Effective Field Theory for Anisotropic Wilson Lattice

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Roadmap for Lattice Spacing Corrections

- Analysis for Mesons at $\mathcal{O}(a)$ and $\mathcal{O}(a^2)$ done by O.Bar, G.Rupak, and N.Shoresh (2002)

- Analysis for Baryons at $\mathcal{O}(a)$ done by S.R. Beane and M.J. Savage (2003)

- Analysis for Baryons at $\mathcal{O}(a^2)$ done by B.C. Tiburzi (2005)

Why the Anisotropic Lattice?

- Greater time resolution without incurring the cost of greater space resolution and vice versa

- Allows us to probe high lying excited states (inverse time spacing of 6 GeV)

- Can give more intermediate states that result in better data interpretation

Anisotropic Wilson Lattice Action

ξ_0 is the bare anisotropy
P_{ij} is the space-space plaquette
P_{ti} is the space-time plaquette
U represents the gauge links

Plaquette Action

$$S_W^\xi = \beta \sum_{n=t,i<j} \frac{1}{\xi_0} P_{ij}(U) + \beta \sum_{n=t,i} \xi_0 P_{ti}(U)$$
$$+ \sum_n \bar{\psi}_n \left[a_t m_0 + W_t(U) + \frac{\nu}{\xi_0} W_i(U) \right] \psi_n - \bar{\psi}_n \left[c_t \sigma_{ti} \hat{F}_{ti}(U) + \sum_{i<j} \frac{c_r}{\xi_0} \sigma_{ij} \hat{F}_{ij}(U) \right] \psi_n$$

Quark Action

m_0 is bare mass
$W_t(U)$ is the time Wilson Lattice derivatives
$W_s(U)$ is the space Wilson Lattice derivatives
ν is a parameter tuned to correct the speed of light

Clover Improvement

At tree level:
$c_t = \frac{1}{2}\left(\frac{1}{\xi} + \nu\right)$
$c_r = \nu$
$\xi = \frac{a_s}{a_t}$

- m_0, ξ_0, and ν are non-perturbatively determined.

Anisotropic Symanzik Action

$$S^{\xi}_{Sym} = \int d^4x \ \mathcal{L}^{\xi}_{Sym}$$

$$\mathcal{L}^{\xi}_{W} = \mathcal{L}^{\xi(4)}_{W} + a_s \mathcal{L}^{\xi(5)}_{W} + a_s^2 \mathcal{L}^{\xi(6)}_{W}$$

The unimproved $\mathcal{O}(a)$ Lagrangian (with ν tuned to remove $O(4)$ breaking in continuum limit) is:

$$\mathcal{L}^{\xi}_{W} \quad = \quad \bar{q}[\not{D} + m_q]q + a_s c_{SW} \bar{q}\sigma_{\mu\nu}F_{\mu\nu}q + a_s c^{\xi}_{SW} \bar{q}\sigma_{ti}F_{ti}q$$

- In isotropic limit (when $a_s = a_t = a$), $c^{\xi}_{SW} = 0$, which removes the anisotropic term.
- With perfect clover improvement, $c_{SW} = c^{\xi}_{SW} = 0$, which leads to first lattice spacing effects at $\mathcal{O}(a^2)$.
- As another notational convenience, we define $u^{\xi}_{\mu} = (1, \mathbf{0})$. Therefore, a term with $\sigma_{ti}F_{ti}$ can be written as $u^{\xi}_{\mu}u^{\xi}_{\nu}\sigma_{\mu\nu}F_{\mu\nu}$

Spurion Analysis for Anisotropic Terms

Isotropic

$\mathcal{O}(a): \quad (a_s \bar{q} c_{SW} \sigma_{\mu\nu} F_{\mu\nu} q)$

Symm: Breaks $SU_L(2) \otimes SU_R(2)$ chiral symmetry and obeys $O(4)$ symmetry.

Promote $a_s c_{SW}$ to spurion

$$a_s c_{SW} \rightarrow L(a_s c_{SW}) R^\dagger$$

$$(a_s c_{SW})^\dagger \rightarrow R(a_s c_{SW})^\dagger L^\dagger$$

Set spurion to $a_s c_{SW} \mathbb{I}$
to explicitly break the chiral symmetry

Anisotropic

$\mathcal{O}(a): \quad (a_s \bar{q} c^\xi_{SW} u^\xi_\mu u^\xi_\nu \sigma_{\mu\lambda} F_{\nu\lambda} q)$

Symm: Breaks $SU_L(2) \otimes SU_R(2)$ chiral symmetry and breaks $O(4) \rightarrow O(3)$.

Promote $a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu$ to spurion

$$a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu \rightarrow L(a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu) R^\dagger$$

$$(a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu)^\dagger \rightarrow R(a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu)^\dagger L^\dagger$$

Additionally:
$a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu \rightarrow a_s c^\xi_{SW} u^\xi_\rho u^\xi_\sigma \Lambda_{\mu\rho} \Lambda_{\nu\sigma}$
where $\Lambda_{\mu\nu}$ represents
an $O(4)$ transformation.

Set spurion to $a_s c^\xi_{SW} u^\xi_\mu u^\xi_\nu \mathbb{I}$
to explicitly break the chiral
and $O(4)$ symmetry

Spurion Analysis for Anisotropic Terms (cont.)

- $\mathcal{O}(a^2)$ terms follow the same process.

Example: $(\bar{q}\sigma_{\mu\nu}q)^2$ → $(\bar{q}\sigma_{\mu\nu}q)^2 + u^\xi_\mu u^\xi_\nu (\bar{q}\sigma_{\mu\lambda}q)(\bar{q}\sigma_{\nu\lambda}q)$

Isotropic | Anisotropic

- New at $\mathcal{O}(a^2)$:

$\bar{q}\gamma_\mu D_\mu D_\mu D_\mu q$ | $\bar{q}\gamma_i D_i D_i D_i q$

Isotropic
Breaks O(4)

Anisotropic
Breaks O(3)

Chiral Perturbation Theory

From the spurion analysis, the resulting meson χPT is:

$$\Sigma = \exp\left(\frac{2i\Phi}{f}\right)$$

$$\mathcal{L}_\phi^\xi = \frac{f^2}{8}\mathrm{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right) - \frac{f^2}{4}\mathrm{tr}\left(m_q\mathbb{B}_0\Sigma^\dagger + \Sigma(m_q\mathbb{B}_0)^\dagger\right) - \frac{f^2}{4}\mathrm{tr}\left(a_s\mathbb{W}_0\Sigma^\dagger + \Sigma(a_s\mathbb{W}_0)^\dagger\right) - \frac{f^2}{4}\mathrm{tr}\left(a_s\mathbb{W}_0^\xi\Sigma^\dagger + \Sigma(a_s\mathbb{W}_0^\xi)^\dagger\right)$$

$\frac{f^2}{8}\mathrm{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right) - \frac{f^2}{4}\mathrm{tr}\left(m_q\mathbb{B}_0\Sigma^\dagger + \Sigma(m_q\mathbb{B}_0)^\dagger\right)$	Continuum χPT
$-\frac{f^2}{4}\mathrm{tr}\left(a_s\mathbb{W}_0\Sigma^\dagger + \Sigma(a_s\mathbb{W}_0)^\dagger\right)$	Isotropic $\mathcal{O}(a)$
$-\frac{f^2}{4}\mathrm{tr}\left(a_s\mathbb{W}_0^\xi\Sigma^\dagger + \Sigma(a_s\mathbb{W}_0^\xi)^\dagger\right)$	Anisotropic $\mathcal{O}(a)$

$$\mathbb{B}_0 = \lim_{m_q\to 0}\frac{|\langle\bar{q}q\rangle|}{f^2} \qquad \mathbb{W}_0 = \lim_{m_q\to 0} c_{SW}\frac{|\langle\bar{q}\sigma_{\mu\nu}F_{\mu\nu}q\rangle|}{f^2} \qquad \mathbb{W}_0^\xi = \lim_{m_q\to 0} c_{SW}^\xi u_\mu^\xi u_\nu^\xi \frac{|\langle\bar{q}\sigma_{\mu\lambda}F_{\nu\lambda}q\rangle|}{f^2}$$

- Anisotropic terms that do not break any additional isotropic symmetries (like the one above) are difficult to differentiate from their isotropic partner.

- Anisotropic terms that do break additional symmetries first appear at $\mathcal{O}(ap^2)$ for meson χPT. Terms like this will lead to corrections in the dispersion relation.

Example: $W_2^\xi\mathrm{tr}\left(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger u_\mu^\xi u_\nu^\xi\left[a_s W_0^\xi\Sigma^\dagger + \Sigma(a_s W_0^\xi)^\dagger\right]\right)$

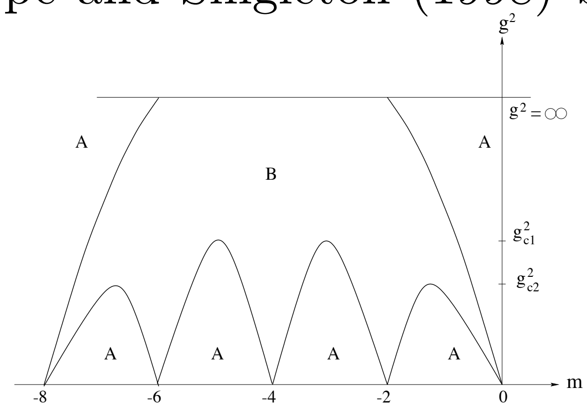
Altered Aoki Regime

• At finite lattice spacing, Aoki noticed regions in g^2 vs m_0 space that spontaneously breaks flavor and parity. When in this region, it results in two masless and one massive pion.

• This effect was extended to χPT by Sharpe and Singleton (1998) by investigating the vacuum state of the potential:

$$\mathcal{V}_\chi = -\frac{c_1}{4}\mathrm{tr}\left(\Sigma + \Sigma^\dagger\right) + \frac{c_2}{16}\left[\mathrm{tr}\left(\Sigma + \Sigma^\dagger\right)\right]^2$$

$$\mathcal{V}_\chi = -c_1 A + c_2 A^2$$



• Result: If minimum A_0 satisfies $-1 < A_0 < 1$, then we are in the Aoki regime. This can possibly occur when $c_1 \sim c_2$ and $c_2 > 0$.

• Since the anisotropic χPT introduces several new undetermined terms to both c_1 and c_2 (at higher order for improved action), the Aoki regime will be altered. This could result in the anisotropic theory in the Aoki regime when the isotropic limit is not, and vice versa.

Conclusion

What is new in the anisotropic Lattice?

- New terms in χPT that break $O(4)$ symmetry first appearing at $\mathcal{O}(a)$.
  - Lead to corrections in the dispersion relations.

- New boundaries of the Aoki regime in phase space.
  - May require additional tuning to remove effects.

For more information and detail, check out our paper on arXiv [hep-lat]: arXiv:0708.2254