MC Studies of Fisher Zeros In Gauge Models

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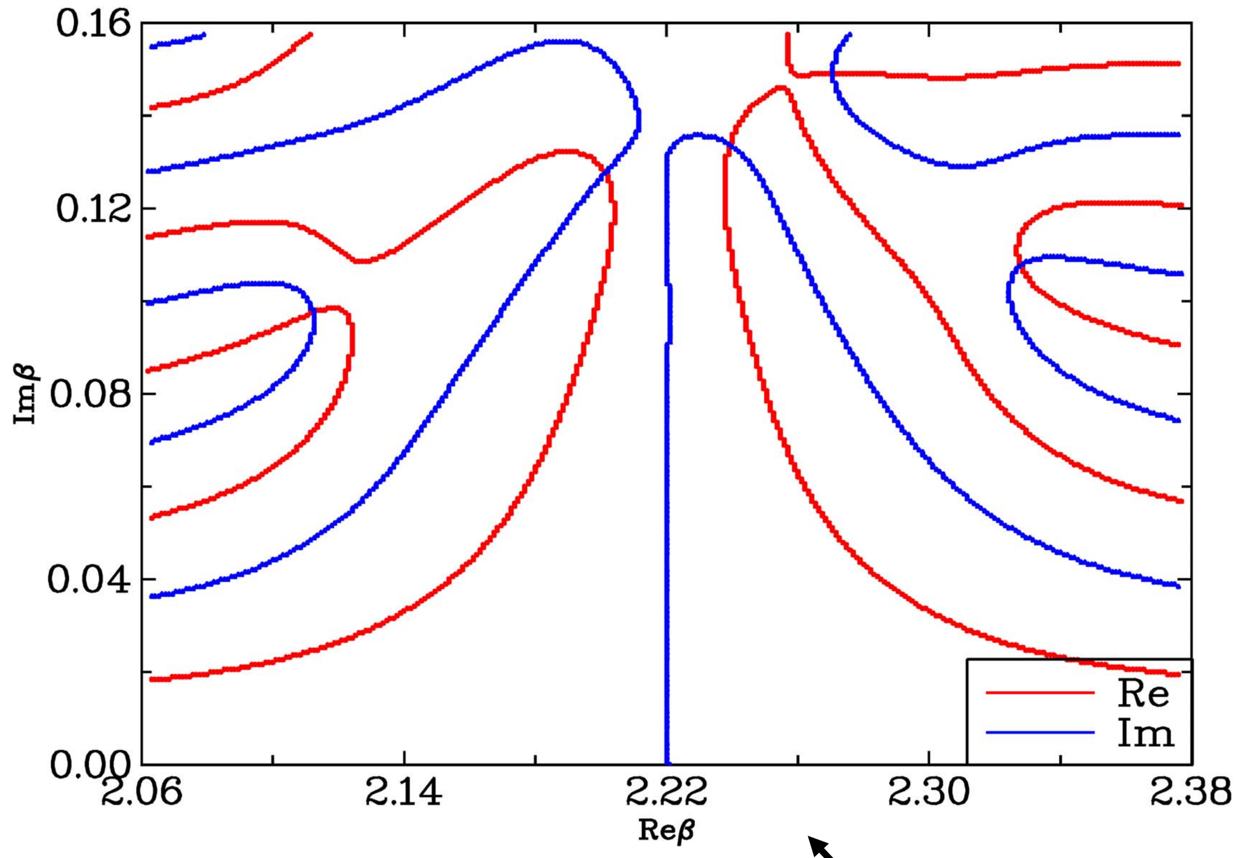
The location of the zeros of the partition function in the complex beta plane (Fisher Zeros) contain important information regarding the behavior of statistical systems at small and large beta but also about possible phase transition or crossover behavior at intermediate values of beta.



The partition function is defined as:

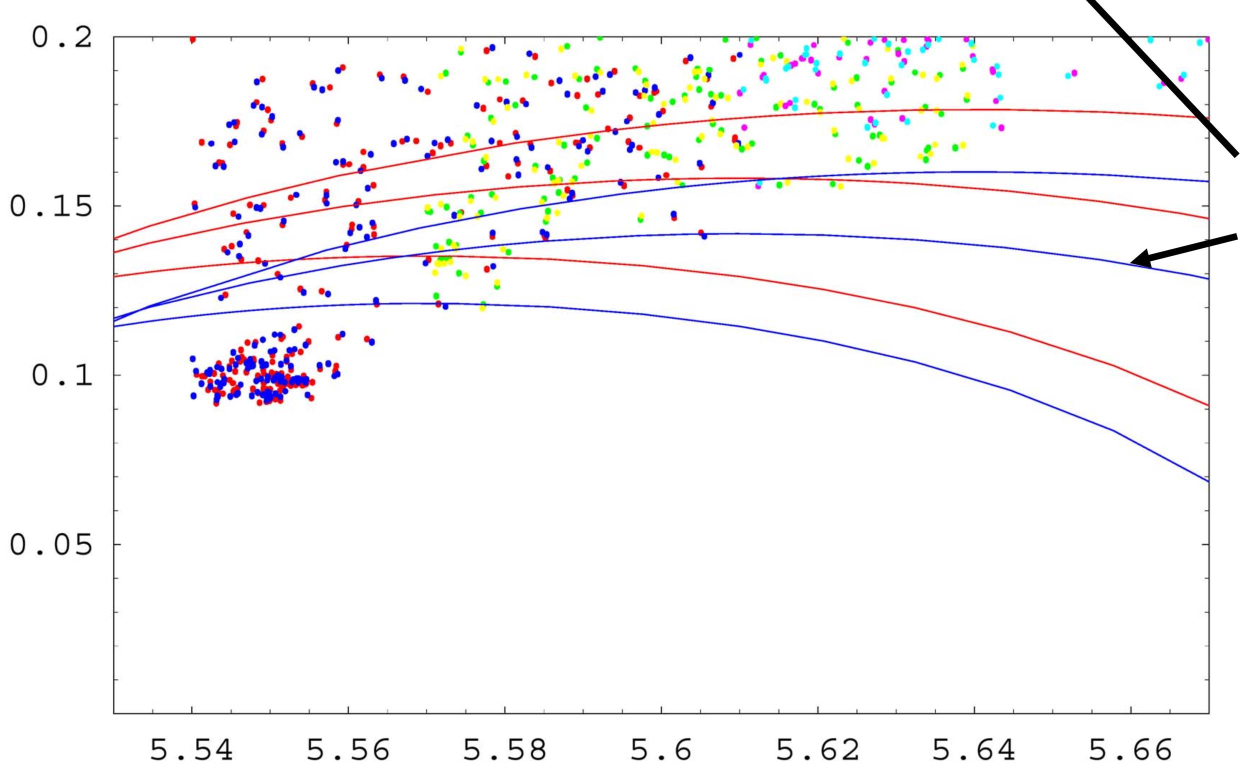
$$Z(\beta_0+\Delta\beta)=Z(\beta_0)\langle\exp(-\Delta\beta S)\rangle_{\beta_0}$$

If we subtract the average in the exponent we get:

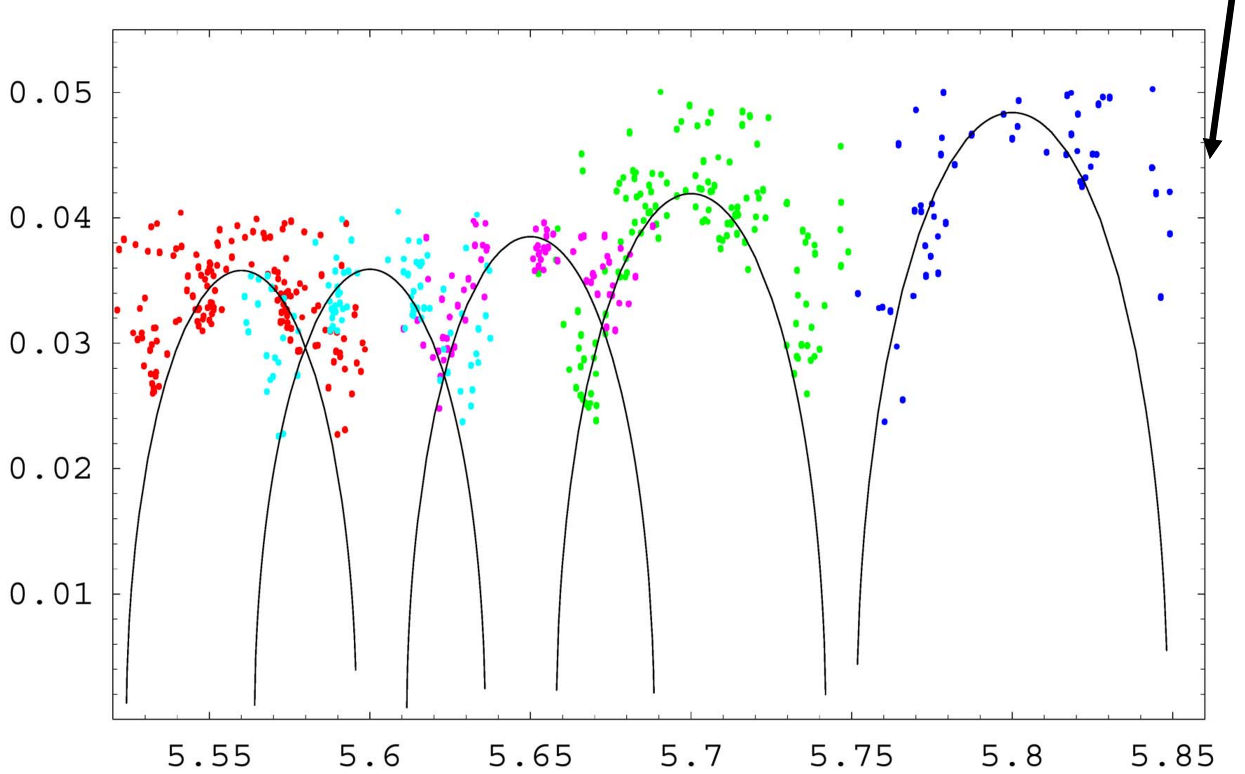
$$\langle\exp(-\Delta\beta(S-\langle S\rangle)\rangle_{\beta_0}=\exp(-\Delta\beta\langle S\rangle)Z(\beta_0+\Delta\beta)/Z(\beta_0)$$

Which has the same zeros as the partition function. Using a Gaussian estimation for what values in this plane are reliable we find:

$$|\Delta\beta|^2<\ln(N_{Conf})/\sigma_S^2$$



Typical imaginary and real curves in SU(2)
Bootstrapped zeros for SU(3) on a 4^4 lattice
And for an 8^4 lattice

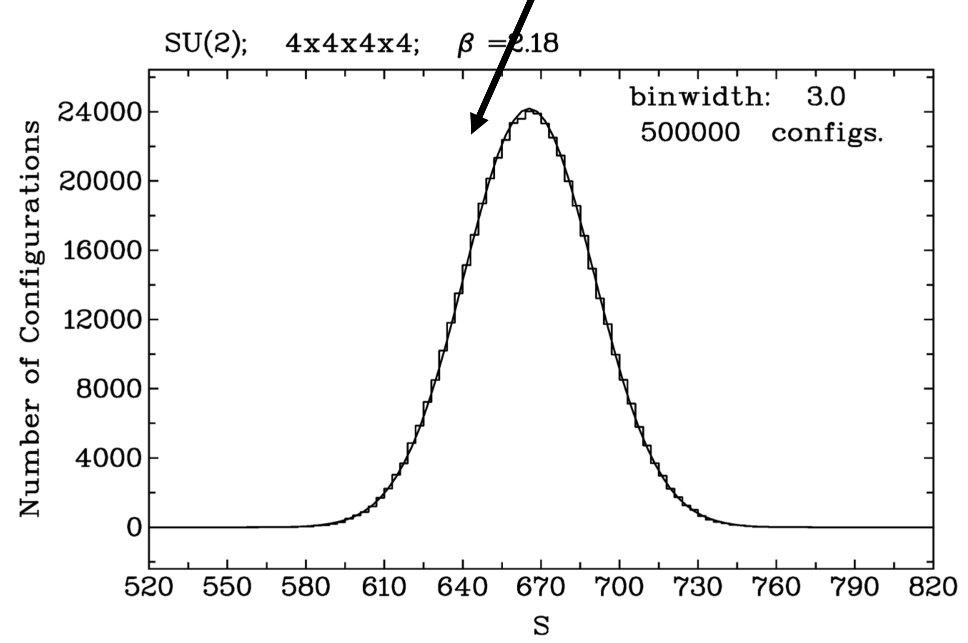


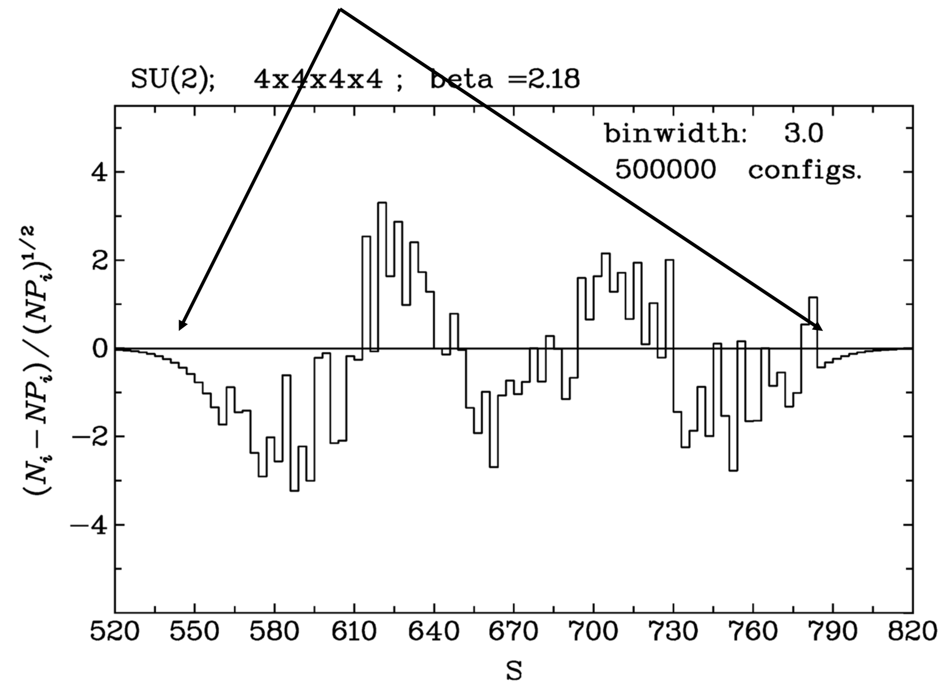
The nice regularities of the difference with the Gaussian approximation (for small lattices) suggests:

$$P(S)\propto\exp(-\lambda_1 S-\lambda_2 S^2-\lambda_3 S^3-\lambda_4 S^4)$$

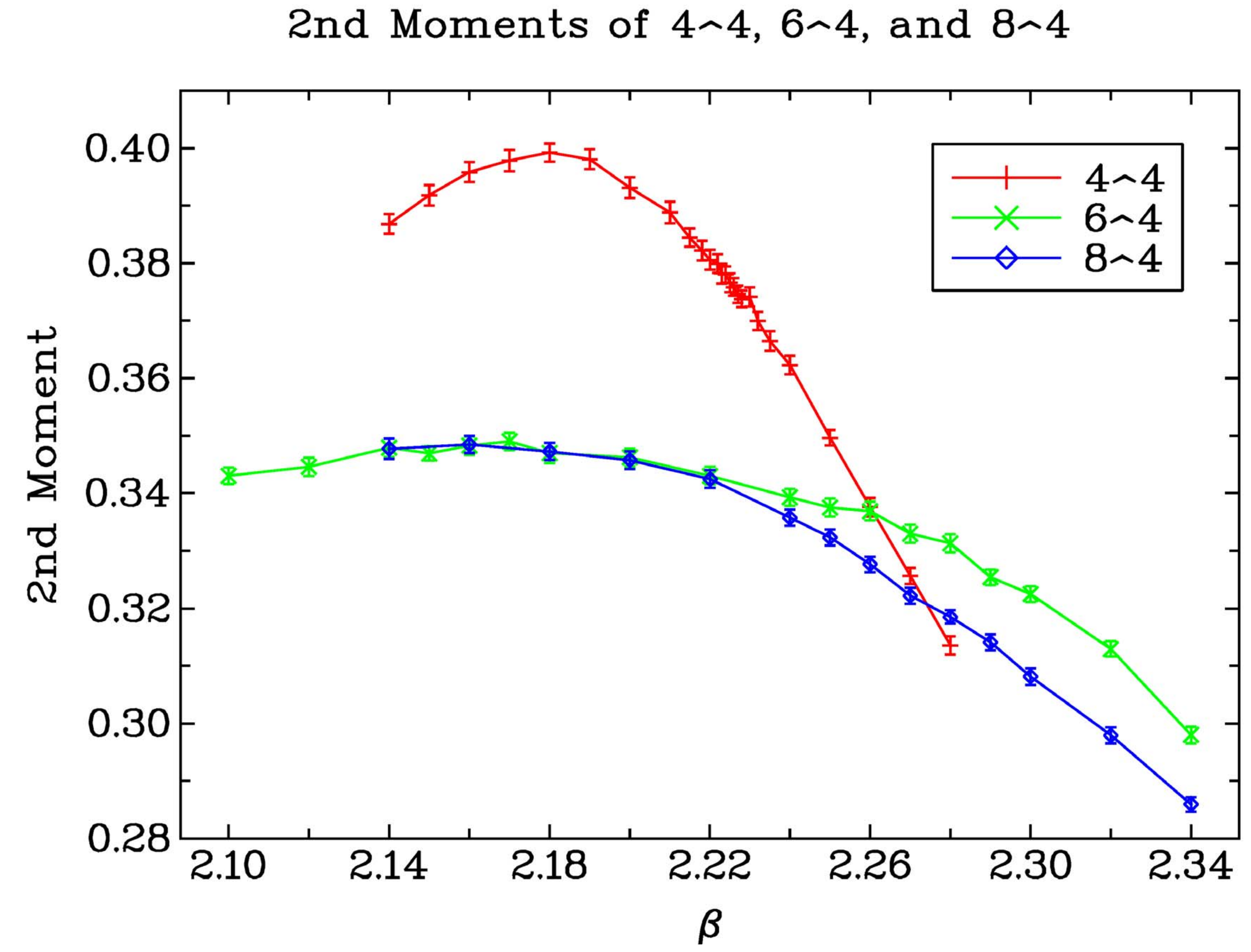
The unknown parameters can be determined from the first four moments (shown on the right) using Newton's Method and Chi squared minimization. These methods along with the direct method covered on the left show good agreement for 4^4 lattices.

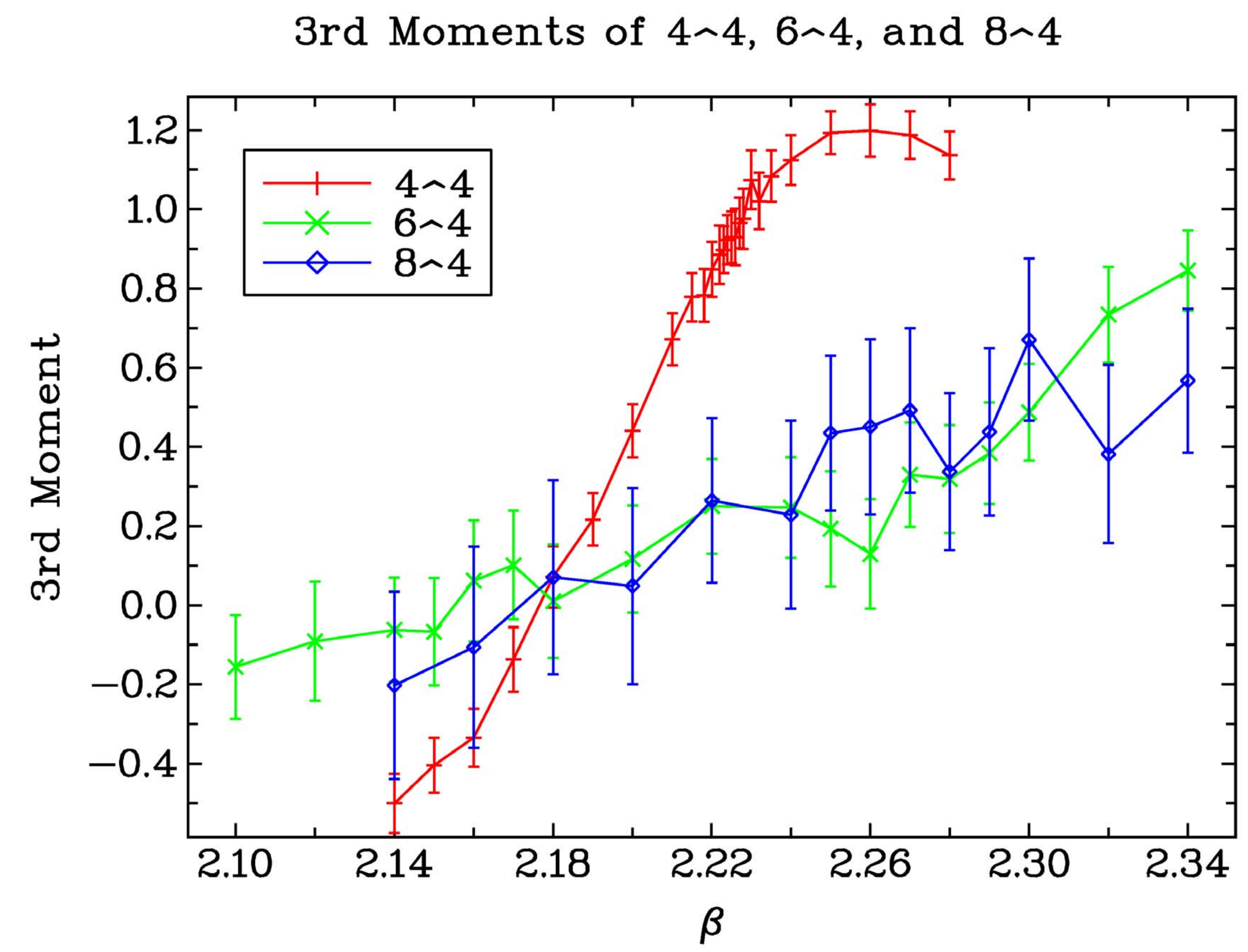
Quasi-Gaussian distribution of S
Its residual, note the low statistics in the tails

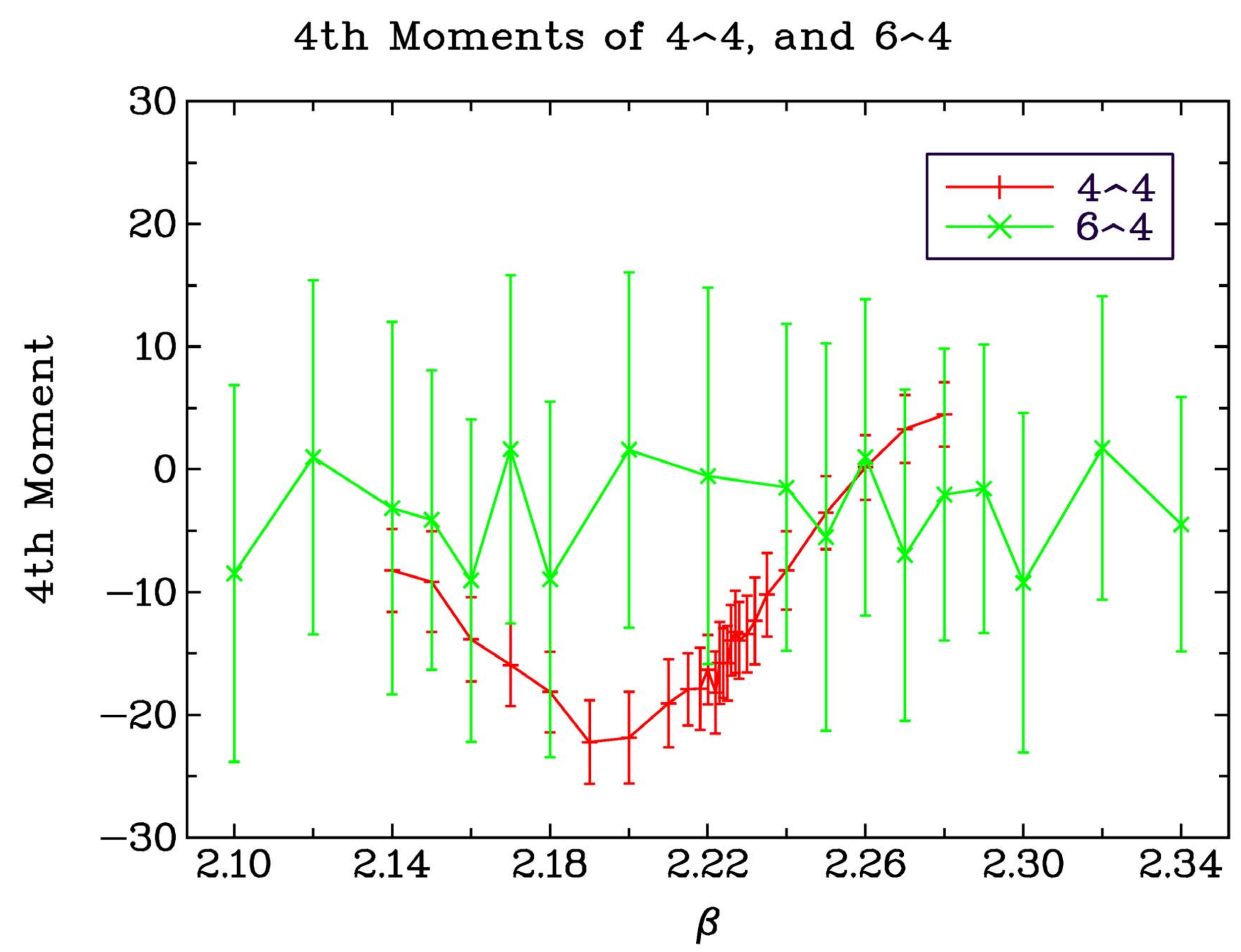




We found that the direct MC method of finding Fisher Zeros can only be used on small lattices and even there the findings are not concrete. To find zeros at larger volumes we will have to refine our current methods or find others.







References:
N. A. Alves, B.A. Berg, and S. Sanielevici, Nucl. Phys. **B376**, 218 (1992), hep-lat/9107002.
M. Falcioni, E. Marinari, M. L. Paciello, G. Parisi, and B. Taglienti, Phys. Lett. **B108**, 331 (1982).