control theory with humans "in the loop"

goal: learn how to model control systems with humans and machines using linear systems theory

topics: 1°. signals & systems

2°. linear time-invariant (LTI) systems

3°. block diagram algebra with LTI systems

4°. feedforward and feedback control systems

1°. signals & systems

- signals are all around us – any measurable quantity that changes in time

ex: the volume of my voice; my Zoom video

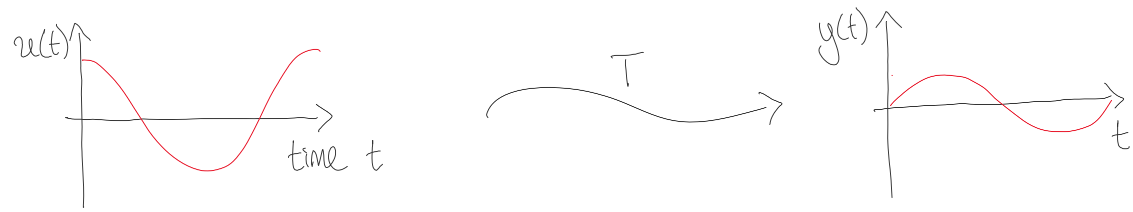
- systems are also ubiquitous – anything that transmits/transforms signals

ex: the Internet (TCP/IP) network that transmits my voice/video data to you; the (de)compression and (de)encoding algorithms that convert data to voice/video

→ what other examples of signals & systems can you think of? e.g. from daily life, or from science or technology you're interested in?

2°. linear time-invariant (LTI) systems

- consider a system that transforms input signal u to output signal y:



def: T is <u>linear</u> if $(\alpha \cdot u)(t) \overset{T}{\longmapsto} (\alpha \cdot y)(t)$ for all $\alpha \in \mathbb{R}$, $y = Tu$

and $(u_1 + u_2)(t) \overset{T}{\longmapsto} (y_1 + y_2)(t)$ for all $y_1 = Tu_1$, $y_2 = Tu_2$

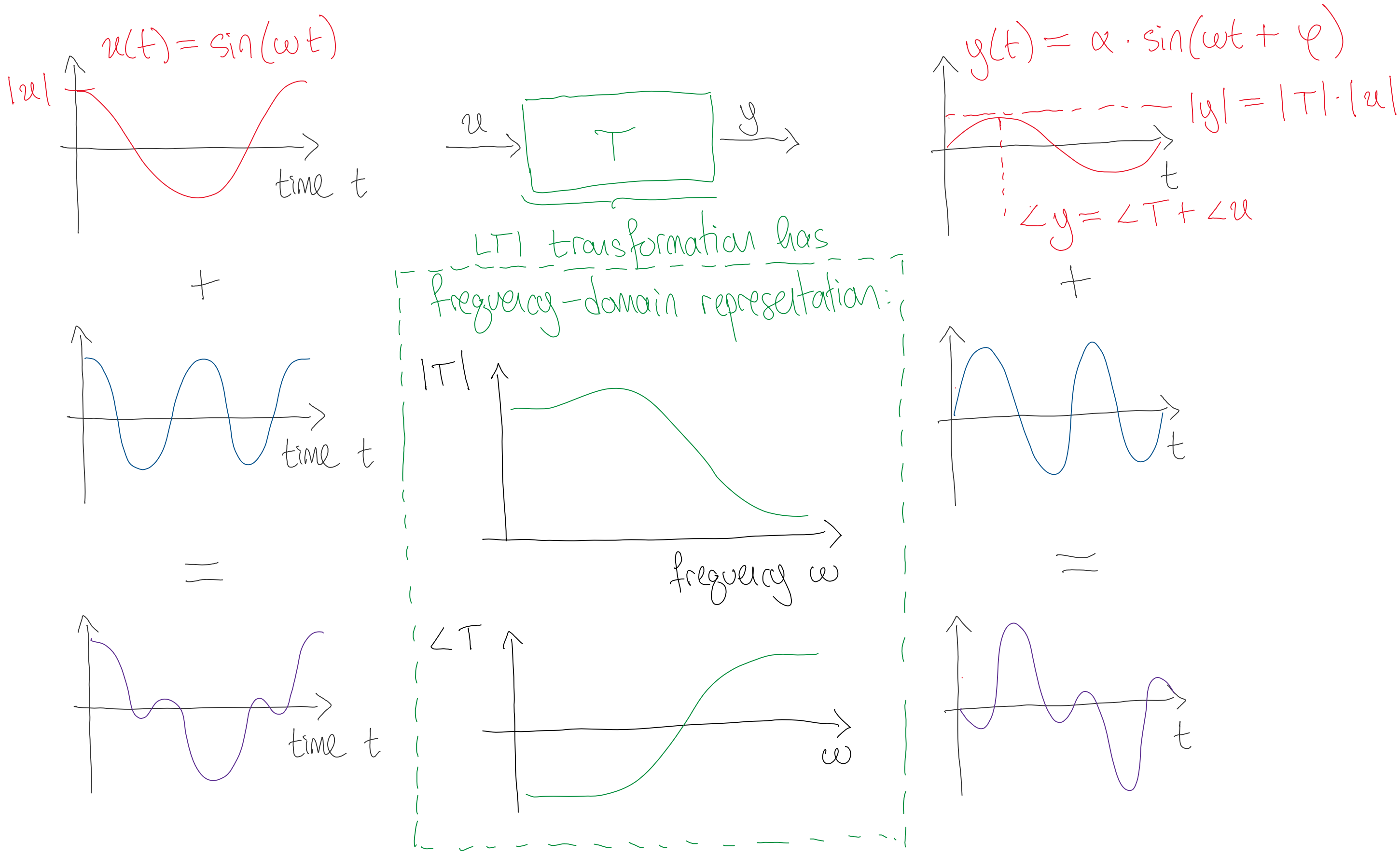
<u>time-invariant</u> if $u(t-\tau) \longmapsto y(t-\tau)$ for all $y = Tu$

ex:
- multiplication by scalar β: $y(t) = Tu(t) = \beta \cdot u(t)$ ☑ L ☑ TI
- multiplication by signal $b(t)$: $y(t) = Tu(t) = b(t) \cdot u(t)$ ☑ L ☒ TI

[◦ convolution by a signal ◦ Fourier / Laplace / Wavelet transform]

* these seemingly-simple properties are extremely powerful, because they ensure we can analyze T in <u>frequency domain</u>:

Fact: an LTI system T simply scales and phase-shifts a sinusoid:

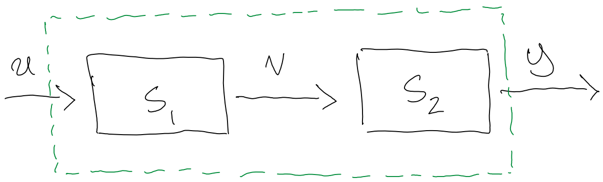


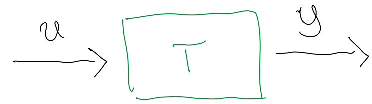
[◦ conversion from time-domain signal to frequency-domain is accomplished with the Fourier transform, which represents a given signal as an (uncountably infinite) linear combination of simple sinusoids]

3°. block diagram algebra with LTI systems

- the special property of LTI systems means we can reason about how interconnected systems transform signals using simple algebra.

CASCADE:

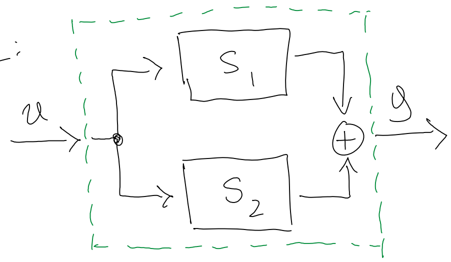




$T = S_2 \cdot S_1$

$$y = S_2 \cdot v, \quad v = S_1 \cdot u \implies y = S_2 \cdot S_1 \cdot u =: T \cdot u$$

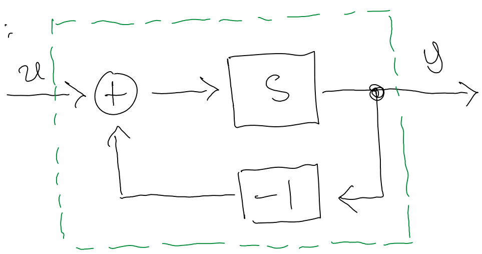
PARALLEL:



$T = S_2 + S_1$

$$y = S_1 \cdot u + S_2 \cdot u \implies y = (S_1 + S_2) \cdot u =: T \cdot u$$

FEEDBACK:



$T = \frac{S}{1+S}$

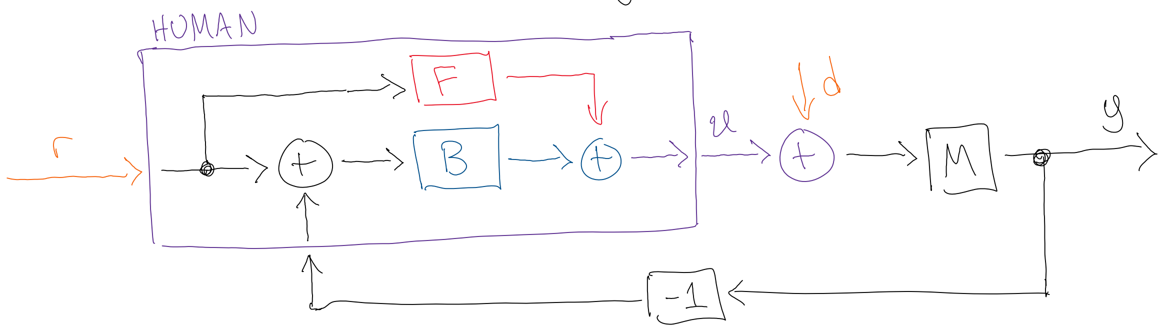
$$y = S \cdot (u - y) \implies y = \frac{S}{1+S} \cdot u =: T \cdot u$$
$$= S \cdot u - S \cdot y$$

* these fundamental rules can be used to transcribe a block diagram into an equivalent set of algebraic equations

→ block diagrams aren't just pretty pictures/conceptual – they're MATH!

4°: feedforward and feedback control systems

- we now return to the block diagram with a human "in the loop":



→ solve for u in terms of r and d: $u = H_{ur} \cdot r + H_{ud} \cdot d$

$$u = Fr + B(r-y), \quad y = M(u+d) \Rightarrow u = \frac{F+B}{1+BM} \cdot r + \frac{-BM}{1+BM} \cdot d$$

→ solve for F and B in terms of H_{ur} and H_{ud}

$$H_{ur} = \frac{F+B}{1+BM}, \quad H_{ud} = \frac{-BM}{1+BM} \cdot d \Rightarrow F = \frac{H_{ur} + M^{-1} H_{ud}}{1+H_{ud}}, \quad B = \frac{-H_{ud}}{M(1+H_{ud})}$$

* importantly, we can measure H_{ur} and H_{ud} experimentally, and then compute <u>feedforward</u> F and <u>feedback</u> B