Proof that
$$N_sP_s - X - (P_s - X)t_p + (P_s - P_e)t_{cg}(1 - N_s) > 0$$
 (8.16)

Recall from the text that if (8.16) > 0 then, if you expect the stock price to appreciate, a better strategy to exercising the NQO and holding the stock, is to invest the borrowed cash that would have been used to pay the exercise price and taxes due at exercise and buy the stock directly while continuing to hold the NQO. Also recall we are addressing the question: Should one exercise NQOs early and hold the stock – so as to convert future stock price appreciation that would be taxed as ordinary income if the NQO is held, to capital gains if the NQO is exercised today and the stock held? The answer is no if (8.16) > 0.

Proof:¹

Recall, N_s, hereafter simply N, = number of shares that option-holder can purchase per option = (exercise price + tax saved)/current stock price. Thus $N = (X + (P_e - X)t_p)/P_e$ (1)

To prove (8.16) > 0, we initially assume (we will prove this later) $1 > N > t_p > 0$

Equation (8.16) can be rewritten as

 $P_{s}(N - t_{p}) - X(1 - t_{p}) + (P_{s} - P_{e})t_{cg}(1 - N)$

Now from proof below that $N > t_p$, we can substitute $X/P_e(1 - t_p)$ for $(N - t_p)$ (see ** below) in the first term above

$$P_{s}(X/P_{e}(1-t_{p})) - X(1-t_{p}) + (P_{s}-P_{e})t_{cg}(1-N)$$

$$= P_{s}(X/P_{e}(1-t_{p}) - X P_{e}/P_{e} (1-t_{p}) + (P_{s}-P_{e})t_{cg}(1-N)$$

$$= (P_{s}-P_{e})(X/P_{e})(1-t_{p}) + (P_{s}-P_{e})t_{cg}(1-N)$$

$$> 0 \qquad > 0 \qquad (2)$$

$$Thus (2) > 0$$

Thus the alternative strategy of holding the NQOs and buying additional stock is always preferred to the strategy of exercising the NQOs and holding the stock. (It is important to note that the comparison is only between these two alternatives – it might be better to exercise the NQO and immediately sell the stock so as to diversify the employee's wealth.)

In showing the above we assumed $1 > N > t_p > 0$

Proof that 1 > N

¹ We thank David Guenther, University of Colorado, for this proof.

From (1) $N = (X + (P_e - X)t_p)/P_e$ $= \dot{X}/P_e + (P_e - \dot{X})/P_e)t_p$ $= X/P_e + (1 - X/P_e)t_p$ And note $X/P_{e} + (1 - X/P_{e}) = 1$, and because $t_{p} < 1$ N < 1. Proof that $N > t_p$ Again from (1) $N = (X + (P_e - X)t_p)/P_e$ $= X/P_e + (1 - X/P_e)t_p$ or $N - t_p = X/P_e + (1 - X/P_e)t_p - t_p$ $= X/P_e + (1 - X/P_e - 1)t_p$ $= X/P_e + (-X/P_e)t_p$ $= X/P_e(1 - t_p) > 0.$

Thus $N > t_p$

(**)