Progress in Calculating Muon Anomalous Magnetic Dipole Moment on The Lattice

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Background & Motivation:

The magnetic dipole moment of muon is one of the most precisely measured and calculated quantities in elementary particle physics. It has been measured to a precision of 0.54 parts per million (ppm) at Brookhaven National Lab (BNL) [1–6]. The measured quantity has reached a comparable level to the Standard Model prediction. The sensitivity between the theoretical and measured quantities can be attributed to possible new physics such as Super-Symmetry (SUSY).

The magnetic dipole moment of muon of mass m and charge e is given by

$$\vec{m} = g\left(\frac{e}{2m}\right)\vec{S} \qquad (1)$$

where g and $\vec{S}$ are the gyromagnetic ratio and muon spin respectively. Quantum Electro-Dynamics (QED) predicts g = 2 at the tree level for an elementary spin-1/2-particle such as muon. Quantum corrections from QED loops diagrams, from strong or weak interactions, or from possible new physics lead to a contribution of

$$a = \left(\frac{g-2}{2}\right) \qquad (2)$$

which is called the anomalous magnetic dipole moment. The theoretical prediction of one-loop QED contribution to the anomalous magnetic moment of a lepton is given by the well-known Schwinger term α/2π [7]. This dominant contribution is then also further subjected to higher-order QED and QCD (Quantum Chromo-Dynamics) corrections. The loop contributions from heavier particles with mass $M_{New\ Phys.}$ are suppressed by $m_\mu^2/M^2_{New\ Phys.}$, where m_μ and $M_{New\ Phys.}$ are muon and SUSY particle masses. Therefore, the anomalous magnetic dipole moment of muon is $(m_\mu/m_e)^2 \approx 40000$ times more sensitive to new physics than that of electron. The anomalous magnetic dipole moment has been measured by g − 2 experiment E821 with a great accuracy at BNL [1–6]

$$a_\mu(EXP) = 11659208\,(6.3)\times 10^{-10} \qquad (3)$$

The Standard Model Theory prediction given in [8] based on e^+e^- cross-section of hadronic contributions is

$$a_\mu(SM) = 11659184.1(7.2)^{Vac.Pol.}(3.5)^{QED/Weak}\times 10^{-10}$$
$$= 11659184.1\,(8.0)\times 10^{-10} \qquad (4)$$

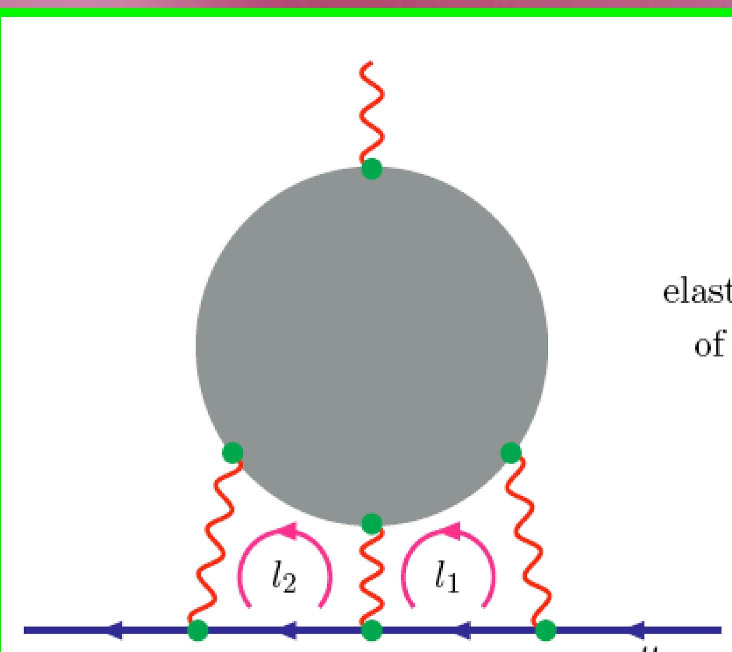
where superscripts correspond to error contributions due to vacuum polarization, light-by-light scattering, Quantum Electro-Dynamics and Weak interactions respectively. The difference between the SM prediction and experimental result is given by

$$\Delta(EXP - SM) = 23.9\;(9.9)\times 10^{-10} \qquad (5)$$

There is 2.4 standard deviation between theory and experimental results, which certainly demand the reduction of uncertainty attributed to QCD contribution. The hadronic light-by-light contribution is very difficult to evaluate. The current estimate varies between model calculations $8.6(3.5)\times 10^{-10}$ [9] to $13.6(2.5)\times 10^{-10}$ [10]. Evaluating the light-by-light contribution with greater accuracy might help reducing the discrepancy currently evident between the SM prediction and experimental result, which may give a hint to the underlying structures of possible new physics. In the next section, the outlines of our proposed research have been elaborated.

Proposed Research:

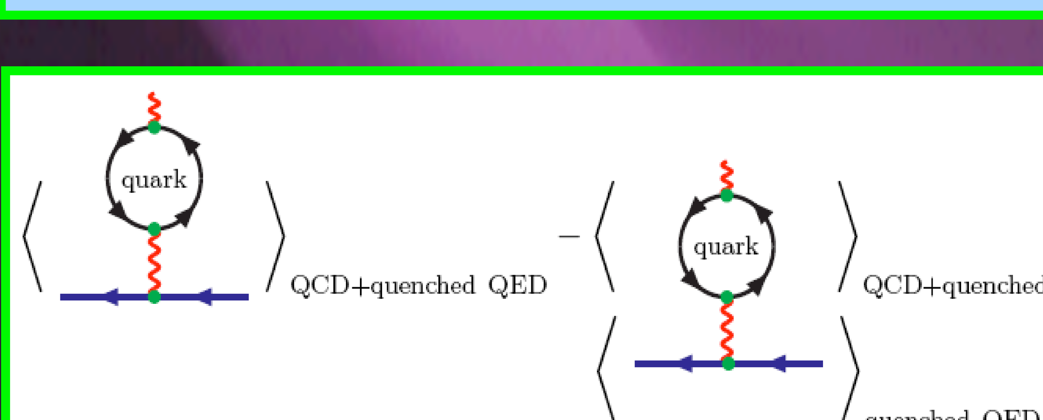
Our proposal here based on [11] is to evaluate one type of QCD contribution, the hadronic light-by-light scattering (h-lbl) contribution to the muon anomalous magnetic dipole moment depicted in Fig. 1, which gives rise to an $O(\alpha_{em}^3)$ contribution to muon g − 2. The diagram can be computed using the following naive approach; the four electro-magnetic currents are calculated repeatedly using lattice QCD techniques for two independent momenta l_1, l_2 of two off-shell photons, and are then integrated over l_1, l_2.



elastic scattering amplitude of two photons by QCD

Figure 1

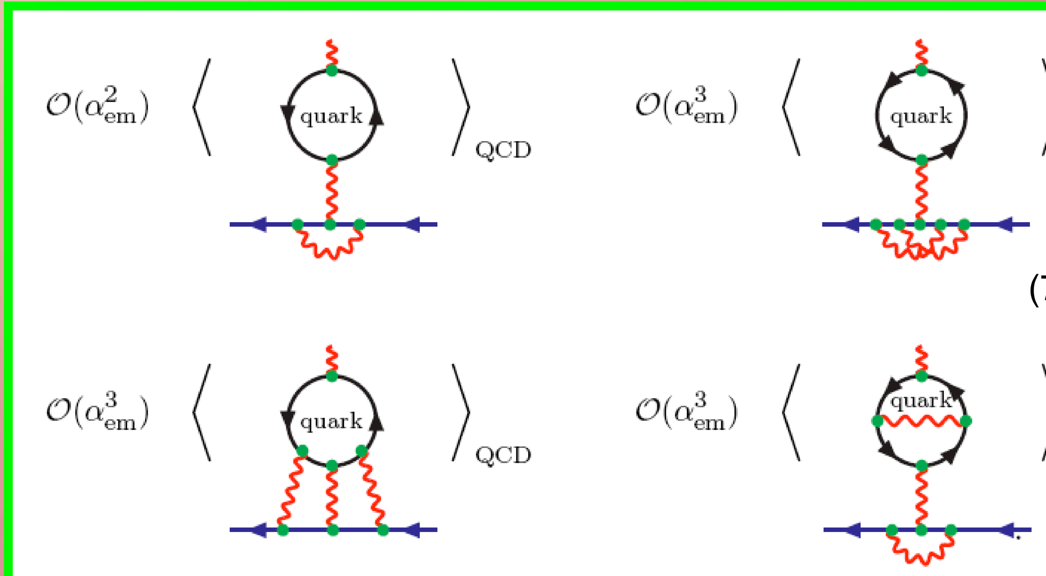
The direct lattice calculations of the above diagram are notoriously difficult. Hence, the following approach is being proposed to calculate the light-by-light contribution through the use of combined (QED + QCD) lattice simulation. The red line in Eq. (6) is the free photon propagator $D_{\mu\nu}(x,y)$ in the non-compact lattice QED solved in an appropriate gauge fixing condition. The black line corresponds to the full quark propagator $S_f(x,y;U,u)$ for a given set of $SU(3)_C$ gauge configuration $\{U_{x,\mu}\}$ and $U(1)_{em}$ gauge configuration $\{u_{x,\mu}\}$. The sum over relevant flavors f is implicitly assumed. The blue line denotes the full muon propagator s(x, y; u). The average <, > means the one over the unquenched $SU(3)_C$ and/or the quenched $U(1)_{em}$ gauge configurations, which are specified by the corresponding subscripts. Eqs. (6) and (7) describe our proposed methodology in order to successfully extract the desired observables.



(6)

Let us look at the first term of Eq. (6) perturbatively with respect to QED in order to explain the underlying mechanism regarding the proposed method. Its magnetic components up to $O(\alpha_{em}^3)$ consist of Feynmann diagrams depicted in Eq. (7). The left diagram in the first line gives the $O(\alpha_{em}^2)$ -contribution. The $O(\alpha_{em})$-corrections to its muon part and to its quark part induce $O(\alpha_{em}^3)$-contributions shown in the right diagram on the first line as well as on the second line respectively. We recall that the QED gauge configurations in the first term of Eq. (6) are commonly shared by the quark part and the muon part. Hence, the photons can be exchanged between the two parts. As a consequence, the left diagram in the second line of Eq. (7) is induced at $O(\alpha_{em}^3)$ which takes the form of our target, Fig. 1. Alternatively, the quark and muon parts in the first and third diagrams in Eq. (7) are connected only by a single photon attached a priori. The second term in Eq. (6) also contains those extra diagrams. Thus, by subtracting the second term from the first term, we may extract the h-lbl contribution.

The quantities evaluated in our method (6) are constructed from two currents for both terms, which are surely less noisy than the case of four currents encountered in the naive approach. The extremely high degree of correlation between first and second terms of (6) makes sure that the proposed method may work. In next section, the progress in our research in calculating these observables has been discussed.



(7)

Research in Progress:

Our goal at the moment is to successfully reproduce the Schwinger term (α/2π) non-perturbatively. In order to accomplish this objective, we have been doing several calculations of the muon two and three point correlation functions to extract the form factors, F1 and F2. Domain Wall Fermions (DWF) are used in these simulations.

Our numerical studies are carried out at QCDOC (Quantum Chromo-Dynamics On-a-Chip) supercomputers at Columbia University and Brookhaven National Lab. The QCDOC architecture has been designed to provide a highly cost-effective, massively parallel computing resources for extremely demanding problems.

Our initial efforts centered on just obtaining a statistically significant signal. We realized that we had to extend the time size of the lattice to 64 from 32 so that a suitable plateau could be observed. That was successful in the context that we had been able to observe suitable plateaus (see Fig. 2), at least for the first five or six non-zero momenta of the injected photon at QED vertex.

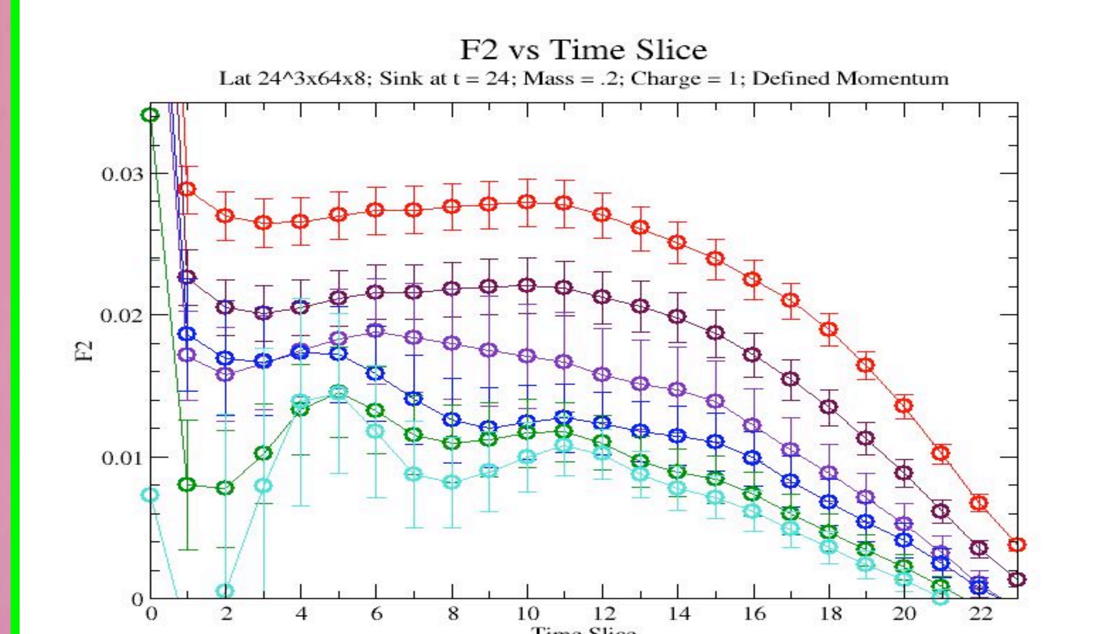


Figure 2

In order to explore the small momentum region, we implemented twisted boundary conditions with a small additive momentum (p = θ/L) where theta is the twist. We tried the simulations with θ = 0.5, 3 and 5. It does seem that the twist is working properly in the context that the measured momentum from the two point function is consistent with the input value of theta.

All the above discussions are basically centered on extracting F2 since F1 is very trivial to calculate. The signal for F2 at the lowest value of transferred momentum, q = θ/L, where θ is 0.5, is seen to be roughly approaching the right order of magnitude compared to expectation from the known continuum value, α/2π with the increasing lattice volume. All of our calculations have been carried out with un-physical charge, e = 1. Since we are not at q^2 = 0, our results may differ due to momentum dependence of the form factor. In reality, F2 is extrapolated to q^2 = 0 limit.

We showed the results coded up with local current in the next section. Since the Ward identity is not satisfied for the local current, it became important for us to use conserved current to eliminate the contaminations of extra terms arising from non-conserved local current. Right now, our results from conserved current are in progress.

Results:

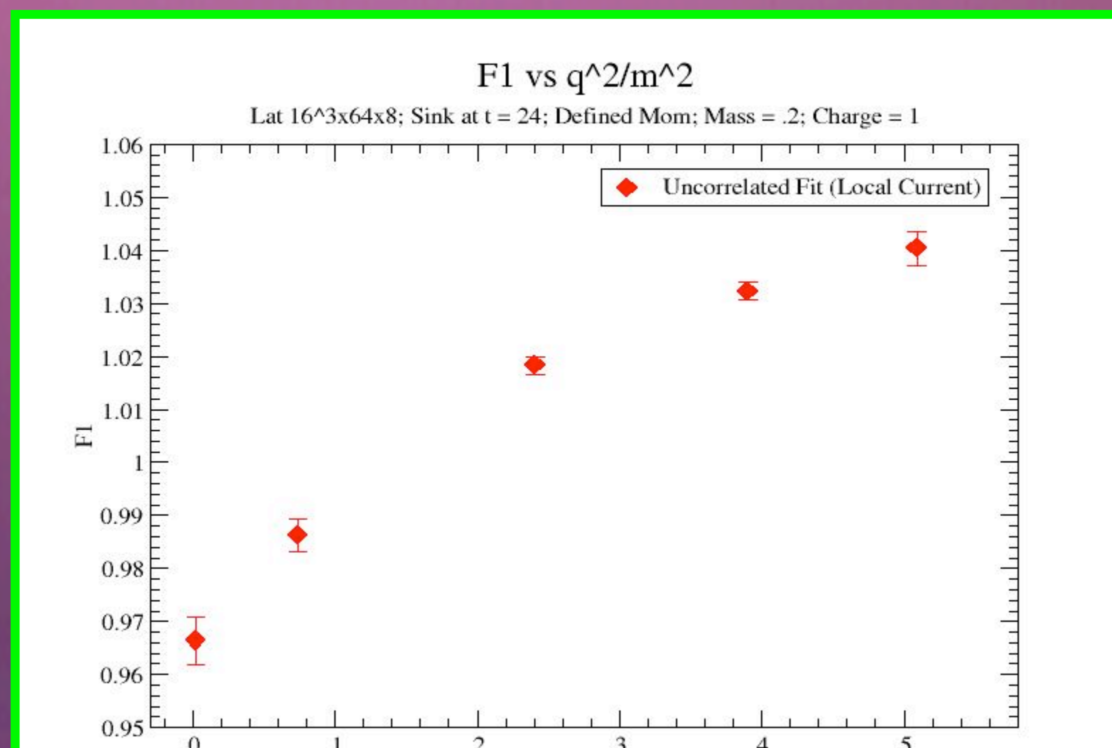


Figure 3

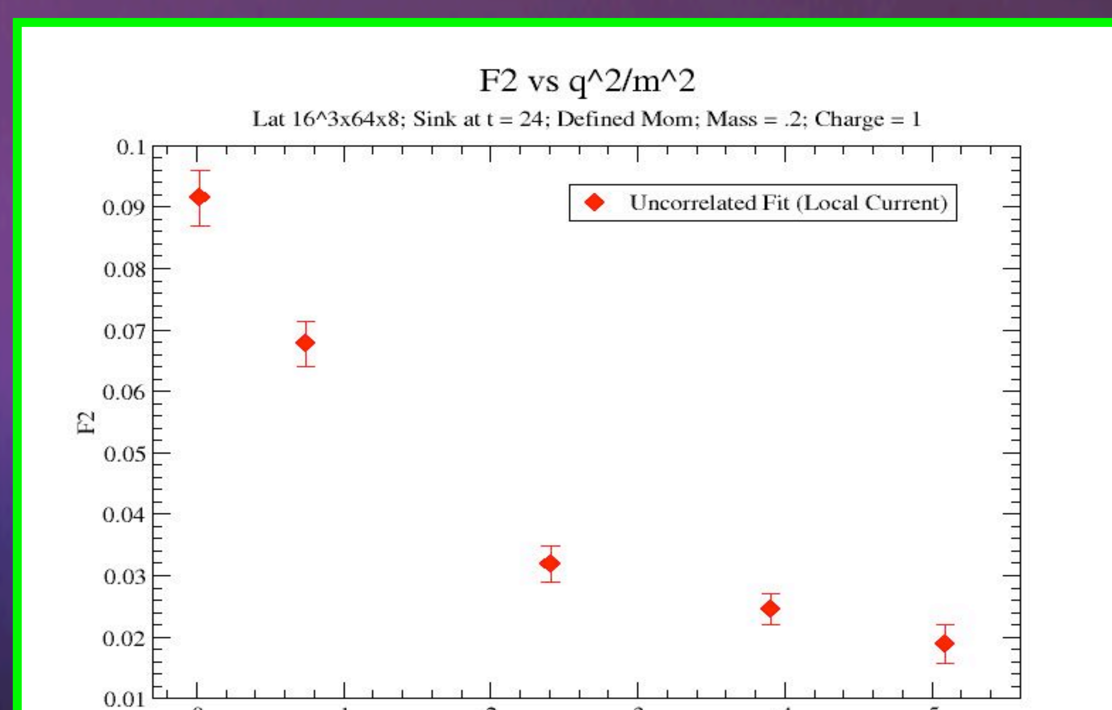


Figure 4

In Fig. 3 and Fig. 4, F1 and F2 have been plotted as a function of q^2/m^2 for 16^3x64 lattices, and it shows that F1 is in the order of 1 as expected though it is slowly increasing as q^2/m^2 increases. On the other hand F2 increases rather faster especially for smaller momentum region. F1 & F2 have been fitted into a constant linear fit with the off-diagonal elements of the covariance matrix set to zeros.

In Fig. 5 and Fig. 6, F1 and F2 have again been plotted as a function of q^2/m^2 for 24^3x64 lattices, and it shows that F1 is in the order of 1 as expected, but is slowly increasing as q^2/m^2 increases. F2 also increases as q^2/m^2 decreases. It is quite evident from the results that with increasing volume, F2 is approaching to the Schwinger expectation (i.e. α/2π).

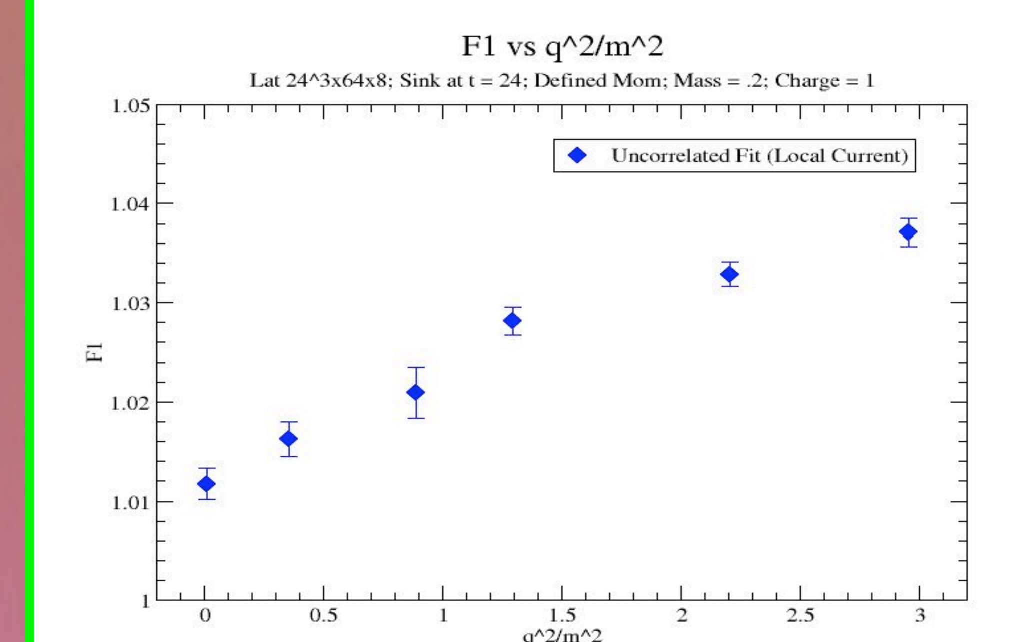


Figure 5

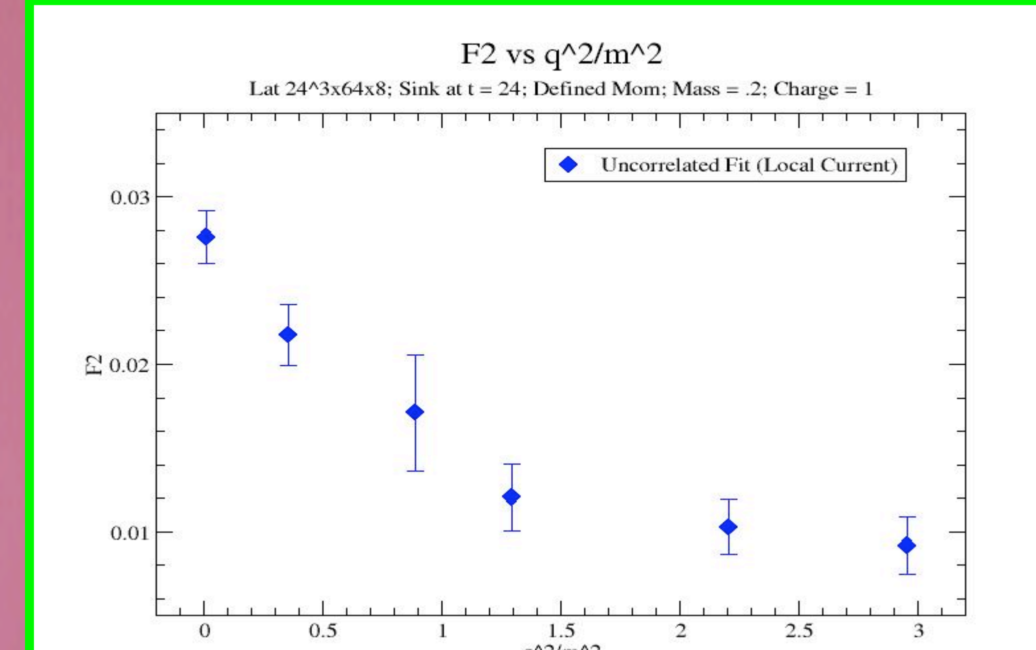


Figure 6

Finally, let me briefly elaborate on the issue of lattice artifacts. It is clear from [12] that at tree level, the three point functions suffer order (am)^2 and (a^2mp) corrections which can be read off the equations for the electromagnetic form factors of [12]. Solving those equations for F1 and F2, it was found that F2 becomes negative and roughly momentum independent. It was also realized that F2 tends to zero as m → 0. This is consistent with our data. After running the job with smaller m = 0.1, an increase in F2 was found in the positive direction. We also need to do more careful tree level analysis to be positive about this notion. In any case, it seems clear that there is a lattice artifact that needs to be controlled to get the continuum limit since these effects are on the same order as the term we are looking for ($O(\alpha_{em})$), or even larger (m = 0.1 or 0.2) in our simulations. Note, we can't take m too small because then we run into finite volume effects. One option to remove these artifacts is to add the clover term to the DWF action as described in [12]. Since we are looking for order alpha effects, such a complication may be unavoidable.

Outlook:

We are hoping to rectify the obstacles to produce the Schwinger term (α/2π) successfully non-perturbatively in the near future. As soon as we are confident about our results, we will get into calculating our target diagram in Fig. 1. Our aim is to first insert fermionic loop (i.e. pure QED calculation) for the injected photon at the vertex before incorporating more harder and tedious hadronic loop. Hopefully, we will be able to successfully determine the h-lbl contribution in the future to investigate whether any underlying new physics such as Super-Symmetry can be revealed. Even if there is no SUSY, it is still very intriguing and challenging task to shrink the gap between existing theoretical (SM) prediction and observed experimental result.

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