# Light-Cone Distribution Amplitudes from Lattice QCD

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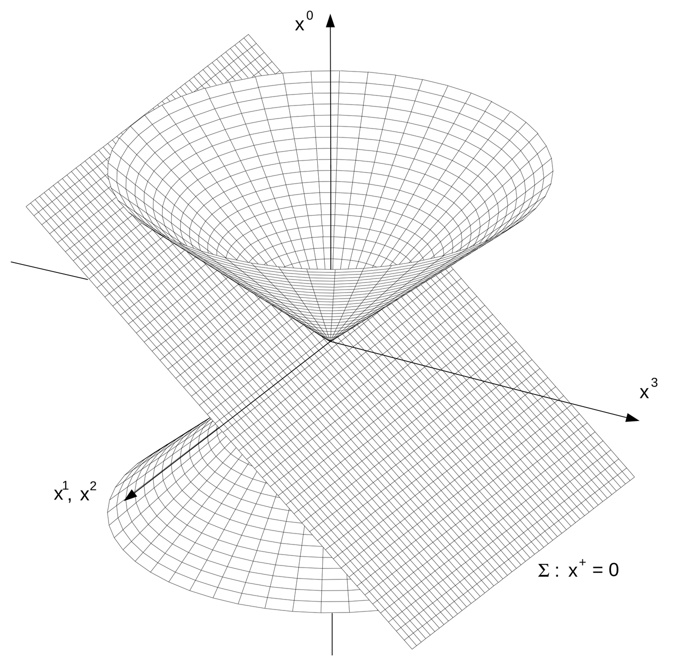
## 1 Light-Cone Quantization

- A motivation for, and interpretation of, Light-Cone Distribution Amplitudes is provided by the formalism of **Light-Cone Quantization** (LCQ) [1].
- In LCQ, we introduce **light-cone coordinates**:

$$x^{\pm} \equiv x^0 \pm x^3 \qquad x_{\perp} = (x^1, x^2).$$

Canonical commutation relations are then defined **on a null-plane** $x^+ = 0$, contrary to the usual Equal-Time Quantization (ETQ):




- Of the few sensible choices for a time parameter in Hamiltonian dynamics, this choice **maximises the number of Lorentz Group generators (7/10) that are kinematical** (don't involve interactions).
- Creation and annihilation operators obtained by projection onto a null-plane create and destroy different states than their ETQ counterparts - so LCQ implies a different **choice of Fock basis for the QFTs Hilbert space**. The relation between the LCQ and ETQ bases involves the full dynamics of the QFT $\Longrightarrow$ complicated enough to be useful.

*Light-Cone Quantization allows us to formulate QCD such that it looks as much as possible like the quark-parton model.*

## 2 Light-Cone Wavefunctions

- The LCQ **Fock vacuum *is* the physical vacuum state** $\Longrightarrow$ Fock-state expansion now tractable.
- LCQ hadronic wavefunctions can be interpreted unambiguously: all quanta in a hadron's wavefunction are **directly connected to that hadron** rather than to vacuum fluctuations.
- As some boosts are kinematical, once the wavefunction is known in one inertial frame it is easily obtained in any other. Wavefunctions depend only on **'internal' longitudinal momentum fractions** $x_i \equiv \frac{k_i^+}{p^+}$.
- Defining the **invariant mass** operator:

$$H_{LC} = P^- P^+ - P_{\perp}^2$$

in **light-cone gauge** $A^+ = 0$, for a hadron H we can write:

$$H_{LC}|H\rangle = M_H^2|H\rangle$$

- As $P^+$, $P_{\perp}$ are conserved, we can construct the matrix elements $\langle n|H_{LC}|m\rangle$ on a complete set $\{|n\rangle\}$ of **eigenstates of the free Hamiltonian** $H_{LC}^0 = H_{LC}(g = 0)$.
- **Hadronic wavefunctions** are then defined by:

$$\langle n; x_i, \mathbf{k}_{\perp i}, \lambda_i|\psi_H\rangle = \psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$$

for example, for the proton:

$$|p\rangle = \sum_n \langle n|p\rangle|n\rangle = \psi^{(\Lambda)}_{3q/p}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|uud\rangle + \psi^{(\Lambda)}_{3qg/p}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|uudg\rangle + \ldots$$

*Light-Cone Wavefunctions are universal, process-independent, frame-independent amplitudes encoding all possible quark and gluon correlations.*

## 3 Light-Cone Distribution Amplitudes

- The Light-Cone Wavefunctions **unify the description of inclusive and exclusive reactions** and provide a physical factorization scheme. Other non-perturbative QCD quantities can be related to them, e.g., the parton distribution functions (pdfs) of Deep-Inelastic Scattering:

$$q(x, Q^2) = \sum_n \int d^2k_{\perp} \sum_{\lambda} |\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)|^2$$

- Another non-perturbative quantity is the **Distribution Amplitude** (DA). For the pion, $\phi_\pi$ is defined by the hadronic light-cone matrix element:

$$\langle 0|\overline{q}(z)\gamma_\rho\gamma_5\mathscr{P}(z,-z)q(-z)|\pi(p)\rangle|_{z^2=0} \equiv f_\pi(ip_\rho)\int_0^1 du\, e^{i(u-\bar{u})p.z}\phi_\pi(u,\mu)$$

- DAs arise in **hard-exclusive processes** to which **collinear factorization** theorems apply. For example, at large $Q^2$ the pion's electromagnetic form-factor $F_\pi(Q^2)$ separates into a hard-scattering kernel $T_H$, and the distribution amplitude $\phi_\pi$[2]:

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\ \phi_\pi(y, Q^2)T_H(x, y, Q^2)\phi_\pi(x, Q^2)$$

- The process is dominated by the **lowest (valence) Fock state**, as all partons must be turned to the final direction.
- The relation to the light-cone wavefunction is then quite intuitive:

$$\phi_\pi(x, Q^2) \equiv \int^{Q^2} d^2k_{\perp}\psi_{q\bar{q}/\pi}(x_i, \mathbf{k}_{\perp i}, \lambda)$$

- DAs are useful in flavour physics too, since collinear factorization has been shown to apply (to leading order in $1/m_b$) to 2-body **nonleptonic** $B$ **decays**.
- DAs also play a crucial role in the **Soft Collinear Effective Theory** (SCET)

*Pdfs tell us about the partonic content of hadrons, but as single-particle probabilities they say nothing about correlations between quarks and gluons and are insensitive to the roles of different Fock states. Distribution amplitudes, however, really tell us about hadronic structure at the amplitude level.*

## 4 Moments and the Lattice

- The main tool for studying DAs has been **QCD Light-Cone Sum Rules**. In the past, the **moments** of DAs were studied:

$$\langle \xi^n \rangle = \int d\xi \xi^n \phi(\xi, Q^2)$$

- Today, it is the *Gegenbauer* moments (based on the **conformal expansion**) that are of interest:

$$\phi(x, Q^2) = \phi_{as}(x)\left[1 + \sum_n a_n(Q^2)C_n^{3/2}(2x-1)\right]$$

where the asymptotic DA $\phi_{as}$ is known:

$$\phi_{as}(x) \overset{Q^2\to\infty}{=} 6x(1-x)$$

- For the lower moments, the two are simply related, e.g.: $a^1 = \frac{5}{3}\langle\xi^1\rangle$
- We can obtain moments **on the lattice** via the following hadronic matrix elements (for the pseudoscalar mesons), which can be evaluated using only 2-point functions:

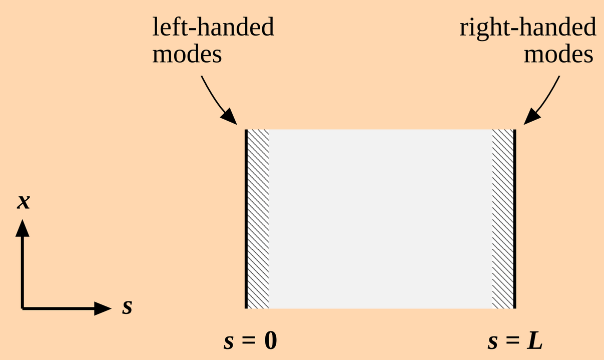
$$\langle 0|\bar{q}\gamma_\rho\gamma_5\overleftrightarrow{D}_\mu q|\pi(p)\rangle = f_\pi\langle\xi\rangle p_\rho p_\mu$$

$$\langle 0|\bar{q}\gamma_\rho\gamma_5\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu q|\pi(p)\rangle = f_\pi\langle\xi^2\rangle p_\rho p_\mu p_\nu$$

- Until quite recently, there were just a few exploratory studies on the lattice for the pseudoscalar mesons.

## 5 Domain-Wall Fermions and QCDOC

- By **introducing a fifth-dimension** we can separate the right- and left-handed fermions, which become exponentially bound to separate domain walls [3].
- With the fifth-dimension infinite, the **Ginsparg-Wilson relation** is satisfied: chiral symmetry is preserved exactly at finite lattice spacing and the fermion action is $O(a)$-improved.
- With a finite fifth-dimension, the chiral properties are tunable, with the symmetry-breaking parameterized by the **residual mass** $m_{res}$ [4].




- DWF are still costly to simulate. The UKQCD/RBC Collaborations use the custom-designed **QCDOC supercomputer** with the **RHMC algorithm**.

## 6 Results

- Recently, results have been obtained for $\langle\xi^1\rangle$ and $\langle\xi^2\rangle$ for the **pseudoscalar mesons** [5, 6], including $\langle\xi^1\rangle_K$ which is an SU(3)-breaking effect.
- The UKQCD results used lattices with the Iwasaki gauge action and 2+1 flavours of DWF. There are two volumes, $16^3\times32(\times16)$ and $24^3\times64(\times16)$, with $a^{-1}$ = 1.6GeV. The quark masses are: $am_s = 0.04$, $am_{u/d} = 0.01, 0.02, 0.03$. Renormalization has so far been carried out perturbatively, but non-perturbative renormalization using the RI/MOM method is in progress.
- Results on the larger lattices for the pseudoscalars and results for the longitudinal DAs of the **light vector mesons** are forthcoming.

## References

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