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# Linking confinement to spectral properties of the Dirac operator

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## Motivation

Confinement and chiral symmetry breaking are two of the central features of QCD. At the QCD finite temperature transition chiral symmetry is restored and the theory deconfines. Numerical simulations in lattice QCD indicate that the critical temperature $T_c$ is the same for both transitions. Thus it is widely believed that there must be a mechanism linking the two phenomena. However, so far there is no generally accepted picture for such a link. For chiral symmetry breaking an important connection between the order parameter, the chiral condensate, and spectral properties of the Dirac operator is known. The Banks-Casher formula [1] links the chiral condensate to the density of Dirac eigenvalues at the origin. Concerning confinement, so far no signature in spectral properties of the Dirac operator is known. On the other hand it is obvious that such signatures must exist. The inverse Dirac operator, i.e., the quark propagator, clearly knows about confinement properties. In this poster we present an attempt to identify spectral signatures of the Dirac operator which are related to confinement.

## Theoretical framework

Our starting point are Polyakov loops on an euclidean lattice. A Polyakov loop is defined as the ordered product of temporal link variables at a fixed spatial position $\vec{x}$.

$$L(\vec{x}) = \text{Tr}_c \left[ \prod_{s=1}^{N} U_4(\vec{x}, s) \right],$$

where the $N$ denotes the number of lattice points in time direction and $\text{Tr}_c$ is the trace over color indices. Working in the lattice regularization, we express the Polyakov loop and its correlators as a spectral sum of eigenvalues and eigenvectors of the Dirac operator with different boundary conditions [2]. Here, we use the staggered Dirac operator,

$$D(\vec{x}, \vec{y}) = \frac{1}{2} \sum_{\mu=1}^{4} \eta_\mu(\vec{x}) \left[ U_\mu(\vec{x})\, \delta_{\vec{x}+\hat{\mu},\vec{y}} \;-\; U_\mu^\dagger(\vec{x}-\hat{\mu})\, \delta_{\vec{x}-\hat{\mu},\vec{y}} \right].$$

The hopping terms of the Dirac operator connect nearest neighbors. When powers of $D$ are considered, these terms combine to chains of hops on the lattice. Taking the $m$-th power will give rise to chains with a maximal length of $m$ steps. Furthermore, we set the two space-time arguments of $D$ to the same value, $\vec{y} = \vec{x}$, such that we pick up only closed loops. Among these are the loops where only hops in time direction occur, such that they close around the compact time.

$$\text{Tr}_c \left[ D^N(\vec{x}, t | \vec{x}, t) \right] = 2^{-N} L(\vec{x}) \;-\; 2^{-N} L^*(\vec{x}) \;+\; \text{other loops}$$

We now explore the fact that the Polyakov loops respond differently to a change of the boundary conditions compared to other, non–winding loops. We can change the temporal boundary condition of the Dirac operator by multiplying all temporal link variables at $t = N$ with some phase factor $z \in \mathbb{C}$, $|z| = 1$, $U_4(\vec{x}, N) \to z\, U_4(\vec{x}, N)$ for all $\vec{x}$. Now using the Dirac operator in the transformed field, denoted as $D_z$, we obtain:

$$\text{Tr}_c \left[ D_z^N(\vec{x}, t | \vec{x}, t) \right] = z\, 2^{-N}\, L(\vec{x}) \;-\; z^*\, 2^{-N}\, L^*(\vec{x}) \;+\; \text{other loops (unchanged!)}$$

Only the two Polyakov loops which wind non–trivially are altered when changing the boundary condition. We consider the Polyakov loop averaged over all of space

$$P \equiv \frac{1}{V_3} \sum_{\vec{x}} L(\vec{x}).$$

Combining the spectral representation of $D$, $D_z$ and $D_{z^*}$ and writing $\text{Tr}_c \left[ D^N \right]$ as $\sum_i \lambda_i^N$ one gets the spectral representation of the Polyakov Loop

$$P = \frac{2^{N-1}}{V} \, \frac{i}{\text{Im}[(1-z)^2]} \left[ (z - z^*) \sum_i \lambda_i^N \;+\; (1-z) \sum_i \lambda_{z,i}^N \;-\; (1-z^*) \sum_i \lambda_{z^*,i}^N \right],$$

where $V = L_3 N$ is the total number of lattice points. The Polyakov loop P thus is represented as a linear combination of spectral sums for the $N$-th power of the eigenvalues computed with three different fermionic boundary conditions in time direction. The boundary conditions for the gauge fields are always kept periodic. This formula relates the vacuum expectation value of the Polyakov loop, which originally is a purely gluonic quantity, to spectral sums of the Dirac eigenvalues.

## Numerical analysis

Having derived the spectral representation of the Polyakov loop, we can analyze numerically various aspects of our formulas. These numerical studies are done on quenched SU(3) ensembles generated with the Lüscher–Weisz action [3]. We use various lattices ranging from $8^3 \times 4$ to $12^3 \times 8$ with several values of the inverse gauge coupling $\beta$ giving rise to configurations on both sides of the QCD phase transition. The lattice spacing was determined with the Sommer parameter in [5]. In our Monte Carlo program also the center symmetry is updated. For each configuration the complete spectrum of the hopping matrices for the staggered Dirac operators was computed using standard libraries. On each configuration this was repeated for different temporal boundary conditions. The statistical errors we quote for the averaged observables are evaluated with single elimination Jackknife.

## Results

Since the massless staggered Dirac operator is an anti–hermitian matrix, it has eigenvalues on the imaginary axis (they come in complex conjugate pairs). For analyzing our data we divide $|\lambda|$ into small bins of size $\Delta|\lambda|$. Each plot is shown in physical units (left) as well as in lattice units (right). We use red for $N_t = 4$, blue for $N_t = 6$ and green lines for $N_t = 8$. To distinguish the spatial extent solid lines are used for $N_s = 12$ and dashed ones for $N_s = 10$.

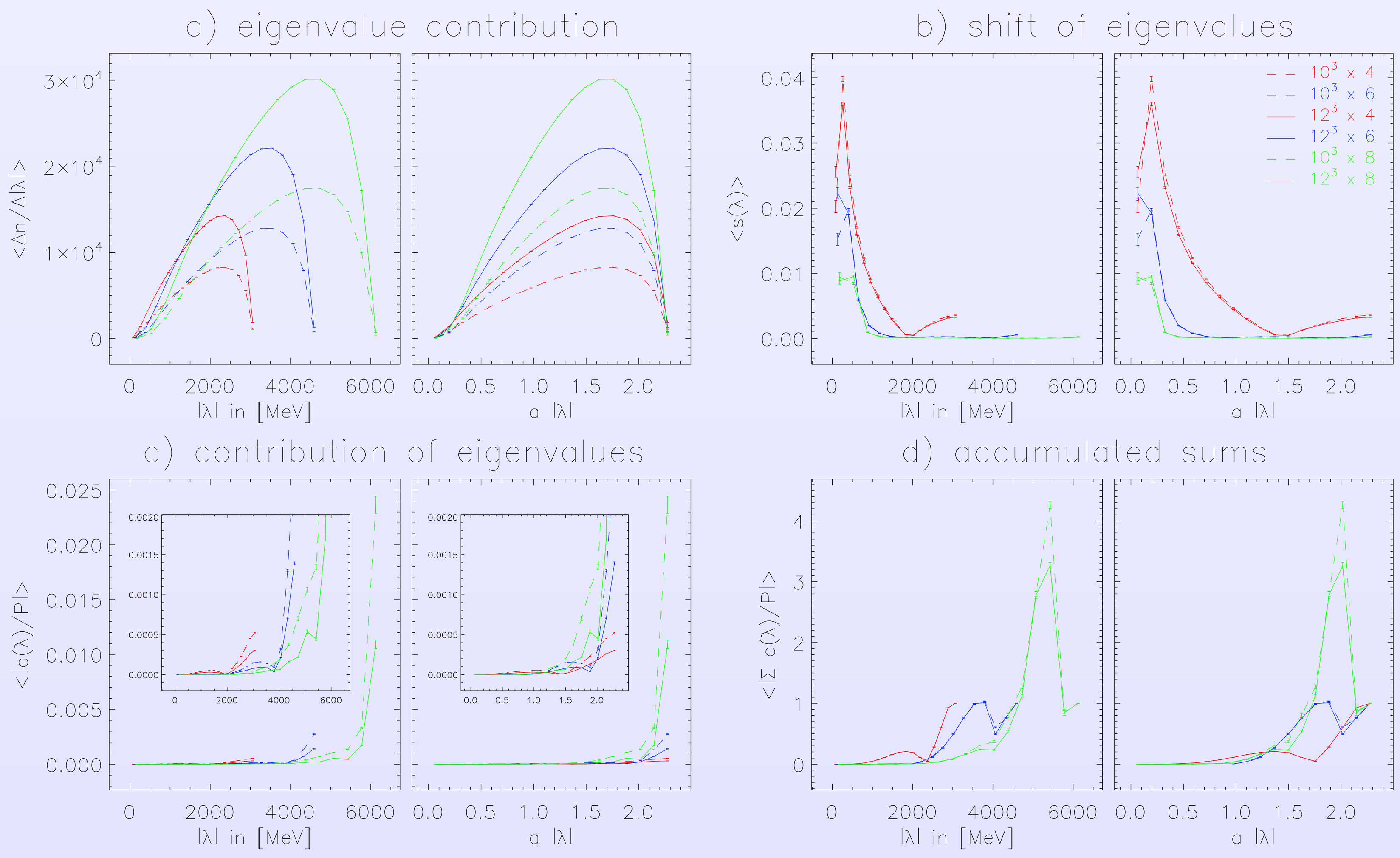


Fig. **a** shows the distribution of eigenvalues as a function of $|\lambda|$. As a first property of the eigenvalues we show how they shift due to a change of temporal boundary condition (Fig. **b**). For the average shift $s(\lambda_i)$ defined in [4], it is obvious that the infrared part is shifted more then the UV–part. Fig. **c** depicts the contribution of each bin to the Polyakov loop. We calculate the contribution for each bin separately and normalize them with the value of the Polyakov loop. One can easily see that the loop is strongly dominated by the UV–part for all lattices and temporal extents. As a last quantity (Fig. **d**) we accumulate the contributions of Fig. **c** up to a certain value of $|\lambda|$ and plot this versus $|\lambda|$. Also here the UV-dominance is large.

## Summary

We numerically analyze spectral sums for Polyakov Loops. For that purpose we computed complete Staggered Dirac spectra with three different fermionic boundary conditions, using quenched ensembles below and above the QCD phase transition. In particular we study the distribution of the eigenvalues and their shift under a change of boundary conditions. Concerning this shift we established that the IR modes are shifted most. For the buildup of the Polyakov loop, we considered the contribution of an individual eigenvalue as well as the accumulated contribution. Both, the individual as well as the accumulated contributions show that mainly the eigenvalues in the UV build up the Polyakov loop. All plots indicate that properties of the eigenvalues are independent of the spatial volume $V_3$.

## References

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