Aristotle on the
Principle of Non-Contradiction

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Not the least among the many puzzling features of the fourth book of Aristotle’s *Metaphysics* is his discussion of the Principle of Non-Contradiction (hereafter ‘PNC’). Even leaving aside the obvious difficulty of determining what his arguments succeed in showing about PNC, we face the more fundamental problem of figuring out what he *takes* them to show. For he proceeds in such a way as to suggest that he is not always completely clear about what he is up to.

Aristotle seems to be offering arguments in support of PNC. Yet to do so would be to try to demonstrate something he considers indemonstrable, to prove a first principle, to treat an ultimate *explanans* as also an *explanandum* — *and* to try to explain it. These maneuvers fly in the face of the teachings of the *Organon*, which allow no room for a demonstration, or proof (*apodeixis*), of PNC.
Yet, PNC, like any other first principle, can (indeed, must) be known to be true. It is not sufficient for a first principle merely to be believed or accepted or assumed. For the Posterior Analytics tells us that the premises of a demonstration must be better known than its conclusion. But if to be known is just to be the conclusion of a demonstrative syllogism, the doctrine of the Posterior Analytics would seem to be in trouble. For if all knowledge were demonstrative, a knowable first principle would have to be a demonstrable first principle. There would thus appear to be a collision between the knowability and the unprovability of first principles.

Aristotle's solution in the Posterior Analytics is to distinguish between epistêmê (scientific knowledge) and nous (intuitive intellect). First principles, such as PNC, are not objects of scientific knowledge — since they are not demonstrable — but are still known, since they are grasped by nous.

However unsatisfying we may find this appeal to the deus ex machina of nous, we must grant, I think, that the Posterior Analytics offers a coherent (even if ultimately indefensible) account of the knowability of first principles. But this account does not seem to mesh at all well with Aristotle's procedure in Metaphysics I, where there is no mention of the role of nous as a faculty which grasps first principles. Rather, we are given an elenctic demonstration of PNC (or so it would seem). Indeed, we are provided with this demonstration by the discipline of First Philosophy, a general science of which there is no mention, and for which there seems to be no room, in the Organon.

There would thus seem to be an inconsistency between the Posterior Analytics and Metaphysics I on the epistemic status of first principles such as PNC, and the commentators have not missed it. T.H. Irwin1 sees the Metaphysics as rejecting several key ideas developed earlier in the Organon; Lukasiewicz2 is apparently prepared to find an inconsistency within Metaphysics I itself, on the grounds that Aristotle there first states that PNC is unprovable and then attempts to prove it.

Alan Code disagrees with both Irwin and Lukasiewicz. Consider his dispute with Irwin. On Irwin's account, the Metaphysics agrees with the Organon in holding that PNC is beyond demonstration, but disagrees

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1 'Aristotle's Discovery of Metaphysics,' Review of Metaphysics 31 (1977) 210-29
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with it in allowing there to be a kind of scientific knowledge (epistēmē) of things that are beyond demonstration. This branch of knowledge is not one of the Organon's departmental sciences, whose methods are demonstrative, but the general science of First Philosophy (the study of being qua being). First Philosophy's method is not demonstration (apodeixis), but elenctic demonstration. Clearly, then, elenctic demonstration cannot be viewed as a species of demonstration, and Irwin does not so view it. Rather, it is a kind of dialectical argument, proceeding in a refutational manner against an interlocutor whose willingness to debate has already been secured. One might take this to be a point of agreement between the two works, since dialectic (indeed, a dialectical treatment of first principles) is part of the Organon's scheme. But Irwin notes that dialectic in the Organon depends on beliefs and hence is not epistēmē-inducing. It can point us in the direction of first principles, but it cannot prove them. So, since the Organon holds that dialectic does not produce knowledge and the Metaphysics holds that elenctic demonstration (= dialectic) does produce knowledge, the Metaphysics does, after all, contradict the Organon on this point.

Although Code's case against Irwin is elaborate and argued in detail, its core is relatively simple. Irwin's account takes Metaphysics Γ to be attempting to establish, by means of elenctic demonstration, the truth of PNC. And this is what Code disputes. According to him, what Aristotle is up to in Metaphysics Γ is to prove certain things about PNC, rather than to prove (elenctically or in any other way) PNC itself. What Aristotle wants to prove about PNC is that it is 'the firmest principle of all.' By this he seems to mean that PNC is indubitable. Everyone believes PNC, Aristotle is maintaining; indeed, no one is even capable of disbelieving it. To use an older terminology, what Aristotle attempts to prove is not that PNC is true, but that it is a Law of Thought.

It cannot be disputed that Aristotle's aim, at least some of the time, is directed toward the indubitability of PNC. The argument of Γ 3 (1005b22-34) that PNC is the firmest of all principles certainly is aimed at indubitability rather than at truth. For Aristotle is very clear that this argument uses PNC in establishing its conclusion, and he is equally clear that its conclusion is not PNC but rather the proposition that PNC is the firmest of all principles (Γ 4, 1006a4-5). So the argument of Γ 3 is not an attempt to prove PNC. The fact that the conclusion of this argument entails that PNC is true does not show, as Code quite correctly points out, that the argument can be construed as a proof of PNC. A proof that q — as opposed to a merely sound argument with q as a conclusion — must have as a premise the 'why' of q. And there is no 'why' of PNC. PNC, as
it turns out, is the 'why' of its own indubitability. PNC itself is the principle which explains why PNC is a Law of Thought.

The argument of Γ 3 thus becomes absolutely central for Code, since it is one of the few places where Aristotle is clearly trying to establish something about PNC rather than arguing for PNC itself. But, as Code also notes, the argument of Γ 3 precedes the introduction, at the beginning of Γ 4, of the notion of an elenctic demonstration. And for Code's conciliatory effort to be successful, it is the elenctic demonstrations of Γ 4, not just the 'contrariety' argument of Γ 3, that must be plausibly construed as supporting the indubitability of PNC. I have some doubts about this. Code also flirts with the idea that the general science of being qua being to which these elenctic arguments belong may be plausibly construed as conforming to the model of the departmental sciences proposed in the Posterior Analytics. I have some doubts about this, as well.

There are, by Kirwan’s count, three of these elenctic demonstrations in Γ 4. Aristotle precedes them with some introductory remarks about his intentions. Some people, he says, assert (i) that it is possible for the same thing to be and not to be and (ii) that it is possible to believe this (1005b35-1006a2). These people, presumably, dispute both PNC itself and the claim that PNC is indubitable. After conceding that we cannot give a demonstration (1006a9), Aristotle proposes to counter these opponents elenctically, and in that way to 'demonstrate this to be impossible' (1006a12). But what does 'this' refer to? Presumably, what the opponents have asserted to be possible. But they have asserted two things to be possible; so by 'demonstrating this to be impossible' Aristotle might mean to demonstrate any of the following:

(a) PNC
(b) The indubitability of PNC
(c) The conjunction of (a) and (b).

Apparently, we will have to look at the arguments themselves to see what Aristotle's intentions are.

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The conclusions of the arguments (in those cases in which the conclusions are plainly stated) are as follows:

1. 'It could not be that everything was so-and-so and not so-and-so' (1006a30-31).
2.(a) 'It is not possible that it should be simultaneously true to say that the same thing is a man and is not a man' (1006b33-34).
2.(c) 'It is impossible to predicate contradictories simultaneously' (1007b18).
5. 'There can be no such thing as simultaneously asserting and denying the same thing truly' (1008a36-b1).
6. 'Nobody is actually in that condition [sc. believing that things both are and are not in a certain state], neither those who state this thesis nor anybody else' (1008b12-13).

Of these five conclusions, only one (argument 6.) is clearly the indubitability thesis. Two are plainly not (1. and 2.[a]), and two can only be tortured into statements of the indubitability thesis by taking 'predicate' to mean 'believe' (2.[c]) and construing 'asserting and denying' as 'believing and disbelieving' (5.). But how plausible is such a construal in a context in which Aristotle explicitly says that people may not believe the things they say (1005b25-26)?

I doubt that Code wishes to indulge in such tortuous reconstruals. Nor do I think he would admit that Aristotle has, in Γ 4, lost sight of his quarry. For Code seems willing to concede that the elenctic demonstrations are aimed at 'refut[ing] somebody who asserts that PNC is false' (356). He even concedes that 'the conclusion of the elenctic argument is supposed to be PNC' (356). Nevertheless, he says, Aristotle is not addressing himself to an audience that needs to be convinced of its truth' (356). On this showing, Aristotle's method of refuting those who deny PNC is to offer arguments whose conclusion is PNC, but his purpose in doing this is to explain to someone who already accepts PNC why he must accept it. That is, Aristotle offers arguments whose conclusion is PNC in order to establish that PNC is indubitable.

The peculiarity of this procedure is somewhat masked by one of Code's formulations of it. Aristotle, he says, 'in showing why I must accept [PNC], ... is not giving me a reason for accepting it .... He is giving me a reason why it must be the case that I ... accept it' (356).

There is a clear sense in which one who gives me an argument whose conclusion is \( q \) is giving me a reason why I must accept \( q \), i.e., why I
should accept it. But that is the sense in which 'must accept' means 'have been given a compelling reason for accepting,' and it is not the sense which Code requires. He requires that 'I must accept $q$' here means 'as a matter of logical necessity, I do (not "should") accept $q$.' But it is less easy to see how an argument whose conclusion is $q$ can be used to establish that I 'must accept $q$' in that sense.

One may therefore have some reservations whether Aristotle in $\Gamma$ 4 is so clearly focused on the indubitability of PNC as Code has suggested. Still, Code's suggestion is appealing. Aristotle's general strategy is to point out that one who denies PNC is at least committed to the possibility of saying something, i.e., to the possibility of significant speech, and to argue that in order to have significant speech, PNC must be accepted. But there is an important ambiguity in this claim. Who must accept PNC in order for a speaker's speech to be significant? The speaker? We theorists? Aristotle's discussion seems to vacillate between these possibilities. Certainly, we cannot describe the speaker's speech as significant unless we think that there is something that he does mean as opposed to various other things that he does not. Our theory of significant speech would disintegrate if it violated PNC. But must the speaker also accept PNC? If he does not, he will be in no position to say what it is that he means as opposed to what it is that he doesn't mean. But that is just to say that if he does not accept PNC, he cannot formulate an adequate theory of his own linguistic success. And it remains to be shown that he cannot be successful as a speaker unless he is successful as a theorist.

This point is worth further investigation, for if it can be resolved, we may be able to construe the conclusions of the elenctic arguments as the opponent's commitments, and hence as second-order statements even though they are put forward as first-order statements. In this way, although PNC itself would be the ostensible conclusion of some of these arguments, we may read Aristotle as saying, in effect, 'therefore, as my opponent plainly assumes, PNC must be true.' The parenthetical 'as my opponent plainly assumes' is omitted presumably because it is an implicit operator governing all of the statements in the elenctic demonstration.

My inclination to agree with Code about Aristotle's aims in $\Gamma$ 4 is strengthened by his compelling case against Irwin's alternative. Code is quite correct in pointing out that Aristotle's elenctic demonstrations seem to take for granted a number of theses concerning Aristotelian essences, theses which are hardly to be considered methodologically suitable to serve as premises on the basis of which to establish PNC. On the other hand, since the claim that PNC is indubitable is not a first principle of
any science, it would seem unobjectionable to appeal to other metaphysical theses in its support. This becomes even clearer when we recall that Aristotle announces in Γ 3 that one and the same science deals with both common axioms and with substance. So the theses concerning essences are certainly appropriate premises to use in attempting to establish the indubitability of PNC.

I turn now to my second doubt mentioned above, viz., whether it is plausible to construe the elenctic arguments of Γ 4 as belonging to a general science of being qua being that conforms to the model of a science presented in the Posterior Analytics. Code's account of the purpose of the elenctic demonstrations certainly fits Aristotle's description in Γ 3 of the subject-matter of First Philosophy. That subject-matter includes the common axioms, of which PNC is one. And the thesis that PNC is indubitable ascribes to PNC one of its necessary (although non-essential) characteristics, as one might expect a thesis of a science to do.

But does anything in the Posterior Analytics prepare us for this turn? That is, is there a basis in Aristotle's description of a science for the notion that a science may have one of its axioms (albeit not one of its proper axioms) as its own subject-matter? There would not seem to be. In the few passages in which Aristotle sets out the ingredients of a science (or of a scientific demonstration), and which Code cites, Aristotle mentions three things: a genus, axioms, and attributes (pathe) (76b11-15), or a genus, axioms, and the conclusion to be proved (to apodeiknumenon to sumperasma) (75a40). I take his idea to be that a science will establish certain theorems by means of, or from, it first principles and common axioms. These theorems will be propositions ascribing necessary attributes, both essential and non-essential, to the subject genus and its various species. There is no suggestion that the theorems themselves may belong to the subject genus, in the sense of being entities whose attributes it is the business of the science in question to establish. Biology studies animals and establishes truths about animals as animals (rather than as, say, physical objects). But Aristotle does not seem to me to be suggesting that it is also the business of biology to study truths about animals, at least not in the sense in which to do so would be to establish truths about such truths. Biology is not meta-biology; physics is not meta-physics. Similarly, it would not seem that the first principles of a given science are among the things it studies and whose attributes it establishes. It arrives at those principles by examining specimens of the various species of its subject genus, and it then uses those principles in establishing the attributes of those species qua species of the subject genus. I take it that it is in this way that a science 'concerns' or is 'about' (peri) its first principles.
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In Posterior Analytics A10 Aristotle says that every demonstrative science is 'about' (peri) the so-called common axioms (ta koina legomena axiōmata, 76b14), but he can scarcely mean that these axioms are part of the subject-matter of every science. Surely frog-ology does not study PNC! How, then, might one show that First Philosophy's inclusion of the common axioms as part of its subject-matter conforms to the model of a science in Posterior Analytics?

One might suppose that it is because First Philosophy is a science whose subject is 'the things that are' (ta onta) that First Philosophy may appropriately study the common axioms. But it is clear that this could not be Code's reason; for, as he points out (354) the axioms are not onta. Ontology's justification for studying PNC must be more sophisticated than this.

Unfortunately, it is not clear to me just what justification Code could provide. I suspect that he would appeal to the respect in which First Philosophy studies being — namely, qua being. For he says that to study being qua being 'is to study the ubiquitous truths about all beings as such' (354). Since PNC is one of those truths, the study of PNC belongs to the study of being qua being. But it seems to me that the Posterior Analytics leaves room only for a study of being qua being which arrives at ubiquitous truths about all beings as such, and not for a study of those truths.

There is, indeed, a tendency in Code's paper to shift, nearly imperceptibly, between the principles of a science and its subject matter. Here is an example: '... the principles that hold of being qua being are not part of the subject matter of biology. If something applies to Kermit the frog qua being, then that something does not apply to Kermit qua frog ...' (353). But the fact that Kermit's attributes qua being are not the same as his attributes qua frog shows that metaphysical principles are not biological principles and that the subject matter of metaphysics is not the same as the subject matter of biology. It does not show that biology is not a study of metaphysical principles, or that metaphysics is not a study of biological principles (although these are both very likely true).

This shift is perhaps fostered by Aristotle's tendency to speak of both propositions and terms as archai, or principles. A term that is an archē might be a fundamental attribute of some kind. If such an archē is a necessary attribute of the subject genus of some science, then that attribute becomes part of the subject matter of the science. But a proposition that is an archē is something else again. Since it is not an attribute, it will not be a necessary attribute of the genus, and hence will not become
part of the subject matter of the science. It is part of the study, to be sure, but not part of what is studied.

I am thus inclined to agree with Irwin that the general science of being qua being that is developed in Metaphysics is a radical departure from the departmental sciences of the Posterior Analytics. For it is self-reflective in a way that they are not. Metaphysics does not just arrive at general principles, it studies general principles. And its study of general principles is itself a general one. So metaphysics, uniquely among the sciences, studies the principles that are shared by all the sciences, and thereby studies its own principles. The theory of the Posterior Analytics was a theory about all (first-order) scientific theories; the theory of Metaphysics is a theory about all theories, including the theory of Metaphysics.

I shall conclude by mentioning some other issues that Code’s paper raises and some other areas that might be discussed profitably.

It has been observed that Aristotle’s argument in P 3 for the indubitability of PNC depends upon the assumption that believing that p and believing that ~p are contrary states. This, indeed, seems to be the crucial move in the argument to show that the indubitability of PNC is established by PNC itself. And one may certainly raise serious questions about the acceptability of this assumption. Another, less often noted, feature of Aristotle’s argument is that its proper conclusion is not quite the indubitability of PNC. For PNC is a generalization, and what the argument shows, if anything, is the indubitability of each instance of that generalization. That is, Aristotle establishes, for arbitrarily selected object a and predicate G, that

1. everyone believes that ~(Ga & ~Ga).

From 1. it is legitimate to generalize to

2. (∀x)(∀F)(everyone believes that ~[Fx & ~Fx]).

But 2. does not assert the indubitability of PNC. That would be represented by a different formula:

3. everyone believes that (∀x)(∀F) ~(Fx & ~Fx).4

4 Properly speaking, 3. does not express the indubitability of PNC, but merely its universal acceptance; the correct formulation should be modalized. This complication is irrelevant for present purposes and has therefore been ignored.
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But it is not obvious that 2. entails 3., for it is not obvious that one may legitimately import external quantifiers into a belief context.

Consider, for example, the generalization that all of my beliefs are true. Now I don't for a minute believe this. But although I disbelieve this generalization about my beliefs, there is no particular belief of mine (as Moore's paradox reminds us) that I can single out as false. So there is an almost certainly false generalization that it is quite rational for me to reject, and which I do reject, although there is no instance of it that I am even capable of disbelieving. In short: with respect to each of my beliefs, I believe that each is true; but I do not believe that all of my beliefs are true. Here the importation of a foreign quantifier into a belief context clearly fails. Hence it is not obvious that Aristotle can count on the indubitability of every instance of PNC as a guarantee of the indubitability of PNC itself.

Although some commentators brush this trouble aside as 'not very serious,' I gather that Code would be more concerned. Indeed, his view seems to be that the property of PNC that Aristotle is trying to establish in the $\Gamma 3$ argument — the property that makes PNC the 'firmest principle of all' — is just the indubitability of each of its instances, rather than the indubitability of PNC itself. If so, then on Code's interpretation Aristotle's argument in $\Gamma 3$ does not have to face this difficulty. But now it will be incumbent upon Code to explain how the dialectical arguments of $\Gamma 4$ are related to $\Gamma 3$'s demonstration that PNC is the firmest principle. Presumably, Code thinks of $\Gamma 4$'s argument as subsidiary to the argument of $\Gamma 3$, for which they provide dialectical support. But that interpretation seems plausible only if the proposition dialectically supported in $\Gamma 4$ is the same one argued to be indubitable in $\Gamma 3$. And since the dialectical

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5 I have found this example useful in discussions of Plato's refutation of Protagoras in the Theaetetus. I thank Alan Code for pointing out to me its application to the present case.

6 More precisely, with respect to each proposition, I believe that, if I believe it, it is true:

$$(\forall p)(I \text{ believe that } (I \text{ believe that } p \supset \text{ it is true that } p)).$$

(This is an acceptable generalization because there is no proposition with respect to which I believe both that I believe it and that it is false.) But the importation of the quantifier into the belief-context yields a falsehood, since I do not take myself to be an infallible believer, i.e.,

I do not believe that $$(\forall p)(I \text{ believe that } p \supset \text{ it is true that } p).$$

arguments of Γ 4 seem to have PNC as their conclusion, there is some reason to suspect that the argument of Γ 3 is, after all, intended to establish the indubitability of PNC, and not just the indubitability of each of its instances. But then, by the same token, the argument of Γ 3 will have to confront the objection that it has illicitly smuggled a pair of quantifiers across an epistemic border.

Finally, one of Code’s objections to Irwin’s interpretation of Γ 4 is that it misdescribes the premises of elenctic demonstrations: they are not pre-theoretical truisms that no one can rationally reject. Rather, Code tells us, they involve a substantial commitment to Aristotelian essentialism. This criticism invites several questions. 1. Does the use of the language of substantial essences, in the form of such expressions as to ἀνθρώποι ἐιναί (‘the essence of man’) show that Aristotle is using theses of Aristotelian essentialism as premises in his elenctic demonstrations of PNC? 2. If Aristotle is using such special metaphysical theses as premises, does this discredit his demonstrations of general metaphysical principles? 3. Why does Aristotle think (if, indeed, he does think) that an elenctic defense of PNC need only consider instances of PNC involving essential attributes?

I put these questions forward for further discussion, for I believe that they will have to be dealt with in any adequate treatment of Code’s interpretation. I have no answers to offer at present, but only suggestions and warnings. Re 1.: The presence of the language of essentialism need not automatically signal an appeal to the metaphysical baggage of Aristotle’s special theory. Aristotle characterizes the elenctic demonstrations he is about to give as attempts to derive the consequences of the fact that one who disputes PNC must at least say something (1006a11-13). But even one who says something without actually asserting or denying anything (and hence without betraying a commitment to PNC) must at least utter a meaningful term (όνομα, 1006a30). But what the term means is being this or not being this. So the very utterance of a meaningful term carries with it a commitment to something’s being (as opposed to not being) so-and-so. The expression to ἀνθρώποι ἐιναί is introduced (1006a33) for whatever it is that the term ‘man’ means. More informatively we might say that ‘man’ means two-footed animal (ζώον διπόν, 1006a32). But then, since two-footed animal = being a man, one who utters ‘man,’ meaning by it two-footed animal, is committed to such a thing as being a man. Nothing special need be assumed about the status of to ἀνθρώποι εἰναί; it is just whatever the term ‘man’ signifies.

Unfortunately, this attempt to absolve Aristotle of the charge of appealing to his own essentialism in behalf of more general metaphysical principles cannot go very far. For one thing, it leaves Aristotle with an
argument that appears to equivocate between the 'is' of assertion and the 'is' of predication. For another, even if it works with the arguments at 100a31-b34 (Kirwan's 2.[a]), it stands little chance of success with the argument of 1007a20-b18 (Kirwan's 2.[c]) where it is argued that denying PNC involves doing away with essential predication, which in turn leads to an infinite regress. Here the appeal to special theses about substances and essential predication seems inescapable.

Re 2. and 3.: Aristotle appeals to the principle that *anthrōpos semainei hen* ('man' means one thing) in his elenctic demonstration of the instance of PNC involving the substance-predicate 'man.' Is this generalizable to other types of predicate? Could we substitute *leukon* for *anthrōpos* and prove an instance of PNC involving the quality-predicate 'pale'? If so, what are we to make of the exclusive appeal to substance-predicates and to such entities as *to anthrōpōi einai*? No doubt one may respond that every predicate is, on Aristotle's account, essential to some subject (*pale* is accidental to Callias but essential to the *pale thing* that 'coincides' with Callias). But an adequate response must go further than this, for PNC should establish not just the impossibility of the pale thing's being both pale and not pale, but the impossibility of Callias's being both pale and not pale. Until we can answer such questions as these, we will not have completely understood Aristotle's intentions in his 'defense' of this most fundamental of all principles of human reason.

Received August, 1984