Making sense of staggered light-quark baryons: Insights from the quark model

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Challenge of staggered baryon spectroscopy

- Extract the masses of the lightest octet and decuplet baryons using rooted staggered QCD

- Success would provide valuable evidence for
  - rooted staggered QCD
  - rooted SχPT

- Taste quantum numbers complicate analysis

- Deducing the lightest staggered baryon multiplets involved but straightforward

Taste quantum numbers

- Four tastes for each physical quark flavor

$$M = \begin{pmatrix} \hat{m}I_4 & 0 & 0 \\ 0 & \hat{m}I_4 & 0 \\ 0 & 0 & m_s I_4 \end{pmatrix}$$

- Larger flavor symmetry group

$$SU(3)_F \to SU(12)_f$$

Consider the quark model

- Lightest octet and decuplet embedded in symmetric irrep of $SU(6)$

$$
\begin{array}{ccc}
SU(6) & \supset & SU(2)_S \times SU(3)_F \\
\mathbf{56_S} & \rightarrow & (\mathbf{\tfrac{1}{2}},\ \mathbf{8_M}) \oplus (\mathbf{\tfrac{3}{2}},\ \mathbf{10_S})
\end{array}
$$

- Success of non-relativistic quark model rests on underlying dynamics

- If rooted staggered QCD is correct, then quark model must describe lightest baryon multiplets

Staggered quark model

- Lightest baryon multiplets embedded in symmetric irrep of $SU(24)$

$$
\begin{array}{ccc}
SU(24) & \supset & SU(2)_S \times SU(12)_f \\
\mathbf{2600_S} & \to & (\mathbf{\tfrac{1}{2}},\ \mathbf{572_M}) \oplus (\mathbf{\tfrac{3}{2}},\ \mathbf{364_S})
\end{array}
$$

Where are the physical octet and decuplet?

- Need to know for operator selection and chiral extrapolation

Identifying physical baryons: Single-taste baryons

- If rooted staggered QCD is correct, then taste $SU(4)_T$ is restored in the continuum limit

- Tastes are just like extra flavors; all tastes are equivalent

- Consider baryons containing only one taste of quark; to have correct $SU(2)_S \times SU(12)_f$ symmetry, $SU(3)_F$ symmetry must be same as $SU(12)_f$ symmetry

$$
\begin{array}{ccc}
SU(24) & \supset & SU(2)_S \times SU(12)_f \\
\mathbf{2600_S} & \rightarrow & (\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{572_M}) \oplus (\tfrac{\mathbf{3}}{\mathbf{2}},\ \mathbf{364_S})
\end{array}
$$

Flavor-taste basis

- Disentangle flavor $SU(3)_F$ and taste $SU(4)_T$ quantum numbers:

$$
\begin{aligned}
SU(12)_f \quad &\supset \quad SU(3)_F \times SU(4)_T \\
\mathbf{572_M} \quad &\to \quad (\mathbf{10_S},\ \mathbf{20_M}) \oplus (\mathbf{8_M},\ \mathbf{20_S}) \oplus (\mathbf{8_M},\ \mathbf{20_M}) \\
&\quad\quad \oplus (\mathbf{8_M},\ \mathbf{\bar{4}_A}) \oplus (\mathbf{1_A},\ \mathbf{20_M}) \\
\mathbf{364_S} \quad &\to \quad (\mathbf{10_S},\ \mathbf{20_S}) \oplus (\mathbf{8_M},\ \mathbf{20_M}) \oplus (\mathbf{1_A},\ \mathbf{\bar{4}_A})
\end{aligned}
$$

- In the continuum limit, all members of a given taste multiplet are degenerate

- All $\mathbf{20_S}$ baryons correspond to physical states

Continuum symmetry

- Continuum symmetry is larger than taste alone

$$M = \begin{pmatrix} \hat{m} I_4 & 0 & 0 \\ 0 & \hat{m} I_4 & 0 \\ 0 & 0 & m_s I_4 \end{pmatrix} = \begin{pmatrix} \hat{m} I_8 & 0 \\ 0 & m_s I_4 \end{pmatrix}$$

$$\Rightarrow SU(8)_{\hat{m}} \times SU(4)_{m_s} \supset SU(4)_T$$

- Baryons transforming within a given irrep of continuum symmetry are degenerate

Continuum irreps

$$SU(12)_f \supset SU(8)_{\hat{m}} \times SU(4)_{m_s}$$

$$\mathbf{364_S} \quad \rightarrow \quad (\mathbf{120_S},\ \mathbf{1}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_S})$$

$$\begin{aligned} \mathbf{572_M} \quad \rightarrow \quad & (\mathbf{168_M},\ \mathbf{1}) \oplus (\mathbf{28_A},\ \mathbf{4}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus \ldots \\ & (\mathbf{8},\ \mathbf{6_A}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_M}) \end{aligned}$$

- Deduce correspondence between continuum irreps and physical states by locating single-taste baryons in each continuum irrep

Correspondence with physical baryons

$$SU(12)_f \supset SU(8)_{\hat{m}} \times SU(4)_{m_s}$$

$$\mathbf{364_S} \quad \rightarrow \quad \overset{\Delta}{(\mathbf{120_S},\ \mathbf{1})} \oplus \overset{\Sigma^*}{(\mathbf{36_S},\ \mathbf{4})} \oplus \overset{\Xi^*}{(\mathbf{8},\ \mathbf{10_S})} \oplus \overset{\Omega^-}{(\mathbf{1},\ \mathbf{20_S})}$$

$$\mathbf{572_M} \quad \rightarrow \quad \overset{N}{(\mathbf{168_M},\ \mathbf{1})} \oplus \overset{\Lambda}{(\mathbf{28_A},\ \mathbf{4})} \oplus \overset{\Sigma}{(\mathbf{36_S},\ \mathbf{4})} \oplus \ldots$$

$$\underset{\substack{\Lambda_s \\ (1400)}}{(\mathbf{8},\ \mathbf{6_A})} \oplus \underset{\Xi}{(\mathbf{8},\ \mathbf{10_S})} \oplus \underset{\substack{N_s \\ (1600)}}{(\mathbf{1},\ \mathbf{20_M})}$$

- All irreps but two correspond to physical states
- Exceptions are partially quenched baryons

Wait a minute!

- Does presence of states with unphysical masses invalidate rooted staggered QCD?

$\Rightarrow$ No. Conservation of $SU(8)_{\hat{m}} \times SU(4)_{m_s}$ quantum numbers forbids mixing of these states with physical ones. This situation is what one encounters in partially quenched theories.

$\Rightarrow$ Key: Is taste $SU(4)_T$ restored in the continuum limit?

Summary

- If rooted staggered QCD is correct, then lightest multiplets of staggered baryons are straightforwardly, accurately described by quark model

- Testing resulting picture means testing rooted staggered QCD

- Analysis can immediately be extended to excited states, heavy-light-light baryons, . . .

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No. of lattice irreps per continuum irrep

$$m_x = m_y = \hat{m} \quad \text{and} \quad m_z = m_s$$

$$SU(12)_f \supset SU(8)_{x,y} \times SU(4)_z$$

$$\mathbf{364_S} \quad \rightarrow \quad (\mathbf{120_S},\ \mathbf{1}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_S})$$

Δ 13 $\qquad \Sigma^*$ 20 $\qquad \Xi^*$ 13 $\qquad \Omega^-$ 7

N 12 $\qquad \Lambda$ 12 $\qquad \Sigma$ 12

$$\mathbf{572_M} \quad \rightarrow \quad (\mathbf{168_M},\ \mathbf{1}) \oplus (\mathbf{28_A},\ \mathbf{4}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus \ldots$$

$$(\mathbf{8},\ \mathbf{6_A}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_M})$$

Λ_s 5 $\qquad \Xi$ 7 $\qquad N_s$ 4

Mixing

- States with the same conserved quantum numbers—i.e., corresponding members of the same type of irrep—mix

$$\begin{array}{ccl} \overset{\Sigma^*}{(\tfrac{\mathbf{3}}{\mathbf{2}},\ \mathbf{36_S},\ \mathbf{4})} & \rightarrow & 3(\mathbf{1},\ \mathbf{8})_{-1} \oplus 3(\mathbf{1},\ \mathbf{8'})_{-1} \oplus 7(\mathbf{1},\ \mathbf{16})_{-1} \oplus \ldots \\ & & (\mathbf{0},\ \mathbf{8})_{-1} \oplus (\mathbf{0},\ \mathbf{8'})_{-1} \oplus 5(\mathbf{0},\ \mathbf{16})_{-1} \end{array}$$

$$\begin{array}{ccl} \overset{\Lambda}{(\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{28_A},\ \mathbf{4})} & \rightarrow & 4(\mathbf{1},\ \mathbf{8})_{-1} \oplus (\mathbf{1},\ \mathbf{16})_{-1} \oplus 4(\mathbf{0},\ \mathbf{8})_{-1} \oplus 3(\mathbf{0},\ \mathbf{16})_{-1} \\ \underset{\Sigma}{(\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{36_S},\ \mathbf{4})} & \rightarrow & 4(\mathbf{1},\ \mathbf{8})_{-1} \oplus 3(\mathbf{1},\ \mathbf{16})_{-1} \oplus 4(\mathbf{0},\ \mathbf{8})_{-1} \oplus (\mathbf{0},\ \mathbf{16})_{-1} \end{array}$$

$\Rightarrow$ For each member of the $(\mathbf{0},\ \mathbf{16})_{\mathbf{-1}}$ there is a 9-d mixing matrix

Swapping degeneracies with physical states

$$m_x = m_y = \hat{m},\ m_z = m_s$$

$$\hat{m} \longleftrightarrow m_s$$

$$m_x = m_y = m_s,\ m_z = \hat{m}$$

$$SU(8)_{x,y} \times U(1)_z \times SU(4)_z \qquad SU(2)_I \times U(1)_z \times \mathrm{GTS}$$

$$\begin{aligned} N &\longleftrightarrow N_s \\ \Lambda &\longleftrightarrow \Lambda_s \\ \Sigma &\longleftrightarrow \Xi \\ \Delta &\longleftrightarrow \Omega^- \\ \Sigma^* &\longleftrightarrow \Xi^* \end{aligned}$$