

Chem 452 – Math Tutorial

This tutorial is intended as a quick way for you to brush up on your math skills. Questions of these types will appear in lectures and exams, so treat this seriously.

This tutorial consists of two “self-tests”. Questions appear on one sheet and the answers are on the next sheet. Please try all of the problems in the first self-test before looking at the answers. If you get everything right on the first self-test, then there is no need to do the second one. Most students improve by the second time through.

Evaluate and/or simplify the expressions without a calculator. Do not convert fractions to decimals. Complete this page entirely before turning to the next page, which contains the answers.

QUESTION SET A

1A) $(a^2bc^{-1})(ab^{-3}) = ?$

2A) $(a^{2/3})^{4/3} = ?$

3A) $\log_{10}100 = ?$

4A) $\log 1 = ?$

5A) simplify $e^{2\ln 3}$

6A) express as one term: $\ln 9 - \ln 4 = ?$

7A) use the quadratic equation to solve $2x^2 - x - 3 = 0$

8A) For values of x which are close to 1, $\ln(x) \approx x - 1$. This problem uses a Taylor expansion to show that this is true. Use a Taylor Series to approximate $\ln(1.05) = ?$ to first order.

9A) $f(x) = (x^{4/3} - 2x^{-2})^5$ $\frac{df}{dx} = ?$ (no need to multiply out)

10A) $f(x) = (1 - x^4)/x$ $\frac{df}{dx} = ?$ (no need to multiply out)

11A) $\prod_{x=3}^5 \left(\frac{1}{x-1} \right) = ?$

12A) find the maximum and minimum of the equation $f(x) = x^3 - 27x$

13A) $\int_{x=1}^3 2x^2 dx = ?$

14A) $\int_{v_i}^{v_f} \frac{1}{3v} dv = ?$

15A) $\frac{\partial}{\partial x} \left(2x^4y^6 + \frac{3}{x^2} \right) = ?$

16A) A courtyard contains 10 identical paving stones and 3 blind mice that scurry around at random. What is the probability that all three mice are found on stone #3 at some instant?

17A) For objects with a radius of 2 meters, what are the a) circle's perimeter, b) circle's area, c) sphere's surface area, and d) sphere's volume? (express as multiples of π . Include units!)

Grade questions 1A-17A. Make sure that you understand the answers before turning to the next page.

<u>ANSWER SET A</u>	<u>REASONS</u>
1A) $a^3 b^{-2} c^{-1}$	$a^x a^y = a^{x+y}$
2A) $a^{8/9}$	$(a^x)^y = a^{xy}$
3A) 2	$\log_a(a^b) = b$
4A) 0	$\log 1 = \log_{10} 10^0 = 0$
5A) $(e^{\ln 3})^2 = (3)^2 = 9$	$e^{\ln x} = x$
6A) $\ln(9/4) = \ln(3/2)^2 = 2\ln(3/2)$	$\ln a - \ln b = \ln a + \ln(b)^{-1} = \ln(a \cdot b^{-1}) = \ln(a/b)$, and $\log a^n = n \log a$
7A) $x = -1, \frac{3}{2}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
8A) $\ln(x) = \ln(a) + (x-a)(1/a)$ $\ln(1.05) = 0 + (1.05 - 1)(1/1) = 0.05$	Taylor Series: $f(x) = f(a) + (x-a)f'(a) + (1/2!)(x-a)^2 f''(a) \dots$ To first order, for $x \approx 1$, set $a=1$ so that $\ln x \approx (x - 1)$
9A) $5(x^{4/3} - 2x^{-2})^4 (4/3 x^{1/3} + 4x^{-3})$	for $f(x) = cx^n$, $df/dx = ncx^{n-1}$, where c is a constant
10A) $-4x^3 (1/x) + (1 - x^4)(-1/x^2)$	$d/dx(g(x) \cdot h(x)) = (d/dx g(x))(h(x)) + (g(x))(d/dx h(x))$ or use the quotient rule instead
11A) $1/2 \cdot 1/3 \cdot 1/4$	$\prod_{i=1}^n (x_i) = x_1 \cdot x_2 \cdot x_3 \dots \cdot x_n$
12A) $x = 3$ is min, $x = -3$ is max	min occurs at $df/dx = 0$ and $d^2f/dx^2 > 0$ max occurs at $df/dx = 0$ and $d^2f/dx^2 < 0$
13A) $\frac{2}{3} x^3 \Big _{x=1}^3 = \frac{2}{3} (27 - 1) = \frac{52}{3}$	$\int ax^n dx = \left(\frac{1}{n+1}\right) ax^{n+1}$ for $n \neq -1$
14A) $\frac{1}{3} \ln v \Big _{V_i}^{V_f} = \frac{1}{3} [\ln V_f - \ln V_i] = \frac{1}{3} \ln \left(\frac{V_f}{V_i}\right)$	$\int \frac{a}{x} dx = a \ln x$
15A) $8x^3 y^6 - 6/x^3$	Partial derivatives treat all other variables as constants.
16A) $1/1000$	Total probability = (prob. of mouse 1) x (mouse 2) x (mouse 3) = $1/10 \cdot 1/10 \cdot 1/10$.
17A) 4π meters, 4π (meters) ² 16π (meters) ² , $32/3 \pi$ (meters) ³	perimeter of circle = $2\pi r$, area of circle = πr^2 (hint: perimeter = $\frac{\partial}{\partial r}$ (area)) surface area of sphere = $4\pi r^2$, volume of sphere = $4/3 \pi r^3$ (hint: surface area = $\frac{\partial}{\partial r}$ (volume))

If you feel comfortable with questions 1A-17A and answered them all correctly on the first try, you may skip questions 1B-17B. If you want more practice or missed any of the previous questions, complete this page of questions. Complete this page entirely before turning to the next page, which contains the answers.

QUESTION SET B

1B) $2^3 \cdot 4 = 2^?$

2B) $(a^{3/5})^3 = ?$

3B) $\ln e^3 = ?$

4B) $\log_3 81 = ?$

5B) simplify $2e^{3\ln 2}$

6B) express as one term: $\ln 3 + \ln 4 = ?$

7B) use the quadratic equation to solve $x^2 + x + 2 = 0$. $x = ?$

8B) For values of x which are close to 1, $\ln(x) \approx x - 1$. This problem uses a Taylor expansion to show that this is true. Use a Taylor Series to approximate $\ln(17/16) = ?$ to first order.

9B) $f(x) = (5x^3 + x^{2/3})^4$ $df/dx = ?$ (no need to multiply out)

10B) $f(x) = (1+x)/(x^2-2)$ $df/dx = ?$ (no need to multiply out)

11B) $\prod_{x=1}^3 x^2 = ?$

12B) Find the maximum and minimum of the equation $f(x) = x^3 - 48x$

13B) $\int_{x=1}^2 2x^3 dx = ?$

14B) $\int_{p_i}^{p_f} \frac{2}{p} dp = ?$

15B) $\partial/\partial t (8st^{-2} - 3s^{-3}t^4)$

16B) In 20 pages of notes I randomly make 4 typos. What is the probability that all 4 typos will be on the first page?

17A) For objects with a radius of 3 meters, what are the a) circle's perimeter, b) circle's area, c) sphere's surface area, and d) sphere's volume? (express as multiples of π . Include units!)

ANSWER SET B

If you need more practice with differentiation and integrals, check out the worked problems at:
<http://www.math.ucdavis.edu/~kouba/ProblemsList.html>

1B) 2^5

2B) $a^{9/5}$

3B) 3

4B) $\log_3(3^4) = 4\log_3 3 = 4$

5B) $2(e^{\ln 2})^3 = 2 \cdot 2^3 = 2^4 = 16$

6B) $\ln 12$

7B) $x = \frac{-1 \pm \sqrt{-7}}{2}$

8B) $\ln(17/16) = \ln(1) + (17/16 - 1)(1/1) = 17/16 - 1 = 1/16$

9B) $4(5x^3 + x^{2/3})^3 (15x^2 + 2/3x^{-1/3})$

10B) $\frac{1}{x^2 - 2} - \frac{2x(1+x)}{(x^2 - 2)^2}$

11B) $1 \cdot 4 \cdot 9 = 36$

12B) $x = 4$ is min, $x = -4$ is max

13B) $\frac{2}{4}x^4 \Big|_{x=1}^2 = \frac{1}{2}(2^4 - 1^4) = \frac{1}{2}(16 - 1) = \frac{15}{2}$

14B) $2 \ln p \Big|_{P_i}^{P_f} = 2[\ln P_f - \ln P_i] = 2 \ln\left(\frac{P_f}{P_i}\right)$

15B) $-16st^{-3} - 12s^{-3}t^3$

16B) $(1/20)^4 = 1/160000$

17A) 6π meters, 9π (meters)², 36π (meters)², 36π (meters)³