Multi-channel/particle scattering

Steve Sharpe
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Resonances from lattice QCD

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Outline

- Lecture 1
  - Motivation/Background/Overview
- Lecture 2
  - Deriving the two-particle quantization condition (QC2)
  - Examples of applications
- Lecture 3
  - Sketch of the derivation of the three-particle quantization condition (QC3)
- Lecture 4
  - Applications of QC3
  - Summary of topics not discussed and open issues

Main references for these lectures

- Briceño, Dudek & Young, "Scattering processes & resonances from LQCD," 1706.06223, RMP 2018
- Hansen & SS, "LQCD & three-particle decays of resonances," 1901.00483, to appear in ARNPS
- Lectures by Dudek, Hansen & Meyer at HMI Institute on "Scattering from the lattice: applications to phenomenology and beyond," May 2018, https://indico.cern.ch/event/690702/
- Lüscher, Commun.Math.Phys. 105 (1986) 153-188; Nucl.Phys. B354 (1991) 531-578 & B364 (1991) 237-251 (foundational papers)
- Kim, Sachrajda & SS [KSS05], hep-lat/0507006, NPB 2015 (direct derivation in QFT of QC2)
- Hansen & SS [HS14, HS15], 1408.5933, PRD14 & 1504.04248, PRD15 (derivation of QC3 in QFT)
- Briceño, Hansen & SS [BHS17], 1701.07465, PRD17 (including 2↔3 processes in QC3)
- Briceño, Hansen & SS [BHS18], 1803.04169, PRD18 (numerical study of QC3 in isotropic approximation)
- Briceño, Hansen & SS [BHS19], 1810.01429, PRD19 (allowing resonant subprocesses in QC3)
- Blanton, Romero-López & SS [BRS19], 1901.07095, JHEP19 (numerical study of QC3 including d waves)
- Blanton, Briceño, Hansen, Romero-López & SS, in progress, poster at Lattice 2019

Outline for Lecture 1

- Background: hadronic resonances
- Further motivation for studying multiparticle states
- Some scattering basics

Background: hadronic resonances

Stable hadrons in isosymmetric QCD

- QCD with $m_u = m_d$, and no EM (or weak) interactions
  - Theory studied in most LQCD simulations
  - Differs from real world at ~1% level

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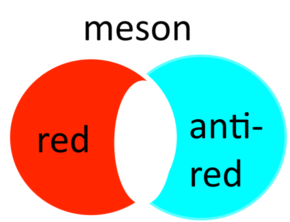

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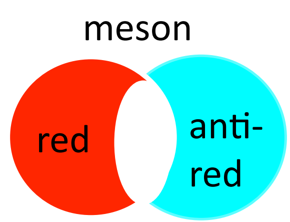

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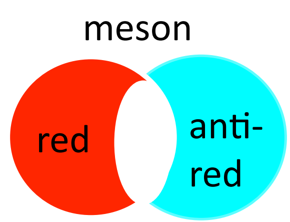

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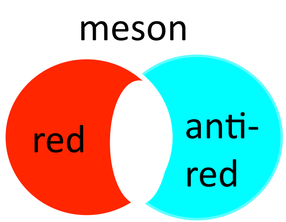
- Mesons 

  - Mesons composed of light quarks: $\pi(q\bar{q}), K(q\bar{s}), \eta(q\bar{q})$
  - Including heavy quarks: $D(c\bar{q}), D_s(c\bar{s}), B(b\bar{q}), B^*(q\bar{b}), B_s(s\bar{b}), B_s^*(s\bar{b}), B_c(c\bar{b})$

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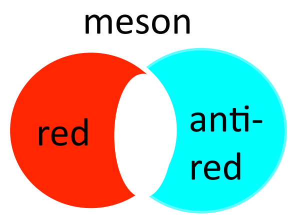

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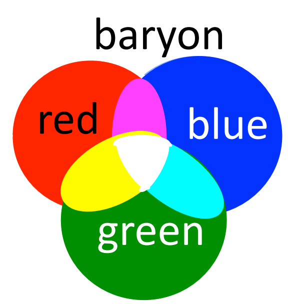

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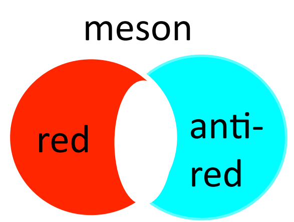

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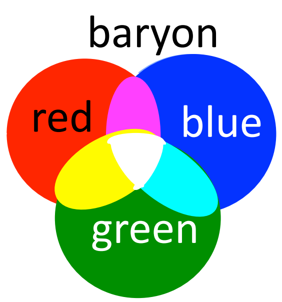

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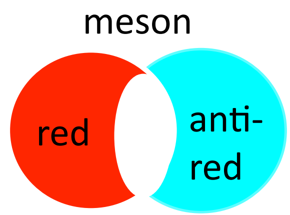

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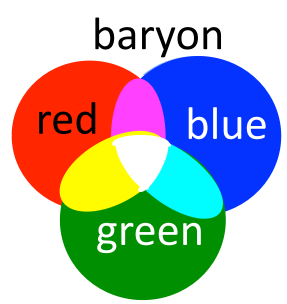

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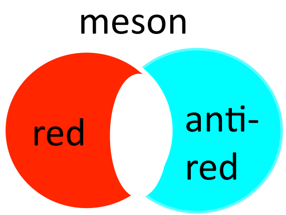

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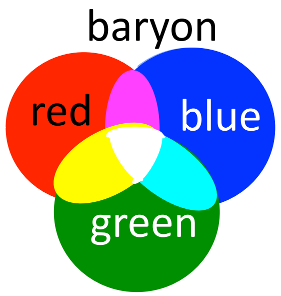

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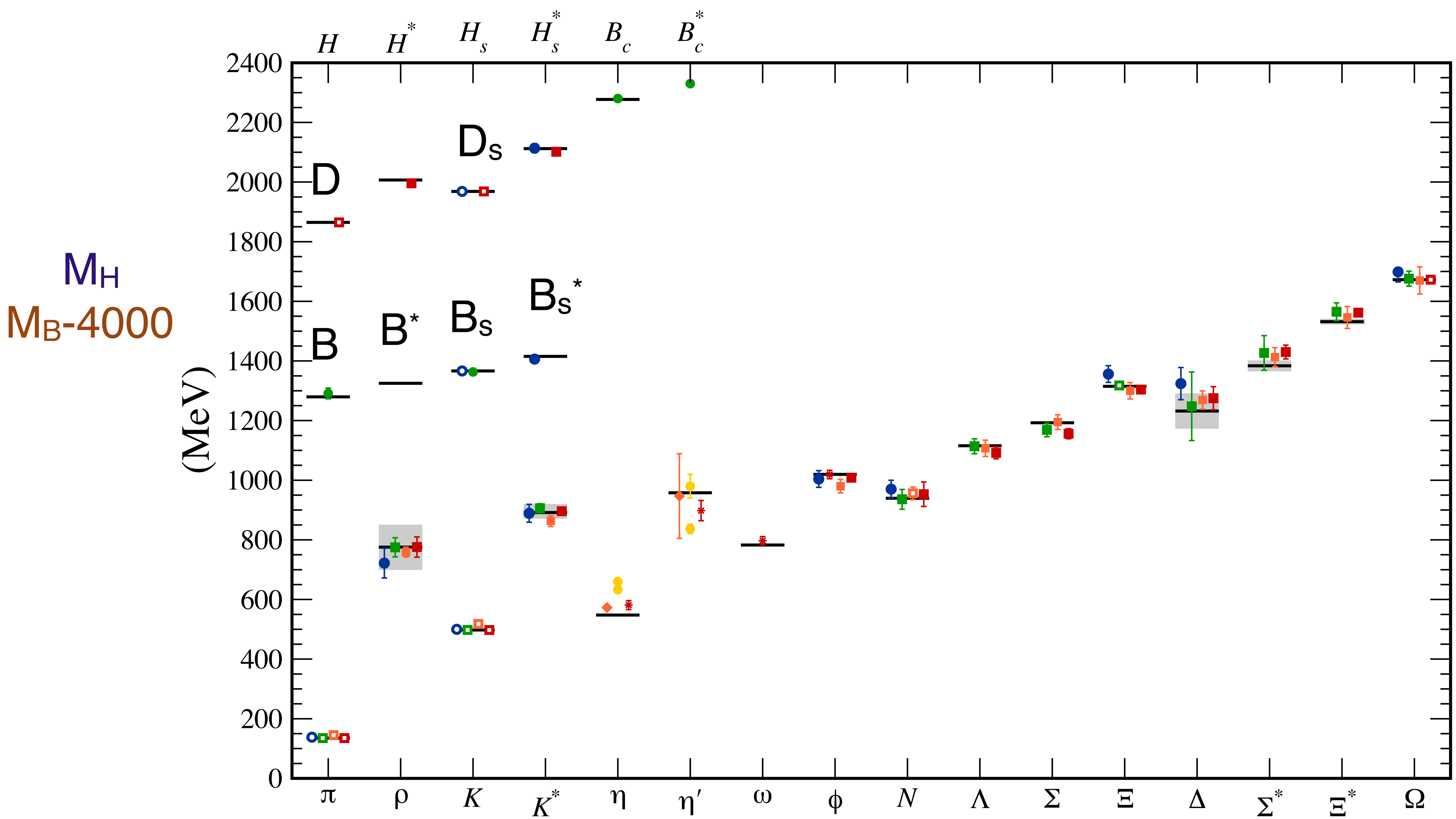

[Kronfeld, 1203.1204]

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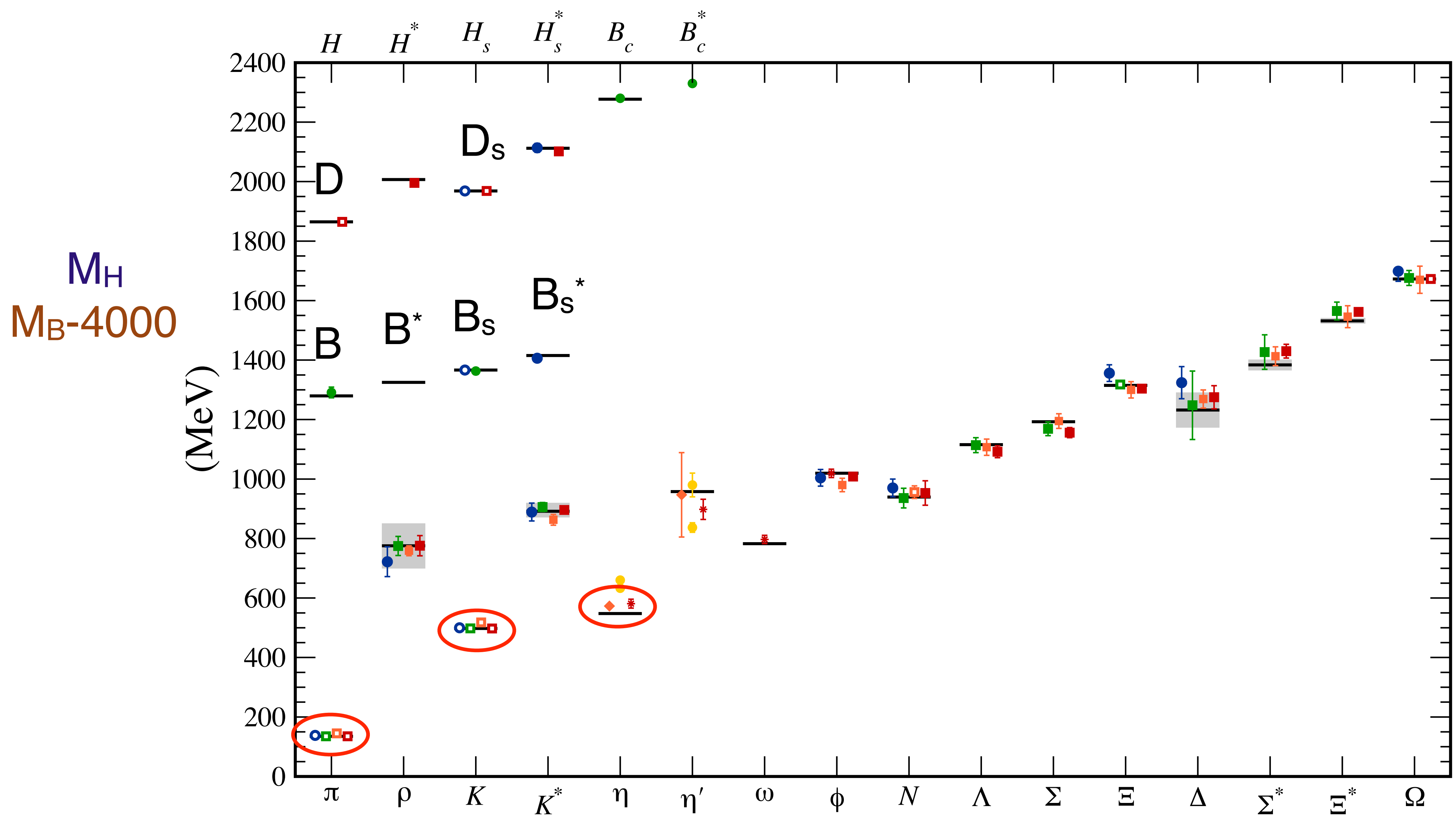

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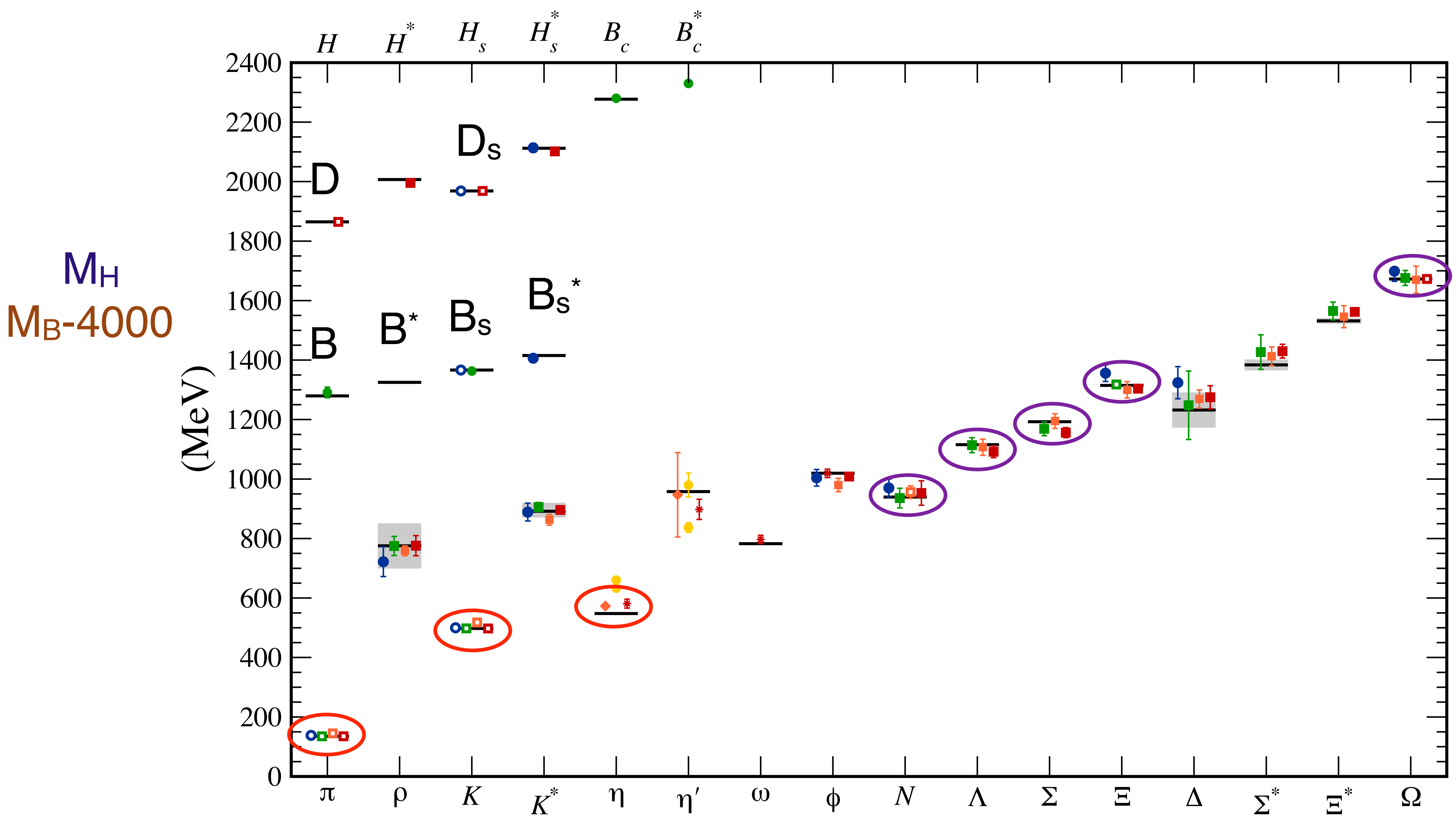

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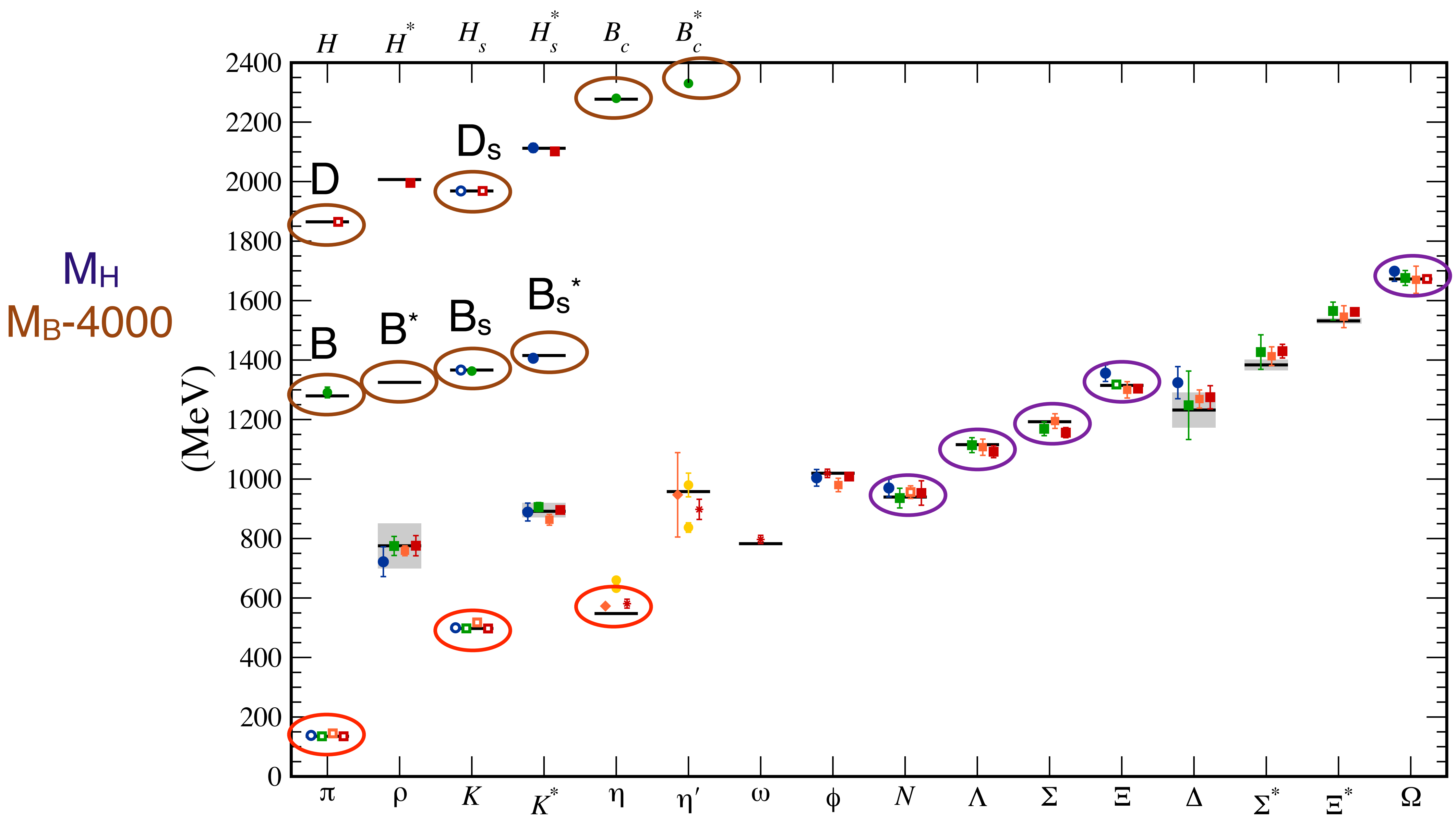

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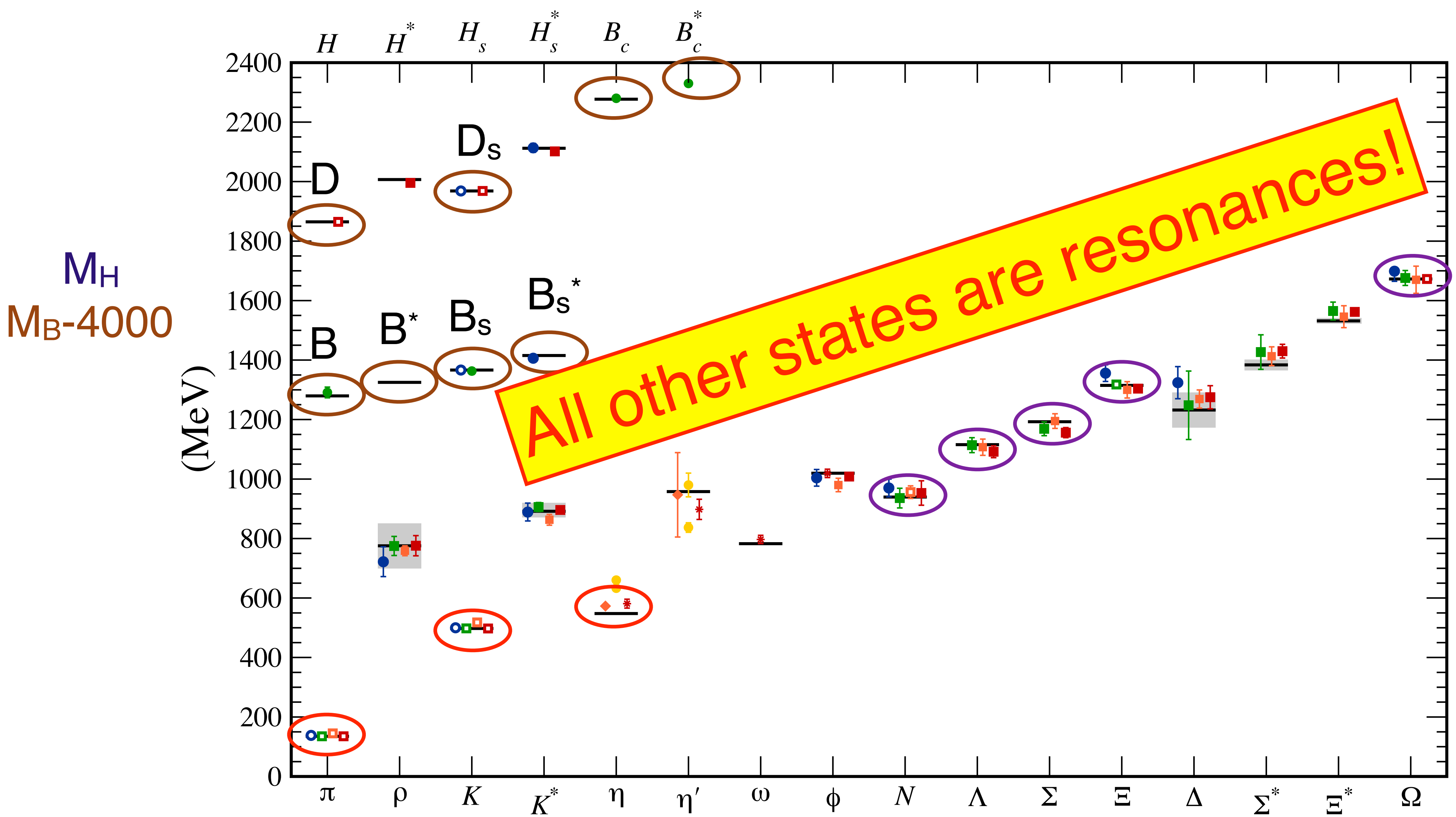

[Kronfeld, 1203.1204]

Plethora of resonances

- Most hadrons are resonances!

pdg meson listings

Stable

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\rho_3(1690)$	$1^+(3^{--})$
• π^0	$1^-(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• η	$0^+(0^{-+})$	$a_2(1700)$	$1^-(2^{++})$
• $f_0(500)$	$0^+(0^{++})$	• $f_0(1710)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\omega(782)$	$0^-(1^{--})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	$f_2(1810)$	$0^+(2^{++})$
• $f_0(980)$	$0^+(0^{++})$	$X(1835)$	$?^?(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$X(1840)$	$?^?(?^{??})$
• $\phi(1020)$	$0^-(1^{--})$	$a_1(1420)$	$1^-(1^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_2(2010)$	$0^+(2^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$f_0(2020)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $a_4(2040)$	$1^-(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2100)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_2(2150)$	$0^+(2^{++})$

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$
$K^*_2(1980)$	$1/2(2^+)$
• $K^*_4(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D^\pm_s$	$0(0^-)$
• $D^{*\pm}_s$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(2^+)$
• $D^*_{s1}(2700)^\pm$	$0(1^-)$
$D^*_{s1}(2860)^\pm$	$0(1^-)$
$D^*_{s3}(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B^*_J(5732)$	$?(?^?)$
• $B^*_2(5747)^+$	$1/2(2^+)$
• $B^*_2(5747)^0$	$1/2(2^+)$

$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi(3823)$	$?^?(2^{--})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)$	$1(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4055)^\pm$	$?(?^?)$
• $X(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4200)^\pm$	$?(1^+)$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$

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Examples of resonances

- Most hadrons are resonances!
- Very short lived, with decays into 2, 3, … stable hadrons

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  - Example 1: single-channel decay of s-wave spin-triplet q q-bar state:

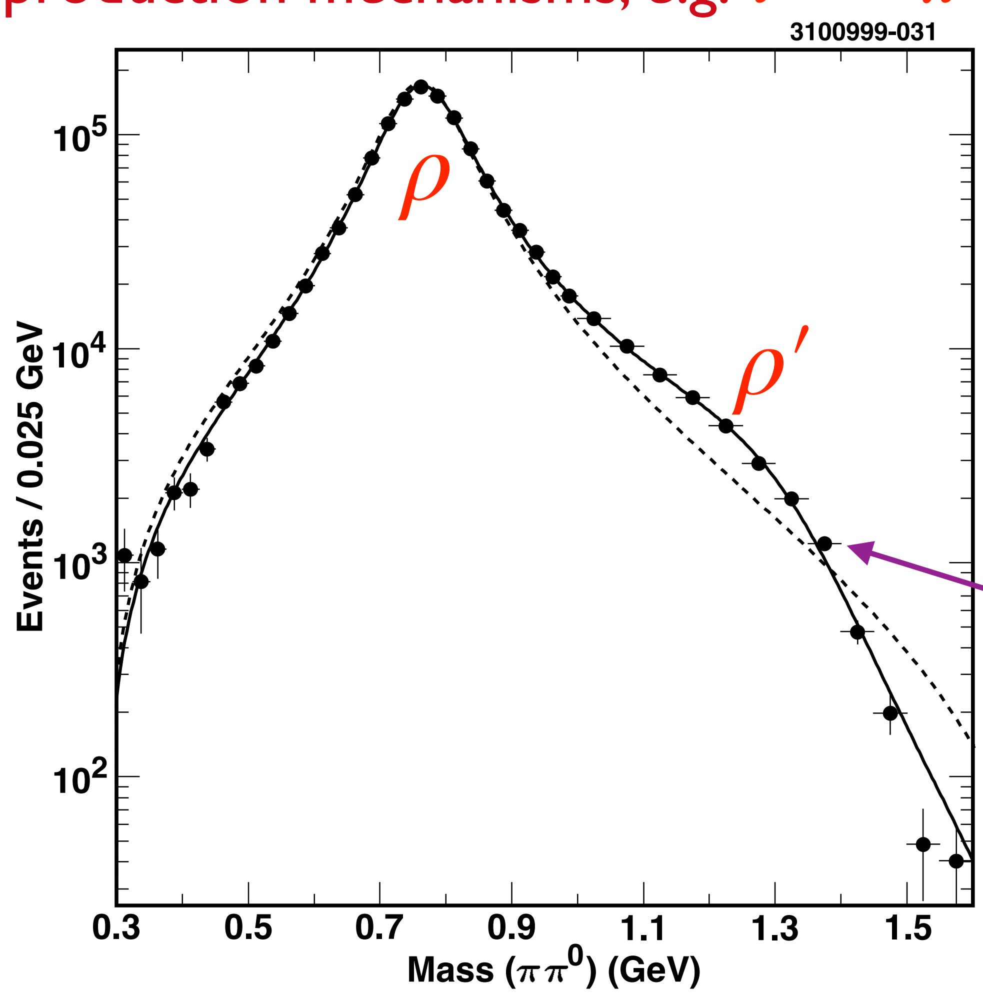
    $$I^G J^{PC} = 1^+1^{--} : \rho \to \pi\pi\,,\ M_\rho \approx 775 MeV\,,\ \Gamma_\rho \approx 150 MeV\ (\tau = 4\times 10^{-23} s)$$

  - Many production mechanisms, e.g. $\tau^- \to \pi^-\pi^0\nu_\tau$

ρ is produced by the vector part of the weak current $\bar{u}\gamma^\mu d$

[CLEO collab., hep-ex/9910046]

$e^+e^- \to \tau^+\tau^- + X$



Fitting the spectrum involves models & uncertainties

Examples of resonances

- Example 2: multi-channel decay of p-wave $q\bar{q}$ state:

pdg summary entry

$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

Mass $m = 1318.3^{+0.5}_{-0.6}$ MeV

Full width $\Gamma = 107 \pm 5$ MeV

$a_2(1320)$ DECAY MODES	Fraction (Γ_i/Γ)
3π	(70.1 ± 2.7) %
$\eta\pi$	(14.5 ± 1.2) %
$\omega\pi\pi$	(10.6 ± 3.2) %
$K\overline{K}$	(4.9 ± 0.8) %
$\eta'(958)\pi$	$(5.5 \pm 0.9) \times 10^{-3}$
$\pi^\pm\gamma$	$(2.91 \pm 0.27) \times 10^{-3}$
$\gamma\gamma$	$(9.4 \pm 0.7) \times 10^{-6}$

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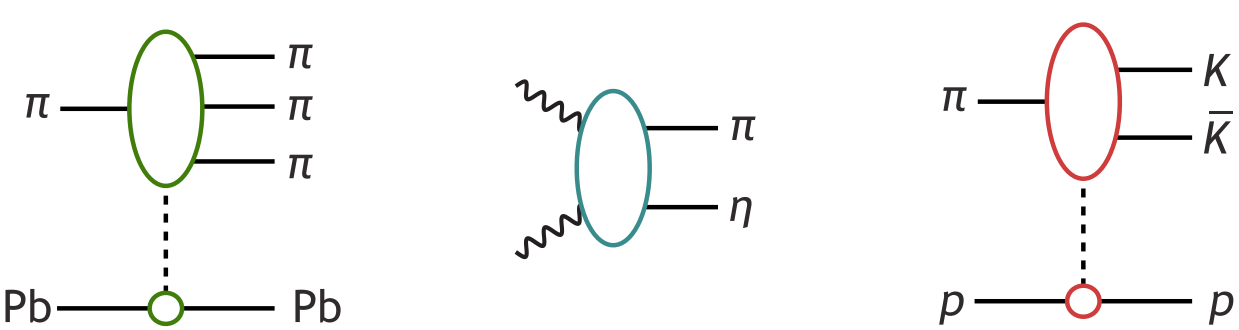
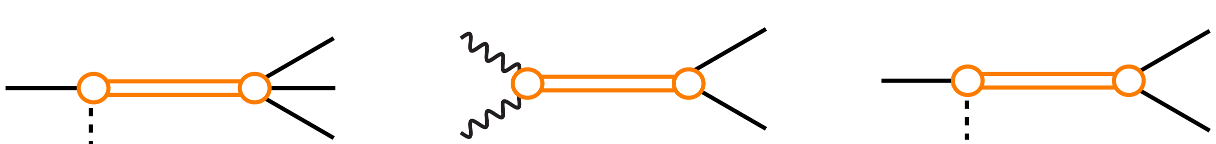
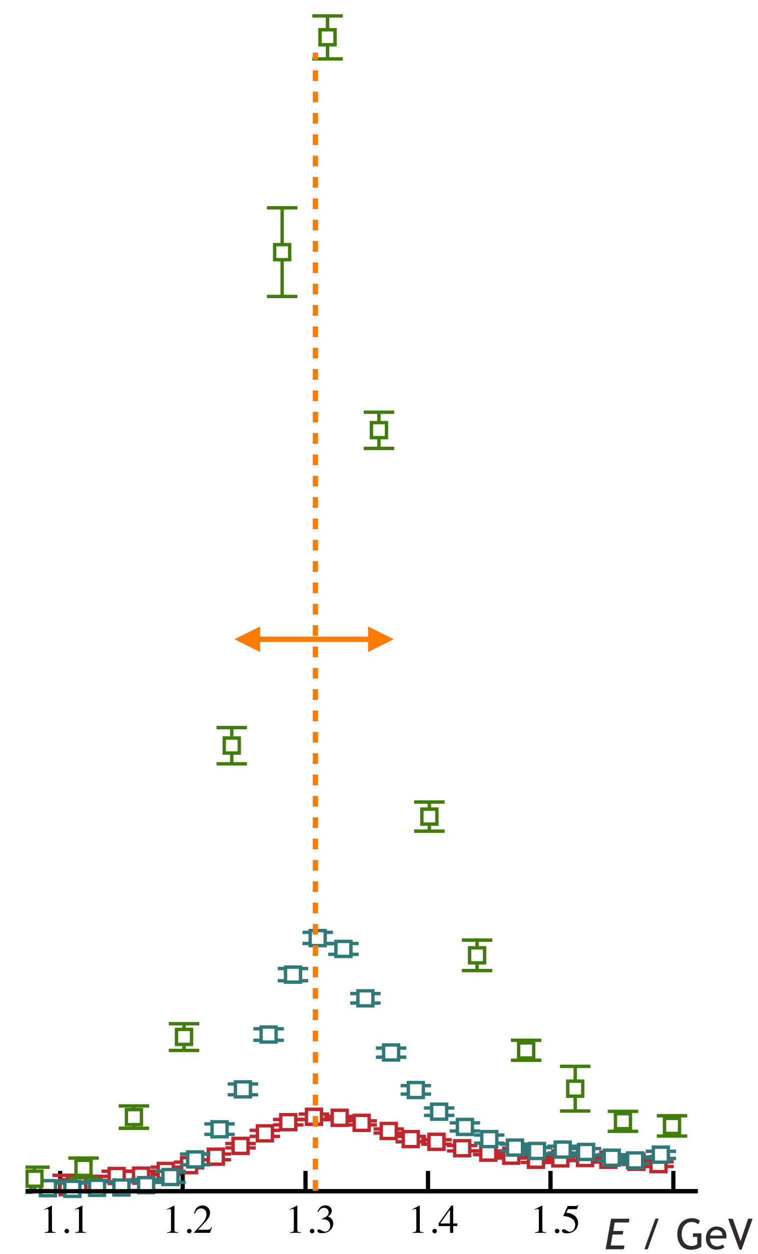
same 'bump' appears in multiple different processes ...



... due to same a_2 resonance



$a_2(1320)$



$\pi\,\mathrm{Pb} \to \pi\rho\,\mathrm{Pb}$	COMPASS
$\gamma\gamma \to \pi\eta$	Belle
$\pi p \to K\bar{K}p$	CERN SPS

[Figures from HMI slides of Jo Dudek]

Examples of resonances

- Example 3: scalar, isoscalars—possible p-wave $q\bar{q}$ states

[PDG]

$f_0(500)$ [g] $I^G(J^{PC}) = 0^+(0^{++})$

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)$-i$(200–350) MeV
Mass (Breit-Wigner) = (400–550) MeV
Full width (Breit-Wigner) = (400–700) MeV

$f_0(500)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	seen	–
$\gamma\gamma$	seen	–

$f_0(980)$ [j] $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$f_0(980)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	seen	476
$K\overline{K}$	seen	36
$\gamma\gamma$	seen	495

Examples of resonances

- Example 3: scalar, isoscalars—possible p-wave $q\bar{q}$ states

[PDG]

$f_0(500)$ [g] $I^G(J^{PC}) = 0^+(0^{++})$

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)$-i$(200–350) MeV
Mass (Breit-Wigner) = (400–550) MeV
Full width (Breit-Wigner) = (400–700) MeV

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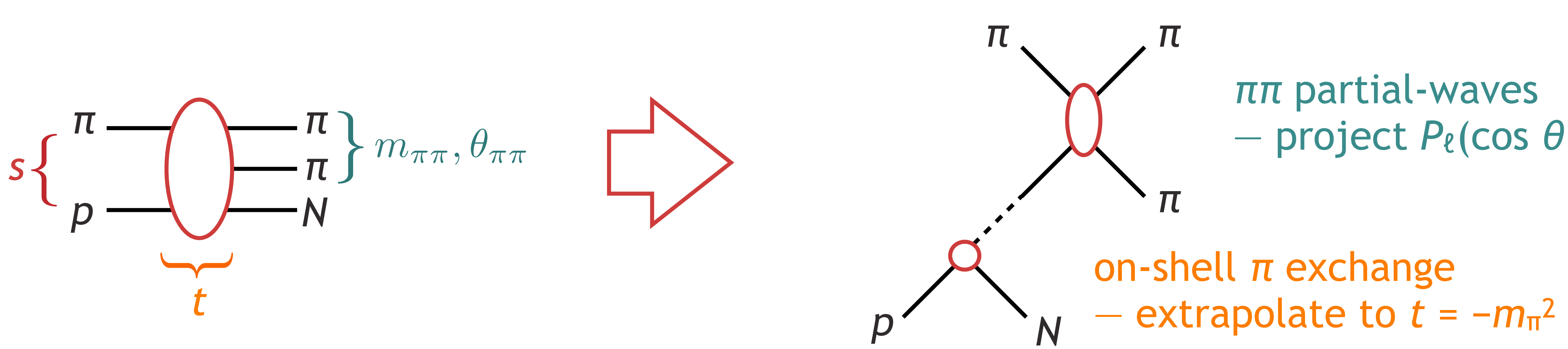
- Large uncertainties because analyses are difficult

extract from charged pion beams on nucleon targets

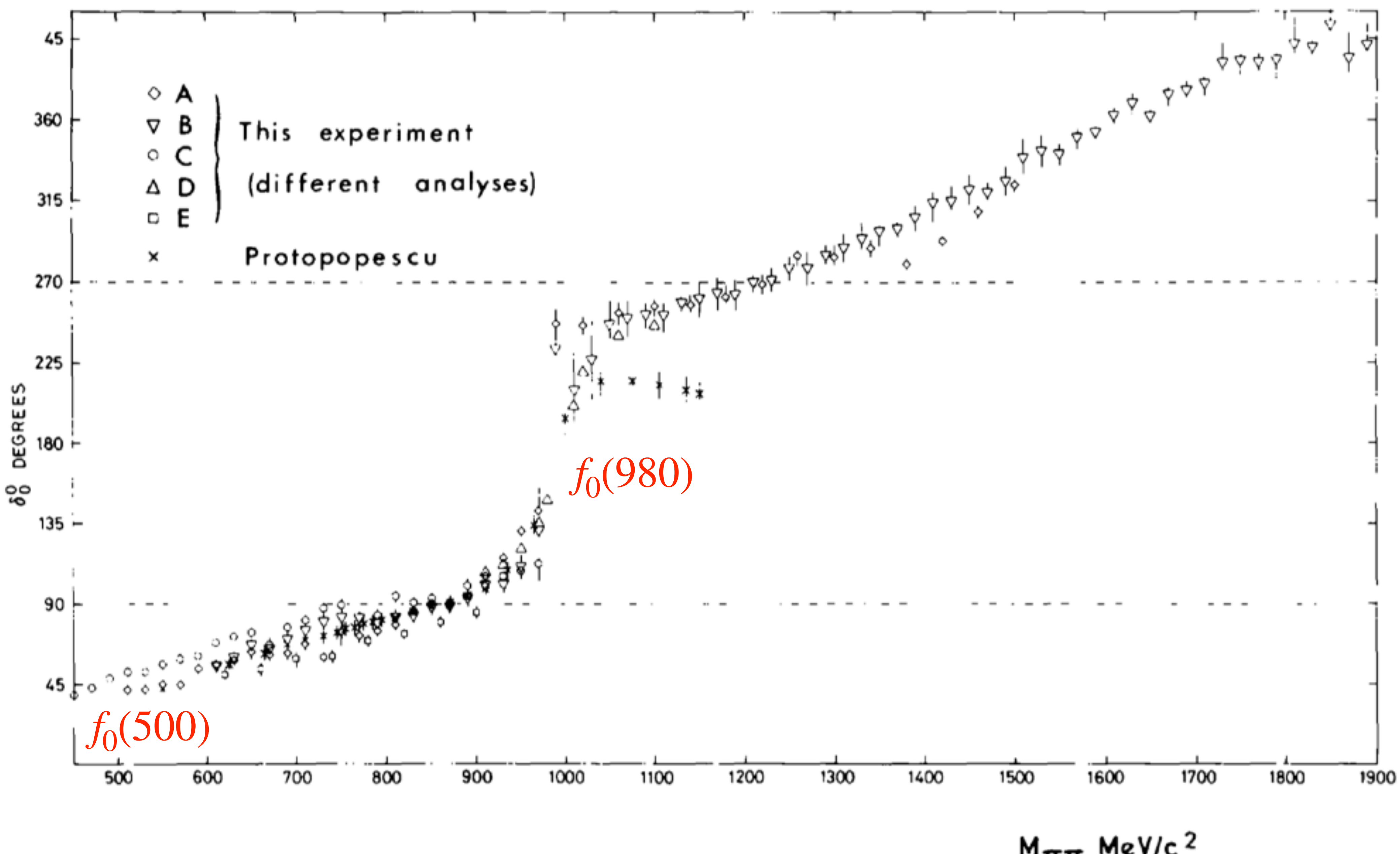


[Figure from HMI slides of Jo Dudek]

Examples of resonances

- Example 3: scalar, isoscalars—possible p-wave $q\bar{q}$ states
  - Extract the phase shift from complicated amplitude analysis

isospin=0

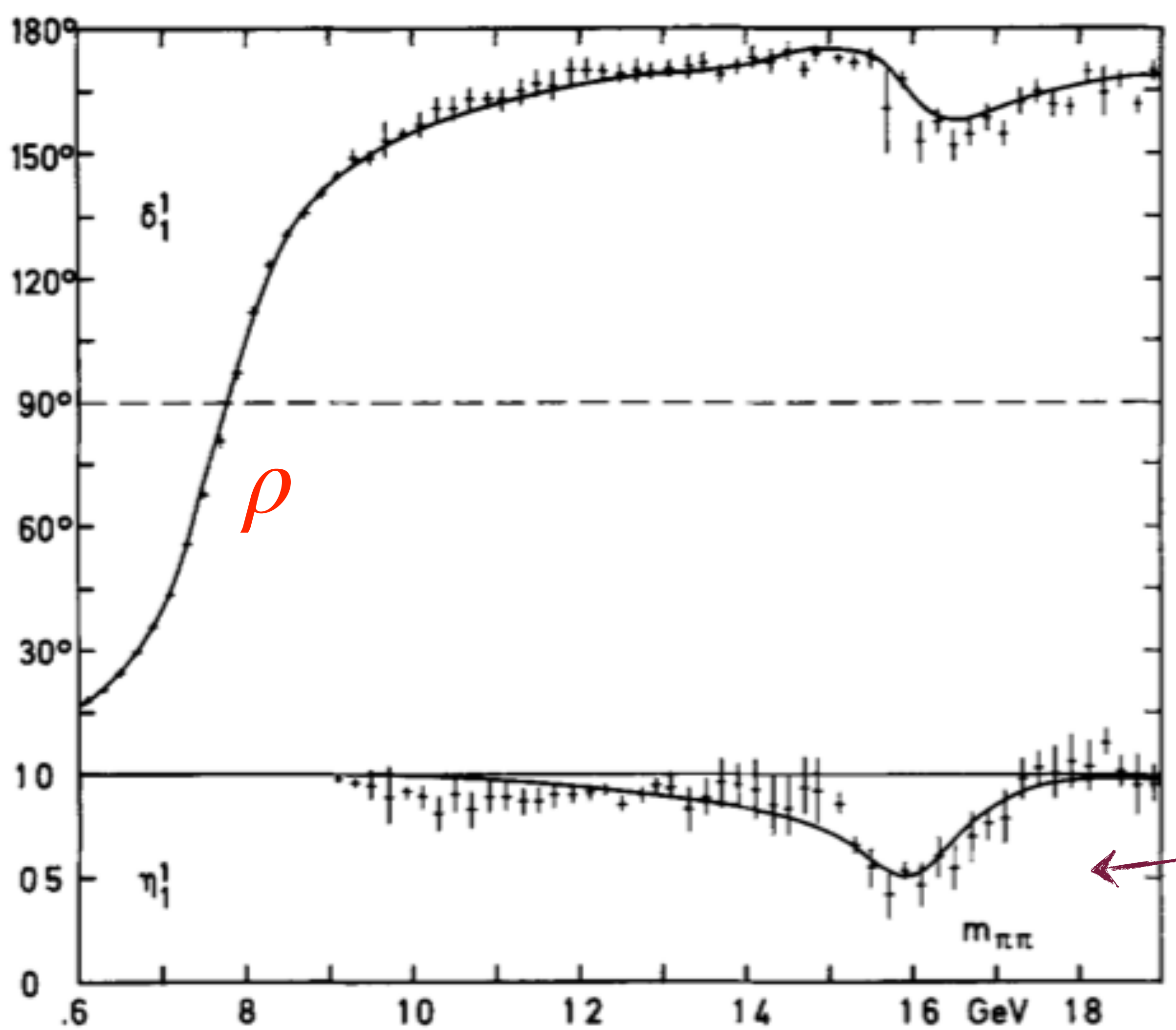


Grayer 1974

Aside on inelasticity

- Phase shift in *I=J=1* ππ channel

isospin=1



Hyams 1973

$1 - |\eta|^2$
gives probability for
scattering into any final state
other than ππ,
e.g. KK-bar, ηη, 4π
Becomes nonzero above
1 GeV

Examples of resonances

- Example 4: Roper (excited nucleon)

[PDG]

$N(1440)\ 1/2^+$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Re(pole position) = 1360 to 1380 (≈ 1370) MeV
−2Im(pole position) = 160 to 190 (≈ 175) MeV
Breit-Wigner mass = 1410 to 1470 (≈ 1440) MeV
Breit-Wigner full width = 250 to 450 (≈ 350) MeV

$N(1440)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	398
$N\eta$	<1 %	†
$N\pi\pi$	17–50 %	347
$\Delta(1232)\pi$, P-wave	6–27 %	147
$N\sigma$	11–23 %	–
$p\gamma$, helicity=1/2	0.035–0.048 %	414
$n\gamma$, helicity=1/2	0.02–0.04 %	413

- Extracted from amplitude analysis of πN scattering
- Lighter than expected from quark model for a radial excitation

Examples of resonances

- Example 5: $Z_c(3900)$—a nonstandard meson

$Z_c(3900)$ $\qquad I^G(J^{PC}) = 1^+(1^{+-})$ [PDG]

Mass $m = 3887.2 \pm 2.3$ MeV (S = 1.6)
Full width $\Gamma = 28.2 \pm 2.6$ MeV

Z_c(3900) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$J/\psi\pi$	seen	699
$h_c\pi^\pm$	not seen	318
$\eta_c\pi^+\pi^-$	not seen	759
$(D\overline{D}^*)^\pm$	seen	–
$D^0 D^{*-}$ + c.c.	seen	153
$D^- D^{*0}$ + c.c.	seen	144
$\omega\pi^\pm$	not seen	1862
$J/\psi\eta$	not seen	510
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$\rho\eta_c$ (now seen at 4.2σ significance, [BESIII])

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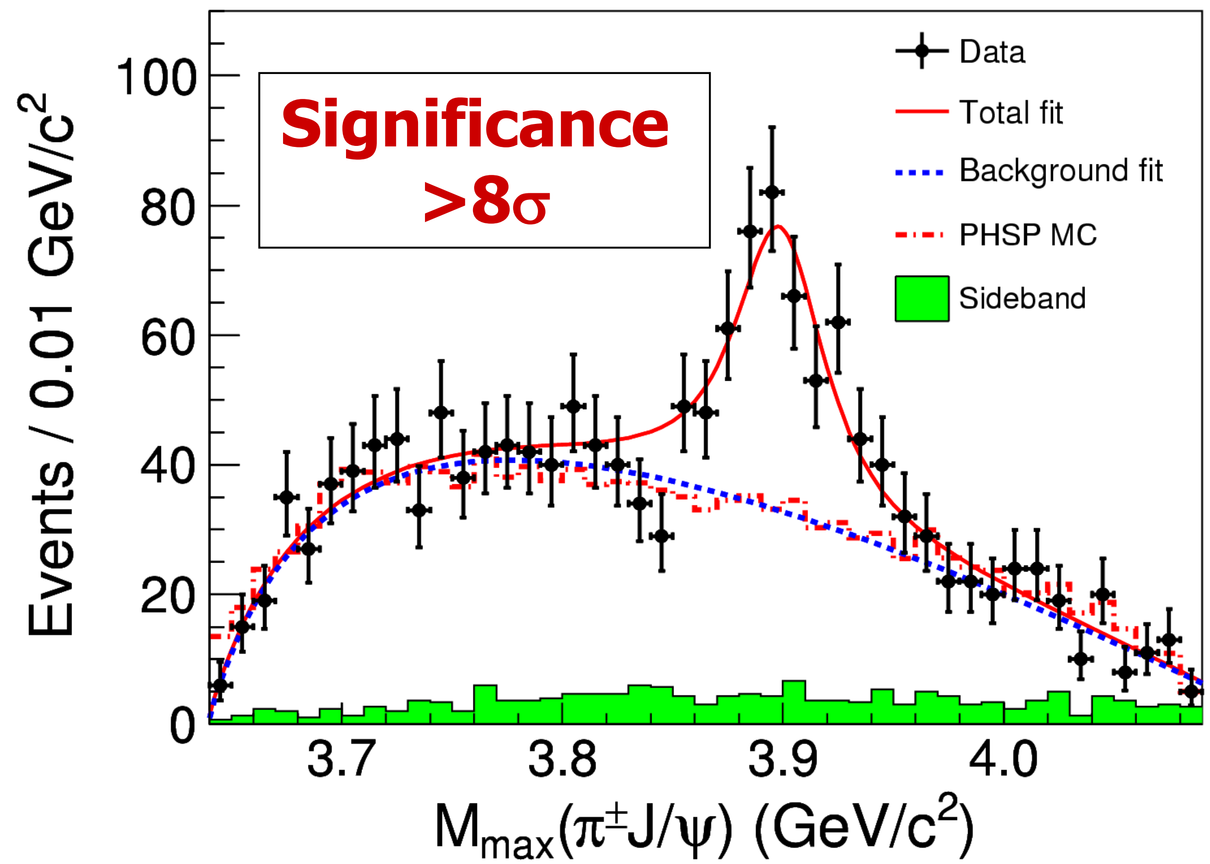
Observed by BESIII, Belle, CLEO-c in 2013

$$e^+e^- \to \pi^\pm Z_c^\mp$$



[BESIII, talk at Lattice 2019 by C.Yuan]

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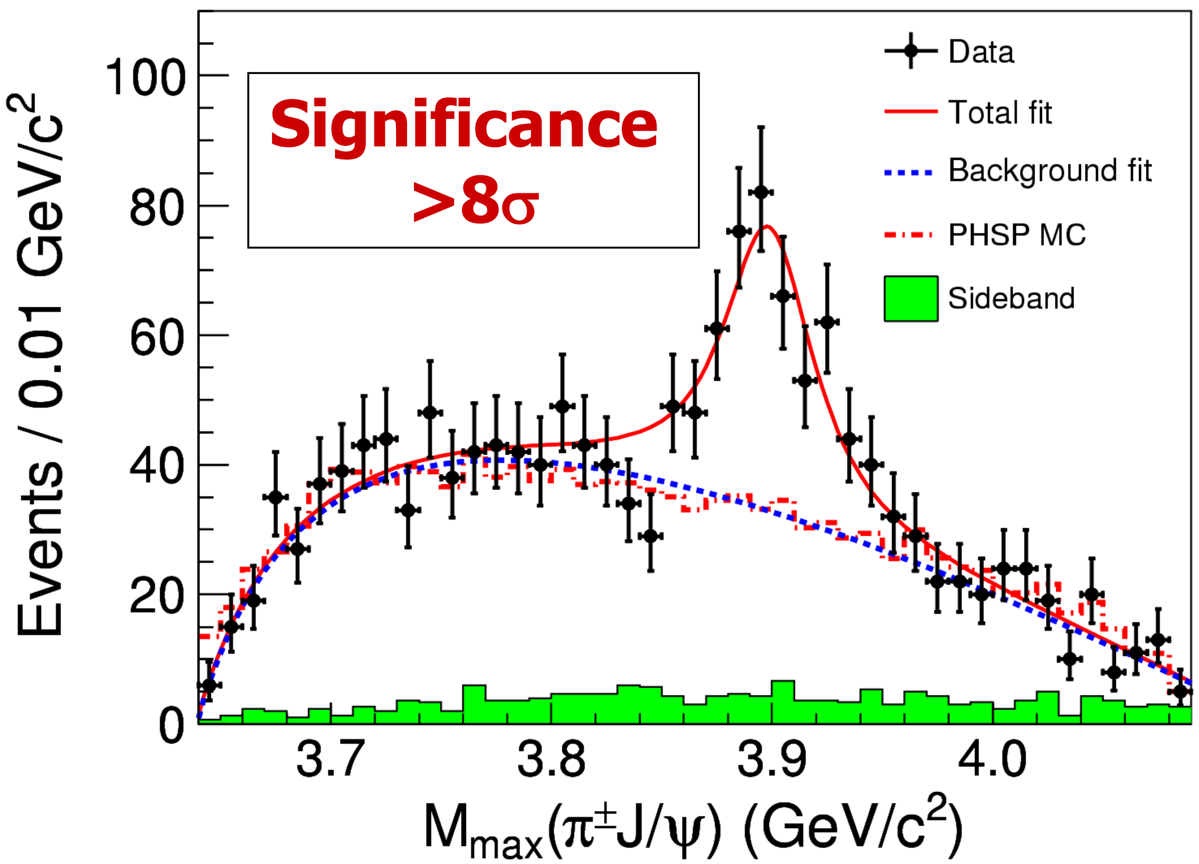

[BESIII, talk at Lattice 2019 by C.Yuan]

- Z_c^+ quark composition: $c\bar{c}u\bar{d}$

Examples of resonances

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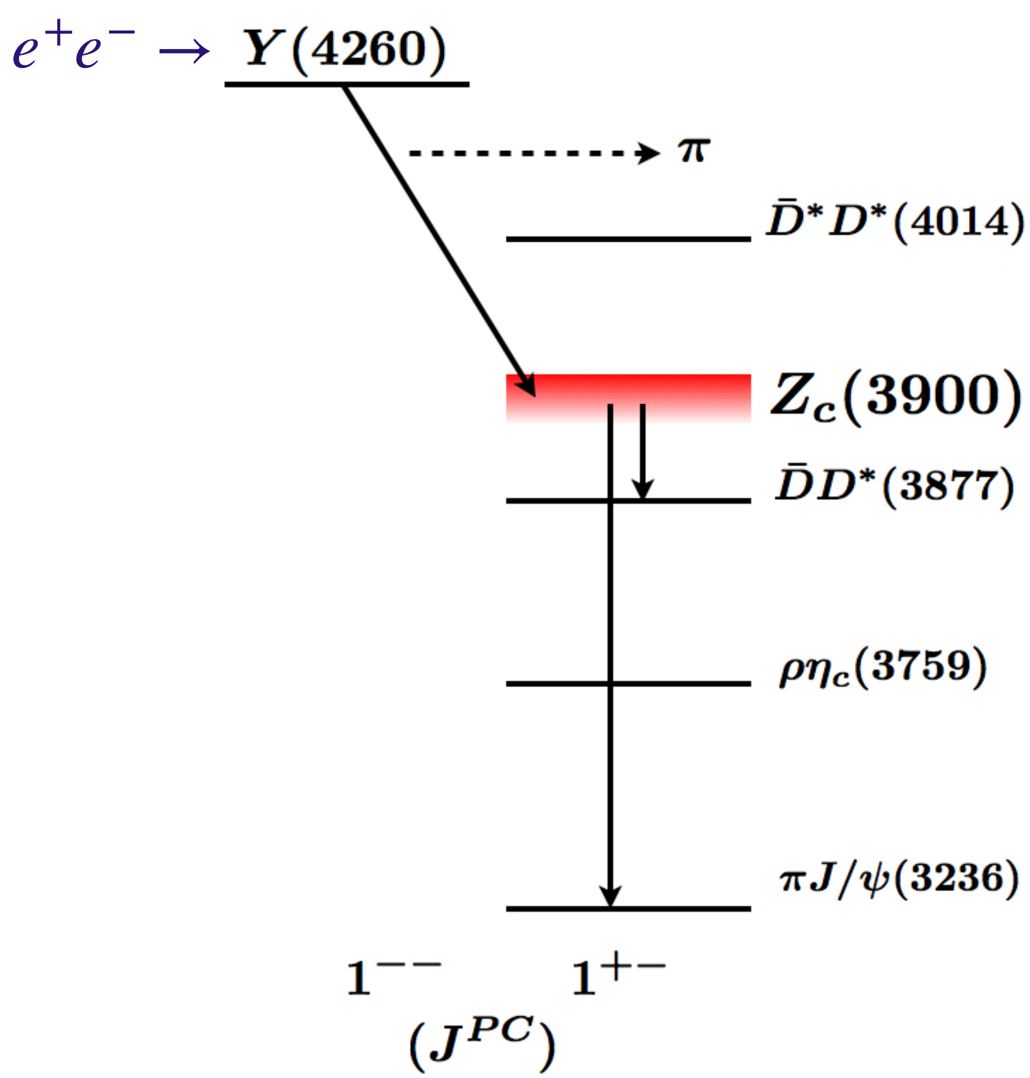
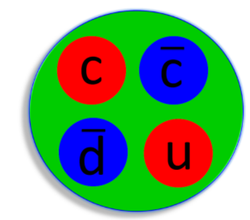
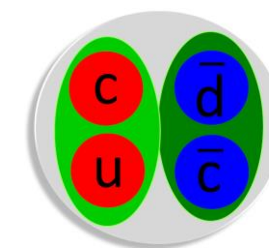


[Ikeda et al., 1602.03465]

- Z_c^+ quark composition: $c\bar{c}u\bar{d}$
- Possible interpretations:
  - Tetraquark



  - Molecule



  - Threshold enhancement—supported by HALQCD study [1602.03465]

Lessons

- Extracting resonance parameters from experiment is indirect & challenging
  - Resonance is defined as a pole in a scattering amplitude—not directly accessible
- Typical resonances have multiple decay channels, each involving 2 or 3 (or more) particles
- Quark model (or other models) fails to explain presence or properties of an increasing number of resonances
  - X, Y, Z resonances, glueballs, hybrids, tetraquarks, pentaquark, …
- Resonances are a largely unexplored frontier in our attempts to understand hadronic physics (i.e. the properties of a strongly-coupled QFT) from first principles

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A major challenge for LQCD!

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  - X, Y, Z resonances, glueballs, hybrids, tetraquarks, pentaquark, …
  - LQCD calculations must use large bases of operators to allow understanding of structure of hadrons—any input is useful!
  - Varying the quark masses can provide additional useful information

Personal note

- As a grad student I used the MIT bag model to predict the masses of "hybrid" mesons—resonances of the form: quark + antiquark + "constituent gluon"

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HYBRIDS: MIXED STATES OF QUARKS AND GLUONS*

Nuclear Physics B222 (1983) 211–244
© North-Holland Publishing Company

Michael CHANOWITZ and Stephen SHARPE

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA

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Submitted for publication

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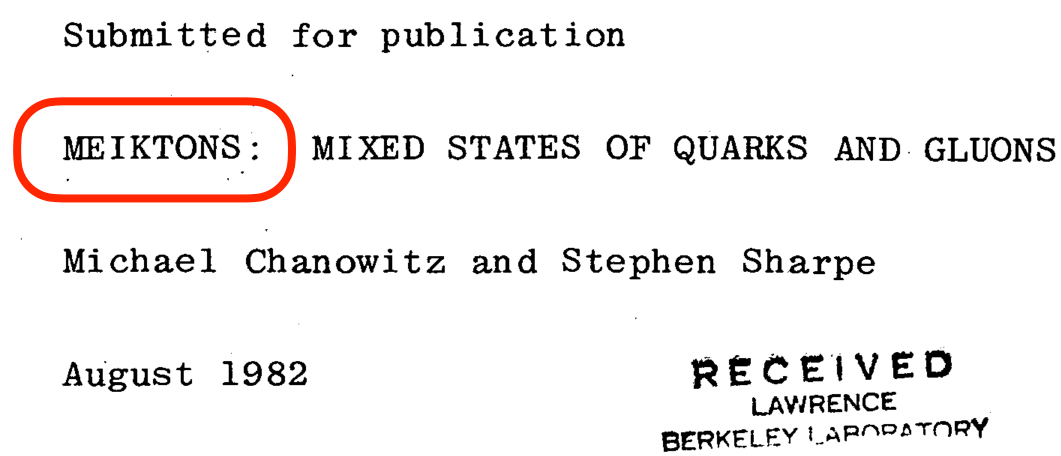
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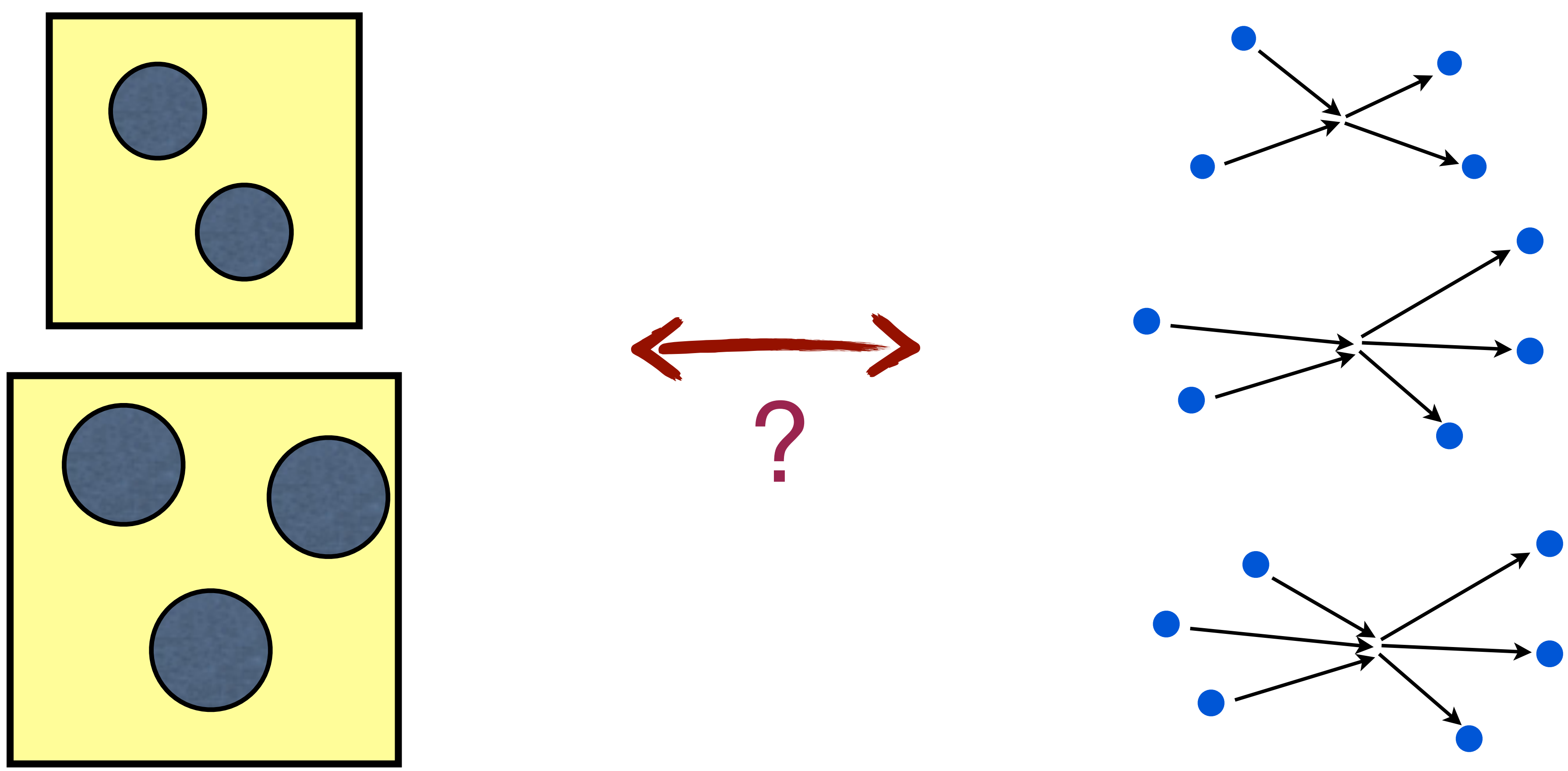
Noise!

- There are now increasingly sophisticated calculations of hybrid meson properties, and these will eventually be based on the formalism I will describe in these lectures

Preview

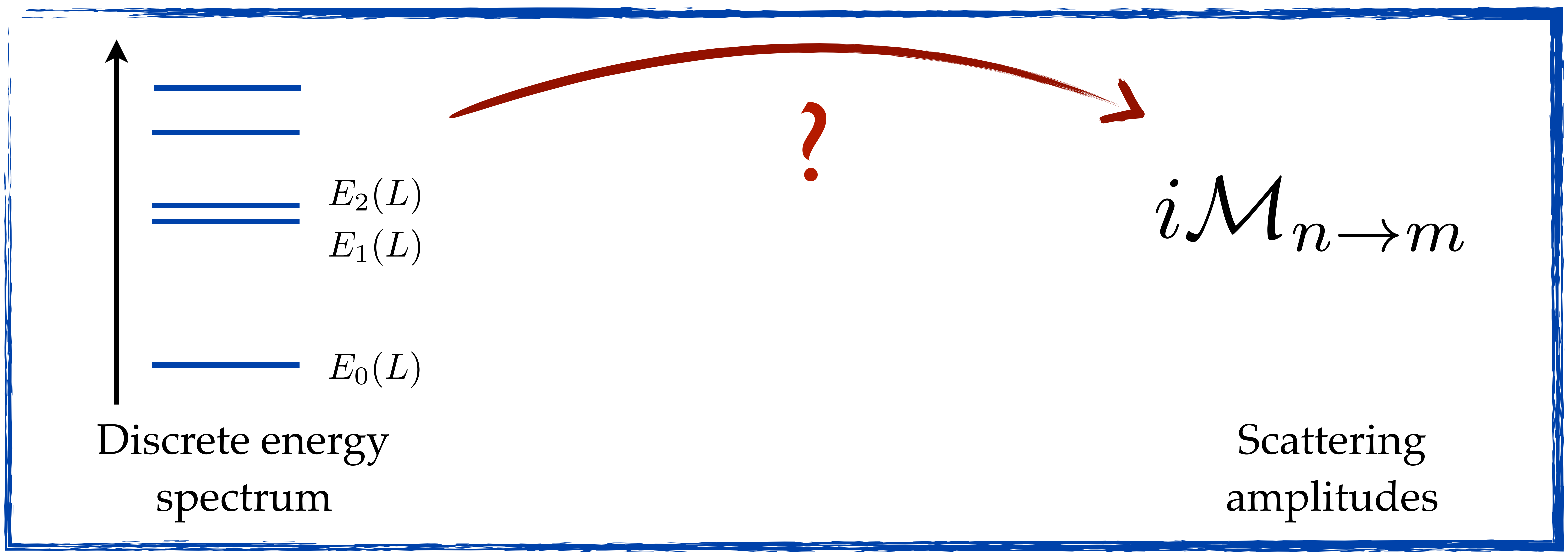
- Fundamental issue:
  - LQCD simulations are done in finite volumes, with imaginary time
  - Experiments are done in infinite volume in real time



How do we connect?

Fundamental Issue

- Lattice QCD can calculate energy levels of multiparticle systems in a box
- How are these related to infinite-volume scattering amplitudes (which determine resonance properties)?



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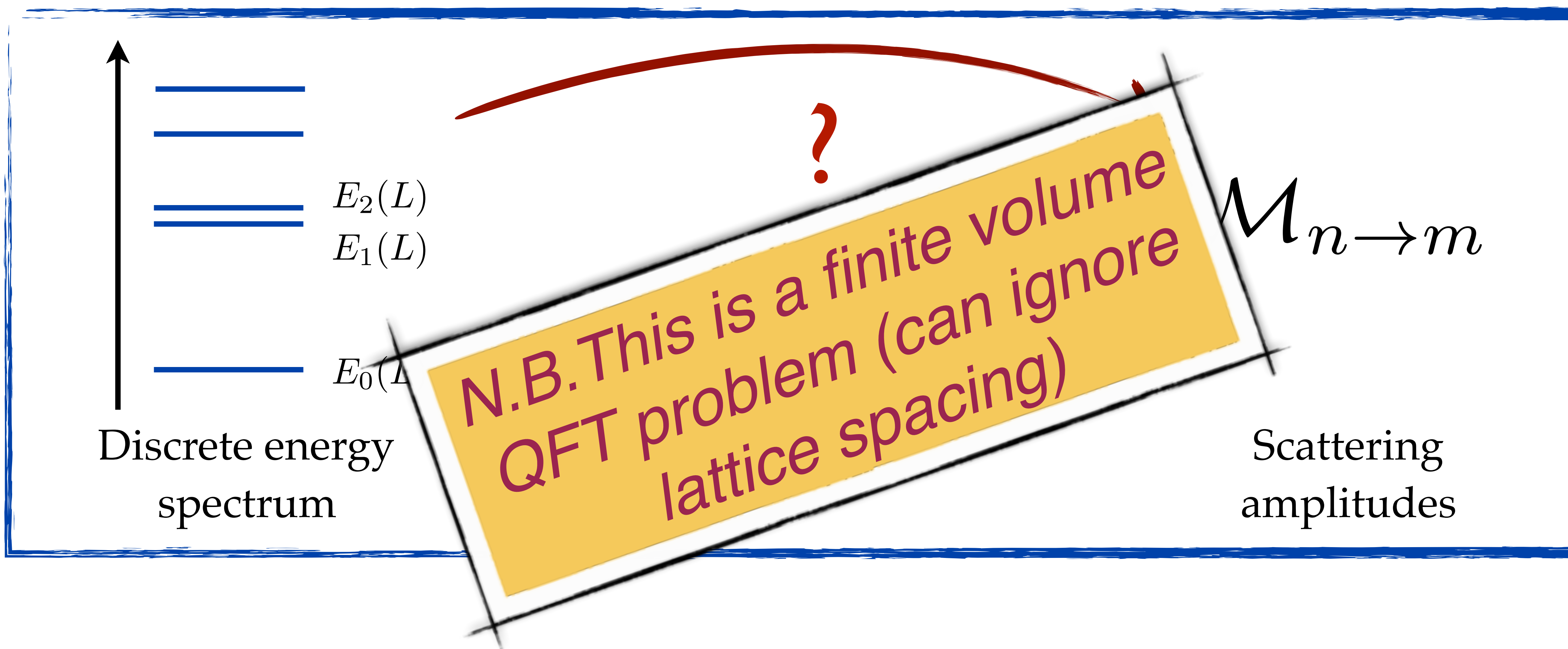

Further motivations for studying multiparticle states

Motivations

- Calculating electroweak decay and transition amplitudes for processes involving multiple particles
- Determining NN and NNN interactions as input for predicting properties of nuclei and nuclear matter

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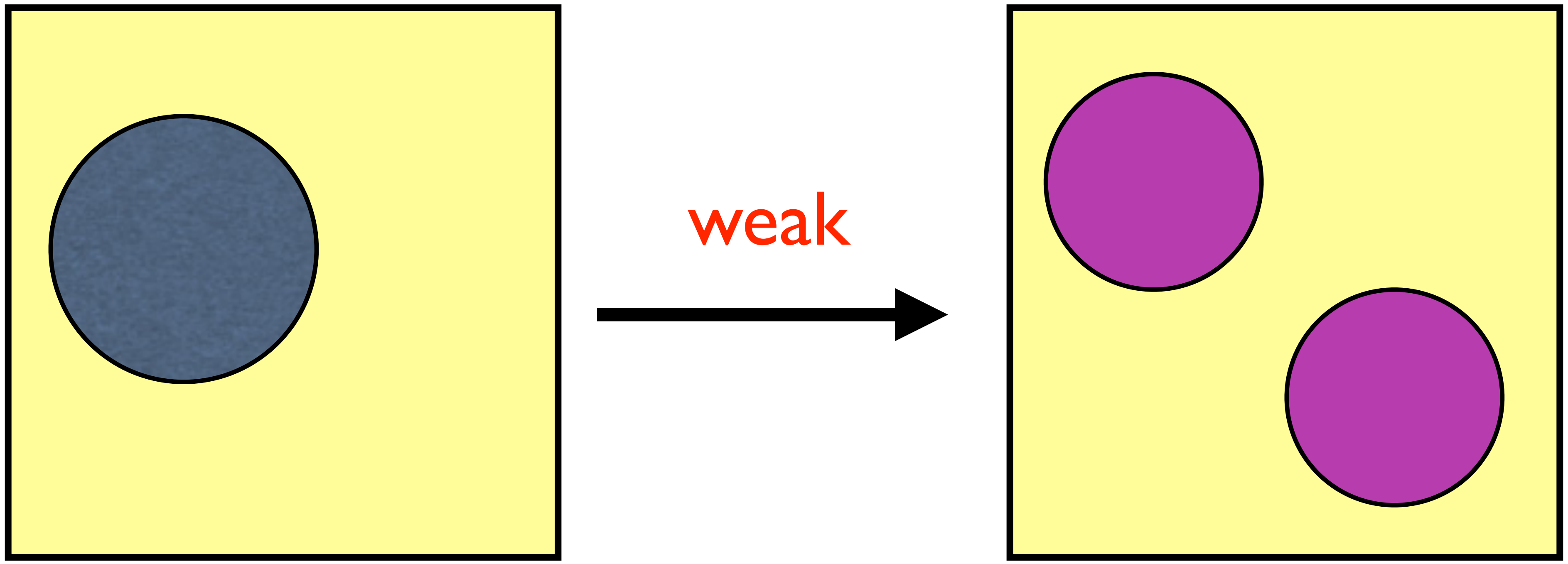
- Determining NN and NNN interactions as input for predicting properties of nuclei and nuclear matter

Will not have time to discuss the required formalism in these lectures, except in passing

Electroweak decays [Sachrajda lectures]

e.g. K→ππ decay amplitudes



- Does the SM reproduce the ΔI=1/2 rule?

$$\Gamma(K_S^0 \to \pi\pi)/\Gamma(K^+ \to \pi\pi) \approx 330$$

- Does the SM reproduce direct CP-violation in K→ππ?

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)} \approx 1 - 6\mathrm{Re}(\epsilon'/\epsilon)$$

$$\epsilon'/\epsilon = 1.63 \pm 0.26 \times 10^{-3}$$ [KTeV & NA48, 1999]

Electroweak decays [Sachrajda lectures]

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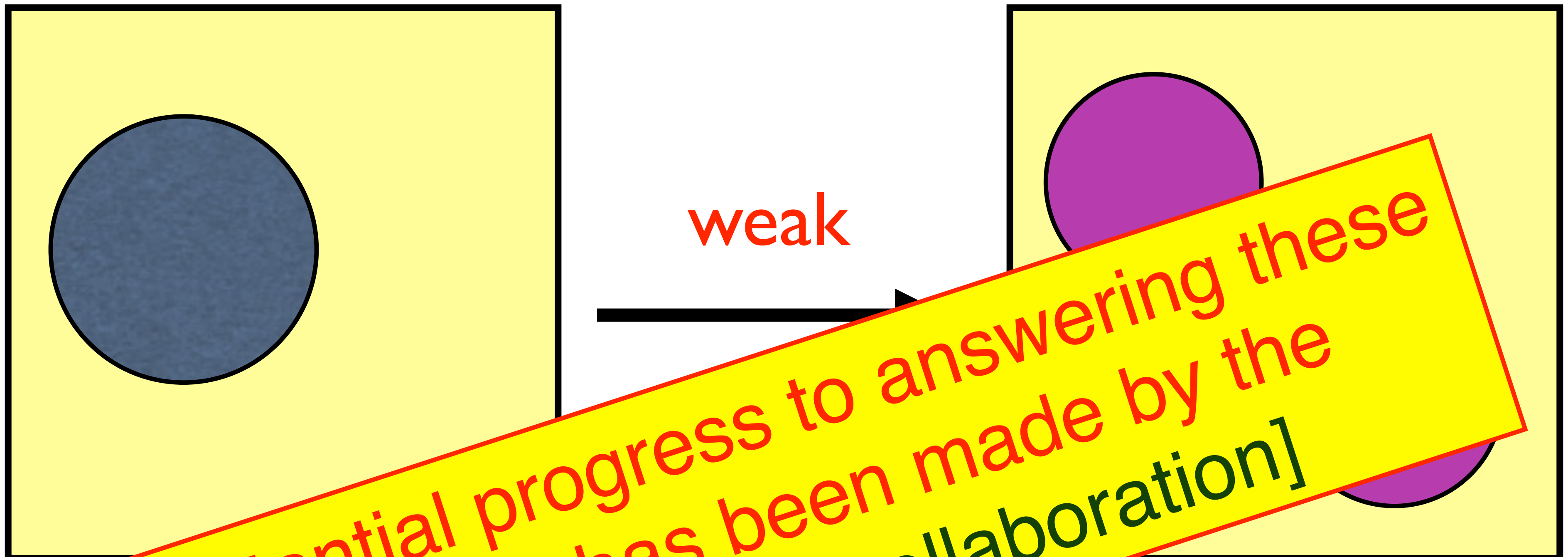

Substantial progress to answering these questions has been made by the [RBC-UKQCD collaboration]

- ... 1/2 rule?

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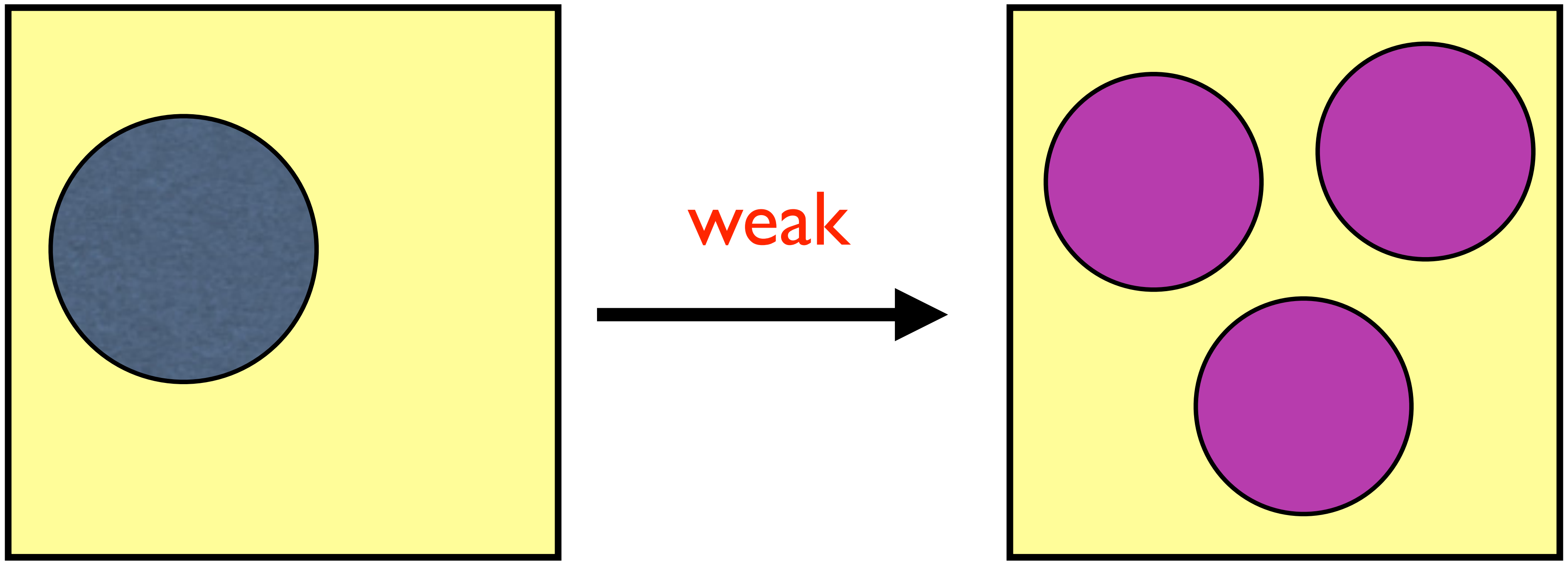

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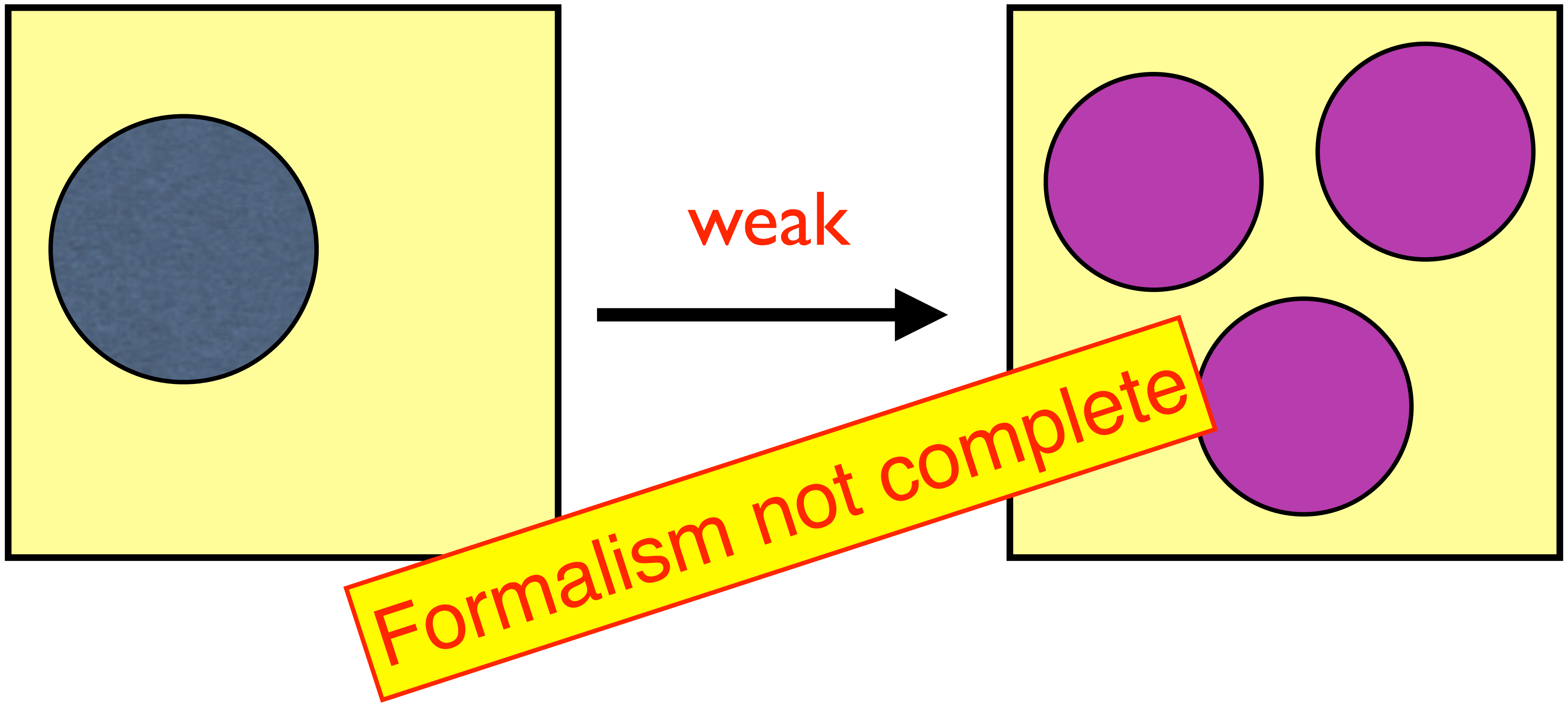

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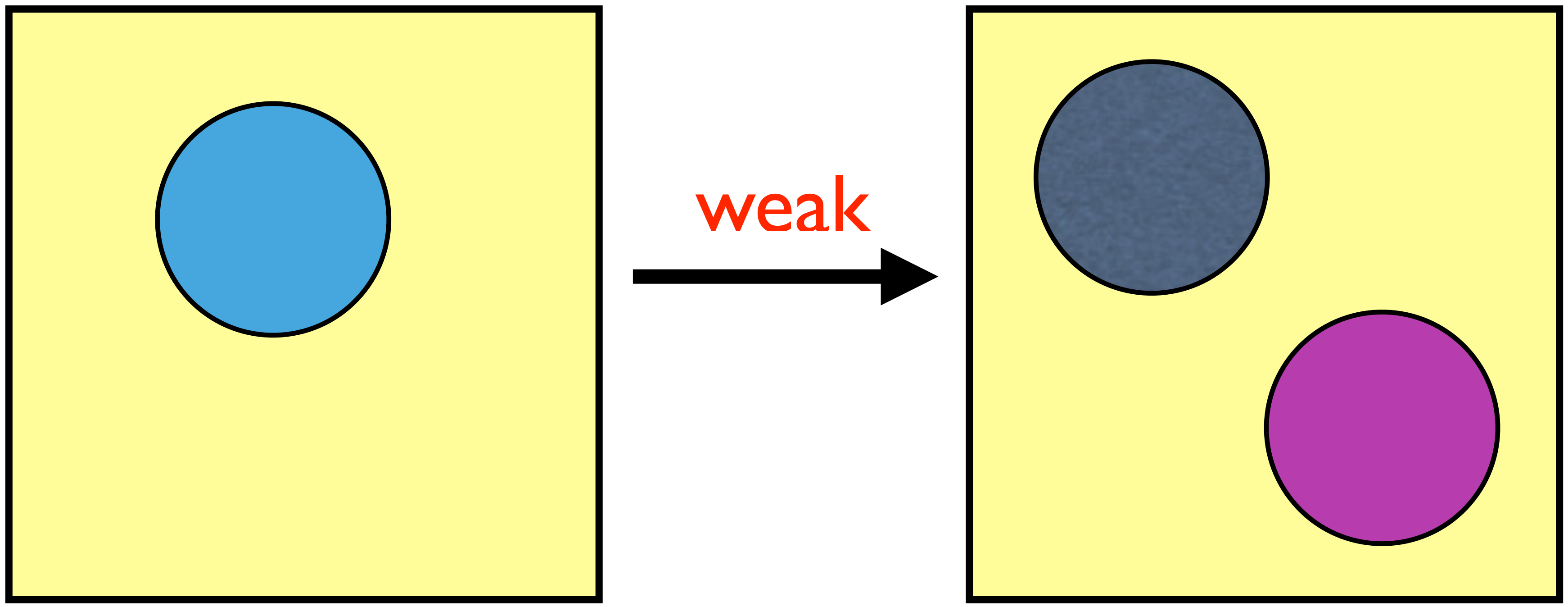
Electroweak transitions

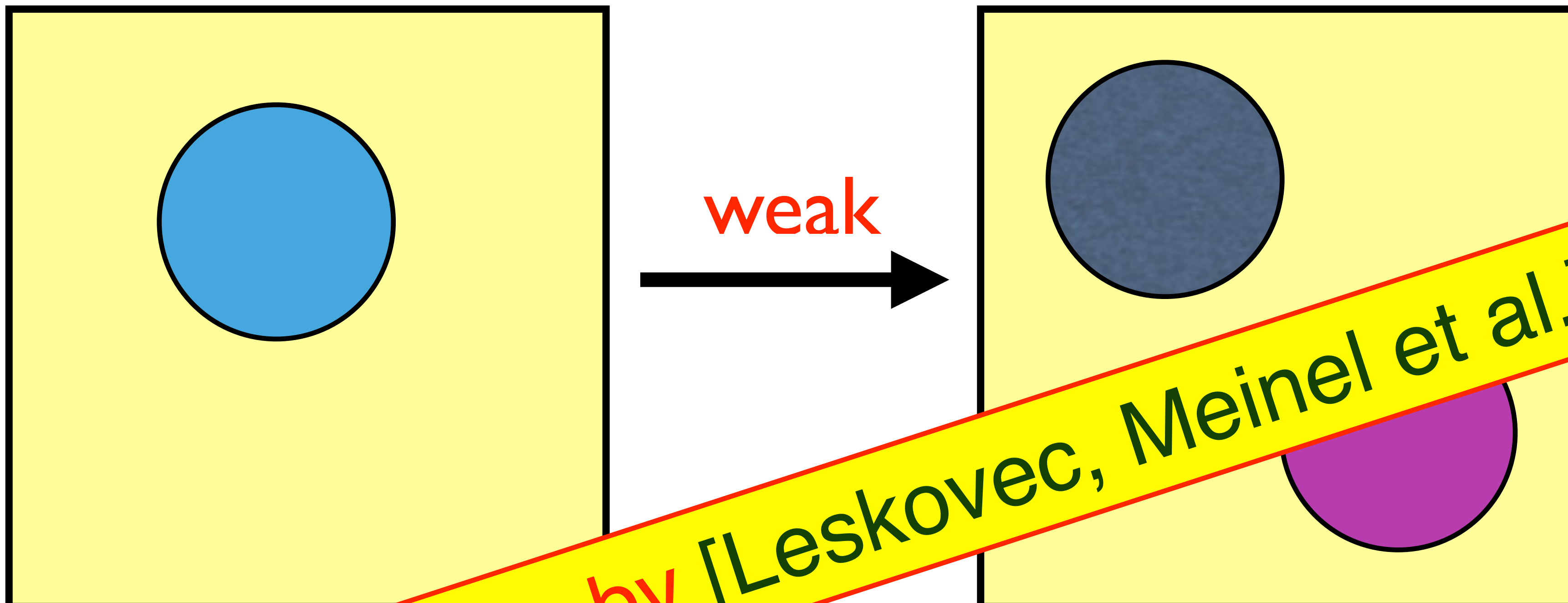
e.g. $B \to K^* \, l \, \nu \to K \, \pi \, l \, \nu$ decay amplitude



- Allows determination of elements of CKM matrix
- LQCD calculation is (much) harder than for $B \to K \, l \, \nu$ & $B \to \pi \, l \, \nu$, but there is lots of experimental data

Electroweak transitions

e.g. B→K* l ν → K π l ν decay amplitude



Work underway by [Leskovec, Meinel et al.]

- …mination of elements of CKM matrix
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A more distant motivation

CERN-EP-2019-042
13 March 2019

Observation of CP violation in charm decays

LHCb collaboration†

Abstract

A search for charge-parity (CP) violation in $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$ decays is reported, using pp collision data corresponding to an integrated luminosity of $6\,\mathrm{fb}^{-1}$ collected at a center-of-mass energy of 13 TeV with the LHCb detector. The flavor of the charm meson is inferred from the charge of the pion in $D^*(2010)^+ \to D^0\pi^+$ decays or from the charge of the muon in $\overline{B} \to D^0 \mu^- \bar{\nu}_\mu X$ decays. The difference between the CP asymmetries in $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$ decays is measured to be $\Delta A_{CP} = [-18.2 \pm 3.2\,(\mathrm{stat.}) \pm 0.9\,(\mathrm{syst.})] \times 10^{-4}$ for π-tagged and $\Delta A_{CP} = [-9 \pm 8\,(\mathrm{stat.}) \pm 5\,(\mathrm{syst.})] \times 10^{-4}$ for μ-tagged D^0 mesons. Combining these with previous LHCb results leads to

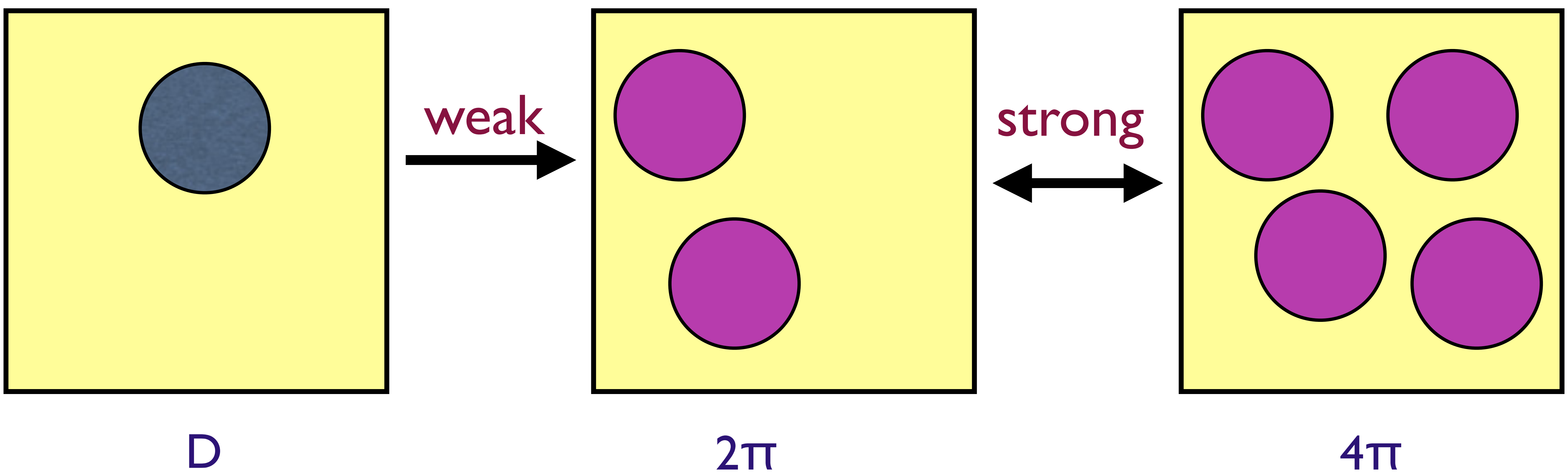
$$\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4},$$

where the uncertainty includes both statistical and systematic contributions. The measured value differs from zero by more than five standard deviations. This is the first observation of CP violation in the decay of charm hadrons.

5.3σ effect

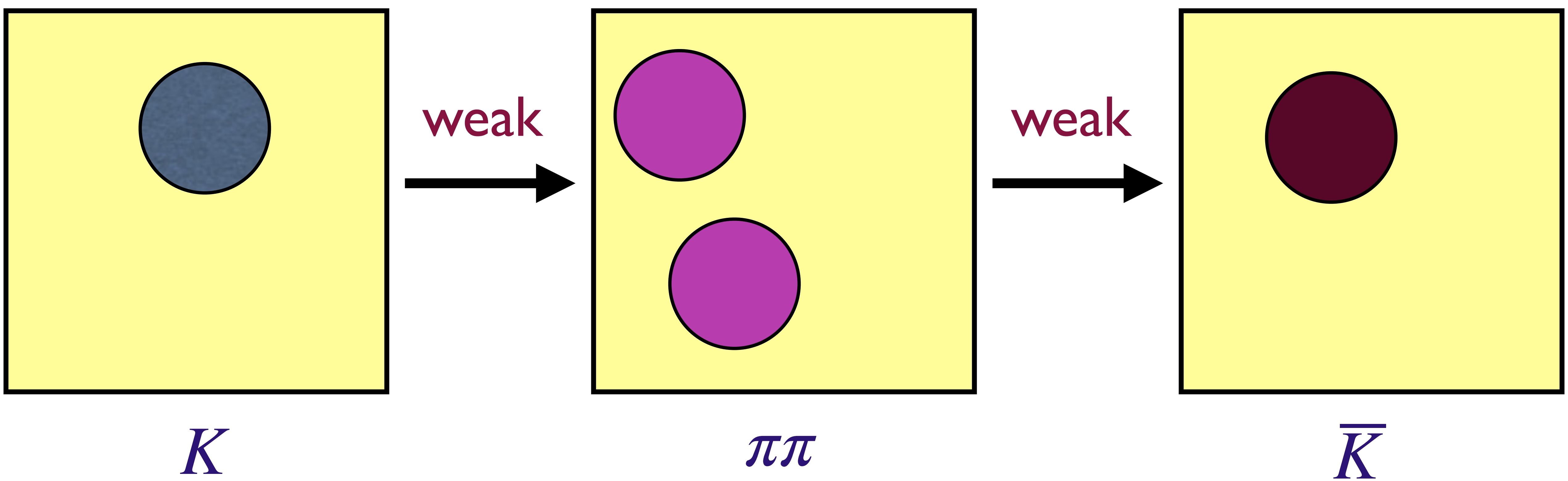
A more distant motivation

- Calculating CP-violation in $D\to\pi\pi, K\overline{K}$ in the Standard Model
- Finite-volume state is a mix of 2π, $K\overline{K}$, $\eta\eta$, 4π, 6π, …
- Need 4 (or more) particles in the box!



ΔM_K [Sachrajda lectures]

- Measured in 1961 by [Fitch *et al.*], but we still do not know whether it is consistent with the standard model
- Dominated by long-distance ππ contribution
- LQCD method, accounting for finite-volume effects, developed by [Christ, Feng, Martinelli & Sachrajda, 1504.01170]
- Numerical calculations underway [RBC-UKQCD]



3-body interactions

3-body interactions

- Determining NN & NNN interactions
  - Input for effective field theory treatments of larger nuclei & nuclear matter
  - NNN interaction important for determining properties of neutron stars
- Similarly, $\pi\pi\pi$, $\pi K\bar{K}$, … interactions needed for study of pion/kaon condensation

3-body interactions

- Determining NN & NNN interactions
  - Input for effective field theory treatment... ar matter
  - NNN interaction im... stars
- Simila... actions needed for study of pion... ation

[HALQCD collaboration] has made significant progress on determining NN potentials from LQCD

Scattering basics (infinite-volume)

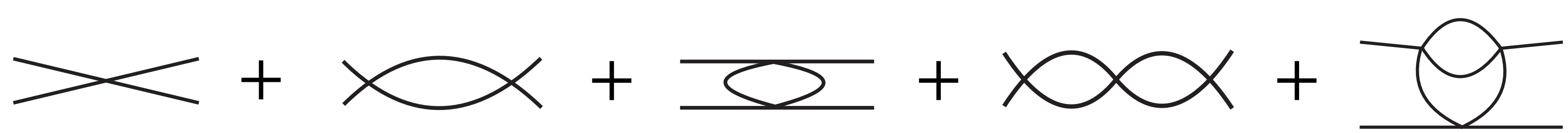
$\mathcal{M}_2$

- Recall some details of the simplest scattering process: 2 → 2
  - We will only discuss scalar (spinless) particles in these lectures, e.g. pions
  - We will also consider only identical particles, e.g. $\pi^+ \pi^+ \to \pi^+ \pi^+$
- Scattering amplitude related to the S matrix

$$S = 1 + iT \qquad \langle f | T | i \rangle = (2\pi)^4 \delta^4(P_f - P_i) \mathcal{M}_{fi}$$

- In a given theory, can calculate in perturbation theory (PT), e.g. in φ^4 theory

$i\mathcal{M}_2 =$



$+ \ldots$

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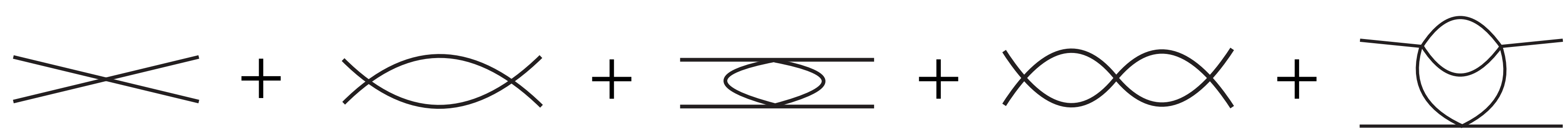
$$S = 1 + iT \qquad \langle f | T | i \rangle = (2\pi)^4 \delta^4(P_f - P_i) \mathcal{M}_{fi}$$

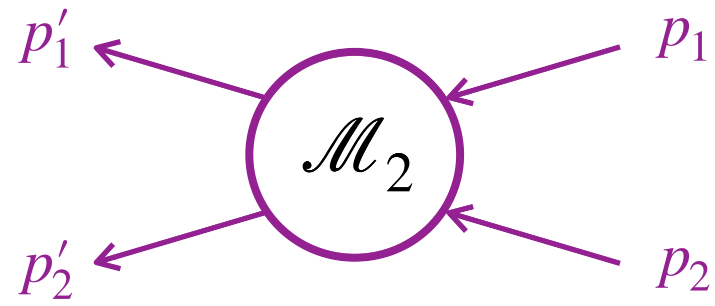
- In a given theory, can calculate in perturbation theory (PT), e.g. in φ^4 theory

$i\mathcal{M}_2 =$  $+ \ldots$

- We will not assume a particular theory, e.g. ChPT or φ^4; instead we use a generic relativistic QFT, with all possible vertices, and work to all orders in PT

Properties of $\mathcal{M}_2$

- Poincaré invariance $\Rightarrow \mathcal{M}_2$ depends on the two independent Mandelstam variables



$$\mathcal{M}_2 = \mathcal{M}_2(s,t), \quad s = (p_1 + p_2)^2,\ t = (p_1 - p_1')^2,\ u = (p_1 - p_2')^2 = 4m^2 - s - t$$

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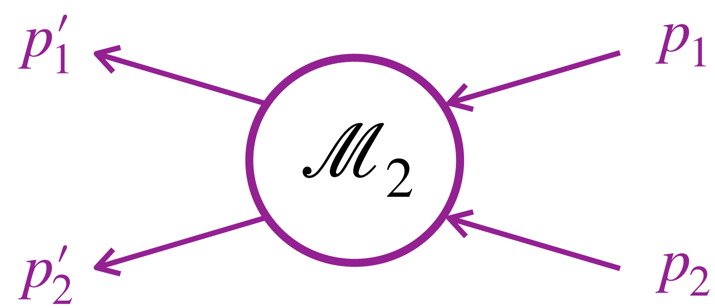

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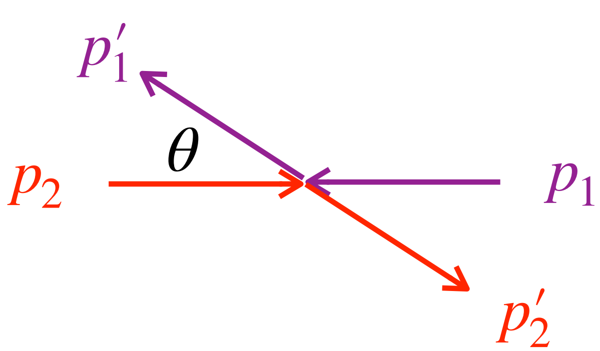
- Partial wave decomposition in CM frame



$$s = E^{*2} = 4(q^2 + m^2),\ t = -2q^2(1 - \cos\theta)$$

$$\mathcal{M}_2(s,t) = \sum_\ell (2\ell+1)\,\mathcal{M}_2^{(\ell)}(s)\,P_\ell(\cos\theta)$$

Only even values of l contribute for identical particles

Properties of $\mathcal{M}_2$

- Unitarity (holds in each partial wave)

$$S^\dagger S = 1 \;\Rightarrow\; \mathrm{Im}(\mathcal{M}_2^{(\ell)}) = \mathcal{M}_2^{(\ell)*}\rho\mathcal{M}_2^{(\ell)} = \rho\,|\,\mathcal{M}_2^{(\ell)}|^2, \qquad \rho = \frac{q}{16\pi E^*} \text{ (phase space)}$$

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- Parametrize K_2 using (real) phase shifts

$$\mathcal{K}_2^{(\ell)} \equiv \frac{1}{\rho}\tan\delta_\ell = \frac{16\pi E^*}{q\cot\delta_\ell} \;\Rightarrow\; \mathcal{M}_2 = \frac{1}{\rho}e^{i\delta}\sin\delta_\ell$$

Properties of $\mathcal{M}_2$

- Threshold behavior (QM)

$$\delta_\ell \sim q^{1+2\ell}\left[1+\mathcal{O}(q^2)\right] \;\Rightarrow\; \mathcal{K}_2^{(\ell)} \sim q^{2\ell}\left[1+\mathcal{O}(q^2)\right]$$

- Effective range expansion (ERE)

$$\frac{1}{\mathcal{K}_2^{(0)}} = \frac{1}{16\pi E_2}\left[-\frac{1}{a_0} + r_0\frac{q^2}{2} + P_0 r_0^3 q^4 + \ldots\right], \quad \frac{1}{\mathcal{K}_2^{(2)}} = -\frac{1}{16\pi E_2}\frac{1}{q^4}\frac{1}{a_2^5} + \ldots$$

  - a_0 is s-wave scattering length, related to threshold scattering amplitude

$$\mathcal{M}_2(q=0) = \mathcal{K}_2(q=0) = 32\pi m a_0$$

  - a_0 is the intercept of the s-wave radial QM wavefunction at q=0 on the r axis, and can have any value: $-\infty < a_0 < +\infty$
  - r_0 is the effective range (typically of order the range of the interaction), P_0 is the "shape parameter" (typically of order unity), and a_2 is the d-wave scattering length

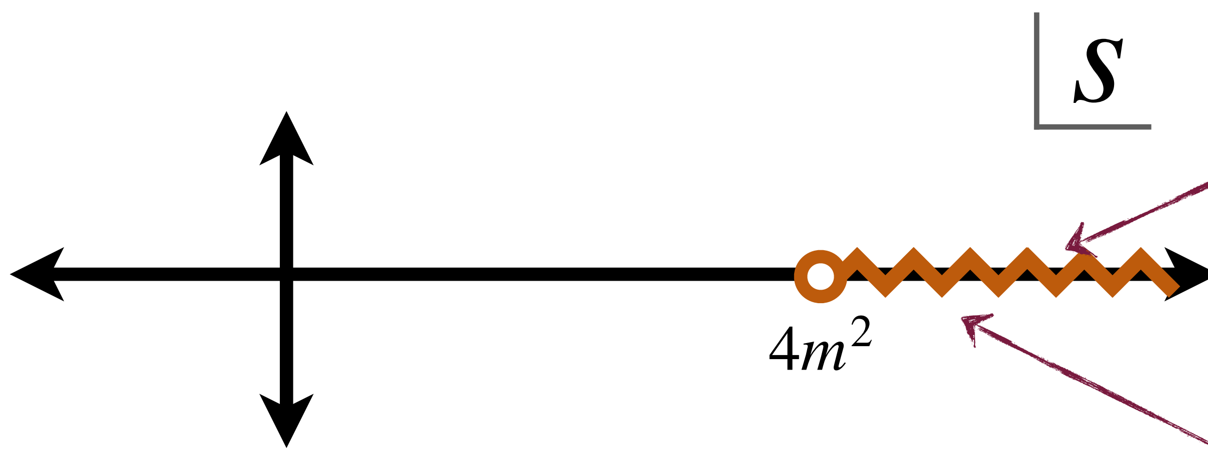
Properties of $\mathcal{M}_2$

- Analytic structure: branch cut along real s axis above threshold, arising from unitarity

$$\mathcal{M}_2^{(\ell)} = \mathcal{K}_2^{(\ell)} + \mathcal{K}_2^{(\ell)} i\rho \mathcal{K}_2^{(\ell)} + \ldots, \qquad \rho = \frac{\sqrt{s-4m^2}}{32\pi\sqrt{s}}$$

gives rise to "right-hand cut"

canceled by factors in $\mathcal{K}_2$



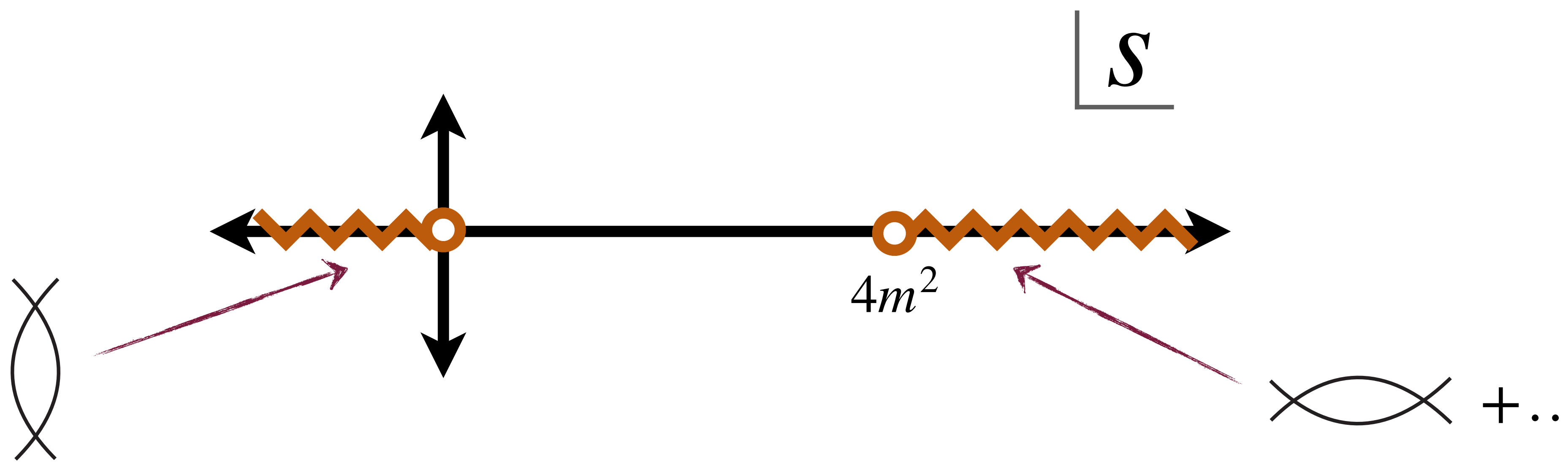
physical amplitude obtained by approaching cut from above

- $\mathcal{M}_2$ has two Riemann sheets, the top one being called the "physical sheet"
- $\mathcal{K}_2$ does not have the right-hand cut; it is analytic at threshold

Properties of $\mathcal{M}_2$

- t- and u-channel exchanges lead to the "left-hand cut"

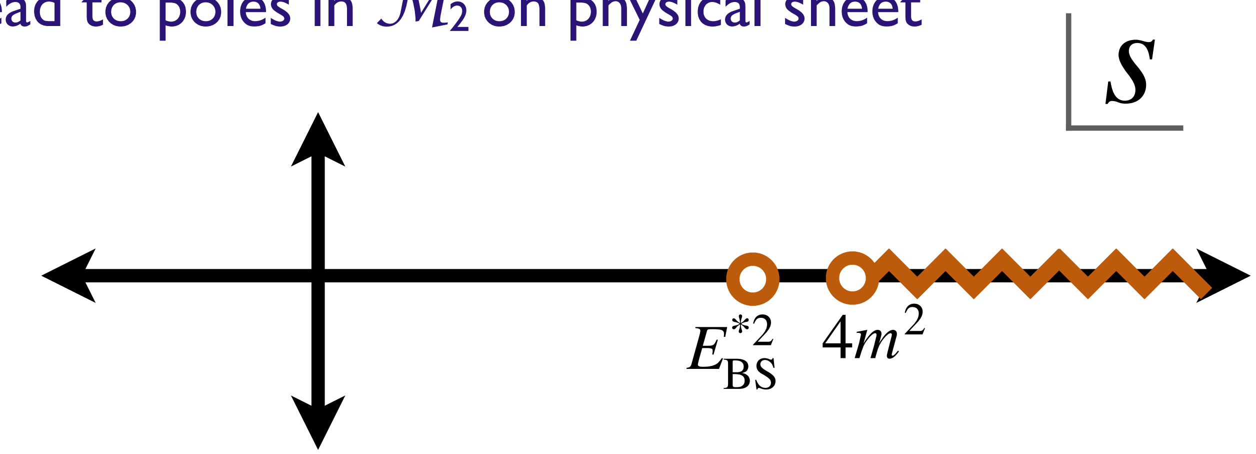
$$\mathcal{M}_2^{(\ell)} = \mathcal{K}_2^{(\ell)} + \mathcal{K}_2^{(\ell)} i\rho \mathcal{K}_2^{(\ell)} + \ldots, \qquad \rho = \frac{\sqrt{s-4m^2}}{32\pi\sqrt{s}}$$



- Left-hand cut is far below threshold, and I will ignore it henceforth
- One does have to worry about it in the 3-particle analysis, but I will not have time to discuss this relatively minor point—see [HS14, HS19]

Bound states

- Bound states lead to poles in $\mathcal{M}_2$ on physical sheet



- $\mathcal{K}_2$ does not have a corresponding pole since ρ is nonzero below threshold

$$1/\mathcal{M}_2^{(\ell)} \equiv 1/\mathcal{K}_2^{(\ell)} - i\rho \text{ where } -i\rho = \frac{|q|}{16\pi E^*} \text{ with } E^{*2}_{\rm BS} = 4(m^2 - |q|^2)$$

- Bound state condition is thus

$$1/\mathcal{M}_2^{(\ell)} = \frac{1}{16\pi E^*}(q\cot\delta_\ell + |q|) = 0$$

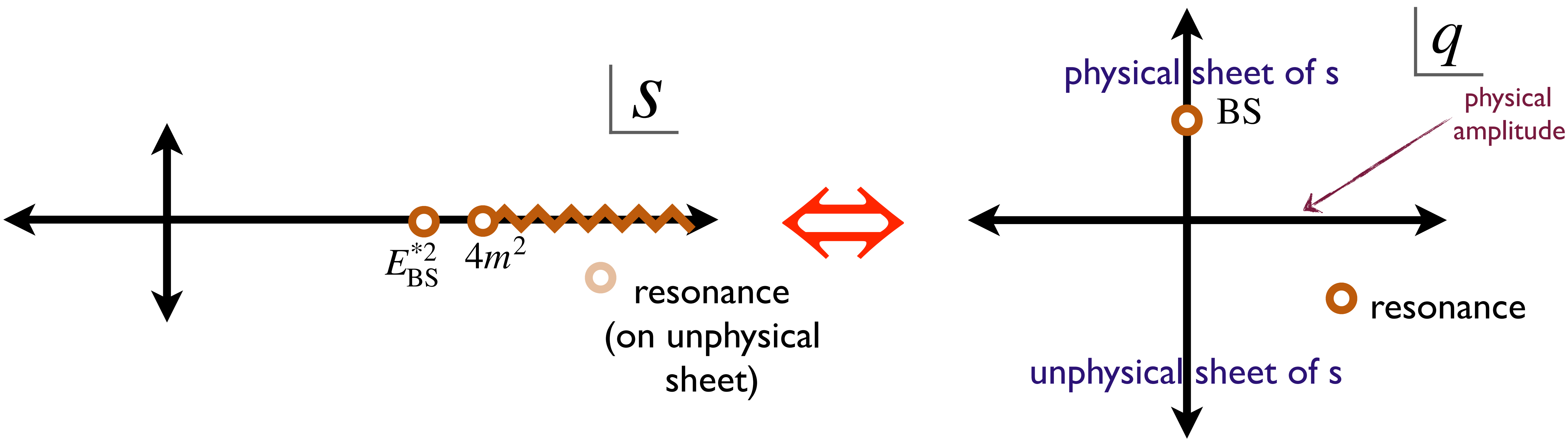
- If keep only the scattering length in the ERE, find bound state for $a_0 > 0$

$$q\cot\delta_0 = -1/a_0 \;\Rightarrow\; |q| = 1/a_0 \;\Rightarrow\; E^*_{BS} = 2\sqrt{m^2 - 1/a_0^2}$$

- Bound state at threshold in unitary limit $a_0 \to \infty$

Resonances

- Resonances lead to poles in M_2 below the real axis on the second (unphysical) sheet
  - Cannot have poles on physical sheet aside from bound states due to causality
  - To display sheets it is better to use single-sheeted variable q



- Resonance with width Γ=1/τ and mass M has pole at

$$E^* = M - i\Gamma/2 \;\Rightarrow\; s = M^2 + (\Gamma/2)^2 - iM\Gamma$$

- Leads to a bump in scattering cross-section ~$|\mathcal{M}_2|^2$ as we saw earlier

Resonances

- Narrow s-wave resonances well described by Breit-Wigner form

$$\tan\delta_{\rm BW} = \frac{E^*\Gamma}{M^2 - E^{*2}} \Rightarrow \mathcal{M}_2 \propto \frac{1}{M^2 - E^{*2} - iE^*\Gamma}$$

- As E^* passes through M from below:
  - Phase shift rises rapidly through 90^0
  - $\mathcal{K}_2 \sim \tan\delta$ has a pole at M (i.e. on the real axis)
- Pole in $\mathcal{K}_2$ does not have any direct physical significance, but does play a role in the finite-volume analysis to follow

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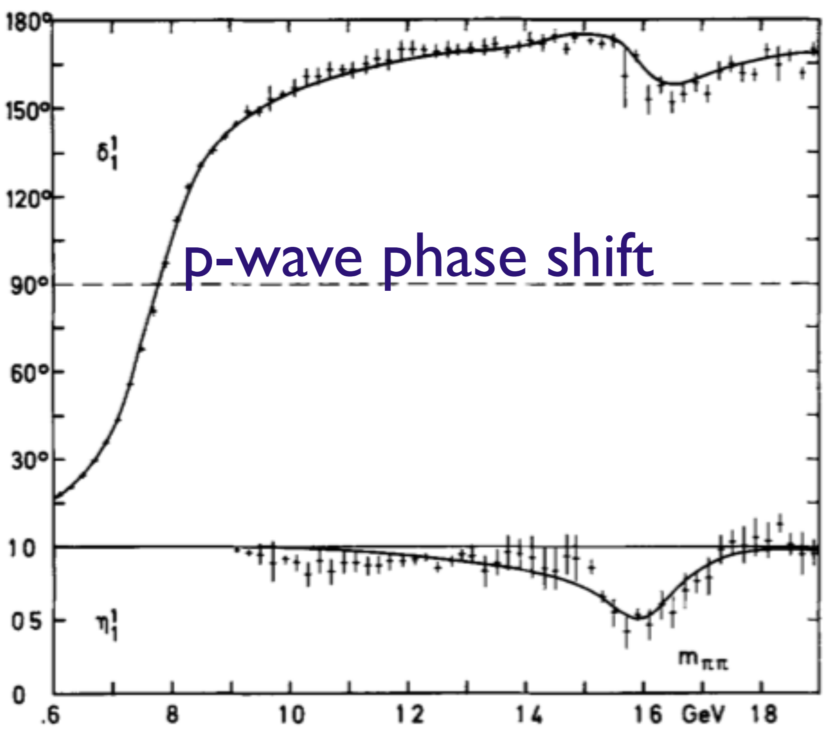
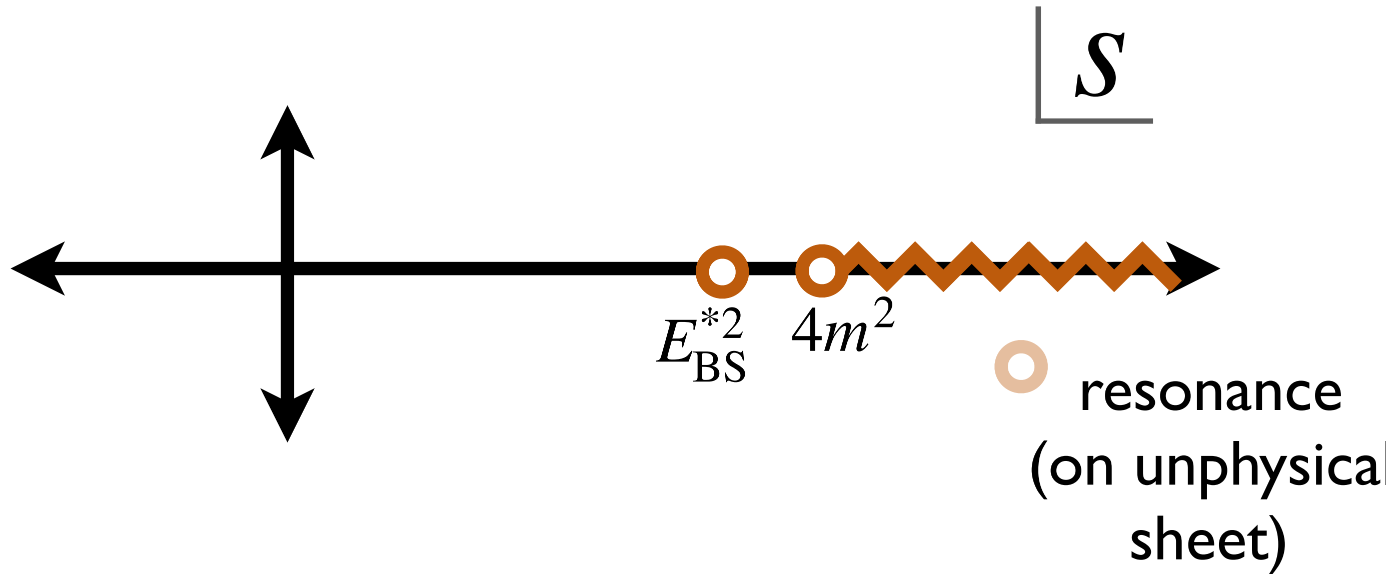


Hyams 1973

Resonances: unavoidable complication



- Neither experiment, nor LQCD calculations, can directly access complex energies
- Thus, in order to study resonances, **both** methods have to parametrize the K matrices with an analytic form that can be continued into the complex plane
- Thus some parametrization dependence is unavoidable
- One should put as much physical knowledge as possible into the parametrization, while minimizing model dependence
- Input from the experimental analysis community can be helpful

G parity

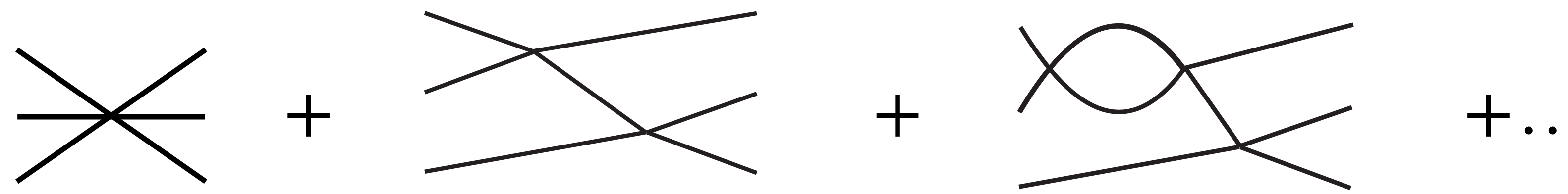
- G parity will come up occasionally in the remaining lectures, so here is a reminder
  - $G = C\, e^{i\pi I_y}$ is an exact symmetry of isosymmetric QCD, and an approximate symmetry of real QCD
  - Eigenstates of G: $\pi(-1),\ \eta(+1),\ \rho(+1),\ \omega(-1), \ldots$
- Relevance for what follows:
  - Restricts decay channels, e.g. $\rho \to \pi\pi,\ \omega \to \pi\pi\pi\ (\eta \to \pi\pi$ forbidden by parity)
  - No interactions involving an odd number of pions, e.g.

$$\pi\pi \leftrightarrow 4\pi, \quad \pi\pi \not\leftrightarrow 3\pi$$

3-particle scattering

- In a theory with a G-parity-like Z_2 symmetry only have 3→3 scattering

$\mathcal{M}_3 \sim$



  - Difficult to measure experimentally, but well defined in QFT
  - 3 particle finite-volume states are accessible to LQCD

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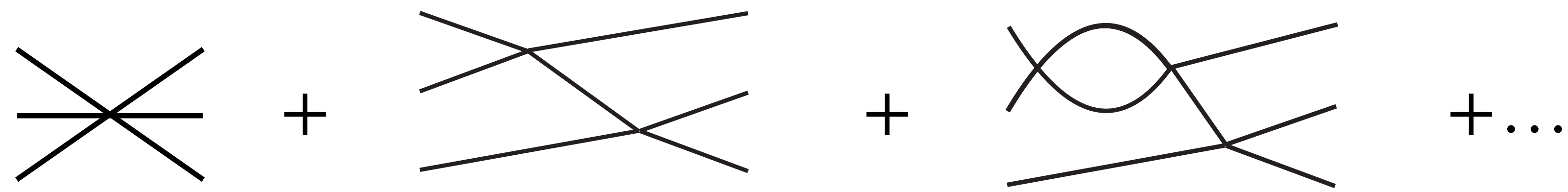

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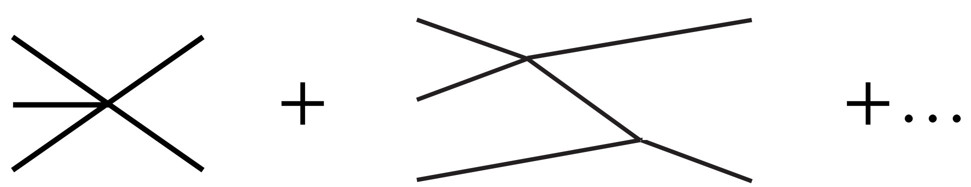
  - 3 particle finite-volume states are accessible to LQCD

- Without the Z_2 symmetry have 2→3, 3→2 & 3→3 scattering, e.g.

$$\mathcal{M}_{23} \sim$$



3-particle scattering

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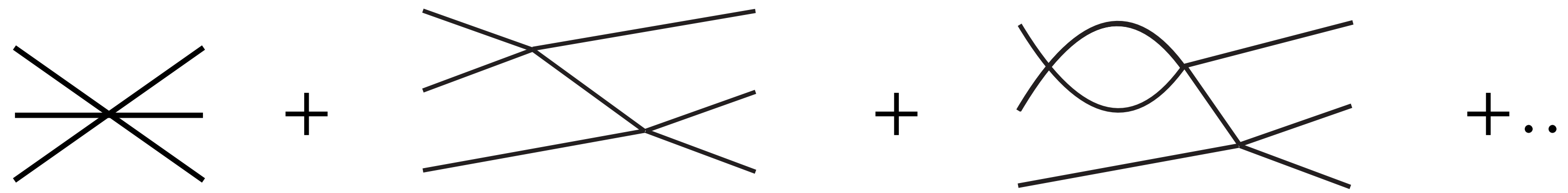
$\mathcal{M}_3 \sim$ 

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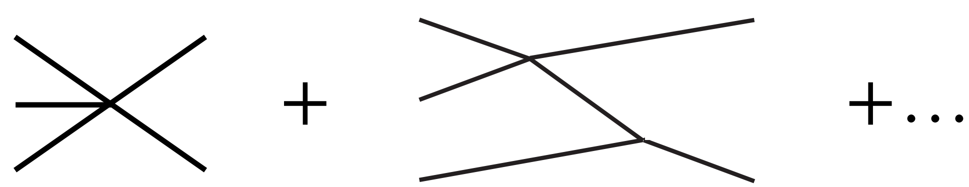
$\mathcal{M}_{23} \sim$ 

- Parametrizing these amplitudes in terms of real K matrices is a nontrivial problem to which the methods I will describe provide, as a spinoff, one solution

Thank you!
Questions?