Effective Field Theories for lattice QCD: Lecture 4

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Outline of Lectures

1. Overview & Introduction to continuum chiral perturbation theory (ChPT)
2. Illustrative results from ChPT; SU(2) ChPT with heavy strange quark; finite volume effects from ChPT and connection to random matrix theory
3. Including discretization effects in ChPT
4. Partially quenched ChPT and applications, including a discussion of whether m_u=0 is meaningful

Outline of lecture 4

- Partial quenching and PQChPT
  - What is partial quenching and why might it be useful?
  - Developing PQChPT
  - Results and status
- m_u=0 and the validity of PQ theories (and the rooting prescription)

Additional References for PQChPT

- A. Morel, "Chiral logarithms in quenched QCD," J. Phys. (Paris) 48 (1987) 111
- C. Bernard & M. Golterman, "Chiral perturbation theory for the quenched approximation of QCD," Phys. Rev. D46 (1992) 853 [hep-lat/9204007]
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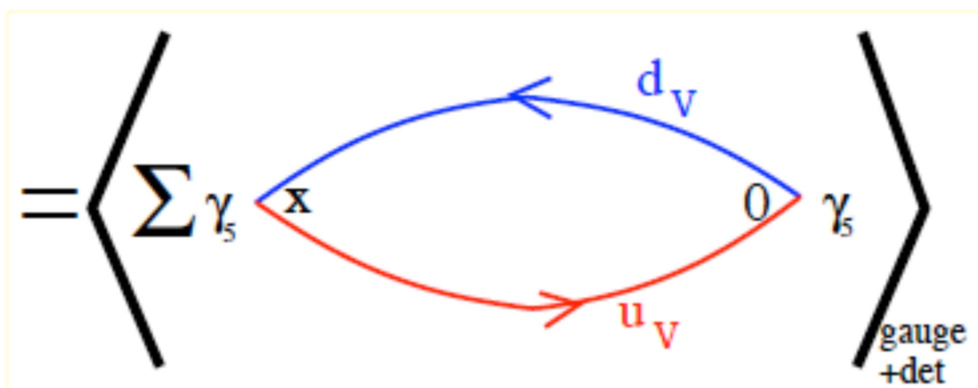
- C. Bernard & M. Golterman [Partially quenched ChPT], Phys. Rev. D49 (1994) 486 [hep-lat/9306005]
- S. Sharpe [Enhanced chiral logs in PQChPT], Phys. Rev. D56 (1997) 7052 [hep-lat/9707018]
- P. Damgaard & K. Splittorff [Replica method for PQChPT], Phys. Rev. D62 (2000) 054509 [hep-lat/0003017]
- S. Sharpe & N. Shoresh, "Physical results from unphysical simulations," Phys. Rev. D 62 (2000) 094503 [hep-lat/0006107]
- S. Sharpe & N. Shoresh [PQChPT general properties], Phys. Rev. D64 (2001) 114510 [hep-lat/0108003]
- S. Sharpe & R. Van de Water [Unphysical LECs], Phys. Rev. D69 (2004) 054027 [hep-lat/0310012]
- M. Golterman, S. Sharpe & R. Singleton [PQ Wilson ChPT], Phys. Rev. D71 (2005) 094503 [hep-lat/0501015]
- C. Bernard & M. Golterman [Transfer matrix for & foundations of PQQCD], arXiv:1011.0184 & in prep.
- P. Damgaard *et al.*, [Constraints on LECS in WChPT], Phys. Rev. Lett. 105 (2010) 162002 [arXiv:1001.2937]
- M. Hansen & S. Sharpe [Constraints on LECs in WChPT], Phys. Rev. D85 (2012) 014503 [arXiv:1111.2404]

What is partial quenching?

- Explain with example of pion correlator:

$$
\begin{aligned}
C_\pi(\tau) &= -\left\langle \sum_{\vec{x}} \bar{u}\gamma_5 d(\vec{x},\tau)\ \bar{d}\gamma_5 u(0) \right\rangle \\
&\equiv -\frac{1}{Z}\int DU \prod_q Dq D\bar{q} e^{-S_{\rm gauge} - \int_x \sum_q \bar{q}(\not{D}+m_q)q} \sum_{\vec{x}} \bar{u}\gamma_5 d(\vec{x},\tau)\ \bar{d}\gamma_5 u(0) \\
&= \frac{1}{Z}\int DU \prod_q \det(\not{D}+m_q) e^{-S_{\rm gauge}} \sum_{\vec{x}} {\rm tr}\left[\gamma_5 \left(\frac{1}{\not{D}+m_d}\right)_{x0} \gamma_5 \left(\frac{1}{\not{D}+m_u}\right)_{0x}\right]
\end{aligned}
$$

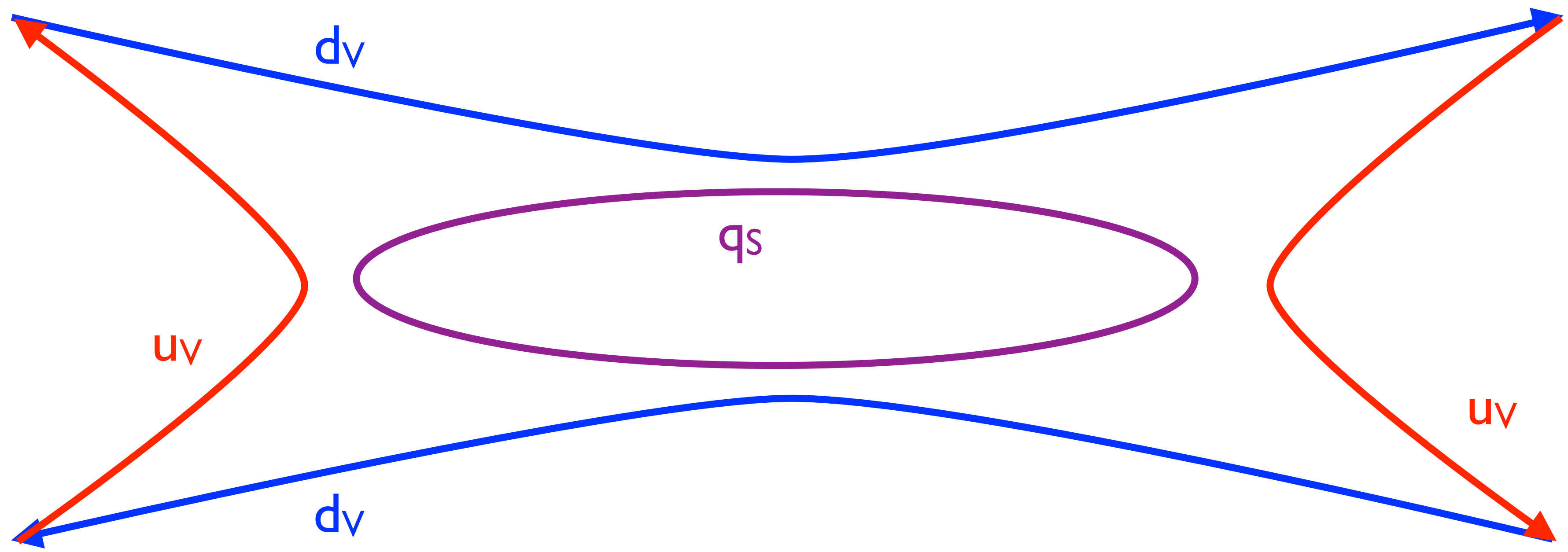


$\propto f_\pi^2 e^{-m_\pi \tau} + \text{exp. suppressed}$

- "sea" quarks in determinant; "valence" in propagators
- **Partial Quenching:** $m_{\rm val} \neq m_{\rm sea}$**—many different** $m_{\rm val}$ **for each** $m_{\rm sea}$
- **Numerically cheap—can we make use of this extra information?**
  - Many (but not all) numerical calculations use PQing

PQQCD is unphysical

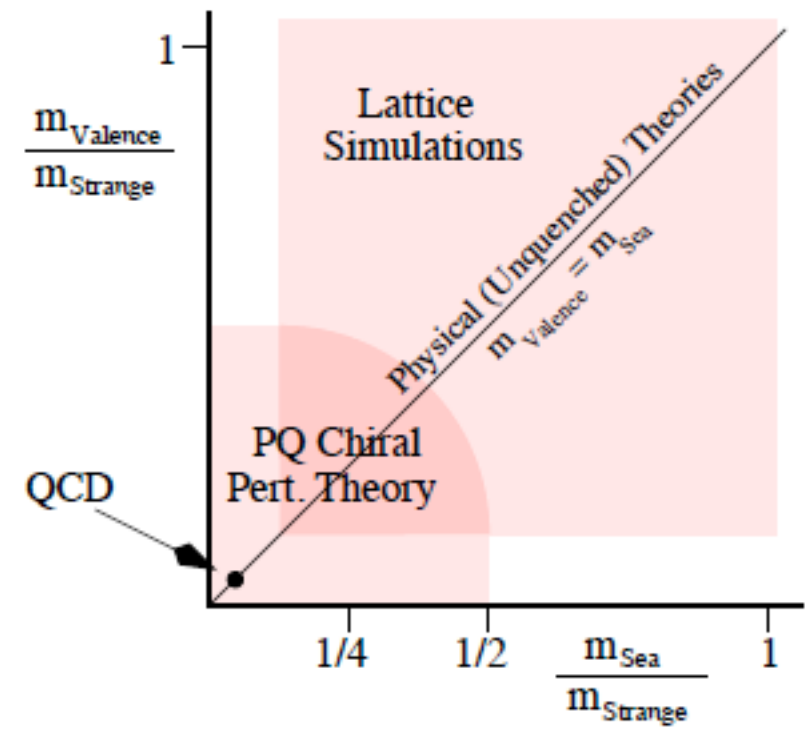
- Intuitively clear that unitarity is violated, since intermediate states differ from external states, e.g. $\pi_{VV}\ \pi_{VV} \to \pi_{VS}\ \pi_{SV} \to \pi_{VV}\ \pi_{VV}$



- Extent and impact of unphysical nature will become clearer when give a formal definition of PQ theory

Why partially quench?

- Use PQQCD to learn about physical, unquenched QCD
- This is possible only within an EFT framework
  - Use partially quenched ChPT (PQChPT)
  - Requires that one works in "chiral regime"
  - PQChPT needs very few extra LECs compared to ChPT
  - Extends range over which can match to ChPT
- Comparison with PQChPT is "anchored" by fact that theory with m_v=m_s is physical
- PQQCD is needed to predict properties of small eigenvalues of Dirac operator & connect with Random Matrix Theory



~ 5 years old

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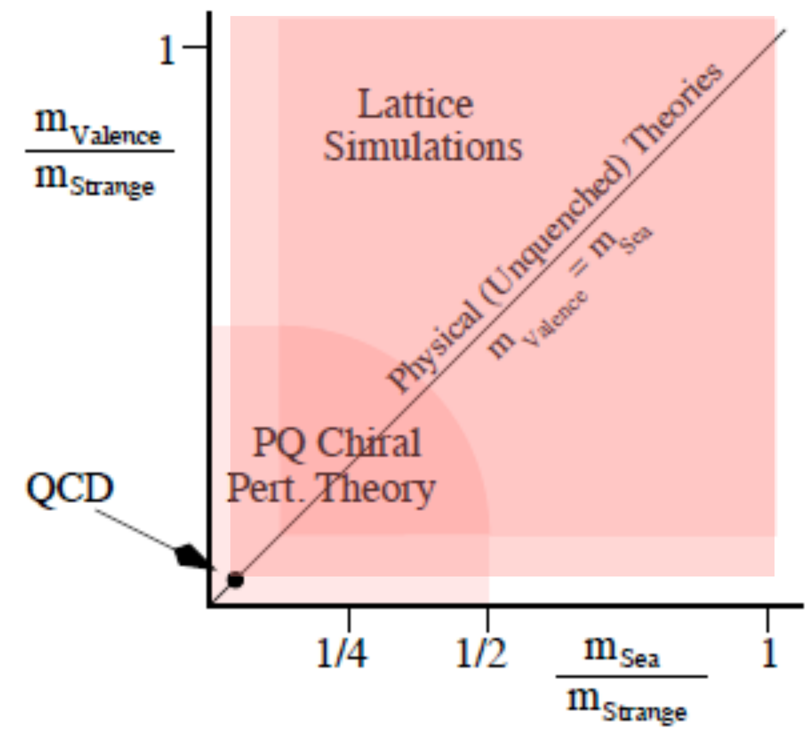


Present status (for some quantities)

Nomenclature

- Why called partially quenched? Why not partially unquenched?
- Bad old days: quenched approximation $m_{\rm sea} \longrightarrow \infty$
  - $\Rightarrow$ $\det(\not{D} + m_q) \to$ constant
  - $\Rightarrow$ **No quark loops**
  - $\Rightarrow$ $Z_{\rm QCD} \to Z_{\rm QQCD} = \int DU e^{-S_{\rm gauge}} = Z_{\rm gauge}$
- Unphysical nature of quenched QCD shows up various ways, e.g. $\langle\bar\psi\psi\rangle \longrightarrow \infty$ as $m_{\rm val} \longrightarrow 0$
- Partial quenching is in one sense a less extreme version of quenching, and thus the name
- If $m_{\rm sea} \gg \Lambda_{\rm QCD}$ then PQQCD, like quenched QCD, only qualitatively related to QCD
- **Consider here only the case when $m_{\rm sea} \ll \Lambda_{\rm QCD}$ so one can use χPT and relate PQCD to QCD quantitatively**

Morels' formulation of (P)QQCD

- **IDEA:** commuting spin-$\frac{1}{2}$ fields (ghosts) $\tilde{q}$ give determinant which cancels that from valence quarks

$$\int D\bar{q}Dq\; e^{-\bar{q}(\not{D}+m_q)q} = \det(\not{D}+m_q)$$

$$\int D\tilde{q}^\dagger D\tilde{q}\; e^{-\tilde{q}^\dagger(\not{D}+m_q)\tilde{q}} = \frac{1}{\det(\not{D}+m_q)}$$

- To formulate PQQCD need three types of "quark"
  - valence quarks $q_{V1}, q_{V2}, \dots q_{VN_V}$ $(N_V = 2, 3, \dots)$
  - sea quarks $q_{S1}, q_{S2}, \dots q_{SN}$ $(N = 2, 3)$
  - ghosts $\tilde{q}_{V1}, \tilde{q}_{V2}, \dots \tilde{q}_{VN_V}$ $(N_V = 2, 3, \dots)$
- Ghosts are degenerate with corresponding valence quarks
- Convergence of ghost integral requires $m_q > 0$ (since $\not{D}$ antihermitian)
  - Some subtleties in extending to non-hermitian lattice Wilson-Dirac operator

Morels' formulation of (P)QQCD

- Partition function reproduces that which is actually simulated

$$
\begin{aligned}
Z_{\rm PQ} &= \int DU e^{-S_{\rm gauge}} \int \prod_{i=1}^{N_V} \left(D\bar q_{V\,i} D q_{V\,i} D\tilde q^\dagger_{V\,i} D\tilde q_{V\,i} \right) \prod_{j=1}^{N} \left(D\bar q_{S\,j} D q_{S\,j} \right) \times \\
&\quad \times \exp\left[-\sum_{i=1}^{N_V} \bar q_{V\,i} (\not{D} + m_{V\,i}) q_{V\,i} - \sum_{j=1}^{N} \bar q_{S\,j} (\not{D} + m_{S\,j}) q_{S\,j} - \sum_{k=1}^{N_V} \tilde q^\dagger_{V\,k} (\not{D} + m_{V\,k}) \tilde q_{V\,k} \right] \\
&= \int DU e^{-S_{\rm gauge}} \prod_{i=1}^{N_V} \left(\frac{\det(\not{D} + m_{V\,i})}{\det(\not{D} + m_{V\,i})} \right) \prod_{j=1}^{N} \det(\not{D} + m_{S\,j}) \\
&= \int DU e^{-S_{\rm gauge}} \prod_{j=1}^{N} \det(\not{D} + m_{S\,j}) \\
&= Z_{\rm QCD-like}
\end{aligned}
$$

- Adding valence fields leads to desired valence propagators

Condensed notation

- Collect all fields into $(N+2N_V)$-dim vectors:

$$Q = \Big(\underbrace{q_{V1}, q_{V2}, \ldots, q_{VN_V}}_{\text{valence}}, \underbrace{q_{S1}, q_{S2}, \ldots, q_{SN}}_{\text{sea}}, \underbrace{\tilde{q}_{V1}, \tilde{q}_{V2}, \ldots, \tilde{q}_{VN_V}}_{\text{ghost}}\Big)$$

$$\overline{Q}^{tr} = \Big(\underbrace{\bar{q}_{V1}, \bar{q}_{V2}, \ldots, \bar{q}_{VN_V}}_{\text{valence}}, \underbrace{\bar{q}_{S1}, \bar{q}_{S2}, \ldots, \bar{q}_{SN}}_{\text{sea}}, \underbrace{\tilde{q}^\dagger_{V1}, \tilde{q}^\dagger_{V2}, \ldots, \tilde{q}^\dagger_{VN_V}}_{\text{ghost}}\Big)$$

$$\mathcal{M} = \Big(\underbrace{m_{V1}, m_{V2}, \ldots, m_{VN_V}}_{\text{valence}}, \underbrace{m_{S1}, m_{S2}, \ldots, m_{SN}}_{\text{sea}}, \underbrace{m_{V1}, m_{V2}, \ldots, m_{VN_V}}_{\text{ghost=valence}}\Big)$$

- Then can write action and partition function as:

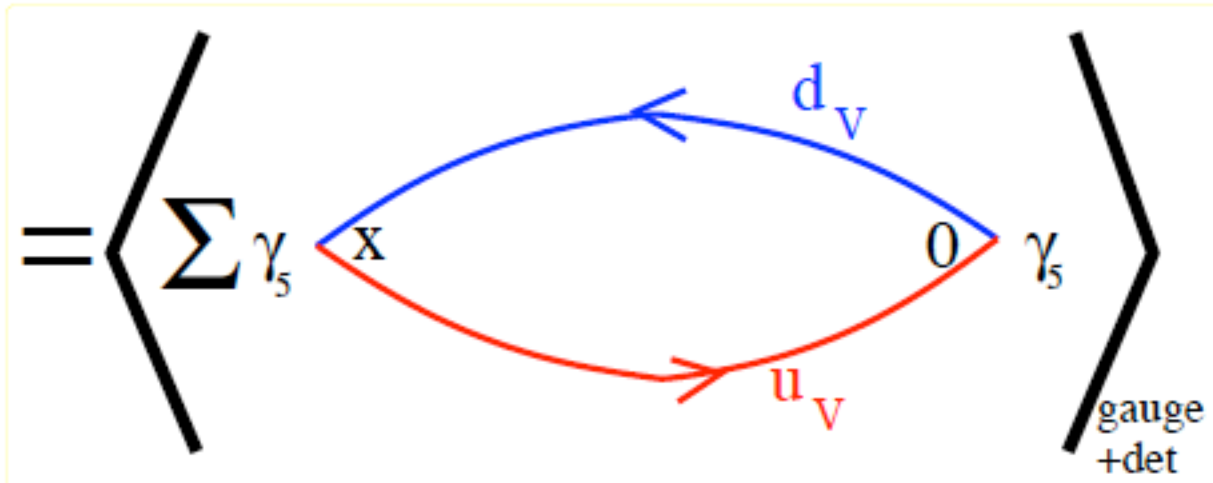
$$S_{\text{PQ}} = S_{\text{gauge}} + \overline{Q}(\not{D} + \mathcal{M})Q$$

$$Z_{\text{PQ}} = \int DU\, D\bar{Q} DQ\; e^{-S_{\text{PQ}}}$$

Formal representation of PQ correlator

$$Q = \Big(\underbrace{q_{V1}, q_{V2}, \ldots, q_{VN_V}}_{\text{valence}}, \underbrace{q_{S1}, q_{S2}, \ldots, q_{SN}}_{\text{sea}}, \underbrace{\tilde{q}_{V1}, \tilde{q}_{V2}, \ldots, \tilde{q}_{VN_V}}_{\text{ghost}}\Big)$$

$C_\pi^{\mathrm{PQ}}(\tau)$



$$= Z_{\mathrm{PQ}}^{-1} \int DU \prod_{j=1}^{N} \det(\not{D} + m_{Sj}) e^{-S_{\mathrm{gauge}}}$$

$$\times \sum_{\vec{x}} \mathrm{tr}\left[\gamma_5 \left(\frac{1}{\not{D} + m_{Vd}}\right)_{x0} \gamma_5 \left(\frac{1}{\not{D} + m_{Vu}}\right)_{0x}\right]$$

$$= Z_{\mathrm{PQ}}^{-1} \int DU D\overline{Q} DQ \; e^{-S_{\mathrm{PQ}}} \sum_{\vec{x}} \bar{u}_V \gamma_5 d_V (\vec{x}, \tau) \; \bar{d}_V \gamma_5 u_V (0)$$

Anchoring to QCD

$$Q = \Big(\underbrace{q_{V1}, q_{V2}, \ldots, q_{VN_V}}_{\text{valence}}, \underbrace{q_{S1}, q_{S2}, \ldots, q_{SN}}_{\text{sea}}, \underbrace{\tilde{q}_{V1}, \tilde{q}_{V2}, \ldots, \tilde{q}_{VN_V}}_{\text{ghost}}\Big)$$

Anchoring to QCD

Set valence and sea masses equal

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Anchoring to QCD

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- If $m_{Vu} = m_{Sj}$ and $m_{Vd} = m_{Sk}$ then valence correlator *is* physical:

$$\begin{aligned} C_\pi^{\rm PQ}(\tau) &= Z_{\rm PQ}^{-1} \int DU D\overline{Q} DQ\, e^{-S_{\rm PQ}} \sum_{\vec{x}} \bar{u}_V \gamma_5 d_V(\vec{x},\tau)\, \bar{d}_V \gamma_5 u_V(0) \\ &= Z_{\rm PQ}^{-1} \int DU D\overline{Q} DQ\, e^{-S_{\rm PQ}} \sum_{\vec{x}} \bar{q}_{Sj} \gamma_5 q_{Sk}(\vec{x},\tau)\, \bar{q}_{Sk} \gamma_5 q_{Sj}(0) \\ &= Z_{\rm QCD-like}^{-1} \int DU \prod_{i=1}^{N} D\bar{q}_{Si} Dq_{Si}\, e^{-S_{\rm QCD-like}} \\ &\qquad \times \sum_{\vec{x}} \bar{q}_{Sj} \gamma_5 q_{Sk}(\vec{x},\tau)\, \bar{q}_{Sk} \gamma_5 q_{Sj}(0) \\ &= C_\pi^{\rm QCD-like}(\tau) \end{aligned}$$

- Example of enhanced (V $\leftrightarrow$ S) symmetry in PQ theory

Summary so far

- PQQCD is a well-defined, local Euclidean statistical theory
    - Describes $m_v \neq m_s$ and allows formal definition of <u>individual</u> Wick contractions
- Morel's formulation restores "unitarity", but at the cost of introducing ghosts
    - Violate spin-statistics theorem, so Minkowski-space theory violates causality & positivity, and may have a Hamiltonian with spectrum unbounded below
    - For $m_v \neq m_s$, can show (under mild assumptions) that flavor-singlet "pion" correlators develop manifestly unphysical double-poles [Sharpe & Shoresh]
- Can generalize to include discretization errors & to mixed actions (different discretizations of valence & sea quarks, e.g. "overlap on twisted mass")
- To make practical use of PQQCD, need to develop PQChPT
    - Is this possible given the unphysical features?
    - Do we need to have a healthy Minkowski theory to justify EFTs?

Outline of lecture 4

- Partial quenching and PQChPT
  - What is partial quenching and why might it be useful?
  - Developing PQChPT
  - Results and status
- m_u=0 and the validity of PQ theories (and the rooting prescription)

Methods for developing PQChPT

- "Supersymmetric" method based on Morel's formulation [Bernard & Golterman]
- "Replica" method adjusting loop contributions by adjusting $N_{\rm sea}$ [Damgaard & Splittorf]
  - Formalizes "Quark-line" method accounting by hand for quarks in loops [Sharpe]
- Give same results to date—likely equivalent
- Use supersymmetric method here

Symmetries of PQQCD

$$Q = \Big(\underbrace{q_{V1}, q_{V2}, \ldots, q_{VN_V}}_{\text{valence}}, \underbrace{q_{S1}, q_{S2}, \ldots, q_{SN}}_{\text{sea}}, \underbrace{\tilde{q}_{V1}, \tilde{q}_{V2}, \ldots, \tilde{q}_{VN_V}}_{\text{ghost}} \Big)$$

- Action of PQQCD looks like QCD

$$S_{\text{PQQCD}} = S_{\text{gauge}} + \overline{Q}(\not{D} + \mathcal{M})Q$$

- Naively, when $M \to 0$ have graded version of QCD chiral symmetry:

$$Q_{L,R} \longrightarrow U_{L,R} Q_{L,R}\,, \qquad \overline{Q}_{L,R} \longrightarrow \overline{Q}_{L,R} U^{\dagger}_{L,R} \qquad U_{L,R} \in SU(N_V + N|N_V)$$

- Apparent symmetry is $SU(N_V + N|N_V)_L \times SU(N_V + N|N_V)_R \times U(1)_V$
- In fact, there are subtleties in the ghost sector, but can ignore in perturbative calculations [Sharpe & Shoresh]

Subtleties have been understood in calculations leading to connection with random matrix theory [Damgaard et al]

Brief primer on graded groups

- U is graded: contains both commuting and anticommuting elements:

$$U = \begin{pmatrix} A & B \\ \underbrace{C}_{N_V+N} & \underbrace{D}_{N_V} \end{pmatrix}, \quad A, D \text{ commuting, } B, C \text{ anticommuting}$$

- If $U \in U(N_V + N|N_V)$ (fundamental representation) then

$$UU^\dagger = U^\dagger U = 1\,, \quad [\text{with } (\eta_1\eta_2)^* \equiv \eta_2^*\eta_1^*]$$

- Supertrace maintains cyclicity:

$$\text{str}U \equiv \text{tr}A - \text{tr}D \quad \Rightarrow \quad \text{str}(U_1U_2) = \text{str}(U_2U_1)$$

- For $U \in SU(N_V + N|N_V)$, superdeterminant is unity:

$$\text{sdet}U \equiv \exp[\text{str}(\ln U)] = \frac{\det(A - BD^{-1}C)}{\det(D)} \quad \Rightarrow \quad \text{sdet}(U_1U_2) = \text{sdet}U_1\text{sdet}U_2$$

Examples of SU(N$_V$+N|N) matrices

$$U = \begin{pmatrix} SU(N_V+N) & 0 \\ 0 & SU(N_V) \end{pmatrix} \quad \Rightarrow \quad \mathrm{sdet} U = 1$$

$$U = \begin{pmatrix} e^{i\theta N_V} & 0 \\ 0 & e^{i\theta(N+N_V)} \end{pmatrix} \quad \Rightarrow \quad \mathrm{sdet} U = \frac{(e^{i\theta N_V})^{N+N_V}}{(e^{i\theta(N+N_V)})^{N_V}} = 1$$

- An overall phase rotation is not in $SU(N_V+N|N)$

$$U = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} \quad \Rightarrow \quad \mathrm{sdet} U = \frac{e^{i\theta(N+N_V)}}{e^{i\theta N_V}} = e^{i\theta N}$$

- Thus $U(N_V+N|N_V) = [SU(N_V+N|N_V) \otimes U(1)]/Z_N$
- Group structure different if $N=0$ (quenched theory)

Constructing the EFT

- Follow the same steps as for standard ChPT as closely as possible
  - Expand about theory with M=0 where symmetry is maximal
    - Strictly speaking, need to keep (arbitrarily) small mass to avoid PQ divergences & to use Vafa-Witten
    - A posteriori find divergences if $m_v \to 0$ at fixed m_s, so must take chiral limit with m_v/m_s fixed
    - Symmetry is $\mathcal{G} = SU(N_V + N|N_V)_L \times SU(N_V + N|N_V)_R$
  - For M real, diagonal, positive [Vafa-Witten] theorem implies that graded vector symmetry is not spontaneously broken [Sharpe & Shoresh; Bernard & Golterman]
    - Quark and ghost condensates equal if $m_v = m_s \to 0$
  - We know chiral symm. breaks spontaneously in QCD with non-zero condensate
    - Since QCD is inside PQQCD ⇒ we know form of PQ condensate & symmetry breaking
    - Order parameter $\Omega_{ij} = \langle Q_{L,i,\alpha,c} \overline{Q}_{R,j,\alpha,c} \rangle_{PQ} \xrightarrow[\mathcal{G}]{} U_L \Omega U_R^\dagger$
    - With standard masses $\Omega = \omega \times 1$ so vacuum manifold is now $SU(N_V + N|N_V)$
    - Symmetry breaking is $\mathcal{G} \to \mathcal{H} = SU(N_V + N|N_V)_V$

Constructing the EFT

- Still following the same steps as for standard ChPT as closely as possible ...
  - Can derive Ward identities in PQQCD, & Goldstone's thm. for 2-pt functions
    - $(N+2N_V)^2-1$ Goldstone "particles" created by $\overline{Q}\gamma_\mu\gamma_5 T^a Q$ with T^a a generator of graded group
  - ***New***: can construct transfer matrix for PQQCD including ghosts & show that, despite not being hermitian, it can be diagonalized and has a bounded spectrum [Bernard & Golterman]
    - Energies can be real or come in complex-conjugate pairs (PT symmetry)
    - Have a complete set of states, although left- and right- eigenvectors are different
    - In free theory, correlators fall exponentially (up to powers from double-poles) but can be of either sign
  - This result, if it holds up to scrutiny, makes the foundation of PQChPT essentially as strong as that of ChPT, since can follow a line of argument due to [Leutwyler] which uses cluster decomposition and does not explicitly rely on unitarity
    - In particular, the existence of a transfer matrix etc. means that the spectrum deduced from 2-pt functions holds also for all other correlators (assuming no other light particles)

Constructing the EFT

- Sketch of [Leutwyler]'s argument
  - Existence of bounded transfer matrix + assumption of unique vacuum implies that PQ theory satifies cluster decomposition
  - Integrating out heavy states (which might have complex energies?) still leads to local vertices which can be connected by Goldstone propagators
  - This leads to the same results as a general effective local Lagrangian in terms of Goldstone fields
  - Implementing local symmetry of generating functional with sources (up to anomalies) leads to result that effective Lagrangian can be chosen to be invariant under local symmetry group
- Bottom line: write down the most general local Lagrangian with sources consistent with local $SU(N_V+N|N_V)_L \times SU(N_V+N|N_V)_R$ symmetry

Generalization of Σ in PQChPT

- Follow method used for QCD:

$$\Omega/\omega \to \Sigma(x) \in SU(N_V + N|N)\,, \qquad \Sigma \underset{\mathcal{G}}{\longrightarrow} U_L\,\Sigma\,U_R^\dagger$$

- For standard masses, $\langle\Sigma\rangle = 1$, so define Goldstones by

$$\Sigma = \exp\left[\frac{2i}{f}\Phi(x)\right]\,, \qquad \Phi(x) = \begin{pmatrix} \phi(x) & \eta_1(x) \\ \eta_2(x) & \widetilde{\phi}(x) \end{pmatrix}$$

  - $\mathrm{sdet}\Sigma = 1 \Rightarrow \mathrm{str}\Phi = \mathrm{tr}\phi - \mathrm{tr}\widetilde{\phi} = 0$

- QCD GBs contained in Φ

$$\Phi(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \pi(x) & 0 \\ \underbrace{0}_{N_V} & \underbrace{0}_{N} & \underbrace{0}_{N_V} \end{pmatrix} \Rightarrow \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Sigma_{\rm QCD} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Building blocks for PQχPT as for χPT, e.g.

$$L_\mu = \Sigma D_\mu \Sigma^\dagger \to U_L L_\mu U_L^\dagger\,, \qquad \mathrm{str}(L_\mu) = 0$$

- Power counting as in χPT

PQ Chiral Lagrangian at NLO

[Bernard & Golterman; Sharpe & Van de Water]

- General form consistent with graded symmetries

$$
\begin{aligned}
\mathcal{L}^{(2)} &= \frac{f^2}{4}\mathrm{str}\left(D_\mu\Sigma D_\mu\Sigma^\dagger\right) - \frac{f^2}{4}\mathrm{str}(\chi\Sigma^\dagger + \Sigma\chi^\dagger) \\
\mathcal{L}^{(4)} &= -L_1\,\mathrm{str}(D_\mu\Sigma D_\mu\Sigma^\dagger)^2 - L_2\,\mathrm{str}(D_\mu\Sigma D_\nu\Sigma^\dagger)\mathrm{tr}(D_\mu\Sigma D_\nu\Sigma^\dagger) \\
&+L_3\,\mathrm{str}(D_\mu\Sigma D_\mu\Sigma^\dagger D_\nu\Sigma D_\nu\Sigma^\dagger) \\
&+L_4\,\mathrm{str}(D_\mu\Sigma^\dagger D_\mu\Sigma)\mathrm{str}(\chi^\dagger\Sigma + \Sigma^\dagger\chi) + L_5\,\mathrm{str}(D_\mu\Sigma^\dagger D_\mu\Sigma)[\chi^\dagger\Sigma + \Sigma^\dagger\chi]) \\
&-L_6\left[\mathrm{str}(\chi^\dagger\Sigma + \Sigma^\dagger\chi)\right]^2 - L_7\left[\mathrm{str}(\chi^\dagger\Sigma - \Sigma^\dagger\chi)\right]^2 - L_8\,\mathrm{str}(\chi^\dagger\Sigma\chi^\dagger\Sigma + \mathrm{p.c.}) \\
&+L_9\,i\mathrm{str}(L_{\mu\nu}D_\mu\Sigma D_\nu\Sigma^\dagger + p.c.) + L_{10}\,\mathrm{str}(L_{\mu\nu}\Sigma R_{\mu\nu}\Sigma^\dagger) \\
&+H_1\,\mathrm{str}(L_{\mu\nu}L_{\mu\nu} + p.c.) + H_2\,\mathrm{str}(\chi^\dagger\chi) + \mathrm{WZW}_{\mathrm{PQ}} \\
&+L_{\mathrm{PQ}}\mathcal{O}_{PQ}
\end{aligned}
$$

- $\chi = 2B_0\mathcal{M}$
- Same form as for QCD with $\mathrm{tr} \to \mathrm{str}$ plus one extra term ($\mathcal{O}_{\mathrm{PQ}}$)
- **How do the LECs relate to those of QCD?**

Anchoring PQChPT to ChPT

- If choose Σ to lie in QCD subspace

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Sigma_{\rm QCD} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

  and sources do not connect subspaces, then

$$\mathcal{L}^{(2,4,\dots)}_{\rm PQ\chi PT}(\Sigma) \to \mathcal{L}^{(2,4,\dots)}_{\chi \rm PT}(\Sigma_{\rm QCD})$$

- If external fields in correlation function are from sea sector, then can show that all valence and ghost contributions cancel in intermediate states
  - $\Rightarrow$ Σ takes the form given above
  - PQχPT calculation collapses to one in χPT
- **Thus LECs in PQχPT are equal to those in χPT**
  - Results in the chiral regime from PQQCD give information about physical LECs

Additional PQ operator: O_{PQ}

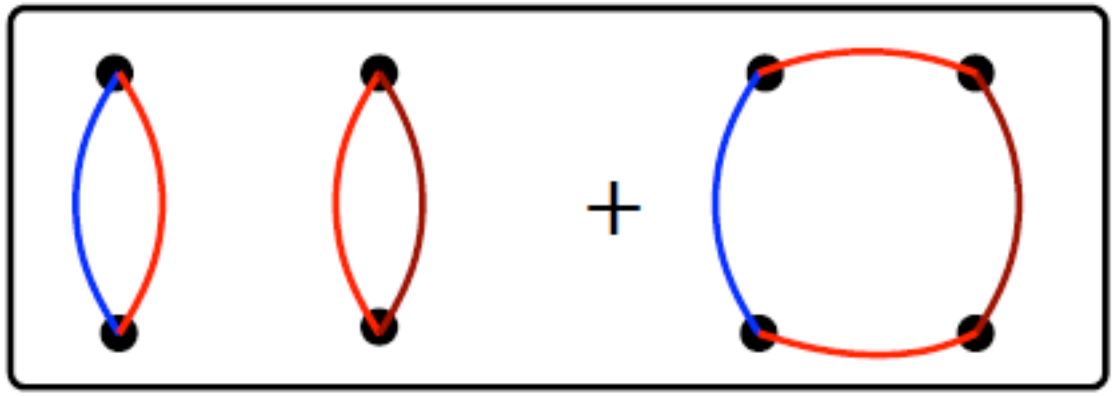
- Starting at NLO, at each order there are an increasing number of PQ operators that vanish on QCD subspace
- At NLO, only one such operator [Sharpe & Van de Water]

$$\begin{aligned}\mathcal{O}_{\rm PQ} &= \mathrm{str}(D_\mu\Sigma D_\nu\Sigma^\dagger D_\mu\Sigma D_\nu\Sigma^\dagger) \\ &\quad -\frac{1}{2}\mathrm{str}(D_\mu\Sigma D_\mu\Sigma^\dagger)^2 - \mathrm{str}(D_\mu\Sigma D_\nu\Sigma^\dagger)\mathrm{str}(D_\mu\Sigma D_\nu\Sigma^\dagger) \\ &\quad + 2\mathrm{str}(D_\mu\Sigma D_\nu\Sigma^\dagger D_\mu\Sigma D_\nu\Sigma^\dagger)\end{aligned}$$

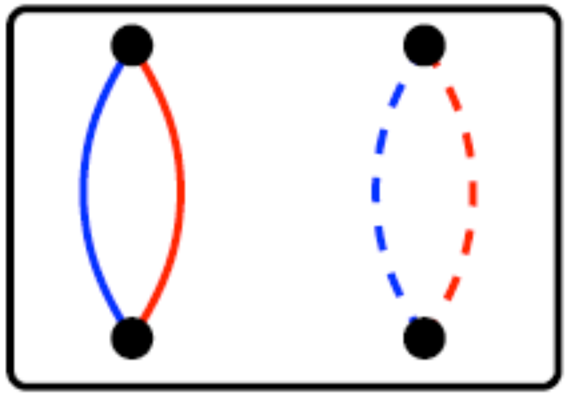
- Vanishes if $\Sigma \to \Sigma_{\rm QCD}$ due to Cayley-Hamilton relations for 3×3 matrices
- Does *not vanish* for general $\Sigma_{\rm PQ}$
- Appears in $\mathcal{L}^{(4)}_{\rm PQ\chi}$ with additional LEC
- Same is true for standard χPT if $N \geq 4$
- $\mathcal{O}_{\rm PQ}$ contributes to $\pi\pi$ scattering at NLO, but to m_π and f_π only at NNLO

Why is O_{PQ} present?

- Because PQQCD allows isolation of individual Wick contractions, unlike QCD
- For example, $\pi^+ K^0$ scattering in QCD has two contractions



- Can separate these contractions in PQQCD, e.g.



- $\mathcal{O}_{\rm PQ}$ contributes to the PQQCD process, but not that in QCD
- Shows how PQQCD differs from QCD even if $m_V = m_S$

Calculating in PQChPT

- PQ Lagrangian at LO:

$$\mathcal{L}^{(2)} = \frac{f^2}{4}\mathrm{str}\left(D_\mu\Sigma D_\mu\Sigma^\dagger\right) - \frac{f^2}{4}\mathrm{str}(\chi\Sigma^\dagger + \Sigma\chi^\dagger)$$

- Insert expansion in Goldstone fields:

$$\Sigma = \exp\left[\frac{2i}{f}\Phi(x)\right], \qquad \Phi(x) = \begin{pmatrix} \phi(x) & \eta_1(x) \\ \eta_2(x) & \tilde{\phi}(x) \end{pmatrix}, \qquad \mathrm{str}\Phi = 0$$

$$\begin{aligned} \mathcal{L}^{(2)} &= \mathrm{str}(\partial_\mu\Phi\partial_\mu\Phi) + \mathrm{str}(\chi\Phi^2) + \ldots \\ &= \mathrm{tr}(\partial_\mu\phi\partial_\mu\phi + \partial_\mu\eta_1\partial_\mu\eta_2 - \partial_\mu\eta_2\partial_\mu\eta_1 - \partial_\mu\tilde{\phi}\partial_\mu\tilde{\phi}) \\ &\quad + \mathrm{tr}\left[(\phi^2 + \eta_1\eta_2)\begin{pmatrix} m_V & 0 \\ 0 & m_S \end{pmatrix}\right] - \mathrm{tr}(\tilde{\phi}^2 m_V) - \mathrm{tr}(\eta_2\eta_1 m_V) \end{aligned}$$

- ϕ part is like in QCD, except includes both valence and sea quarks
  - Propagator for "charged" meson $\bar{q}_1 q_2$ (either valence of sea) is $1/(p^2 + m_{12}^2)$, $m_{12}^2 = (\chi_1 + \chi_2)/2$

Calculating in PQChPT

$$\mathcal{L}^{(2)} = \mathrm{tr}(\partial_\mu\phi\partial_\mu\phi + \partial_\mu\eta_1\partial_\mu\eta_2 - \partial_\mu\eta_2\partial_\mu\eta_1 - \partial_\mu\widetilde{\phi}\partial_\mu\widetilde{\phi})$$
$$+\,\mathrm{tr}\left[(\phi^2+\eta_1\eta_2)\begin{pmatrix} m_V & 0 \\ 0 & m_S \end{pmatrix}\right] - \mathrm{tr}(\widetilde{\phi}^2 m_V) - \mathrm{tr}(\eta_2\eta_1 m_V)$$

- $\widetilde{\phi}$ terms have wrong signs
  - Naively, propagator for "charged" ghost mesons $\bar{\widetilde{q}}_1\widetilde{q}_2$ is $-1/(p^2+m_{12}^2)$, $m_{12}^2 = (\chi_1+\chi_2)/2$
  - But potential not minimized and functional integral not convergent!
  - More careful treatment of symmetries of PQQCD, maintaining convergence of ghost functional integral, concludes that naive result is OK in perturbation theory (but not non-perturbatively, e.g. in ϵ-regime, where should change $\widetilde{\phi}\to i\widetilde{\phi}$, $\Sigma^\dagger\to\Sigma^{-1}$) [Sharpe & Shoresh]
- Goldstone fermion propagators can have either sign (no convergence problems); actual signs important for cancellations

Implementing stracelessness

- How implement $\mathrm{str}(\Phi) = \mathrm{tr}(\phi) - \mathrm{tr}(\widetilde{\phi}) = 0$?
  1. Use a basis of generators which is straceless:

     $\Phi = \sum_a \Phi_a T^a$ with $\mathrm{str}(T^a) = 0$
     - Analagous to not including the η' in QCD χPT

  2. Include identity component but then "integrate out"

     $\Phi \to \Phi + \Phi_0/\sqrt{N}$ so that $\mathrm{str}\Phi = \sqrt{N}\Phi_0$

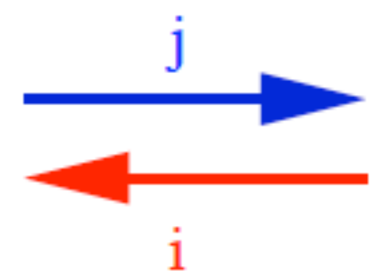
     $\mathcal{L}_{\mathrm{PQ}\chi} \to \mathcal{L}_{\mathrm{PQ}\chi} + m_0^2 \mathrm{str}(\Phi)^2/N$
     - Calculate propagators, then send $m_0^2 \to \infty$ within them
     - To make formally correct, must regularize with a cut-off (e.g. lattice) so that $(\partial_\mu \Phi_0)^2 < m_0^2 \Phi_0^2$ (trivial decoupling)
     - Really just a trick to implement stracelessness

- Introducing Φ_0 has advantage of allowing use of "quark line" basis: $\Phi_{ij} \sim Q_i \overline{Q}_j$ for all i, j

Quark lines & double poles

- "Charged" particle propagators are simple:

$$\langle \Phi_{ij}\Phi_{ji}\rangle = \pm \frac{1}{p^2+(\chi_i+\chi_j)/2} =$$

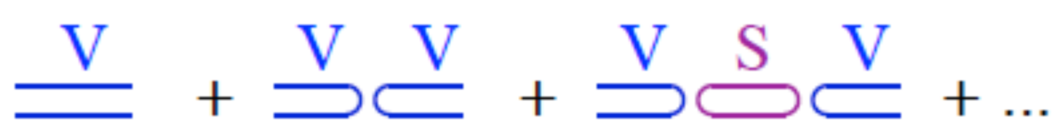


- Neutral propagators have double poles:

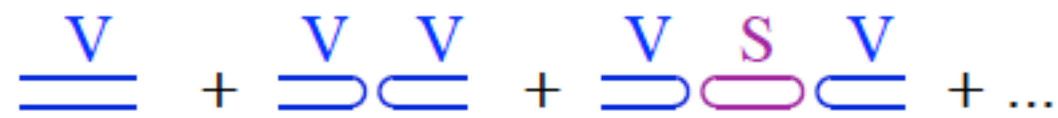
$$\mathcal{L}^{(2)} = \sum_{j=1}^{N+2N_V} \epsilon_j(\partial_\mu\Phi_{jj}\partial_\mu\Phi_{jj} + m_j\Phi_{jj}^2) + (m_0^2/N)(\sum_j \epsilon_j\Phi_{jj})^2$$

$$\epsilon_j = \begin{cases} +1 & \text{valence or sea quarks} \\ -1 & \text{ghosts} \end{cases}$$

- Can simply invert with linear algebra tricks. Schematically, for external valence quarks have "hairpin" sum:



Quark lines & double poles



- Result after $m_0^2 \to \infty$ for $N = 3$ [Bernard & Golterman; Sharpe & Shoresh]

$$\langle \Phi_{ii} \Phi_{jj} \rangle = \frac{\epsilon_i \delta_{ij}}{p^2 + \chi_i} - \frac{1}{N} \frac{1}{(p^2 + \chi_i)(p^2 + \chi_j)} \frac{(p^2 + \chi_{S1})(p^2 + \chi_{S2})(p^2 + \chi_{S3})}{(p^2 + M_{\pi_0}^2)(p^2 + M_\eta^2)}$$

- Simplifies for degenerate sea quarks:

$$\langle \Phi_{ii} \Phi_{jj} \rangle = \frac{\epsilon_i \delta_{ij}}{p^2 + \chi_i} - \frac{1}{N} \frac{(p^2 + \chi_S)}{(p^2 + \chi_i)(p^2 + \chi_j)}$$

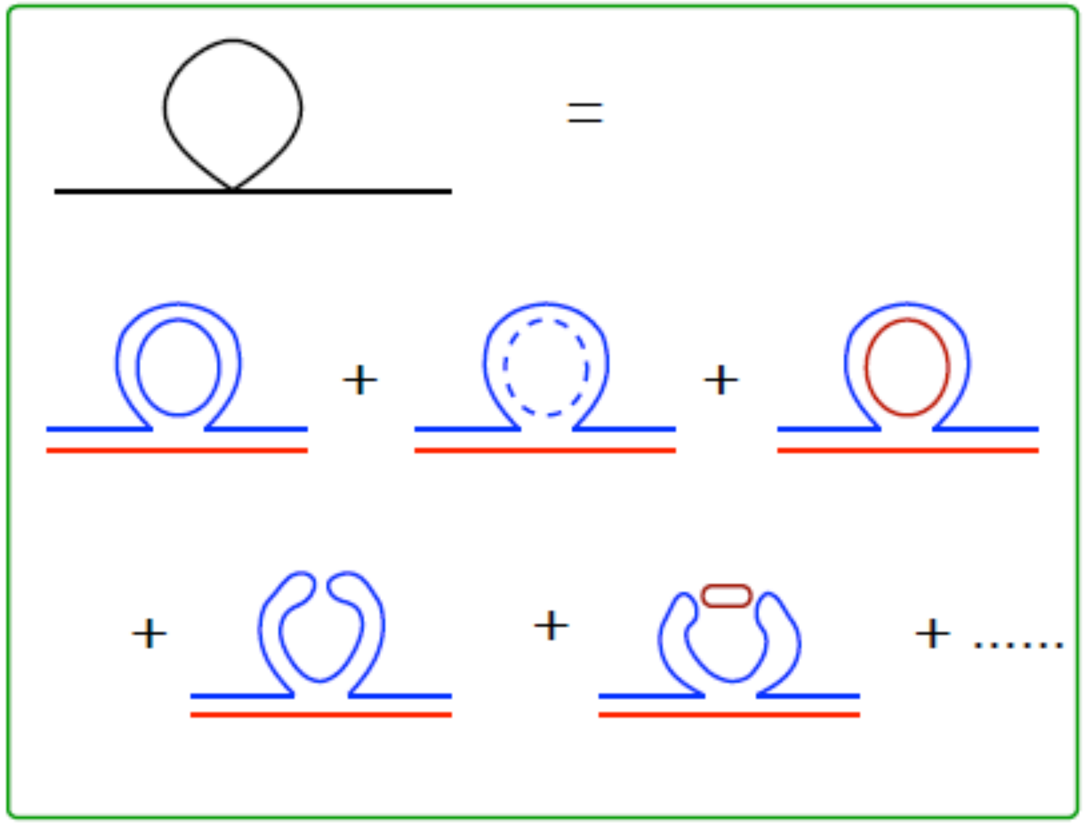
  - Manifestly unphysical double pole for $\chi_i = \chi_j$
  - Residue is then $(\chi_i - \chi_S)/N$, so vanishes for physical subspace
  - Can show *from symmetries of PQQCD* that if charged propagators have single poles, then neutral have double (and no higher) poles [Sharpe & Shoresh]

Outline of lecture 4

- Partial quenching and PQChPT
  - What is partial quenching and why might it be useful?
  - Developing PQChPT
  - Results and status
- m_u=0 and the validity of PQ theories (and the rooting prescription)

Sample calculation: m_π

- Calculations are straightforward extension of standard χPT
- Mass-squared of "pion" composed of valence quarks $V1, V2$
- Quark-line diagrams for 1-loop contributions



- LO four-pion vertices have single strace, so are "connected"
- Manifest cancellation between contributions from commuting and anticommuting particles

Sample calculation: m_π^2

- To simplify expression for loop contributions, assume N degenerate sea quarks and $m_{V1} = m_{V2} \neq m_S$

$$m_{VV}^2 = \chi_V \left(1 + \frac{1}{N} \frac{2\chi_V - \chi_S}{\Lambda_\chi^2} \ln(\chi_V/\mu^2) + \frac{\chi_V - \chi_S}{N\Lambda_\chi^2} \right.$$
$$\left. + \frac{8}{f^2} \left[(2L_8 - L_5)\chi_V + (2L_6 - L_4) N \chi_S \right] \right)$$

  - Reduces to QCD-like result when $\chi_V \to \chi_S$
  - χ_V and χ_S provide separate dials for determining $2L_8 - L_5$ and $2L_6 - L_4$
  - Result in PQ mass-plane depends on physical LECs
  - Unphysical nature of result clear from divergence in $\chi_S \ln \chi_V$ as $\chi_V \to 0$
  - In practice, expansion breaks down only for very small χ_V

Status of PQChPT calculations

- It is now standard to extend any χPT calculation to PQχPT
  - Many quantities considered at NLO: pions, baryons, vector mesons, scalar mesons, heavy-light hadrons, weak matrix elements (B_K, $K \to \pi\pi$), NEDM, pion scattering, ...
  - First calculations at NNLO for pion properties
  - PQ effects also included in tmχPT, staggered χPT and mixed action χPT
  - Most non-trivial example is baryons, where need to use a set-up in which all three quark lines are explicit
  - Most striking result is for scalar meson correlators, where hairpin propagators lead to unphysical *negative* contributions at long distances
- In general, can use PQχPT to determine form of expected results for individual contractions (e.g. connected and disconnected contributions to π_0 propagators in tmLQCD) [Hansen & Sharpe]
- Most extensive practical use is in MILC improved staggered simulations
- PQChPT can be used to estimate size of disconnected contribs, e.g. g-2 [Juettner]
- Generalization to ε-regime allows predictions for small eigenvalues & connection with RMT including discretization errors
  - Recent discovery of constraints on signs of some LECs in WChPT [Damgaard, Splittorff, Verbaarschot; Kieburg et al.; Hansen & Sharpe]

Outline of lecture 4

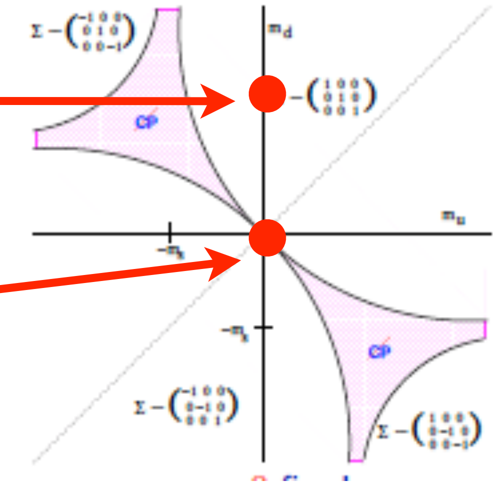
- Partial quenching and PQChPT
  - What is partial quenching and why might it be useful?
  - Developing PQChPT
  - Results and status
- m_u=0 and the validity of PQ theories (and the rooting prescription)

Some additional references for $m_u=0$

Including some on the rooting controversy

- M. Creutz, "Ambiguities in the up-quark mass," Phys. Rev. Lett. 92 (2004) 162003 [hep-ph/0312018]
- K. Choi, C. Kim & W. Sze ['t Hooft vertex gives additive mass renorm], Phys. Rev. Lett. 61(1988) 794
- T. Banks, Y. Nir & N. Seiberg [additive mass renorm & strong CP problem], hep-ph/9403203
- M. Creutz, "One flavor QCD," Annals. Phys. 322 (2007) 1518 [hep-th/0609187]
- T. DeGrand *et al.*, [N_f=1 condensate], Phys. Rev. D74 (2006) 054501 [hep-th/0605147]
- M. Creutz, "The 't Hooft vertex revisited," Annals. Phys. 323 (2008) 2349 [arXiv:0711.2640]
- M. Creutz, "Chiral anomalies and rooted staggered fermions," Phys. Lett. B649 (2007) 230 [hep-lat/0603020]
- C. Bernard, M. Golterman, S. Sharpe & Y. Shamir [Comment on previous paper], Phys. Lett. B649 (2007) 235 [hep-lat/0603027]
- M. Creutz [Comment on comment], Phys. Lett. B649 (2007) 241 [arXiv:0704.2016]
- C. Bernard, M. Golterman, S. Sharpe & Y. Shamir, "'t Hooft vertices, partial quenching & rooted staggered QCD," Phys. Rev. D77 (2008) 114504 [arXiv:0711.0696]
- M. Creutz [Comment on previous paper], Phys. Rev. D78 (2008) 078501 [arXiv:0805.1350]
- C. Bernard *et al.* [Comment on comment], Phys. Rev. D78 (2008) 078502 [arXiv:0808.2056]
- S. Sharpe, "Rooted staggered fermions, good, bad or ugly?" PoS Lat2006 (2006) 22 [hep-lat/0610094]
- M. Golterman, "QCD with rooted staggered fermions," arXiv:0812.3110
- M. Creutz, "Confinement, chiral symmetry & the lattice," arXiv:1103.3304
- S. Durr & C. Hoelbling, "Scaling tests with dynamical overlap and rooted staggered quarks,", Phys. Rev. D71 (2005) 054501 [hep-lat/0411022]

Ambiguity in $\mathrm{m_u}$=0?



- Consider QCD with m_d and m_s fixed (e.g. at their physical values), but send $m_u \to 0$
  - No increase in symmetry
  - $m_\pi^2 \propto (m_u + m_d) + \mathrm{NLO}$ does not vanish
- Contrast this with sending both $m_u, m_d \to 0$:
  - $SU(2)_L \times SU(2)_R$ becomes exact, and $m_\pi^2 \to 0$
- But doesn't $m_u \to 0$ have unambiguous meaning at the level of the lattice action?
  - Naively would seem so if use fermions with exact chiral symmetry (e.g. overlap)
  - But there are (infinitely) many choices for overlap kernel, which assign different topological charges to "rough" configurations

Also with Wilson fermions using PCAC masses [Sommer's lectures]

- If we set $m_u = 0$ using two different kernels, will we obtain, in the continuum limit, the same value for mass ratios, e.g. $m_{\pi_0}/m_{\mathrm{proton}}$?
  - The standard answer is **YES**
  - [Creutz, PRL 92, 162003 (2004)] argues **NO!**
- This is the potential ambiguity.

Restatement in N_f=1 QCD

- Can formulate the issue also in $N_f = 1$ QCD, a simpler setting
- No PGBs: spectrum consists of "η", "Δ", etc.
- With two overlap operators having different kernels, if one sets $m = 0$, and takes the continuum limit (not an easy task in practice!) will one get the same value for m_η/m_Δ?
  - The standard answer is **YES**
  - [Creutz, PRL 92, 162003 (2004)] argues **NO**
  - Note that for $a \neq 0$ will certainly have "kernel-dependent" discretization errors—the issue is what happens when $a \to 0$.
- Use this formulation in subsequent discussion:
  - Note that $\langle\bar{\psi}\psi\rangle \neq 0$, although this breaks no symmetry

Well defined if use overlap fermions

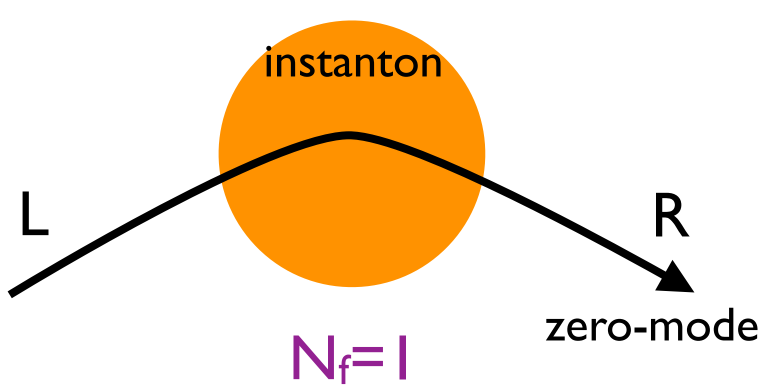
Non-vanishing checked by comparing eigenvalues to RMT (which also checks PQChPT) [DeGrand *et al.*]

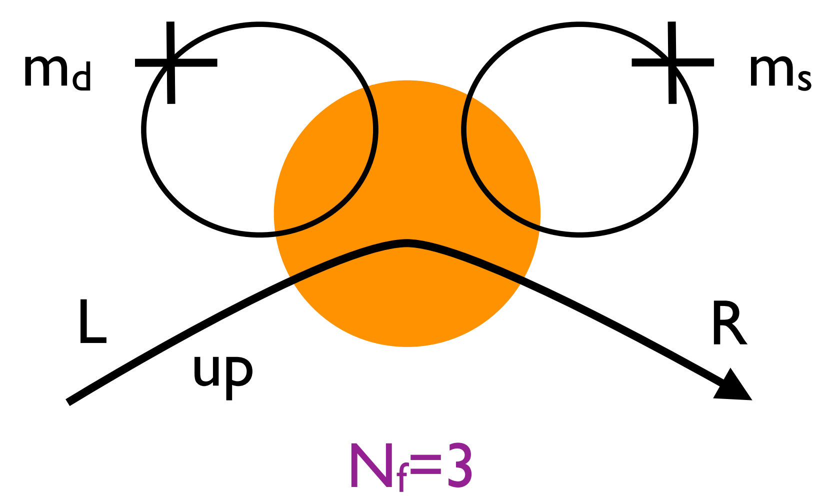
The standard argument

- In perturbation theory, if have chiral symmetry (as with overlap), quark mass is renormalized multiplicatively, to all orders

$$
\begin{aligned}
m(a) &= M g(a)^{\gamma_0/\beta_0}[1+O(g^2)] \\
a\Lambda &= e^{-1/(2\beta_0 g^2)} g^{-\beta_1/\beta_0^2}[1+O(g^2)] \\
\beta_0 &= (11-2N_f/3)/(16\pi^2)
\end{aligned}
$$

- This is uncontroversial. If it were the whole story, it would imply that, once $g(a)$ is small enough (so the universal parts of the β-function and anomalous dimension dominate) setting $M=0$ ($\Rightarrow m(a)=0$) leads to universal long-distance physics, irrespective of the overlap kernel.
  - Just as different gauge actions give a Symanzik effective action that differs by $a^2\times$ irrelevant dim-6 operators, so two different $m=0$ theories will differ by irrelevant dim >4 operators
- What about non-perturbative contributions to the running?
  - The 't Hooft vertex!





The standard argument

- In one flavor QCD, the 't Hooft vertex is bilinear, and leads to additive shift of quark mass
- Instanton calculations are not reliable when instantons are large, since $g(\rho)$ is not small
- However, what is needed for the RG evolution between scale $1/a$ and $1/(a+da)$ are instantons of size $\rho \sim a$
- If a is small enough, the semi-classical result should be reliable:

$$\frac{dm}{d\ln a} \approx m\gamma_0 g^2 + \text{const} \times (1/a) e^{-8\pi^2/g^2} g^n$$
$$\approx m\gamma_0 g^2 + \text{const} \times \Lambda (a\Lambda)^{28/3}$$

For N_f=3

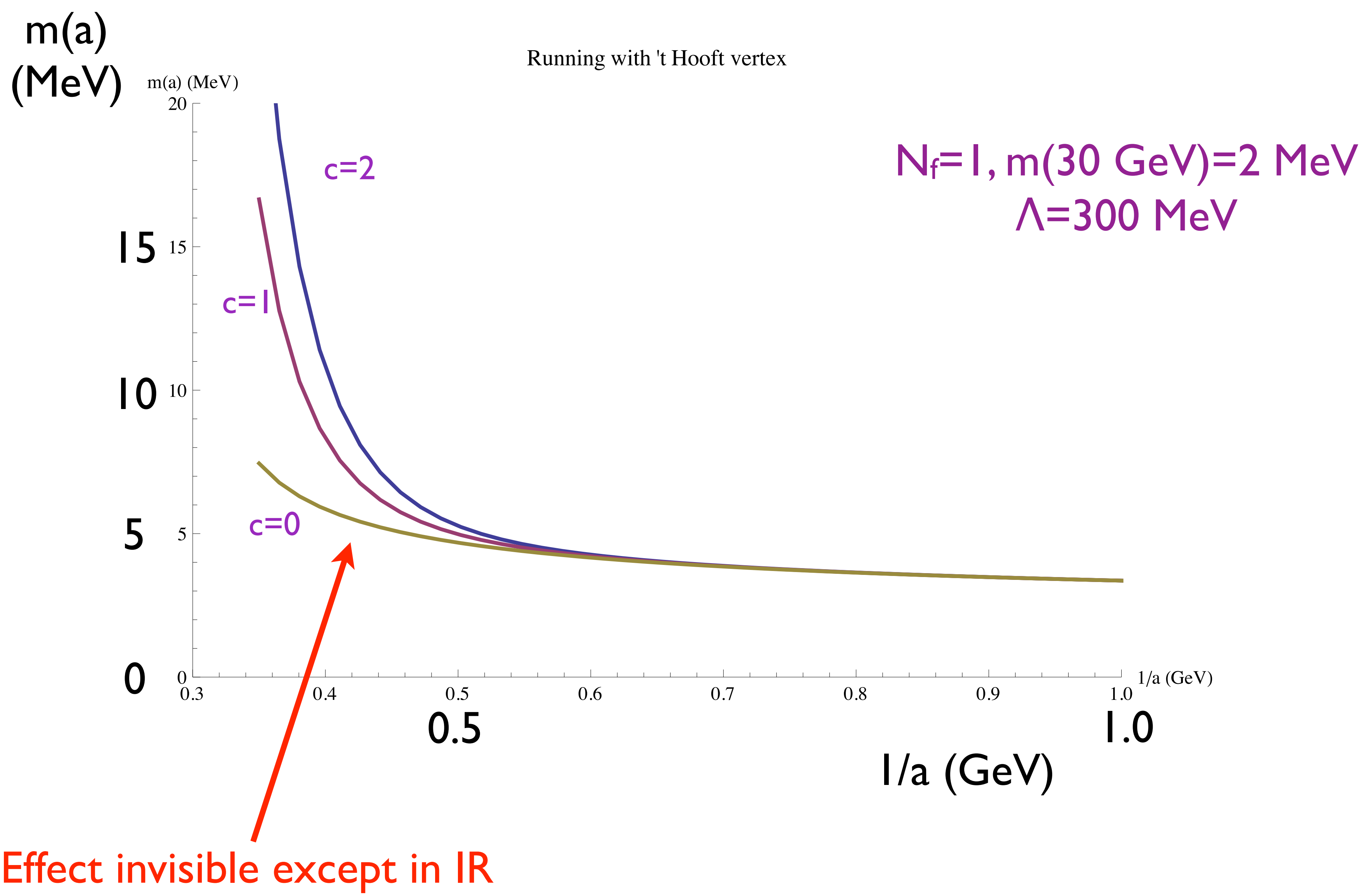
$$\frac{m_d m_s}{\Lambda}(a\Lambda)^{10}$$

[Georgi & Macarthy 1981] [Choi, Kim, Sze, PRL 61, 794 (1988)] [Banks, Nir & Seiberg, hep-ph/9403203]

- Additive contribution present, which can only calculate approximately
  - However, it vanishes as $a^{\sim 9}$

Example of running

$$\frac{dm}{d\ln a} \approx m\gamma_0 g^2 + \text{const} \times \Lambda(a\Lambda)^{28/3}$$



The standard argument

$$\frac{dm}{d\ln a} \approx m\gamma_0 g^2 + \text{const} \times \Lambda(a\Lambda)^{28/3}$$

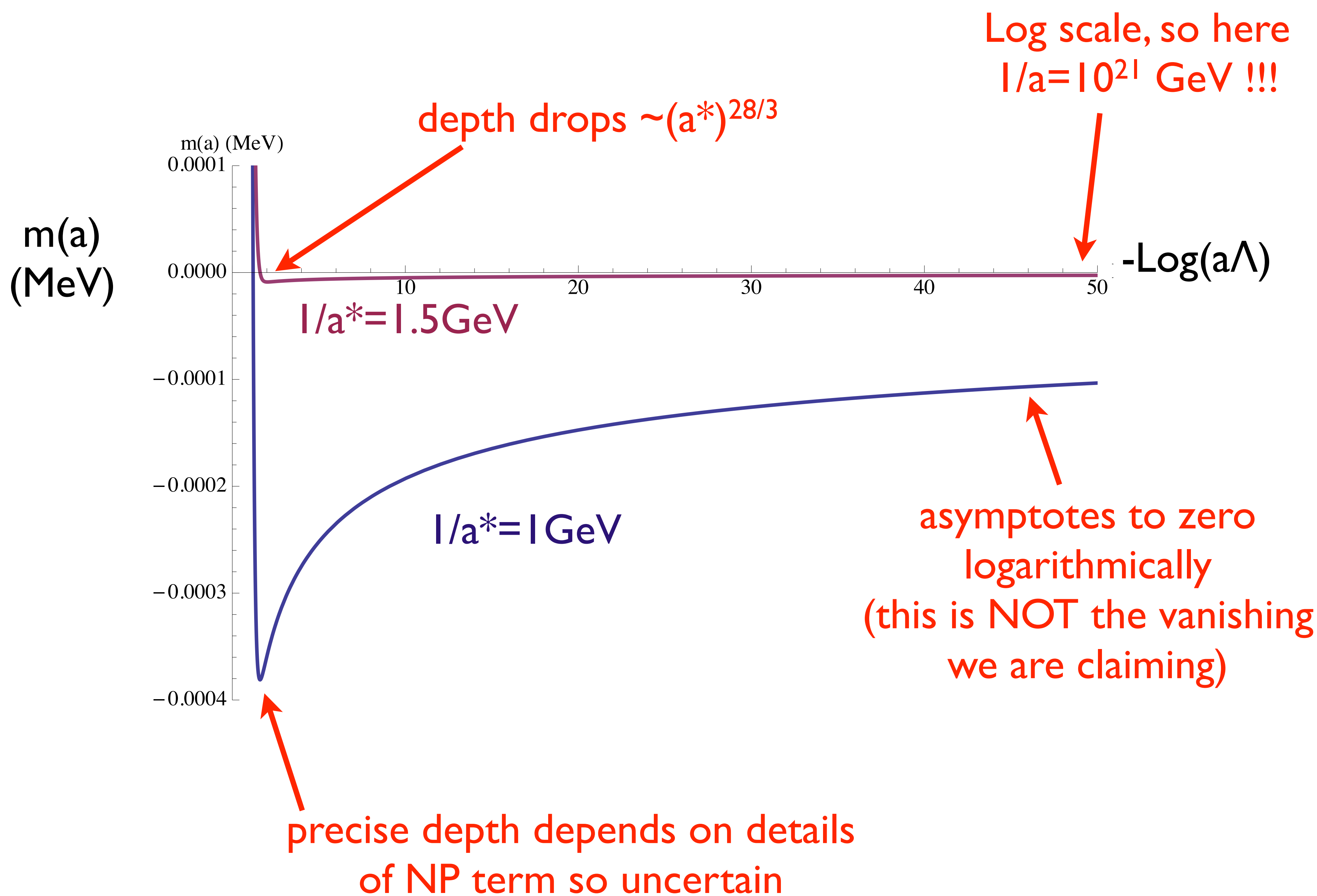
- There is an uncertainty in the running of m
  - At a given a, for

$$|m(a)| \gtrsim m_{cr} \approx \frac{(a\Lambda)^{28/3}\Lambda}{g(a)^2\gamma_0}$$

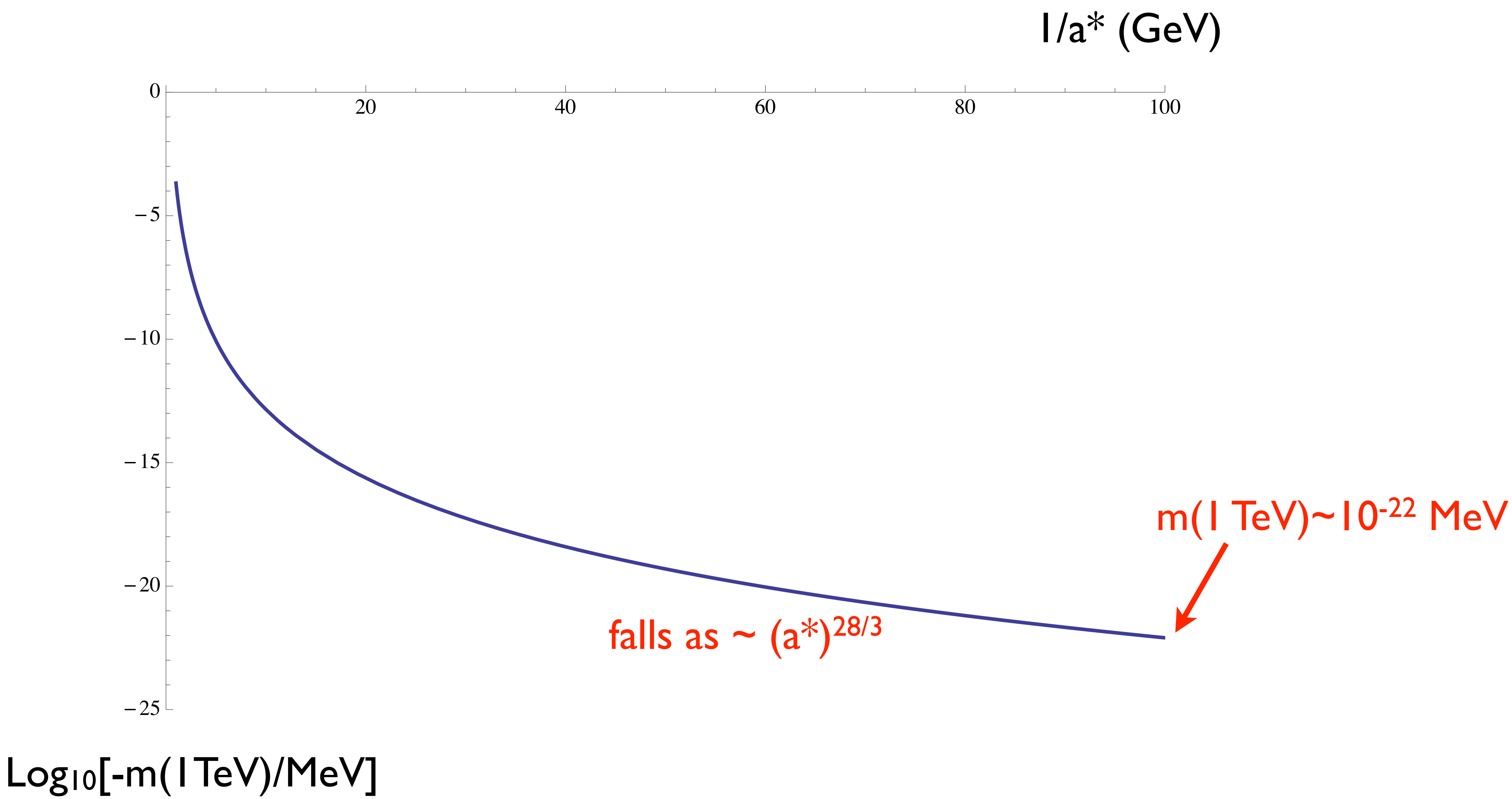
    the RG evolution to smaller a will be essentially unaffected by the additive term, and thus unambiguous
  - For $|m(a)| \lesssim m_{cr}$ evolution to smaller a is not controlled
  - In this sense there is an ambiguity in $m(a)$ of size m_{cr}
- As $a \to 0$, however, this ambiguity shrinks rapidly to zero, much faster than the standard logarithmic decrease of $m(a)$ and faster than other disc. errors
- Thus, in the standard view, we do know, in a regularization invariant way, what $m = 0$ means in the continuum limit
  - In particular, we can simply take $a \to 0$ holding $m(a) = 0$

Running if set $m(a^*)=0$

$N_f=1$, $\Lambda=300$ MeV, $c=1$



m(1TeV) given that m(a*)=0



Mike Creutz's view (my summary)

- [Creutz, PRL 92, 162003 (2004)] finds this argument unconvincing
- The argument certainly relies on the assumption that we know the form of the non-perturbative terms at short distances
  - Note that the value of $m(a)$ for the massless theory at $a \approx \Lambda_{\rm QCD}^{-1}$ (the "constituent quark mass") is unknown, since the additive term certainly dominates by this scale
  - But this is irrelevant for $m(a)$ as $a \to 0$
- Creutz makes some qualitative arguments, but does not directly address the standard argument given above
  - Please read and draw your own conclusions
- It would be very interesting to test Creutz's proposed breakdown in universality numerically

Relation to PQQCD

- PQ extensions of QCD-like theories provide a way of using symmetries to unambiguously define "$m_u = 0$" [Farchioni *et al.*, 0706.1131,0710.4454]
- Consider the PQ $N_f = 1$ theory, with N_V valence quarks (and corresponding ghosts) *degenerate* with the sea quark
  - Enlarged theory now has an approximate chiral symmetry $SU(N_V+1|N_V)_L \times SU(N_V+1|N_V)_R$
  - This symmetry becomes exact when $m \to 0$
  - The fact that $\langle\bar\psi\psi\rangle \neq 0$ in $N_f = 1$ QCD implies that the chiral symmetry of the PQ extension is spontaneously broken
  - One can thus write down the corresponding PQχPT, and $m = 0$ at quark level unambiguously maps to $m = 0$ at the chiral level in order to match the symmetries
  - There are thus PG bosons and fermions with $m_\pi^2 \propto m$
  - Thus $m = 0$ is unambiguously selected by vanishing PQ pion mass, just as $m_u = m_d = 0$ is picked out by vanishing physical pion mass (both requiring $L \to \infty$)
  - Used in practice by [Farchioni, 0710.4454]

Relation to PQQCD

- Other (closely related) ways of picking out $m = 0$
  - Vanishing of topological susceptibility, which is defined using PQ correlators [Giusti *et al*, hep-lat/0402027; Lüscher, hep-lat/0404034]
  - $1/m$ divergences in certain *finite volume* PQ correlation functions [Bernard et al, 0711.0696]
- **CONCLUSION:** If $m = 0$ is ambiguous, then the PQ extension of $N_f = 1$ QCD does not have a universal continuum limit
  - For $m = 0$ the PQ pions are massless but m_η, etc. are regularization dependent
- Same argument would apply to other N_f if one of the quark masses vanishes
- These results seem to me to imply that, if $m = 0$ is ambiguous, PQQCD is ill-defined in general (even when $m \neq 0$), and thus that extrapolations using PQχPT are invalid!

Consequences for rooting

- Staggered fermion simulations use the "$\det^{1/4}$" trick to remove extra tastes
- $\det([D+m]^4)^{1/4} = \det(D+m)$ is trivial (assuming $m>0$)
- $D_{stag}+m \rightarrow [D+m]^4$ only in continuum limit
- Using $\det(D_{stag}+m)^{1/4}$ leads to an unphysical theory for $a\neq0$
- Key question: Do the unphysical features vanish when $a\rightarrow0$?
- Variety of analytic arguments (with assumptions) and numerics suggest YES
- If rooting staggered fermions are in the correct universality class, then they necessarily give PQQCD in the continuum limit (e.g. for one staggered fermion, end up with 4 valence and 1 sea quark)
- If PQ theories are ill-defined, so is this continuum limit, and thus so are rooted staggered fermions

Summary of m_u=0 part

- There are several related theoretical issues

1. Is m_u=0 ambiguous?
2. Is m=0 ambiguous in the N_f=1 theory?
3. Are PQ theories well defined in the continuum limit?
4. Does rooted staggered LQCD have the correct continuum limit?
5. Does N_f=1QCD have a non-zero (Banks-Casher) density of microscopic (λ~1/V) eigenvalues?
6. Does m_u=0 solve the strong CP problem?

Summary of $m_u=0$ part

I have argued

- There are several related theoretical issues

1. Is m_u=0 ambiguous? NO
2. Is m=0 ambiguous in the N_f=1 theory? NO
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For #4, I think that the main issues lie elsewhere, & that the answer is "very likely" (another lecture)

Summary of m_u=0 part

I have argued

- There are several related theoretical issues

1. Is $m_u=0$ ambiguous? NO
2. Is $m=0$ ambiguous in the $N_f=1$ theory? NO
3. Are PQ theories well defined in the continuum limit? YES
4. Does rooted staggered LQCD have the correct continuum limit?
5. Does $N_f=1$QCD have a non-zero (Banks-Casher) density of microscopic ($\lambda\sim1/V$) eigenvalues?
6. Does $m_u=0$ solve the strong CP problem?

- These issues deserve further study, including by numerical simulations
- Key issue is whether hadron mass ratios are unambiguous in continuum limit

BACKUP SLIDES

Kaplan-Manohar ambiguity

[D. Kaplan and A. Manohar, Phys. Rev. Lett 56 (1986) 2004]

- Ambiguity in determination of quark mass ratios from comparison of ChPT with experiment
  - Unrelated to fact that cannot determine masses themselves because they are not RG invariant
- Chiral Lagrangian is constructed using symmetries alone
- M and $(M^\dagger)^{-1}\det(M)$ transform identically under $SU(3)_L \times SU(3)_R$
- Chiral Lagrangian invariant under $m_u \to m_u + \alpha\, m_d m_s$, $m_d \to m_d + \alpha\, m_s m_u$, $m_s \to m_s + \alpha\, m_u m_d$, as long as change LECs appropriately
- Cannot determine whether $m_u=0$ using ChPT
- However, QCD is NOT invariant under Kaplan-Manohar transformation, so it does not prevent determination of m_u using LQCD
- Similarity of form to 't Hooft vertex due to underlying chiral symmetry

Solving the strong CP problem?

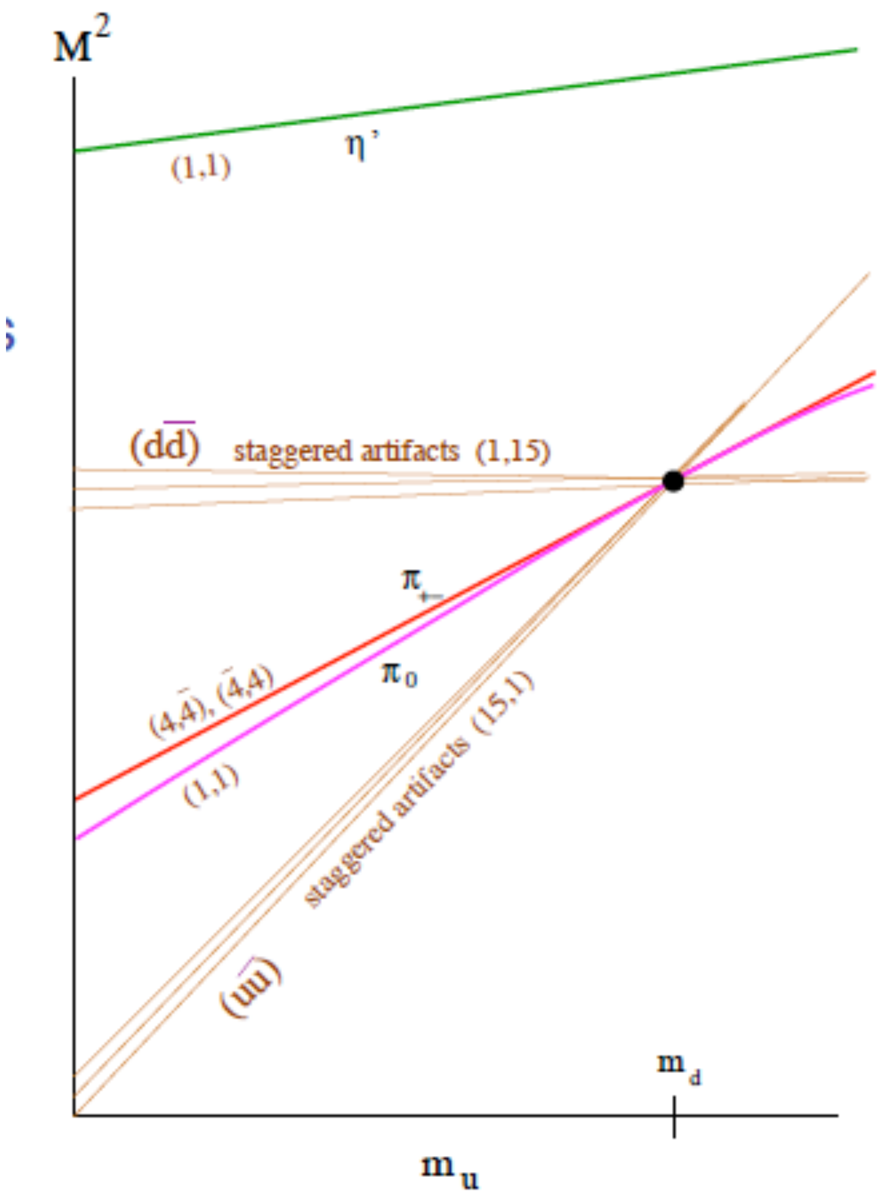
- Full QCD Lagrangian includes $\theta F\tilde{F}$ term which violates CP
- Formally, can rotate into mass matrix because of axial anomaly & bring entire phase onto m_u

$$M = \mathrm{diag}(m_u e^{i\bar{\theta}}, m_d, m_s)$$

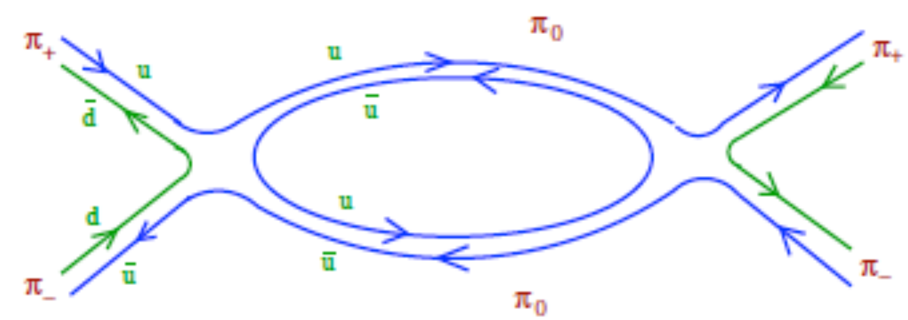
- $|\theta\text{-bar}| \lesssim 10^{-10}$ to agree with bounds on electric dipole moments
- Could have avoided, apparently, with $m_u=0$ (not, in fact, true in nature)
- Theoretically, could $m_u=0$ have worked? If m_u ambiguous, clearly not
- [Srednicki: hep-ph/0503051] notes that additive mass renormalization only affects $\mathrm{Re}(m_u)$: if $\mathrm{Im}(m_u)=0$ at any scale, then true at all scales
- More generally, solve strong CP problem if $\mathrm{Im}[\det(M)]=0$ at any scale
- Another solution is the axion (make θ dynamical)---does this work?

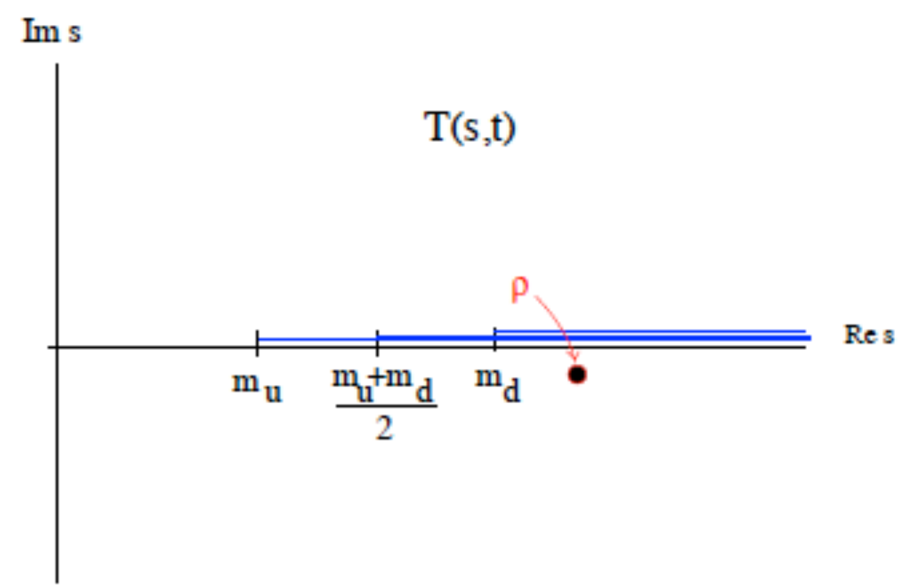
Spurious cuts?

- Rooted staggered theory has spurious, unphysical cuts in pion scattering amplitude



[Creutz' lectures]





Spurious cuts?

- Rooted staggered theory has spurious, unphysical cuts in pion scattering amplitude
- Answer (obtained using rSChPT): Unphysical cuts are present for $a \neq 0$ but have discontinuities ("strengths") which vanish like a^2
- Also, if one wanted to study the $m_u=0$ issue with staggered fermions, one must take the $a \to 0$ limit before $m_u \to 0$ (otherwise, e.g., the condensate will vanish)
- Numerical checks of these properties in Schwinger model by [Durr & Hoelbling]
- Related issues arise in scalar two-point correlator where unphysical cuts lead to negative contributions that vanish like a^2, and which have been observed and found to be consistent with rSChPT by the [MILC collaboration]