Gl(N_s+N_v|N_v)$ in both methods.

7 p & ε-regimes in PQ theory

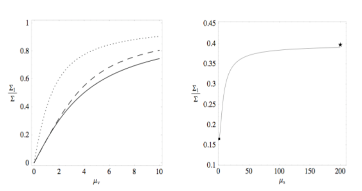
In the p regime it is trivial to take the limit $N_v \to 0$.



Dashed: PQ, $N_s=N_v=2$
Full: Full, $N_v+N_s=2$, $m_s=m_v$

NB: PQ with $m_s=m_v$ equivalent to Full with with $m_s=m_v$ $m_s=m_v$ and $N=N_s$ (S&S conjecture)

In the ε regime the change in σ_v is non trivial.



LEFT-Dashed: PQ, $N_s=N_v=2$
Full: Full, $N_v+N_s=2$, $m_s=m_v$
Dotted: Q
NB: non trivial test of S&S
RIGHT- PQ tends to Q when m_s decouples

8 mixed regime in PQ theory

The limit $N_v \to 0$ is well defined for the left correlator.
In general: NO! Consider the generator T^η:

$$T^\eta = \sqrt{\frac{N_v N_s}{2N}} \mathrm{diag}\{\underbrace{\frac{1}{N_v}, \dots, \frac{1}{N_v}}_{N_v}, \underbrace{-\frac{1}{N_s}, \dots, -\frac{1}{N_s}}_{N_s}\} \quad M^2_\eta = \frac{N_v M^2_{ss} + N_s M^2_{vv}}{N}$$

So: $\boxed{N_v \neq 0 \Rightarrow M^2_\eta \approx O(\varepsilon^2) \quad N_v = 0 \Rightarrow M^2_\eta \approx O(\varepsilon^4)}$

It's known that the SU(N) singlet cannot be decoupled in QChPT.
Here T^η plays a similar role in PQChPT being a $SU(N_v)$ singlet.
We need a different factorization:

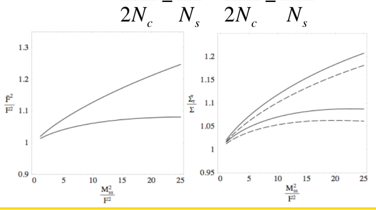
$$U = \begin{pmatrix} U_0 e^{\frac{2i\eta}{N_v F}} & 0 \\ 0 & e^{-\frac{2i\eta}{N_s F}} \end{pmatrix} \exp\left(\frac{2i\xi}{F}\right) \quad \int dx\, \mathrm{Tr}[T^a \xi] = 0 \quad T^a \in su(N_v) \cup T^\eta$$

If θ is absorbed in m_s $\mathcal{M} = \begin{pmatrix} m_v & 0 \\ 0 & m_s e^{\frac{i\theta}{N_s}} \end{pmatrix}$ we showed that the θ−η behaves as a quantity of O(ε). Consequently at NLO the faxtorization zero VS non zero modes occurs like in the full theory case.

9 Decoupling in the PQ mixed regime

The sea quarks cannot decouple because the η remains light.
However we should be able to match to a $SU(N_v)$ Quenched theory
If m_0 is the singlet mass and α the relative field strength:

$$\frac{\alpha}{2N_c} = \frac{1}{N_s} \quad \frac{m_0^2}{2N_c} = \frac{M^2_{ss}}{N_s}$$



LEFT- $\tilde{F}^2$ do not change with quenching.

RIGHT- Dash:PQ,Full:Full

Dependence on m_s of renormalized quantities

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Acknowledgements:
F.B. is supported by the FPU grant AP2005-5201. This work has been supported in part by Spanish CiCYT (Project No. FPA2004-00996 and FPA2005-01678), by Generalitat Valenciana (Project No. GRUPOS03-13), by EU (MRTN-CT-2006-035482 (FLAVIAnet)) by and the integrated grant HA2005-0066.