Uncertainty in Human Capital Investment and Earnings Dynamics

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Abstract

There is a literature that examines the statistical properties of earnings dynamics by testing heterogeneous growth against random walk. This test is of great consequence because rejection of heterogeneous growth has often been interpreted as rejection of a key role for heterogeneity in human capital investment over the life-cycle. This paper shows that optimal life-cycle investment behavior implies the presence of a permanent component in earnings levels as well as the individual heterogeneity in earnings slopes. Permanent shocks are induced by the response of individuals to human capital investments due to transitory shocks to the rental rate of human capital. We incorporate uncertainty about future rental rates for human capital into an optimal life-cycle human capital investment model and obtain an earnings equation implied by the solution to the worker's optimal investment decision. Using the National Longitudinal Survey of Youth 1979 (NLSY79), we confirm that heterogeneity in earnings slopes, permanent errors, and transitory shocks all play a significant role in earnings dynamics. We also learn that a worker's earnings are more affected by shifts in human capital accumulation path than by individual difference in the ability to produce human capital.

Keywords: Earnings Equation, Human Capital Theory, Random Growth, Random Walk, Rental Rate of Human Capital

JEL Classification Number: J22, J24, J31

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1 Introduction

Human capital theory, as a model of life-cycle investment, provides predictions about the dynamics of earnings over a lifetime. The theory, accompanied by heterogeneity in a worker's ability, generates human capital accumulation paths that are specific to individual workers. In consequence, residuals for an earnings equation will include a random growth component, which is a growth component with random coefficients, and the earnings distribution will span out with experience even after observable attributes of the workers are controlled for. This dispersion of earnings distribution, however, is not uniquely implied by random growth models. An alternative hypothesis where the residuals have a random walk process generates a similar implication about the dispersion of earnings distribution. This exploration is motivated by the statistical consideration that a unit root process provides a better fit to the covariance structure of earnings residuals than the heterogeneous growth hypothesis.

There has been a continuous attempt in labor economics to identify and test the statistical properties of the residuals of an earnings equation. Lillard and Weiss (1979), Baker (1997), Lillard and Reville (1999), Haider (2001), and Guvenen (2007) support heterogeneous growth models. MaCurdy (1982), Abowd and Card (1989), and Meghir and Pistaferri (2004) favor random walk models. In this literature, tests of model specification usually take a form of testing random growth against random walk. This test is of great consequence because rejection of heterogeneous growth has often been interpreted as rejection of a key role for heterogeneity in human capital investment over the life-cycle. Although most of these studies focus on rejecting one of the two competing hypotheses, a more fundamental question that has to be answered prior to testing the hypotheses would be what the sources of randomness in earnings residuals are. Especially the theoretical foundation for permanent errors in the earnings residual are less clear.

This paper shows that optimal human capital investment theory induces both random growth and random walk errors. Heterogeneous growth stems from individual heterogeneity in the ability to produce human capital. Permanent errors are induced by the response of individuals in human capital investments to transitory shocks to the rental rate of human capital. When the future return to human capital is uncertain, transitory shocks to the rental rate of human capital affect optimal working hours, which is closely related to the optimal amount of investment. As human capital theory predicts, any shift in investment will have a permanent impact on the stock of human capital. We derive the implications of the theory for the earnings process by approximating the exact solution to the worker's optimization problem. It turns out that the implied model of the earnings residual nests the random growth and the random walk models.

We quantify the contribution of heterogeneity in wage intercepts, experience slopes, permanent errors, and transitory earnings shocks to the variance of wages over the life-cycle. Using the National Longitudinal Survey of Youth 1979 (NLSY79), we conclude that heterogeneity in earnings slopes, persistent errors, and transitory errors all play a significant role in the dispersion of earnings distribution. We find that the share explained by individual heterogeneity in the variance of earnings residuals drops as workers get more experienced. Variance of persistent errors accumulates with experience and explains much of the total variation. We also find that a worker's earnings are more affected by shifts in human capital accumulation path than by individual difference in the ability to produce human capital.

This paper has the following structure. In section 2, we review the previous literature on earnings residuals and provide a sketch of how it can be improved. Section 3 develops an optimal human capital investment decision rule when the future return to human capital is uncertain. Workers choose the optimal level of input to maximize their expected discounted disposable lifetime income. We show that the implied earnings dynamics includes both heterogeneous growth and persistent errors. In section 4, we estimate the covariance structure of earnings residuals and decompose the total variation into the effects of heterogeneous growth, random walk, and transitory errors components. Section 5 concludes.

2 Previous Literature on Earnings Residuals

The relationship between education, experience, and earnings is modeled by Mincer (1974). The earnings equation specification depends on the form of the life-cycle investment function. If the investment ratio is assumed to decline linearly, the gross log-earnings function becomes parabolic and the net log-earnings function can be specified by a polynomial approximation. In consequence, the Mincer's earnings equation is given by

$$\log w_i(s,t) = \gamma_0 + \gamma_s s + \gamma_1 t + \gamma_2 t^2 + \omega_{it}, \tag{1}$$

where $w_i(s,t)$ is the wage or earnings of a worker i with schooling s and work experience t. By construction, the residual term, ω_{it}, includes pure errors and unobserved individual attributes, where the latter may be interacted with experience. The existing literature provides two rival views on the nature of the earnings dynamics: random growth and random walk models. Both models, however, provide observationally equivalent prediction about the experience profile of the dispersion of earnings.

The random growth model developed by Lillard and Weiss (1979) is specified by

$$\omega_{it} = \widetilde{\gamma}_{0i} + \widetilde{\gamma}_{1i} t + u_{it},$$

where $\widetilde{\gamma}_{0i}$ and $\widetilde{\gamma}_{1i}$ are mean zero individual-specific random variables and u_{it} is a stationary process. The model reflects individual variations in the level and the growth of earnings as well as the transitory but serially correlated errors. The theoretical motivation of the random growth specification is the life-cycle human capital investment model. The increase in the variance of earnings with experience is mainly attributable to individual-specific earnings growth paths. Every worker has a unique earnings profile with an intercept and a slope that may be systematically related. Lillard and Weiss use panel earnings data on American scientists to find spreading earnings distributions over time among observationally similar workers.

The random walk model originates from MaCurdy (1982). He takes the 10-year earnings panel sample of prime-age, white, married males, from the Panel Study of Income Dynamics (PSID) and tests whether workers differ systematically in their income growth rates. He rejects the random growth hypothesis and concludes that the error structure has the form

$$\omega_{it} = \widetilde{\gamma}_{0i} + \sum_{s=1}^{t} \xi_{is} + u_{it},$$

where $\widetilde{\gamma}_{0i}$ is an individual-specific random variable, ξ_{is} is an independent zero mean innovation, and u_{it} is a stationary process. This model implies that every worker is ex ante identical up to the constant and that the increase in the variance of earnings distribution with experience comes from random walk errors. The model does not explicitly provide theoretical background why the error term is an integrated process, but subsequent studies, such as Abowd and Card (1989) and Topel and Ward (1992), confirm that the random walk specification for earnings provides a better fit to the covariance structure of earnings residuals than the random growth model.

Baker (1997) constructs a model that nests both the random growth and the random walk models. He assumes that

$$\omega_{it} = \widetilde{\gamma}_{0i} + \widetilde{\gamma}_{1i} t + \sum_{s=1}^{t} \rho^s \xi_{is} + u_{it},$$

where $\widetilde{\gamma}_{0i}$ and $\widetilde{\gamma}_{1i}$ are mean zero individual-specific random variables, ξ_{is} and u_{it} are stationary processes, and $\rho \in (-1, 1]$. He uses a 20-year panel of earnings data for adult men from the PSID. He shows that $Var(\widetilde{\gamma}_{0i})$, $Var(\widetilde{\gamma}_{1i})$, and $Cov(\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i})$ are significantly different from zero while the absolute value of ρ is strictly less than unity. He demonstrates that ρ remains different from unity unless both $\widetilde{\gamma}_{0i}$ and $\widetilde{\gamma}_{1i}$ are dropped.[2] Monte Carlo results, however, show that the power of this test of the random growth versus the random walk is very small.

While many papers have focused on finding statistical evidence of heterogeneity under a standard earnings equation, there has been an effort to inform the functional form of an earnings equation by revisiting human capital theory. Lillard and Reville (1999) develop a new testable model based on the prediction of the optimal life-cycle human capital investment theory. They argue that individuals with high earnings growth paths must also have greater curvature because human capital investment becomes zero at the end of the working life. Therefore a random coefficient on the linear term in experience alone is not sufficient. Instead, they propose a modified random growth model:

$$\omega_{it} = \widetilde{\gamma}_{0i} + \widetilde{\gamma}_{1i}\left(t + \gamma_2 t^2\right) + u_{it},$$

where $\widetilde{\gamma}_{0i}$ and $\widetilde{\gamma}_{1i}$ are mean zero individual-specific random variables, u_{it} is a stationary process, and γ_2 is a constant. They call this specification the random profile model because it assumes a random coefficient on the entire age profile. They use a 25-year sample of the PSID to find significant $Var(\widetilde{\gamma}_{0i})$, $Var(\widetilde{\gamma}_{1i})$, and $Cov(\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i})$ estimates.

This paper goes beyond the approach taken by Lillard and Reville. We investigate the optimal investment choice problem by adding uncertainty in the rental rate of human capital. Uncertainty in rental rates of human capital may have implications for human capital investment and its subsequent future stock. A transitory shift in human capital investment will have a permanent impact on human capital accumulation. In this way, we prove that permanent errors in earnings residual may also result from human capital investment. Both random growth and random walk errors can be implied by the optimal lifetime human capital investment hypothesis, which is different from existing literature that supports only one of the two rival hypotheses.

[2] To be consistent with the prediction from the human capital hypothesis, a more precise test is dropping $\widetilde{\gamma}_{1i}$ only. This is not carried out in his paper.

3 Optimal Human Capital Investment

3.1 Human Capital Stock, Input, and Earnings

Consider a worker i with t years of work experience in calendar year c. Workers enter the labor market immediately after completion of schooling at $t = 0$ and retire at $t = T_i$. The worker's earnings potential, Y_{itc}, is determined by the product of a stochastic rental rate per unit of human capital, R_c, and the total stock of human capital, H_{itc}:

$$Y_{itc} = R_c H_{itc}.$$

The dynamics of the stock, H_{itc}, are determined by the amount of new human capital produced, q_{itc}, less the depreciated stock, $\delta_i H_{itc}$:

$$\frac{d}{dt} H_{it,c(t)} = q_{itc} - \delta_i H_{itc}, \tag{2}$$

where δ_i is a depreciation rate. In studying dynamics, we use the notation of $c(t)$ to indicate that time and experience move together. The new human capital, q_{itc}, is produced by

$$q_{itc} = f_i(I_{itc}),$$

where $f_i(\cdot)$ is a human capital production function.[3] Its nonnegative argument, I_{itc}, cannot exceed the stock, H_{itc}. Workers may possess different levels of efficiency in production, so the production function is indexed by an i. The human capital production function, $f_i(\cdot)$, is a twice differentiable nonstochastic function, with $f_i' > 0$, $f_i'' < 0$, and $f_i(0) = 0$. If a worker decides to devote I_{itc} units to produce new human capital, the stock, H_{itc}, can be obtained using (2).

The stochastic rental rate, R_c, is determined in a market where the services of human capital are traded. The rental rate is stochastic to incorporate unexpected permanent and temporary shocks in the future rental rate. We specify R_c as a product of the following independent processes:

$$R_c = R_c^{mg}(1 + \varepsilon_c),$$

where R_c^{mg} is a martingale process and ε_c is an expectation zero stationary process with $\Pr(\|\varepsilon_c\| > 1) = 0$.[4]

[3] This is an assumption of non-self-productive human capital production function. Details are in the appendix.

[4] Due to the martingale property of R_c and stationarity of ε_c, the expectation of future rental rate, $R_{c(s)}$, for a worker i in calendar year c is given by $E_c\left[R_{c(s)}\right] = E_c\left[R_{c(s)}^{mg}\left(1 + \varepsilon_{c(s)}\right)\right] = R_c^{mg} = R_c/(1 + \varepsilon_c)$. A typical example of R_c^{mg} is a geometric

Note that we do not presume individual heterogeneity in this specification.

We treat the cost of the particular investment in human capital as the opportunity cost of working following Becker (1964). In this model, a worker chooses how much input of human capital to sacrifice in order to produce human capital in the next period. In the data, we observe human capital indirectly in the form of disposable earnings, W_{itc}:

$$W_{itc} = R_c \left(H_{itc} - I_{itc} \right).$$

If I_{itc} is used to produce new human capital, such as through additional training, a worker trades $(H_{itc} - I_{itc})$ in the market and earns W_{itc}.

3.2 The Optimal Input Decision and the Human Capital Stock

A worker allocates time between the production of goods and the production of human capital. We assume that workers are risk-neutral and maximize the expected discounted disposable lifetime earnings. A worker i's optimal input at experience t in calendar year c is the solution to

$$\begin{aligned} I_{itc}^{*} &= \arg\max E_c \left[\int_t^{T_i} e^{-r_i(s-t)} R_{c(s)} \left(H_{is,c(s)} - I_{is,c(s)} \right) ds \right], \qquad (3) \\ \text{s.t. } \frac{d}{ds} H_{is,c(s)} &= f_i \left(I_{is,c(s)} \right) - \delta_i H_{is,c(s)}, \qquad t \leq s \leq T_i, \\ H_{itc} &: \textit{given}, \end{aligned}$$

where r_i is a discount rate. We assume that a fixed amount of time is allocated to activities that produce earnings (or the stock of human capital), which is the same as in Ben-Porath (1967).[5]

The maximization problem in (3) can be obtained by equalizing the difference between the marginal cost of human capital production and the marginal gain of a unit of human capital. In each period, the cost of producing q_{itc} units of human capital is given by $R_c f_i^{-1}(q_{itc})$. Thus, the marginal cost is given by

$$MC_{itc} = R_c \frac{d}{dq} f_i^{-1}(q_{itc}) = \frac{R_c}{f'(I_{itc})}.$$

Brownian motion, $R_c^{mg} = \exp\left[-\frac{1}{2}\sigma^2 c + \sigma B_c \right]$, where σ is a positive constant and B_c is a standard Brownian motion.

[5] Referring to Ben-Porath (1967), we assume that $I_{it}^{*} < H_{it}$ for all t. The choice $I_{it}^{*} = H_{it}$ corresponds to the period when individual i is in formal education and utilizes the entire earning capacity as an input. Immediately after leaving school the individual devotes less of the earnings capacity to produce human capital. Hence, we assume that at time 0 when an individual finishes schooling, the stock of human capital H_{i0} is large enough that it is strictly greater than I_{i0}^{*}. Completing education implicitly means that full-time input is not optimal any more.

The demand price of human capital at time t is given by

$$P_{itc} = E_c\left[\int_t^{T_i} R_{c(s)} e^{-(r_i+\delta_i)s} ds\right] = \frac{E_c\left[R_{c(s)}\right]}{r_i+\delta_i}\left[1-e^{-(r_i+\delta_i)(T_i-t)}\right].$$

Equate $MC_{itc} = P_{itc}$ and get

$$f_i'\left(I_{it}^*\right) = \frac{R_c}{E_c\left[R_{c(s)}\right]}\frac{r_i+\delta_i}{1-e^{-(r_i+\delta_i)(T_i-t)}}, \qquad 0 \le t < T_i. \tag{4}$$

As f_i' is strictly decreasing and $E_c\left[R_{c(s)}\right]$ is a fixed number at time c, there exists a unique I_{itc}^* for any experience level t. If the rental rate is fixed, the optimal input is the solution to $f_i'\left(I_{it}^*\right) = \left(r_i+\delta_i\right)/1 - e^{-(r_i+\delta_i)(T_i-t)}$. At $t = T_i$, the level of optimal investment becomes 0.

There are a couple of important properties about the optimal input implied by (4). First, suppose that earnings shocks are always permanent. As the expectation of a martingale process at time c equals the spot value at time c, the optimal input is not affected by uncertainty. Hence the corresponding optimal stock of human capital is not affected by uncertainty. Second, transitory shocks affect the optimal input level. Suppose that there are positive transitory shocks. Then the spot rate is higher than expected future rate. Workers invest less and work more because the return to working goes up. A transitory shift in optimal input, however, leads to a permanent shift in the corresponding stock of human capital. Positive transitory shocks permanently shift the human capital accumulation path downward.

Once we calculate the optimal input, the optimal stock is the solution to the functional differential equation (2):

$$\log H_{itc}^* = \log H_{i,0} + \int_0^t \frac{q_{is,c(s)}^*}{H_{is,c(s)}^*} ds - \delta_i t,$$

where $H_{i,0}$ denotes the initial stock upon entering the labor market and the calendar year c increases as experience t increases.[6] Thus, the dynamics of disposable earnings can expressed by

$$\begin{aligned} \log R_c\left(H_{it,c(t)}^* - I_{it,c(t)}^*\right) &= \log R_{c(t)} + \log H_{it,c(t)}^* + \log\left(1 - \frac{I_{it,c(t)}^*}{H_{it,c(t)}^*}\right) \\ &\approx \log R_{c(t)}^{mg} + \varepsilon_{c(t)} + \log H_{i,0} - \delta_i t + \int_0^t \frac{q_{is,c(s)}^*}{H_{is,c(s)}^*} ds - \frac{I_{it,c(t)}^*}{H_{it,c(t)}^*}. \end{aligned} \tag{5}$$

Using (5), we can specify a regression model. The regression model includes an individual-specific

[6] Note that (2) can be rewritten by $H_{itc}^* = H_{i,0}\lim_{n\to\infty}\prod_{m=1}^n\left[1+\frac{1}{n}\frac{q_{i,tn/m,c(tn/m)}^*}{H_{i,tn/m,c(tn/m)}^*} - \frac{1}{n}\delta_i\right]$.

intercept, $\log H_{i,0}$, and an individual-specific growth term, $-\delta_i t$. There is an error process, $\log R_c + \varepsilon_c$, which is common to all workers. Hence it can be controlled by including time dummy variables. In order to explore the statistical properties of the last two terms, assume that the human capital production function is given by $f_i(I_t) = a_i I_t^{b_i}$, as in Haley (1973, 1976). For simplicity, we suppress the subscript i. Then, $\int_0^t \left[q^*_{is,c(s)} / H^*_{is,c(s)} \right] ds$, can be decomposed into two terms:

$$
\begin{aligned}
\int_0^t \frac{q^*_{s,c(s)}}{H^*_{s,c(s)}} ds &\approx \left(\frac{a^{1/b} b}{r+\delta} \right)^{b/(1-b)} \int_0^t \frac{1}{H^*_s} \left[1 - e^{-(r+\delta)(T-s)} \right]^{b/(1-b)} \left[1 - \frac{b}{1-b} \varepsilon_{c(s)} \right] ds \\
&= \left(\frac{a^{1/b} b}{r+\delta} \right)^{b/(1-b)} \int_0^t \frac{1}{H^*_s} \left[1 - e^{-(r+\delta)(T-s)} \right]^{b/(1-b)} ds \qquad (6) \\
&\quad - \frac{b}{1-b} \left(\frac{a^{1/b} b}{r+\delta} \right)^{b/(1-b)} \int_0^t \frac{1}{H^*_s} \left[1 - e^{-(r+\delta)(T-s)} \right]^{b/(1-b)} \varepsilon_{c(s)} ds, \qquad (7)
\end{aligned}
$$

where (6) becomes an individual-specific earnings growth term and (7) represents a permanent error term with individual-specific variance. In general, (7) is likely to be an integrated process. Finally, $I^*_{it,c(t)} / H^*_{it,c(t)}$, can be decomposed into two terms:

$$
\begin{aligned}
\frac{I^*_{t,c(t)}}{H^*_{t,c(t)}} &\approx \frac{1}{H^*_t} \left(\frac{ab}{r+\delta} \right)^{1/(1-b)} \left[1 - e^{-(r+\delta)(T-t)} \right]^{1/(1-b)} \left[1 - \frac{1}{1-b} \varepsilon_{c(t)} \right] \\
&= \frac{1}{H^*_t} \left(\frac{ab}{r+\delta} \right)^{1/(1-b)} \left[1 - e^{-(r+\delta)(T-t)} \right]^{1/(1-b)} \qquad (8) \\
&\quad - \frac{1}{1-b} \frac{1}{H^*_t} \left(\frac{ab}{r+\delta} \right)^{1/(1-b)} \left[1 - e^{-(r+\delta)(T-t)} \right]^{1/(1-b)} \varepsilon_{c(t)}, \qquad (9)
\end{aligned}
$$

where (8) becomes an individual-specific time varying intercept and (9) represents a transitory error term with individual-specific variance.

4 Estimation of Wage Residual Covariance Structure

4.1 Earnings Dynamics and the Human Capital Theory

The logarithm of disposable earnings in (5) can be approximated by

$$
\log R_c \left(H^*_{itc} - I^*_{itc} \right) \approx d_c + d\left(x_{it}, t \right) + \widetilde{d}_i(t) + \sum_{s=1}^{t} \xi_{is} + u_{it}, \qquad t = 1, 2, ..., T, \qquad (10)
$$

where d_c is a time fixed effect, x_{it} is the worker's observable attributes, $d(x_{it}, t)$ is a nonstochastic function of x_{it} and experience, $\widetilde{d}_i(t)$ is a nonstochastic function that reflects unobserved individual heterogeneity, $\sum_{s=1}^{t} \xi_{is}$ is a unit root process, and u_{it} is a stationary process. We assume that all workers retire at T. Worker's observable attributes include gender, race, education, AFQT score, as well as all these variables interacted with experience.

Let ω_{it} be the error term defined by

$$\omega_{it} = \widetilde{d}_i(t) + \sum_{s=1}^{t} \xi_{is} + u_{it}. \tag{11}$$

This specification nests the random growth and random walk models. We observe three different sources of uncertainty. The first component, $\widetilde{d}_i(t)$, reflects random growth; the second component, $\sum_{s=1}^{t} \xi_{is}$, reflects random walk. The last component, u_{it}, is a stationary process, which reflects transitory shocks and other errors. Depending on the estimates, we can test the existence of random growth and/or random walk. When we approximate $\widetilde{d}_i(t)$ by $\widetilde{\gamma}_{0i} + t\widetilde{\gamma}_{1i} + t^2\widetilde{\gamma}_{2i}$, the error covariance structure implied by (11) is given by

$$\begin{aligned} Cov\left[\omega_{it}, \omega_{is}\right] &= \sigma_0^2 + \sigma_1^2 ts + \sigma_2^2 t^2 s^2 \\ &\quad + \rho_{01}\sigma_0\sigma_1 (t+s) + \rho_{02}\sigma_0\sigma_2 \left(t^2 + s^2\right) + \rho_{12}\sigma_1\sigma_2 ts (t+s) \\ &\quad + \sigma^2 \min(t,s) + Cov\left[u_{it}, u_{is}\right], \end{aligned} \tag{12}$$

where

$$\begin{aligned} \left(\sigma_0^2, \sigma_1^2, \sigma_2^2\right) &= \left(Var\left[\widetilde{\gamma}_{0i}\right], Var\left[\widetilde{\gamma}_{1i}\right], Var\left[\widetilde{\gamma}_{2i}\right]\right), \\ \left(\rho_{01}, \rho_{02}, \rho_{12}\right) &= \left(corr\left[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i}\right], corr\left[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{2i}\right], corr\left[\widetilde{\gamma}_{1i}, \widetilde{\gamma}_{2i}\right]\right), \\ \sigma^2 &= Var\left[\xi_t\right]. \end{aligned}$$

We use the NLSY79 to estimate the model. It is a longitudinal data set with a nationally representative sample of 12,686 young men and women who were between the ages of 14 and 21 on January 1, 1979. These individuals were interviewed annually through 1994 and are currently interviewed on a biennial basis. We limit the analysis to workers who have worked at least a certain number of consecutive years. We need consecutive years of observations because human capital, by assumption, is accumulated while a person is employed and working.

We exclude extremely high or low wage observations in the analysis. The process of assigning outliers is as follows. First, when a reported real wage is less than one dollar, we regard this observation as an outlier. This corresponds to the bottom 2.89% of the valid working observations. In the second step, we assign the top 2.89% as outliers by applying an outlier-identifying scheme proposed by Altonji and Doraszelski (2005). We run a median regression of log wage on year dummy variables, a vector of a constant, gender, race, education, AFQT score, and the vector interacted with a quadratic polynomial of experience. Then we take the wage observations that correspond to the top 2.89% residuals and categorize them as outliers.[7] We collect 2,925 individuals with at least 10 observations of consecutive years of working between 1979 and 2002. After excluding the outliers, we end up with 38,097 valid observation points.[8] Summary statistics are reported in Table 1.

Table 1. Summary Statistics

	White		Black		Hispanic		Total
	Male	Female	Male	Female	Male	Female	
Hourly Rate of Pay	8.98	7.66	7.65	6.91	8.50	7.32	8.12
	(4.07)	(3.45)	(3.50)	(2.91)	(4.03)	(3.22)	(3.76)
Education	12.99	13.22	12.65	13.22	12.13	12.53	12.91
	(2.11)	(1.93)	(1.98)	(1.67)	(1.95)	(1.86)	(2.00)
Standardized AFQT	0.60	0.64	–0.43	–0.21	–0.17	0.01	0.28
	(0.82)	(0.73)	(0.93)	(0.75)	(0.95)	(0.77)	(0.92)
Experience	10.88	10.44	11.37	11.14	11.59	11.41	10.98
	(5.54)	(5.48)	(5.41)	(5.38)	(5.54)	(5.45)	(5.50)
Sample Size	14,028	8,313	4,921	4,515	3,814	2,506	38,097
Number of Individuals	1,045	650	386	353	293	198	2,925

Standard deviations are reported in parentheses.

[7] The unobserved heterogeneity is partly controlled by adding a measure of ability (AFQT score) as one of the regressors.

[8] In sum, 2,440 observations are dropped as bottom outliers and 2,437 observations as top outliers.

4.2 Geometry of the Wage Residual Covariance Structure

We take the OLS residuals from (10) as a proxy for the residual term, ω_{it}. The estimated covariance matrix is illustrated in Figure 1.[9] The height represents the covariance and the two horizontal axes are the years of experience. For presentation purposes, the covariance matrix is truncated at the experience level $t = 18$. All the covariance estimates are significantly different from zero implying that there is significant heterogeneity in initial earnings. The error covariance matrix is a combination of random growth, random walk, and other stationary errors as given in (11). These effects are highlighted when the 3-diminsional empirical error covariance matrix is sliced into 2-dimensional graphs. Figures 1(a)-(c) are these slices.

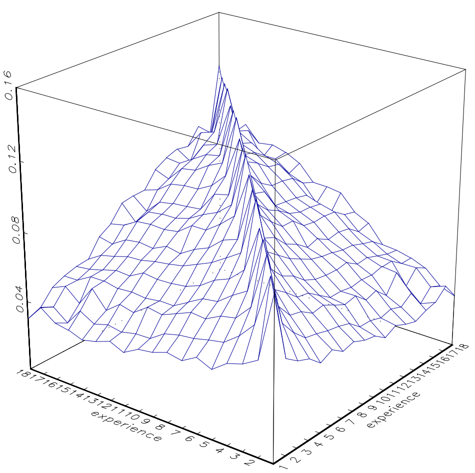

Figure 1. Earnings Residual Covariance Structure

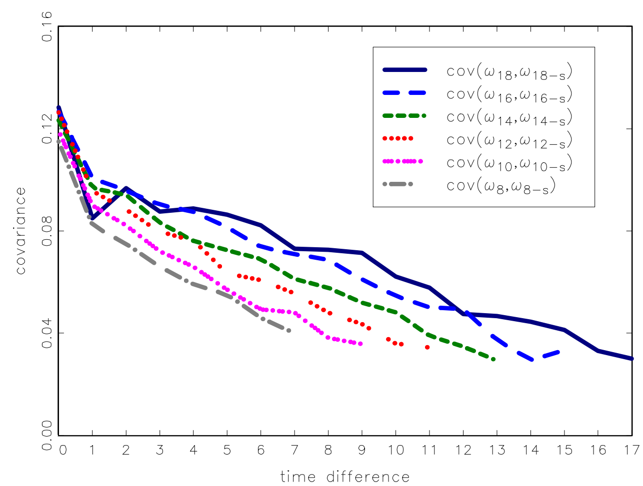

Figure 1(a)

[9] To obtain the covariance structure of an unbalanced panel, we adopt the method that is introduced in Farber and Gibbons (1996). Define a dummy variable $d_{i,ts}$ if $\widehat{\omega}_{it}$ and $\widehat{\omega}_{is}$ are both observed. Let $N_{ts} = \sum_{i=1}^{N} d_{i,ts}$ denote the number of observations for which both $\widehat{\omega}_{it}$ and $\widehat{\omega}_{is}$ are observed. A natural estimator of the covariance between ω_{it} and ω_{is} using the unbalanced data is $\widehat{\sigma}_{ts} = \frac{1}{N}\sum_{i=1}^{N} \frac{d_{i,ts}N}{N_{ts}} \widehat{\omega}_{it}\widehat{\omega}_{is}$. To obtain standard errors, we need to compute an estimator for the covariance of $\widehat{\sigma}_{ts}$ and $\widehat{\sigma}_{qr}$. A consistent estimator of this covariance is $V_{ts,qr} = \frac{1}{N}\sum_{i=1}^{N} \frac{d_{i,ts}N}{N_{ts}} \frac{d_{i,qr}N}{N_{qr}} \left(\widehat{\omega}_{it}\widehat{\omega}_{is} - \widehat{\sigma}_{ts}\right)\left(\widehat{\omega}_{iq}\widehat{\omega}_{ir} - \widehat{\sigma}_{qr}\right)$.

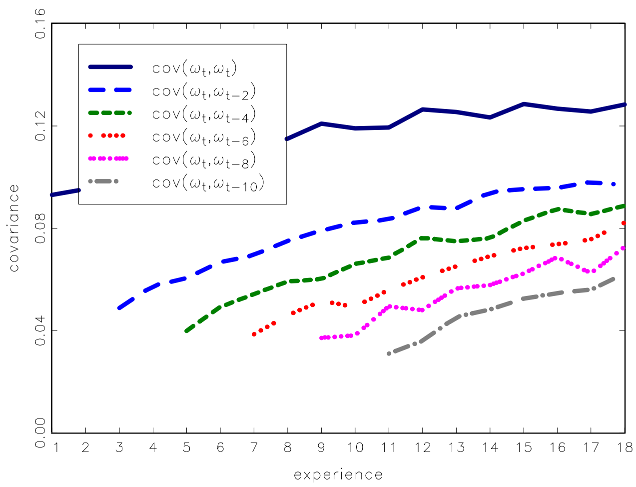

Figure 1(b)

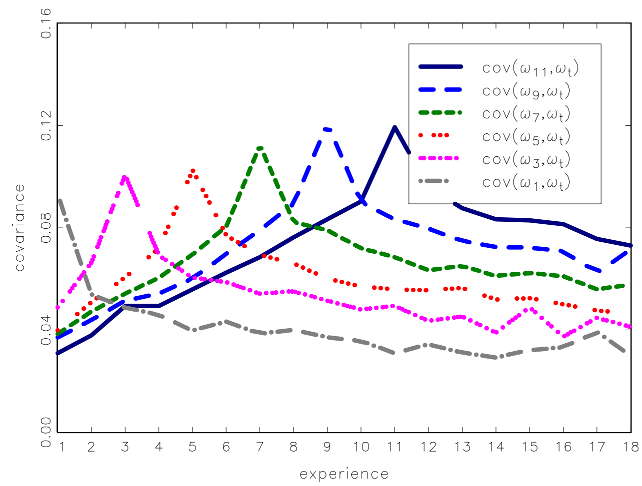

Figure 1(c)

Figure 1(a) corresponds to autocovariance functions in a stationary case. The solid line with $cov\,(\omega_{18}, \omega_{18-s})$ indicates the covariance of ω_{18} and ω_{18-s} from $s = 1$ to $s = 18$ and the broken line with $cov\,(\omega_{16}, \omega_{16-s})$ the covariance of ω_{16} and ω_{16-s} from $s = 1$ to $s = 16$. These lines are not autocovariance functions, because ω_{it} includes an integrated series. If ω_{it} is a pure unit root process, $cov\,(\omega_{t+k}, \omega_{t+k-s}) - cov\,(\omega_t, \omega_{t-s})$ should depend on k but not on s. However, this is not the case in Figure 1(a) as the lines are different from parallel and are not equi-spaced. The existence of a stationary process will only distort the parallel prediction around $s = 0$. Consequently, Figure 1(a) implies the existence processes other than unit root, such as random growth.

Figure 1(b) slices the covariance matrix by the 45° degree lines. In this slice, the difference of the corresponding two experience levels in horizontal axes are constants. Each line describes the time path of the covariances over time. For instance, the solid line of $cov\,(\omega_t, \omega_t)$ indicates the time path of variances from $t = 1$ to $t = 18$ and the broken line of $cov\,(\omega_t, \omega_{t-2})$ indicates the time path of covariances from $t = 3$ to $t = 18$. The large jump from $cov\,(\omega_t, \omega_{t-2})$ to $cov\,(\omega_t, \omega_t)$ implies the existence of transitory shocks. In addition, we expect that a random walk component makes the lines increasing with a constant slope. A random growth component makes the lines increasing within a higher order. In Figure 1(b), the lines are close to linear or concave, which implies that random walk effects seem to dominate random growth effects.

In Figure 1(c), each line represents the covariance with one element fixed and the other element running from 1 to 18. The solid line with $cov\,(\omega_{11}, \omega_t)$ indicates the covariance of ω_{11} and ω_t from $t = 1$

to $t = 18$ and the broken line with $cov(\omega_9, \omega_t)$ indicates the covariance of ω_9 and ω_t from $t = 1$ to $t = 18$. A general pattern of this diagram is that the lines rise, peak, and level out. The peak is definitely due to the stationary errors. The rise could be due to a random growth component, a random walk component, or both. The leveling out part suggests a random walk process: unit root errors are persistent.

In sum, the empirical covariance matrix provides evidence of the three sources of randomness in the earnings residual. Figure 1(a) hints at the existence of heterogeneous growth. Figures 1(b) and 1(c) indicate the existence of permanent and transitory errors. The following section quantifies the contribution of these three sources of randomness.

4.3 Covariance Estimates and Variance Decomposition

We estimate the parameters in (12) using minimum distance estimators: the equally weighted minimum distance (EWMD) estimator and the optimally weighted minimum distance (OWMD) estimator.[10]

[10] Let m_i represent the vector of the $T(T+1)/2$ unique elments in the cross-product matrix of residuals for worker i. In general, some elements in m_i are missing values. Therefore, define a vector, d_i, of indicators of whether each element of m_i is missing or not. Now, let m represent the vector of means of the elements of m_i. Thus, m is defined by

$$m = \left(\frac{\sum_{i=1}^N m_{i1}}{\sum_{i=1}^N d_{i1}}, \frac{\sum_{i=1}^N m_{i2}}{\sum_{i=1}^N d_{i2}}, ..., \frac{\sum_{i=1}^N m_{i,T(T+1)/2}}{\sum_{i=1}^N d_{i,T(T+1)/2}} \right)'.$$

Define

$$\Omega = \sum_{i=1}^N (m_i^* - m^*)(m_i^* - m^*)'.$$

where

$$m_i^* = \left(\frac{m_{i1}}{\sum_{i=1}^N d_{i1}}, \frac{m_{i2}}{\sum_{i=1}^N d_{i2}}, ..., \frac{m_{iS}}{\sum_{i=1}^N d_{iS}} \right)' \text{ and } m^* = \left(\frac{m_1}{\sum_{i=1}^N d_{i1}}, \frac{m_2}{\sum_{i=1}^N d_{i2}}, ..., \frac{m_S}{\sum_{i=1}^N d_{iS}} \right)'.$$

Then the EWMD estimator minimizes the quadratic form

$$\frac{1}{2}(m - g(\theta))'(m - g(\theta)),$$

and the OWMD estimator minimizes the quadratic form

$$\frac{1}{2}(m - g(\theta))'\Omega^{-1}(m - g(\theta)),$$

where $\theta = (\sigma_0^2, \sigma_1^2, \sigma_2^2, \rho_{01}, \rho_{02}, \rho_{12}, \sigma^2, \sigma_e^2, m_1, m_2)'$ and $g(\theta)$ is the model of the covariance structure in (12).

Table 2. Earnings Residual Covariance Estimates

	[1E] EWMD	[1O] OWMD	[2E] EWMD	[2O] OWMD	[3E] EWMD	[3O] OWMD
$Var[\widetilde{\gamma}_{0i}]$	434	381	557	578	482	510
	(27)	(21)	(44)	(33)	(60)	(48)
$Var[\widetilde{\gamma}_{1i}]$			0.469	0.683	0.184	0.343
			(0.516)	(0.354)	(5.02)	(3.58)
$Var[\widetilde{\gamma}_{2i}]$					4.8×10^{-4}	3.1×10^{-4}
					(1.1×10^{-2})	(7.8×10^{-3})
$Cov[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i}]$			−16.0	−19.6	−9.32	−13.1
			(11.6)	(7.1)	(138.5)	(76.1)
$Cov[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{2i}]$					−0.474	−0.395
					(5.2)	(4.7)
$Cov[\widetilde{\gamma}_{1i}, \widetilde{\gamma}_{2i}]$					-9.3×10^{-3}	-1.0×10^{-2}
					(4.5×10^{-1})	(3.4×10^{-1})
$Var[\xi_t]$	31.1	38.5	52.1	52.1	65.3	63.7
	(3.1)	(2.2)	(7.3)	(5.2)	(13.2)	(10.2)
$Var[e_{it}]$	353	314	328	295	314	286
	(9)	(7)	(10)	(8)	(14)	(11)
m_1	0.363	0.247	0.284	0.216	0.226	0.198
	(0.016)	(0.012)	(0.017)	(0.014)	(0.021)	(0.019)
m_2	0.329	0.139	0.216	0.119	0.157	0.107
	(0.022)	(0.011)	(0.018)	(0.013)	(0.018)	(0.015)
χ^2-statistic	811	396	412	289	310	262
($p-$ value)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)

$$\omega_{it} = \widetilde{\gamma}_{0i} + t\widetilde{\gamma}_{1i} + t^2\widetilde{\gamma}_{2i} + \sum_{s=1}^{t} \xi_s + u_{it}$$

$$u_{it} = e_{it} + m_1 e_{i,t-1} + m_2 e_{i,t-2}$$

unit: 100 × log wage (standard errors in the parentheses)

Columns [$1E$] and [$1O$] estimate the model with random $\widetilde{\gamma}_{0i}$, but with nonrandom $\widetilde{\gamma}_{1i}$ and $\widetilde{\gamma}_{2i}$. This specification has a unit root process along with a random intercept, but does not include random growth components. The AR coefficient of the $ARMA(1,2)$ process is assumed to be unity. Hence the transitory errors are given by $MA(2)$. The variance of the random intercept, $Var[\widetilde{\gamma}_{0i}]$, is significant in both columns [$1E$] and [$1O$]. The variance of the increment of a unit root process, $Var[\xi_t]$, is significant in both columns, too. This implies the existence of permanent earnings errors. From the fact that $Var[e_{it}]$ and MA coefficients are significant, we confirm that transitory errors are present. These findings are consistent with those of previous studies that support random walk models.

Columns [$2E$] and [$2O$] estimate the model with random $\widetilde{\gamma}_{0i}$ and $\widetilde{\gamma}_{1i}$, but with nonrandom $\widetilde{\gamma}_{2i}$. This specification includes random growth and random walk components. Transitory errors are $MA(2)$. In this setting, human capital theory predicts $Var[\widetilde{\gamma}_{1i}] > 0$ because the wage dynamics exhibit heterogeneous growth; and $Cov[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i}] < 0$ because high investors have initially low wages. All the coefficients that overlap with models [$1E$] and [$1O$] are significant. The OWMD estimates confirm the prediction implied by human capital theory, $Var[\widetilde{\gamma}_{1i}] > 0$ and $Cov[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i}] < 0$. In sum, heterogeneous growth, random walk, and transitory errors are all present in the wage residual term.

Columns [$3E$] and [$3O$] estimate the model with random $\widetilde{\gamma}_{0i}$, $\widetilde{\gamma}_{1i}$, and $\widetilde{\gamma}_{2i}$. This specification is a combination of the random profile model suggested by Lillard and Reville (1999) and the random walk model. In this setting, however, the variance-covariance estimates for $\widetilde{\gamma}_{1i}$ and $\widetilde{\gamma}_{2i}$ are too noisy to distinguish the two. It suggests that all three sources of randomness are present in the wage process. Consequently, we do not adopt the results of [$3E$] and [$3O$]. The models of [$2E$] and [$2O$] are parsimonious than the models of [$3E$] and [$3O$], but are general enough to allow for heterogeneous growth. We favor the model [$2O$] over the others.

Using the results [$2O$], we conduct two exercises. First, we decompose the total variance in the earnings distribution conditional on experience level into the contributions from the heterogeneous growth, the random walk, and the transitory errors at each experience level. Figure 2(a) shows the variance decomposition results. The solid decreasing line is the variance of the heterogeneous growth, $Var[\widetilde{\gamma}_{0i}] + 2Cov[\widetilde{\gamma}_{0i}, \widetilde{\gamma}_{1i}] \times t + Var[\widetilde{\gamma}_{1i}] \times t^2$; the dashed increasing line is the variance of the permanent earnings shocks, $Var[\xi_t] \times t$; and the short dashed flat line depicts the variance of the transitory errors, $\left(1 + m_1^2 + m_2^2\right) Var[e_{it}]$. The variance of the heterogeneous growth component is declining with experience. This is because workers with high initial income exhibit smaller earnings growth than workers with low initial income. The variances of the unit root process and the transitory errors are linearly increasing and flat, respectively, because of their nature. For comparison purposes, we draw the empirical variance (dotted and broken line) and estimated total variance (dashed and dotted line) in Figure 2(a).

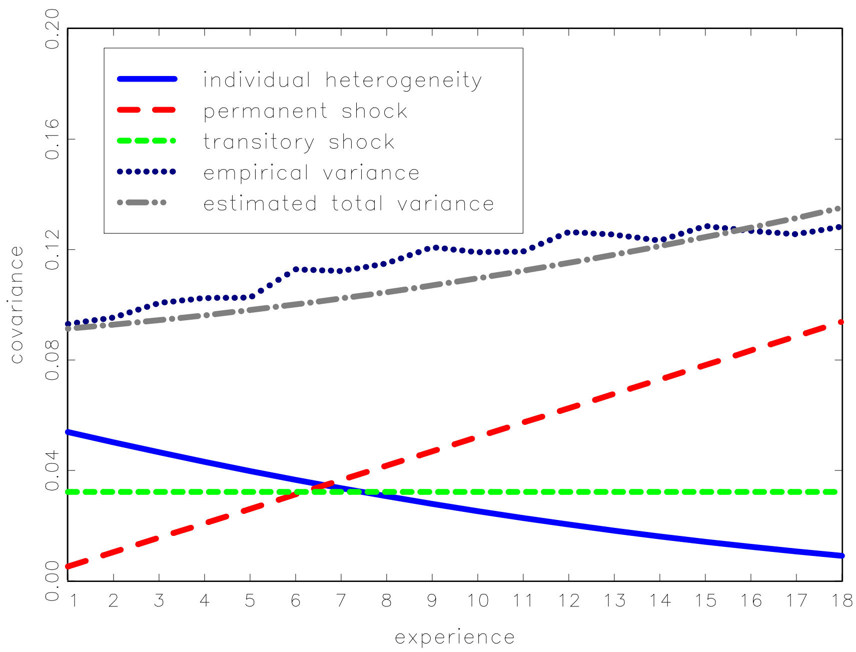

Figure 2(a). Explained Variations

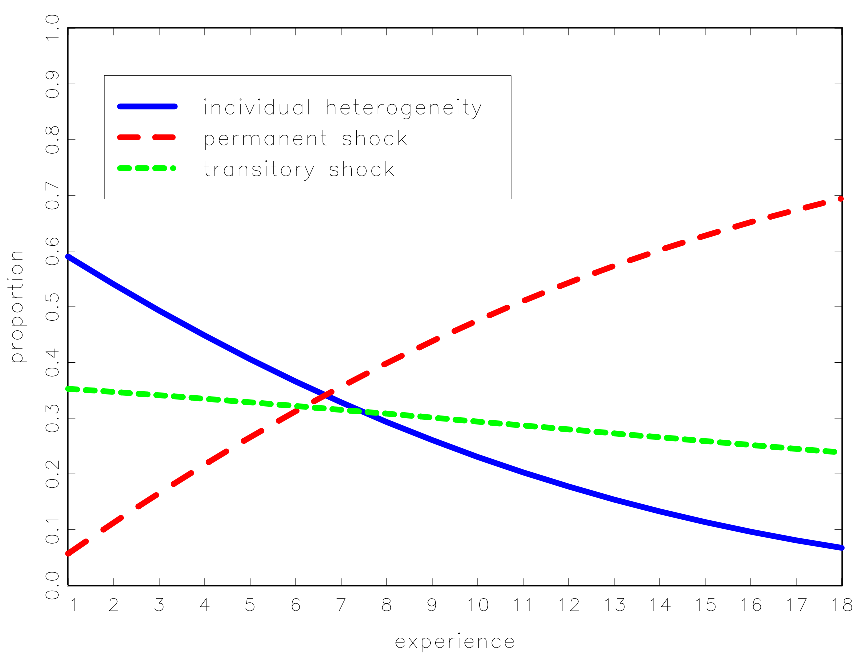

Figure 2(b). Explained Variations (Ratio)

It is also instructive to examine the fraction of the total variance accounted for by component over the life-cycle. In Figure 2(b), individual heterogeneity explains about 60% of the variance of the earnings distribution early in the career, but the share drops to 30% after 8 years and becomes less than 10% when the experience reaches 18 years. Variance of permanent errors accumulates with experience by its nature and explains about 70% of the total variance at the experience level of 18 years. Transitory shocks initially explain about 30% of the variation, dropping to around 20% at the experience level of 18 years.

A second analysis, depicted in Figure 3, looks at the variance of earnings growth by following a given person. The solid line represents the variance of the heterogeneous growth, $Var\left[\widetilde{\gamma}_{1i}\right] \times t^2$. It reflects individual heterogeneity in the ability of human capital production. The dashed line represents the variance of the permanent earnings shocks, $Var\left[\xi_t\right] \times t$. It reflects individual heterogeneity in the amount of human capital investment due to transitory rental rate shocks. From Figure 3, we learn that a worker's earnings are more affected by shifts in the human capital accumulation path than by individual differences in the ability to produce human capital.

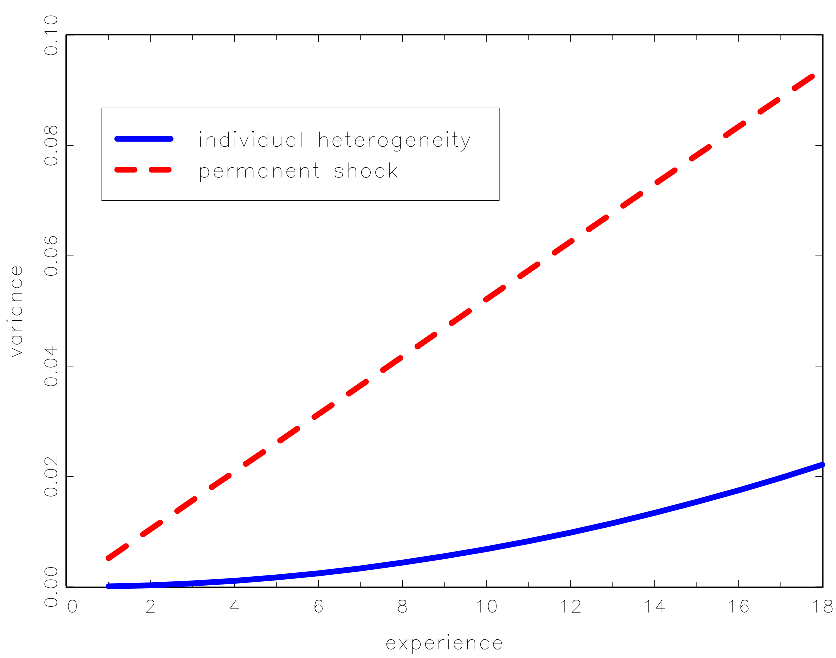

Figure 3. Fraction of the Variance in Earnings Growth Explained by Individual Heterogeneity and Persistent Errors

5 Concluding Remarks

This paper presents that both heterogeneous growth and permanent errors can be implied by human capital theory. By exploring implications of uncertainty about the return to human capital for the earnings process, we are able to bridge the literature emphasizing heterogeneity in growth and papers which feature purely stochastic models. Our approach is distinct from that of previous research, which aims to support one of the two rival hypotheses: the random growth and the random walk models. We start off with the optimal human capital investment theory, and let workers face stochastic rental rates of human capital. We derive the implications of the human capital theory under uncertainty for the earnings process. It turns out that the implied model nests both random growth and random walk models. This provides a human capital explanation for both the random growth and random walk components. Heterogeneous growth stems from individual heterogeneity in the ability of human capital production. Permanent errors stem from individual heterogeneity in the amount of human capital investment due to transitory rental rate shocks.

Empirical findings suggest that both heterogeneous growth and permanent errors exist in earnings residuals. We use the estimates of the earnings residual covariance matrix to decompose the variance into three parts that are explained by heterogeneous growth, permanent errors, and transitory shocks. Individual heterogeneity is more important early in the career, and permanent shocks become dominant as a worker becomes experienced. The variance explained by transitory errors remains stable and is relatively large. We also learn that a worker's earnings are more affected by changes in human capital production than by individual differences in the ability to produce human capital. The next step would be to derive the mapping from the structural parameters to the parameters of the earnings process, and estimate the structural parameters. This is left for future research.

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7 Appendix

7.1 Data: NLSY79

We use the NLSY79 to estimate the model. It is a longitudinal data set with a nationally representative sample of 12,686 young men and women who were between the ages of 14 and 21 on January 1, 1979. These individuals were interviewed annually through 1994 and are currently interviewed on a biennial basis. We collect observations from 1979 to 2002. We exclude the oversampled economically disadvantaged and the military samples because they are not representative of the civilian population. We keep oversampled black and Hispanic samples because race is exogenous and can be controlled in the analysis.

A person is classified as working if he (1) reports the hourly rate of pay, (2) has a valid job, and (3) works at least 30 weeks a year with a minimum of 30 hours/week. From 1994, we modify the third criterion to include individuals working at least 60 weeks per two years with a minimum of 30 hours/week. We use work history data which provides a week-by-week longitudinal work record of each NLSY79 respondent from January 1, 1978, through the current survey date. The hourly rate of pay data for the current or most recent job is used for wage information. It is deflated by the 1982-1984=100 CPI.

Education is measured by the highest grade completed, reported after one year of school completion. If school completion information is missing or the years of education is less than 8, the corresponding person is dropped from the sample before running the outlier identifying regression. Missing data in the annually reported highest grade completed, however, are filled up under the two assumptions: the initially reported education level is correct and the educational attainment remains constant or increases after that point.

The AFQT is a general measure of trainability and a primary criterion of enlistment eligibility for the Armed Forces. The AFQT score is calculated from the Armed Services Vocational Aptitude Battery (ASVAB). The ASVAB was administered in 1980 to the 1979 sample of NLSY79 respondents. About 94 percent of the 1979 sample completed this test. A composite raw score derived from select sections of the battery can be used to construct an approximate and unofficial AFQT score. To control the effect of age to the ability measure, the AFQT scores are standardized by subtracting the mean and dividing by the standard deviation for each age group. We eliminate the persons with no AFQT scores from our analysis.

7.2 Discussion on Human Capital Production Functions

Generally, a human capital production function may have a form of

$$q_t = f\left(I_t, H_t\right).$$

This model implies that the production is determined by both the input and the stock of human capital as opposed to depending on the input only. This production function is called self-productive because the stock, H_t, itself enters as an argument for production.[11] For instance, a production function may be defined by

$$q_t = aI_t^b H_t^c, \qquad a > 0,\ 0 < b < 1.$$

In this case, the production function is self-productive if $c \in (0, 1-b)$, and is non-self-productive if $c = 0$.

This paper considers non-self-productive human capital production functions only. This class of functions is parsimonious, but is flexible enough to capture the general feature of wage dynamics. In addition, the class of non-self-productive production functions is much easier to handle in practice. When a function is non-self-productive, workers at the same experience level with different amount of human capital stock will exhibit the same amount of input. This is not contradictory because same amount of input does not imply an identical share of human capital input. We can write an input, as a part of the stock of human capital, as:

$$I_t = k_t H_t.$$

We expect that a worker with greater stock will use a smaller portion of human capital in production. Consequently, a worker with higher stock of human capital has a higher earnings growth path.

The shape of the optimal path is directly related to the choice of which class of production functions we choose. For instance, the optimal input is always downward sloping with experience in non-self-productive cases, while self-productive production functions yield hump-shaped optimal input paths. This is because input is more productive if incorporated with higher stock when the production function is self-productive. Intuitively, we expect that workers put more effort into human capital production when they are young. In this sense, the class of non-self-productive production functions is general enough for our purposes.

[11] Heckman (1973) supports this specification. He develops a general model of life-cycle consumption, investment, and labor supply and finds that human capital exhibits negative productivity in its own production. Given this result, he argues that a production function without H_t as an argument is not flexible enough to capture the path of investment over the life-cycle.