New Vista on

Excited States in Lattice Gauge Theories

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Contents:

- Monte Carlo Hamiltonian
- Spectrum of excited states
- Temperature windows
- Outlook

A critical review:

Lagrangian LGT (based on the Wilson action, ...):

- **Standard approach and very successful**

Difficulties:

- **Excited states spectrum**
- **Wave functions**

Hamiltonian LGT **(based on the Kogut-Susskind Hamiltoni):**

- **Allows to compute the excited states spectra and wave functions.**

Big problem:

- **To find a set of basis states which are physically relevant**

Monte Carlo Hamiltonian

Transition amplitude in the Lagrangian method (in 1-D):

$$\langle \mathbf{O} \rangle = \frac{\int [\mathbf{dx}]\, \mathbf{O}[\mathbf{U}]\, \mathbf{exp}\left(-\frac{\mathbf{1}}{\hbar} \mathbf{S}[\mathbf{x}] \right)}{\int [\mathbf{dx}]\, \mathbf{exp}\left(-\frac{\mathbf{1}}{\hbar} \mathbf{S}[\mathbf{x}] \right)}$$

Transition amplitudes between position states via Hamiltonian:

$$< \mathbf{x_{fi}}, \mathbf{T} | \mathbf{x_{in}}, \mathbf{0} > = < \mathbf{x_{fi}} | \mathbf{exp}(- \mathbf{HT}/\hbar) | \mathbf{x_{in}} >$$

MC with imp. sampling :

$$\langle \mathbf{O} \rangle \approx \frac{\mathbf{1}}{\mathbf{N_C}} \sum_{\mathbf{C}} \mathbf{O[C]}$$

$$< \mathbf{x_{fi}}, \mathbf{T} \mid \mathbf{x_{in,0}} > \approx < \mathbf{x_{fi}} \mid \exp(-\mathbf{H_{eff}}\ \mathbf{T}/\hbar) \mid \mathbf{x_{in}} >$$ **?**

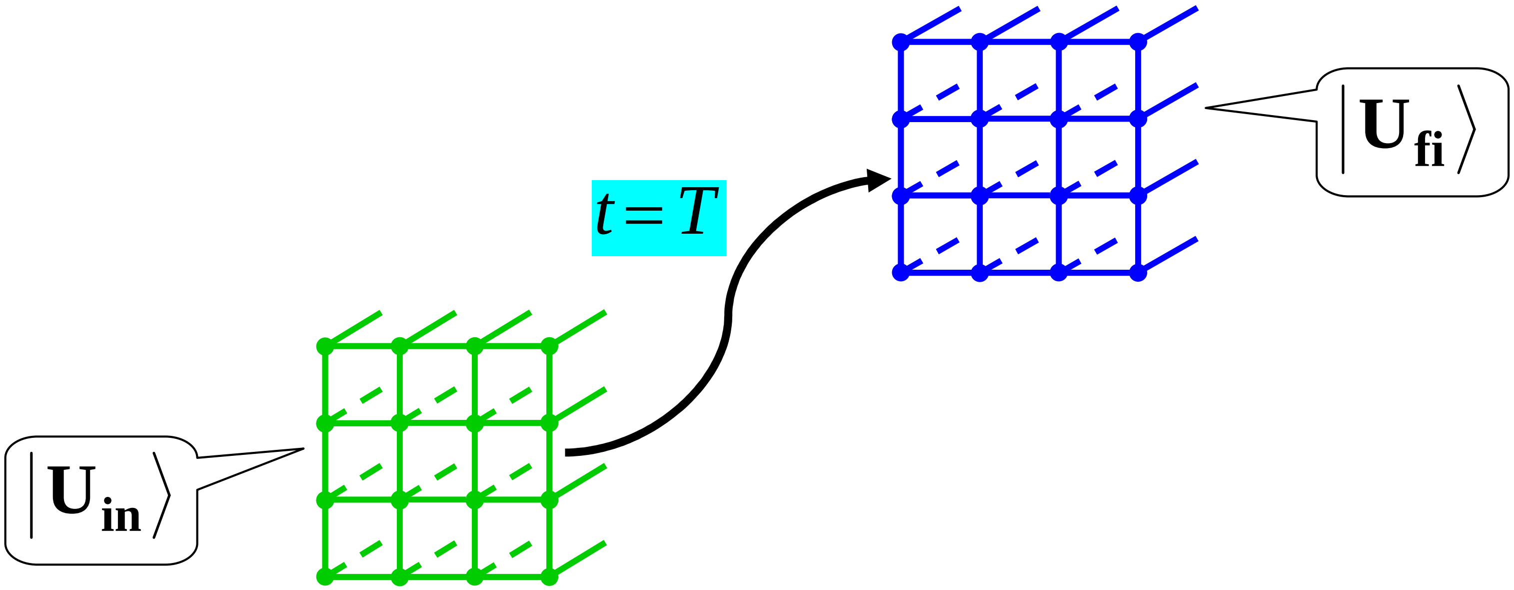
$\mathbf{H}$ — Infinite degree

$\mathbf{H_{eff}}$ — Finite degree

In lattice gauge theories:

Euclidean transition amplitude between two physical states:

$$\langle U_{fi}, t = T \,|\, U_{in}, t = 0 \rangle = \langle U_{fi} \,|\, exp(-\mathbf{H}_{ks} T/\hbar) \,|\, U_{in} \rangle$$



Bargmann link states: $|U\rangle = |U_{12}, U_{23}, \ldots\rangle$

|U>'s are gauge invariant projection of the Bargmann link states.

$$< U_{fi}, t = T \mid P_{inv} \mid U_{in}, t = 0 > = \int [dU] \, exp(-S[U]) \big|_{U_{in}}^{U_{fi}}$$

Transfer matrix for a finite T

Transfer matrix corresponding to a finite T for the purpose of reconstruct the spectrum in some finite low energy domain.

$$M(T) = [M_{ij}(T)]_{N \times N}$$

$$M_{ij}(T) = < U_i \,|\, exp(-H_{ks} T / \hbar) \,|\, U_j > \quad ,i, j \in 1, 2, ..., N$$

M(T) is a positive and Hermitian matrix.

$$\mathbf{M(T)}$$

Diagonalization

$$\mathbf{M(T) = U_0^{+}\, D(T)\, U_0}$$

Spectrum

Wave functions

Effective Hamiltonian:

$$H_{eff} = \sum_{k=1}^{N} | E_k^{eff} > E_k^{eff} < E_k^{eff} |$$

Lesson:

an effective H can be found via MC, Such that transition amplitudes become a finite sum over N eigenstates, where N is in the order of N_c.

Kröger et al, Phys. Lett. A258 (1999) 6.
Huang et al, Phys.Lett. A299 (2002) 483.

Computation of matrix elements

$$\mathbf{M_{ij}(T)} = < U_j \,|\, exp(-HT/\hbar) \,|\, U_i >$$

$$= \frac{< U_j \,|\, exp(-HT/\hbar) \,|\, U_i >}{< U_j \,|\, exp(-H^{kin} T/\hbar) \,|\, U_i >} \times < U_j \,|\, exp(-H^{kin} T/\hbar) \,|\, U_i >$$

Ratio of path integrals

Kinetic T.A.

Ratio

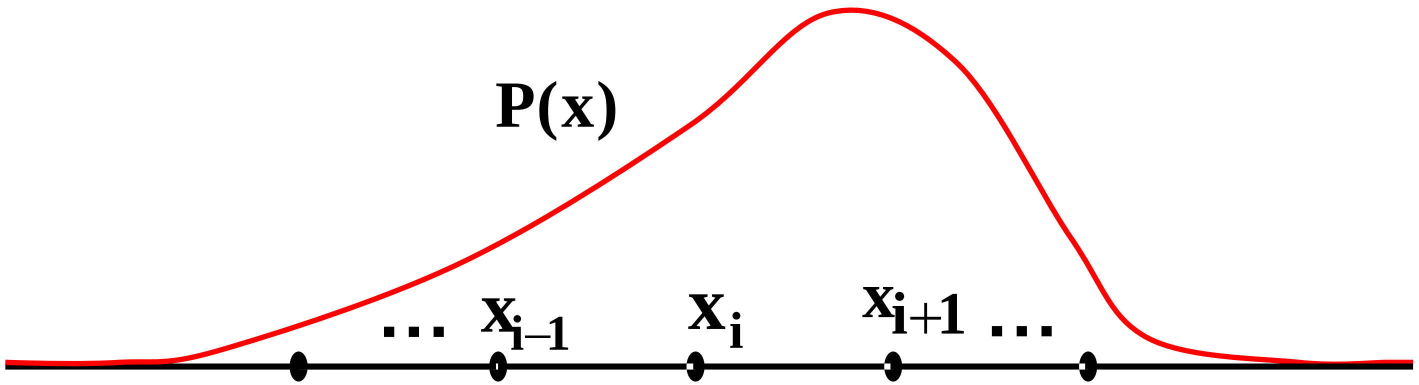
MC with importance sampling

Kinetic amplitude

Analytic methods using group theory

Distribution of configurations:

Regular configurations:

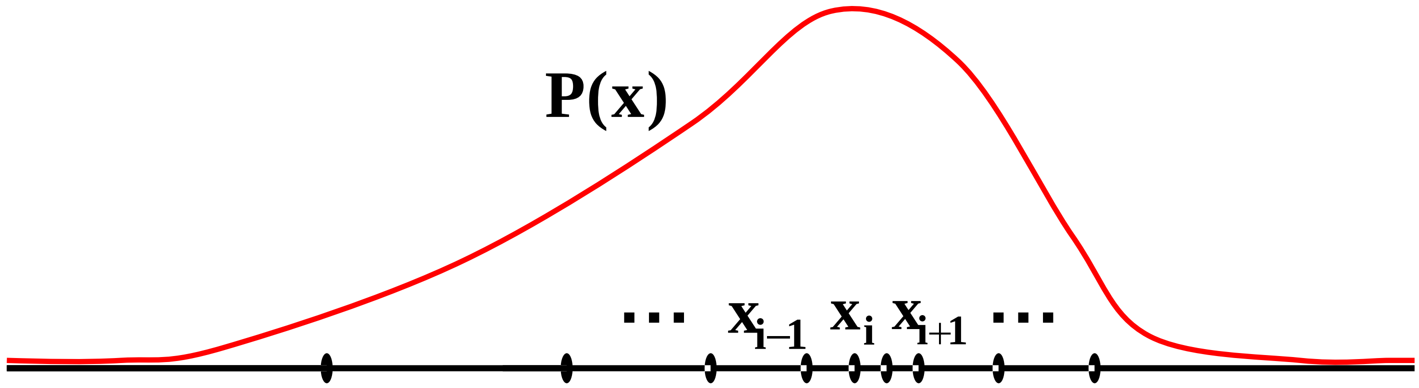


is not easy to do!

Solution: **Stochastic configurations**

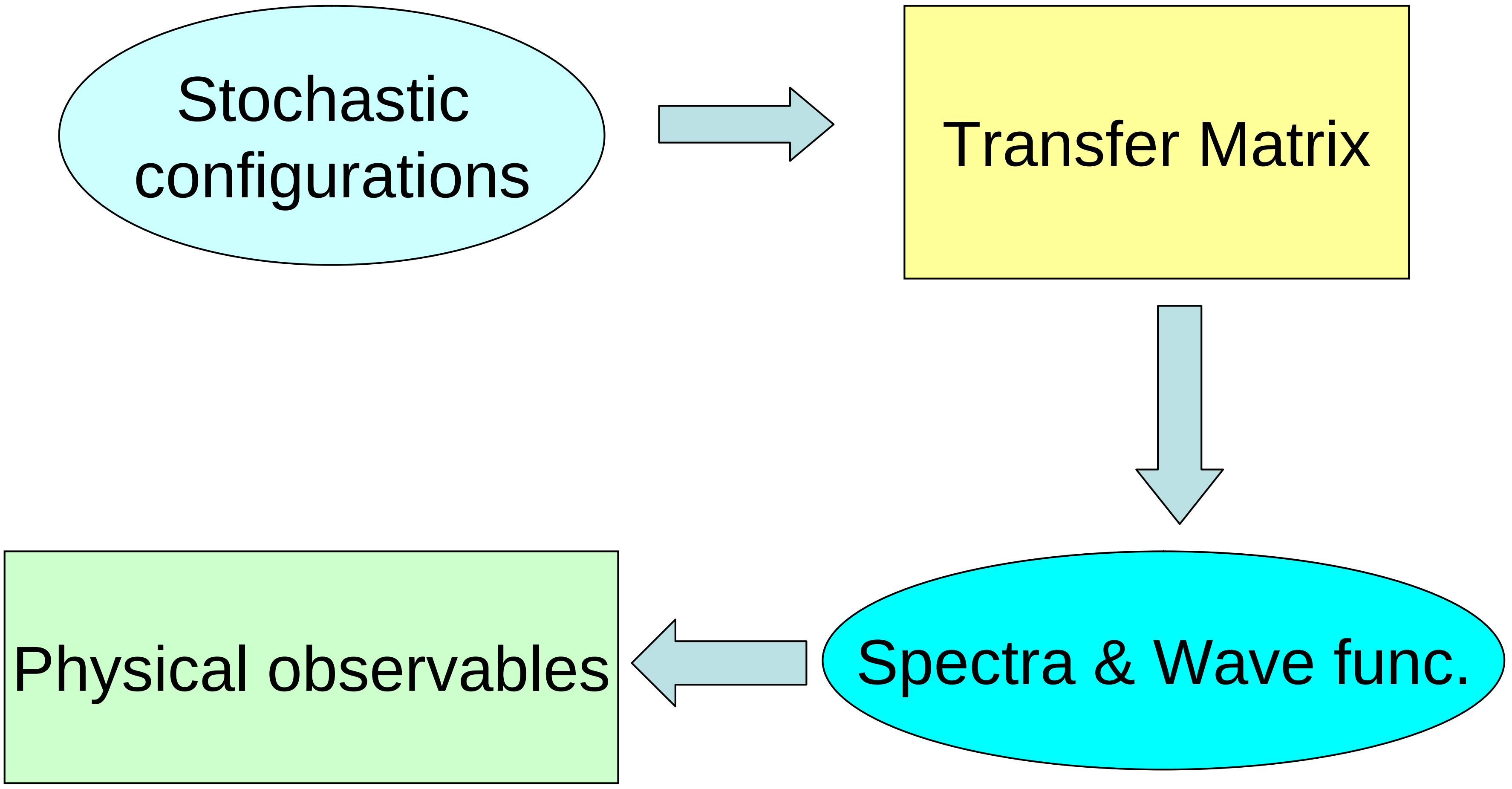
Guidance by physics: **Let physics tell us which states are important.**

Lesson: **Apply Monte Carlo using action weight factor.**



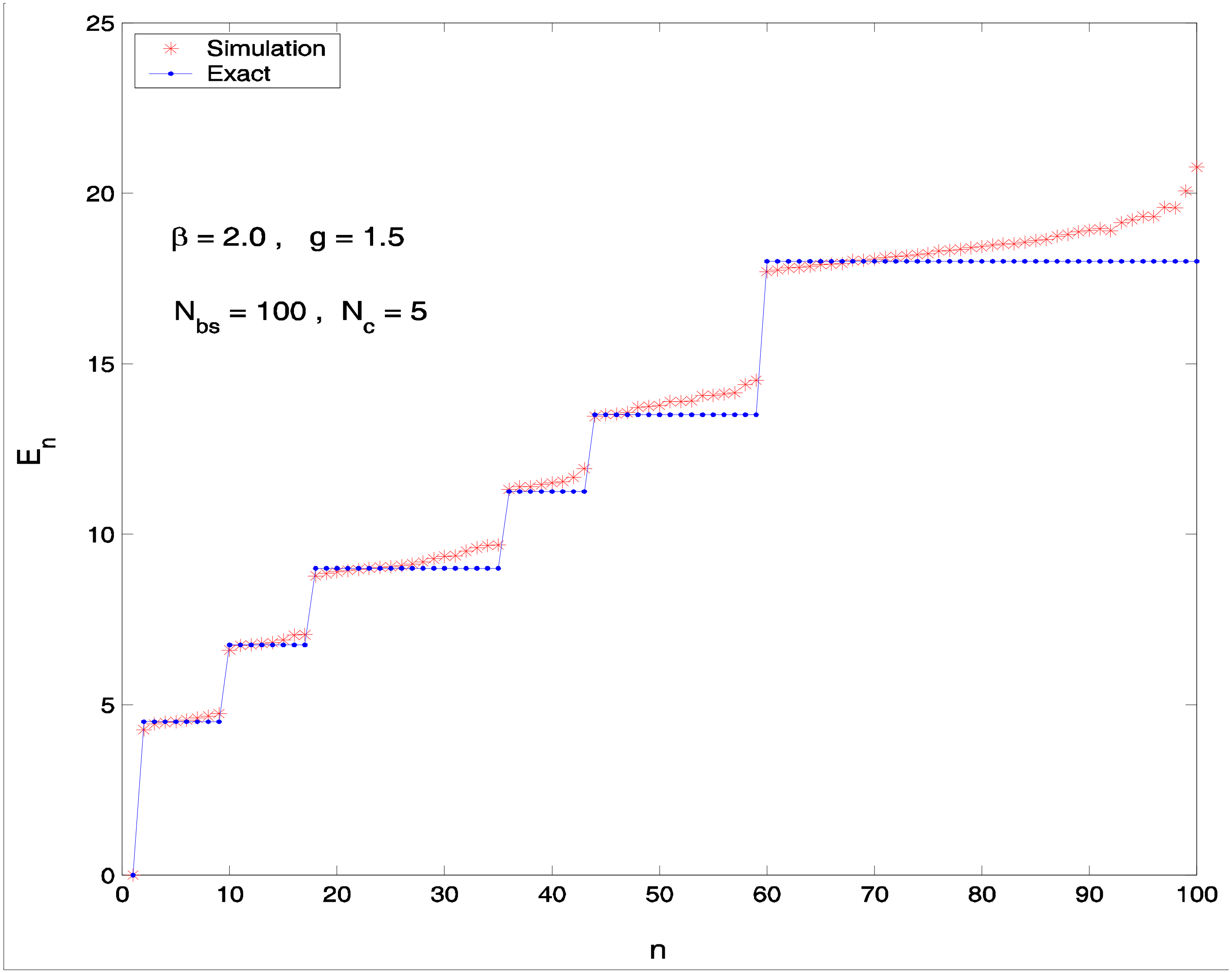
Note: **stoch. states must be normalizable.**

A short story of MC Hamiltonian:



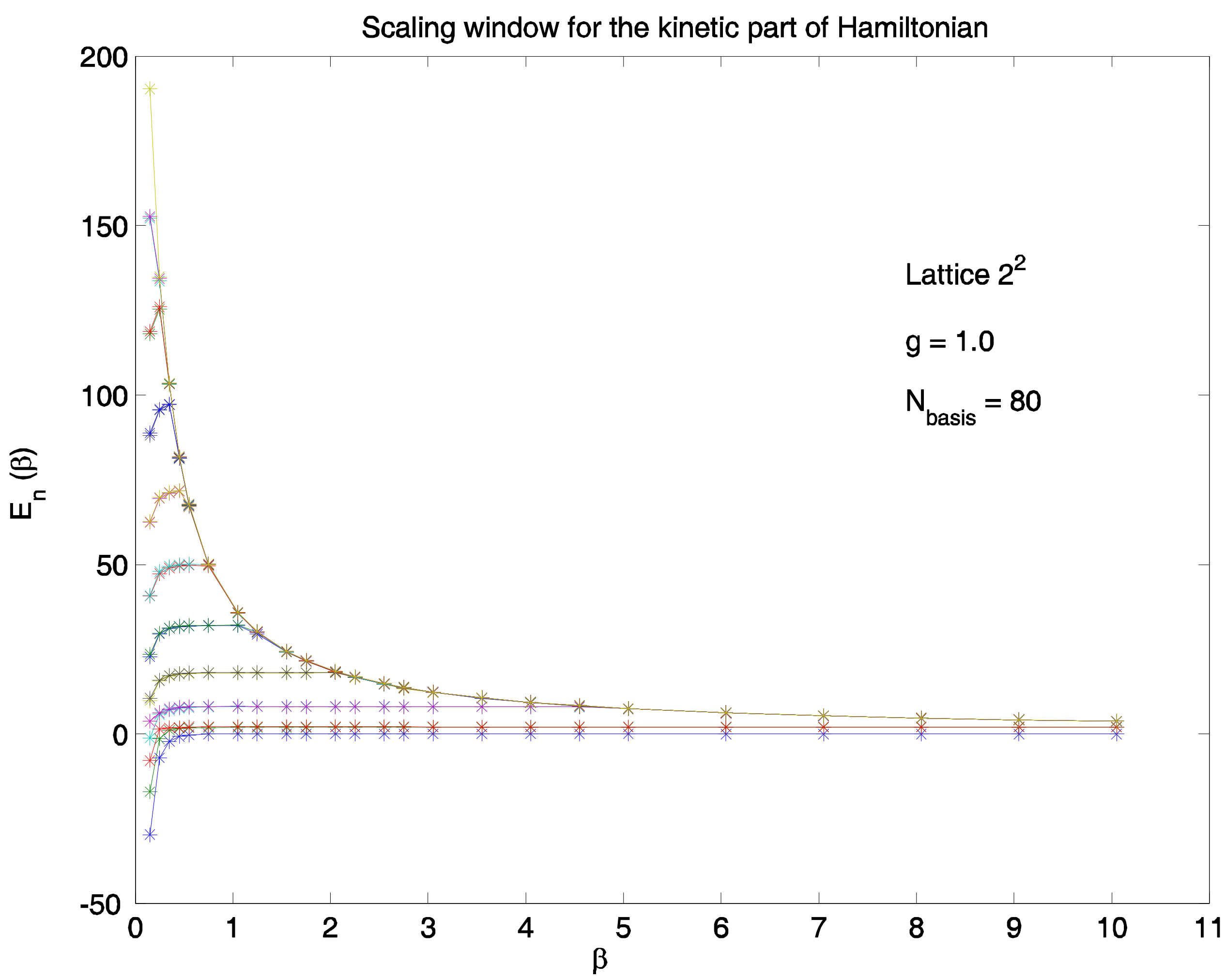
Spectrum of excited states

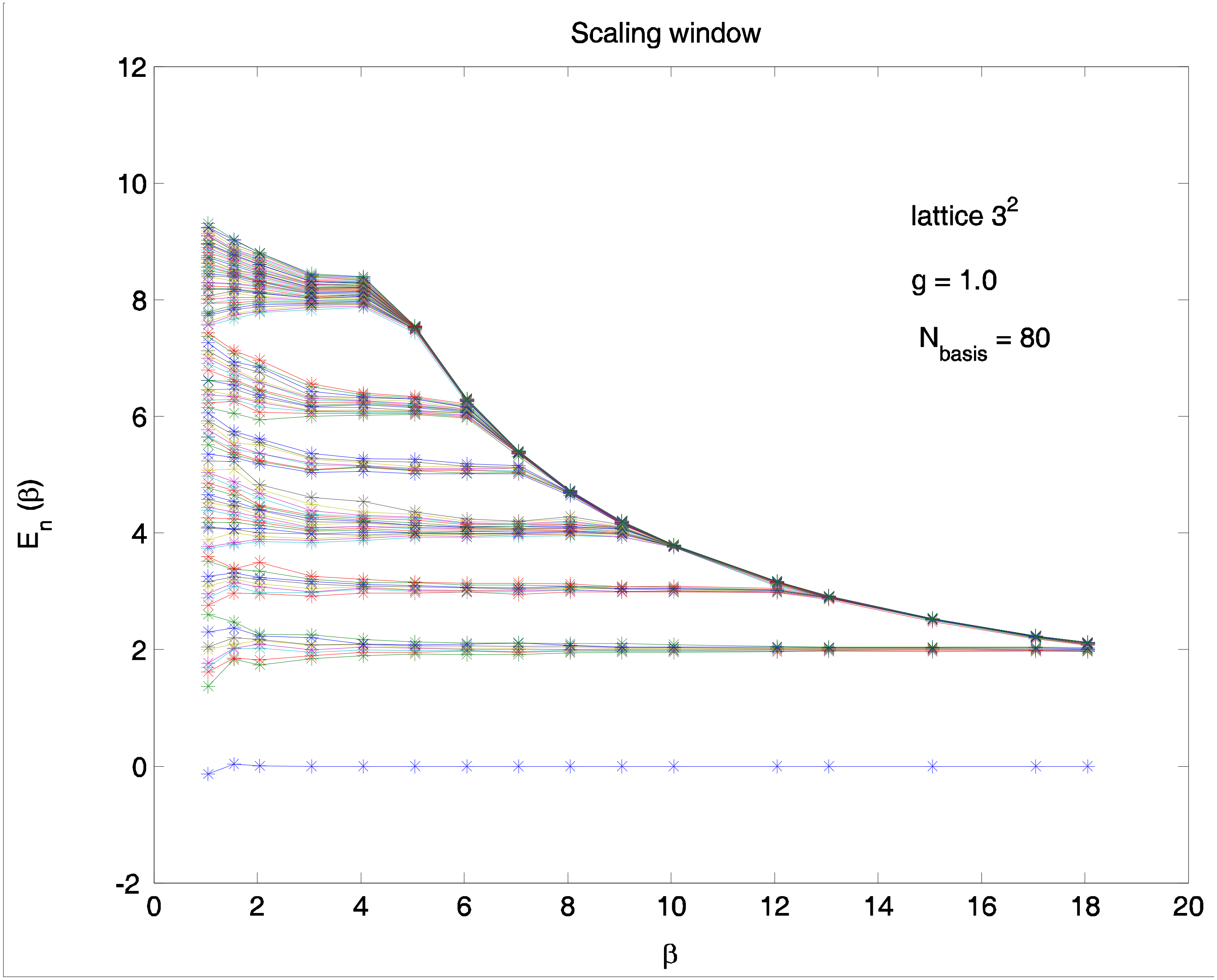
Example: Four plaquettes



Temperature window

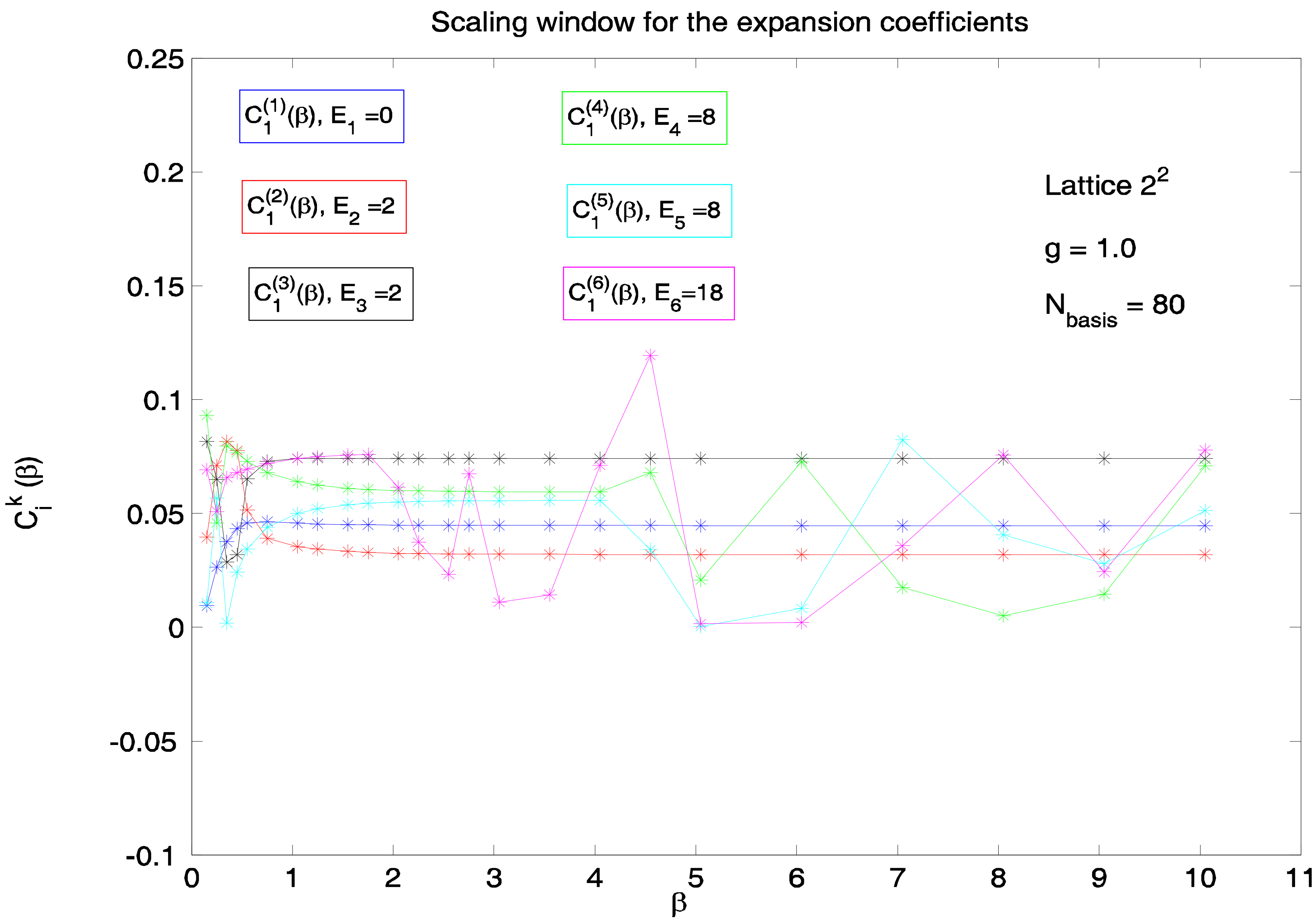
Monte Carlo Hamiltonian has a window of validity. Beyond this window, the effective Hamiltonian becomes unphysical.





Scaling window
lattice 3^2
g = 1.0
N_{basis} = 80
E_n (β)
β
12
10
8
6
4
2
0
-2
0
2
4
6
8
10
12
14
16
18
20

Temperature Window: Wave Functions



Thermodynamical functions

Definition:

$$Z(\beta) = Tr[\exp(-\beta H)],$$

$$U(\beta) = -\frac{\partial \log Z}{\partial \beta}$$

In lattice:

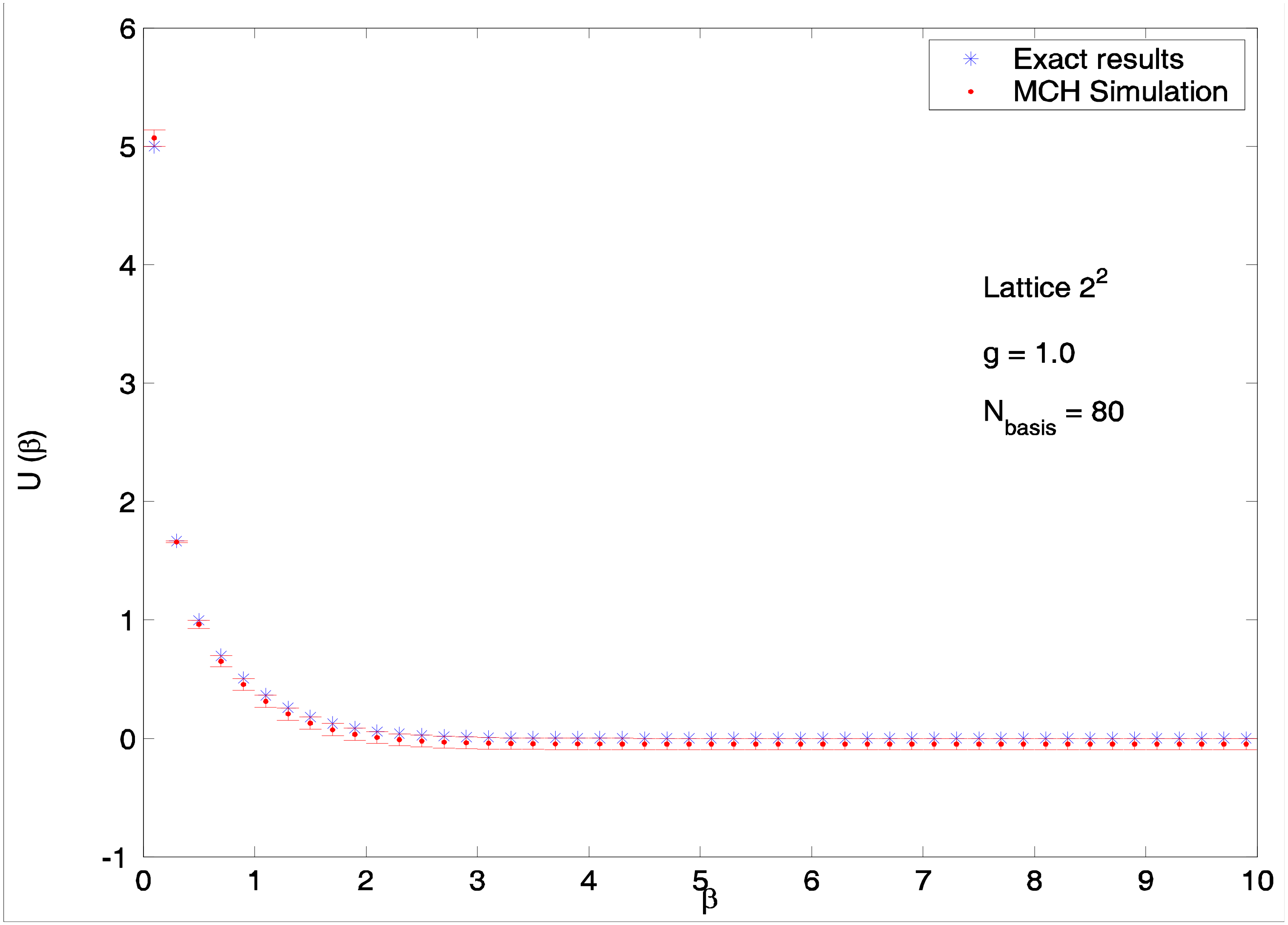
$$U(\beta) = \frac{N_s}{2a_t} + \frac{1}{N_t}\left\langle \frac{\partial}{\partial a_t} S \right\rangle$$

By MC Hamiltonian:

$$Z^{eff}(\beta) = \sum_{n=1}^{N} \exp[-\beta E^{eff}{}_n],$$

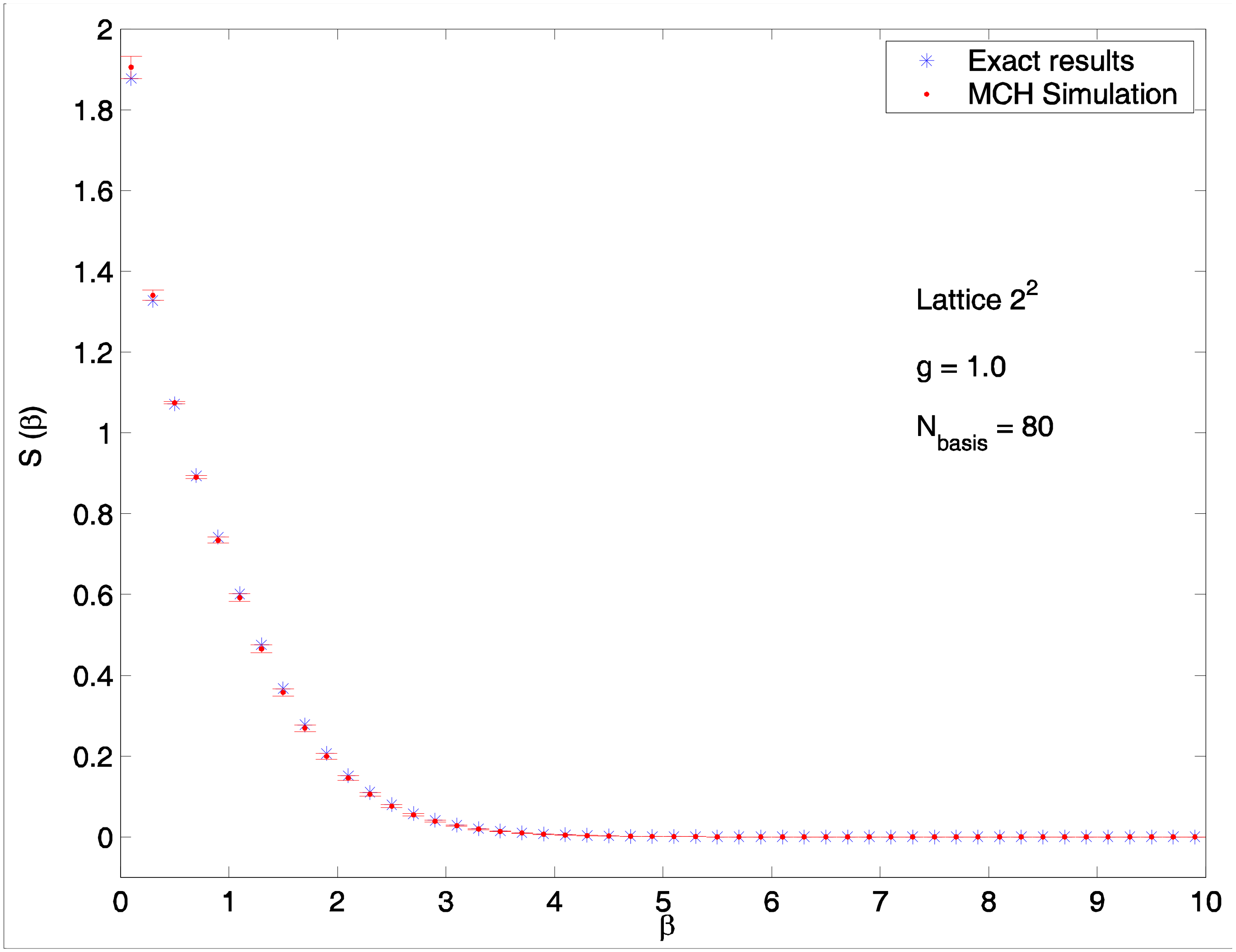
$$U^{eff}(\beta) = -\frac{1}{Z^{eff}(\beta)} \sum_{n=1}^{N} E^{eff}{}_n \exp[-\beta E^{eff}{}_n]$$

Average energy U



Temperature window

Entropy S



Outlook:

- Application of Monte Carlo Hamiltonian:

- Idea works well in QM and scalar field theory.
- Hadronic wave and structure functions in QCD (for small x_B and Q^2)
- S-matrix, scattering and decay amplitudes
- Finite density QED and QCD