Reduction and Identification for Hybrid Dynamical Models of Terrestrial Locomotion

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SPIE May 2013

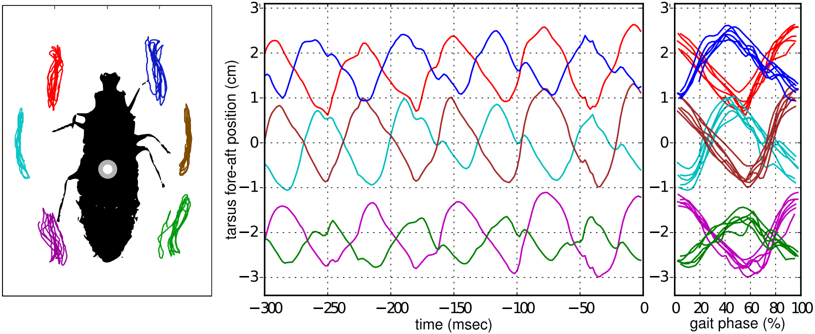
Dynamics of terrestrial locomotion

americana

Periplaneta americana

video courtesy of Poly-PEDAL Lab, UC Berkeley

Empirically, animals use few degrees-of-freedom



Cockroach dynamics ∼ 7 dimensional (Revzen & Guckenheimer 2011)

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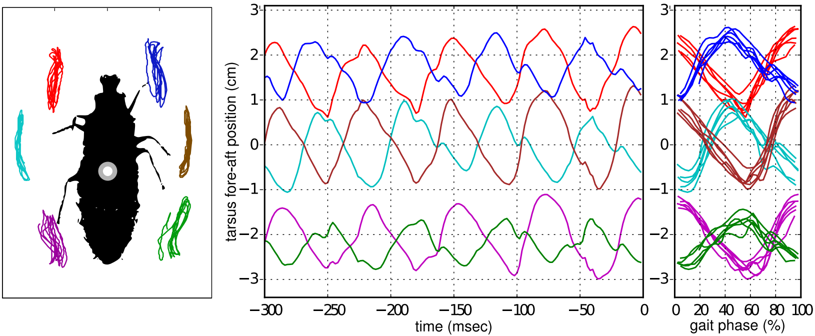


Cockroach dynamics $\sim$ 7 dimensional (Revzen & Guckenheimer 2011)

Mechanisms:

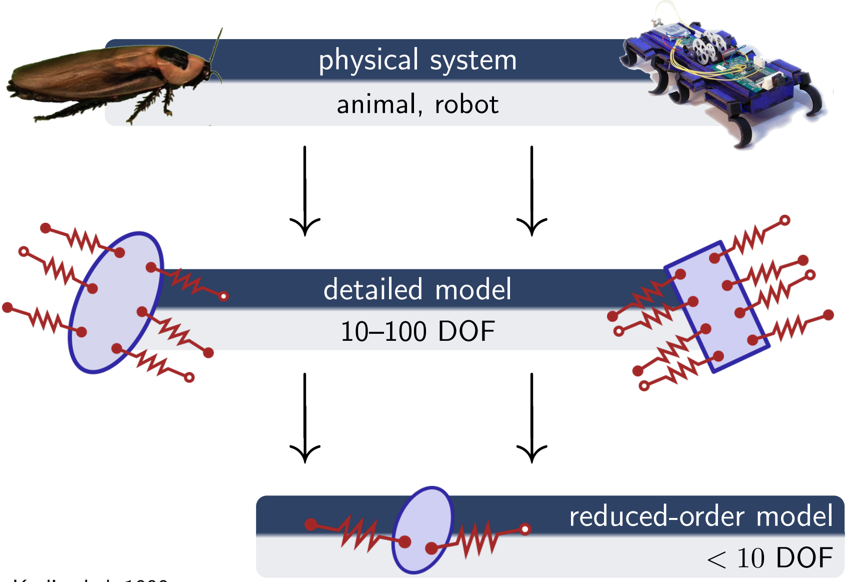
Neural synchronization
Cohen et al. 1982

Physiological symmetry
Golubitsky et al. 1999

Muscle activation synergy
Ting & Macpherson 2005

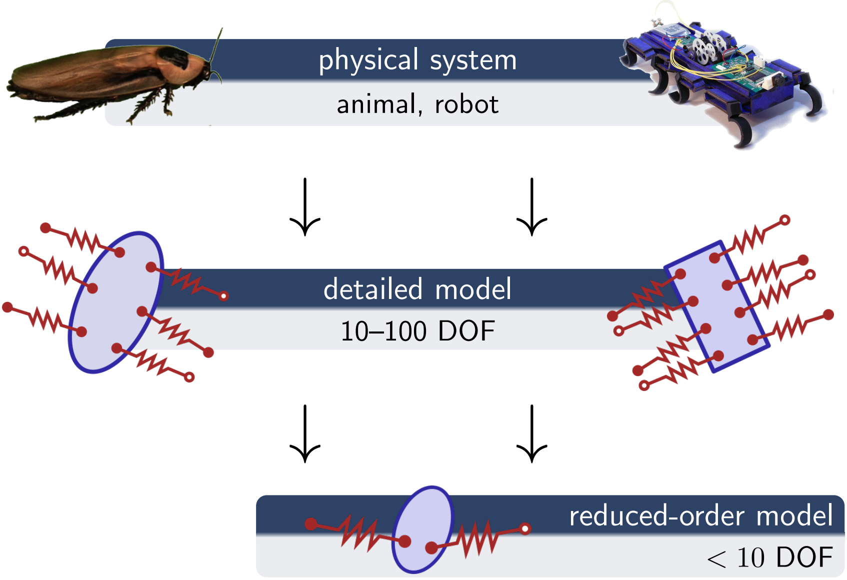
Granular media solidification
Li et al. 2009

Reduced-order model describes dynamic locomotion



Full & Koditschek 1999

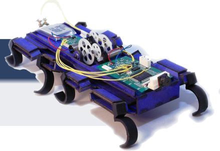
Obstacles to using reduced-order models



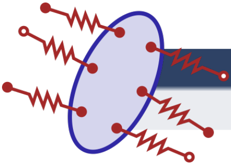
Obstacles to using reduced-order models

physical system

animal, robot

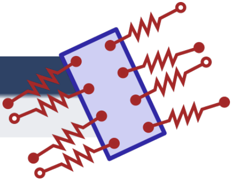


?



detailed model

10–100 DOF



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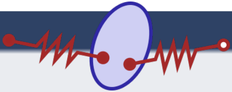

reduced-order model

< 10 DOF

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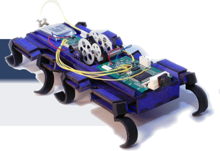

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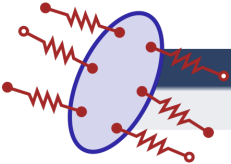

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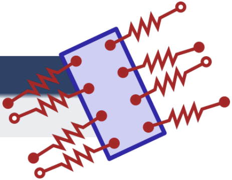
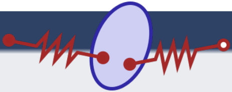


reduction



reduced-order model

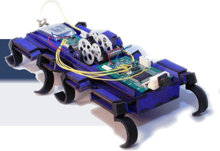
< 10 DOF

reduction
Grizzle et al. 2002
Ames et al. 2006
Proctor et al. 2010

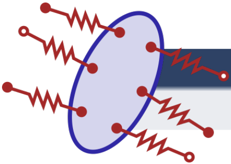
Obstacles to using reduced-order models

physical system

animal, robot



↓ **identification** ↓



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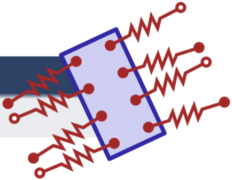

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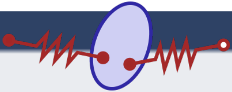


reduced-order model

< 10 DOF

identification

Mazor et al. 1998

Ferrari-Trecate et al. 2003

Vidal 2008

reduction

Grizzle et al. 2002

Ames et al. 2006

Proctor et al. 2010

Overview

Motivation
reduced-order models describe dynamic locomotion

Reduction
hybrid dynamics reduce dimensionality near periodic orbits

Identification
reduction enables scalable algorithm for parameter estimation

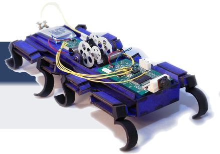
Conclusion
novel quantitative predictions for biomechanics

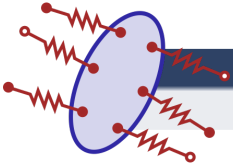
model-based design and control of dynamic robots

Reduction

physical system

animal, robot





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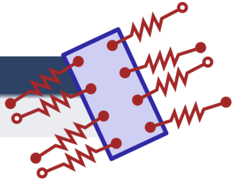

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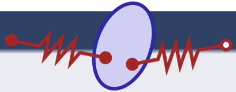
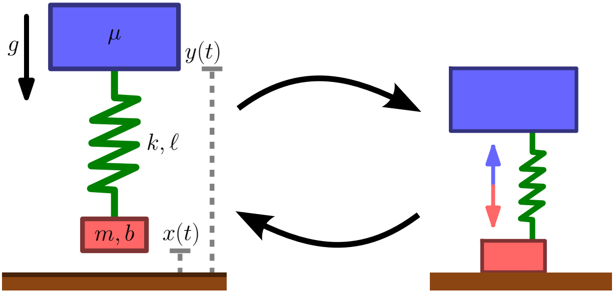


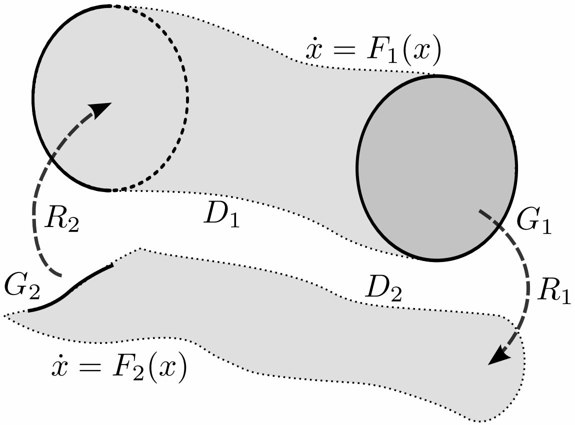
reduced-order model

< 10 DOF

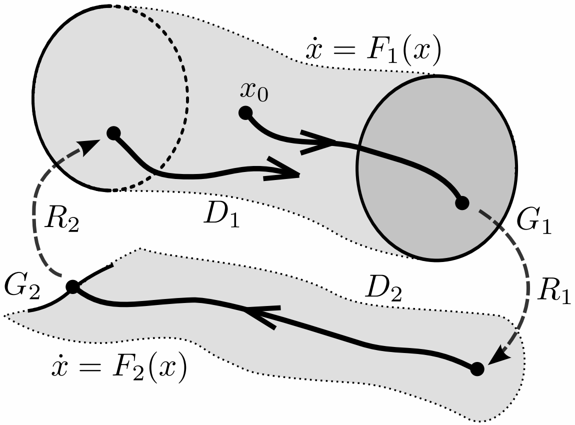
Example (vertical hopper)



Hybrid dynamical system



Trajectory for a hybrid dynamical system



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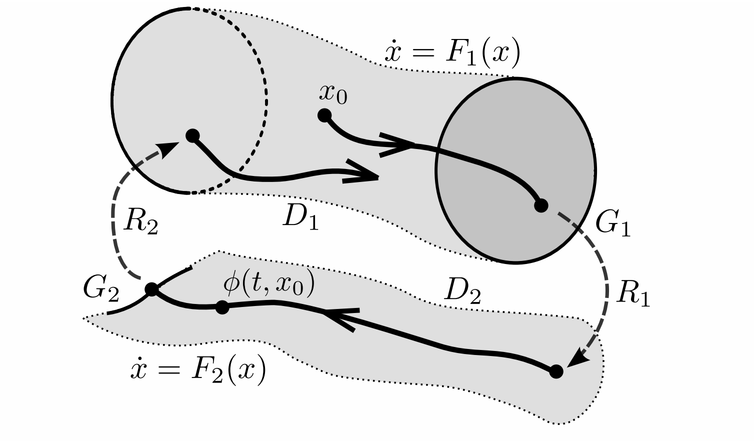
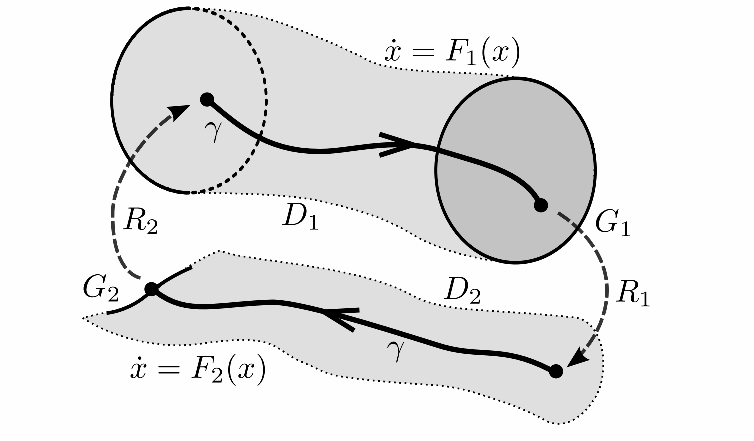
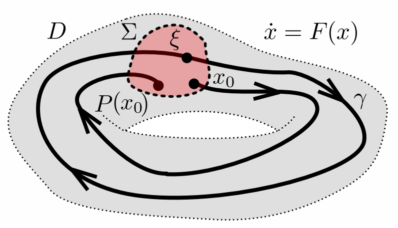


Periodic orbit γ for a hybrid dynamical system



Poincaré map for periodic orbit γ

smooth dynamical system



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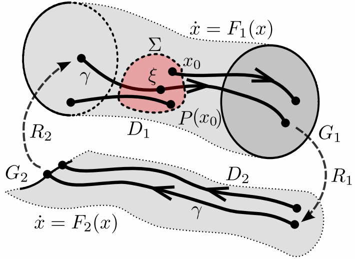

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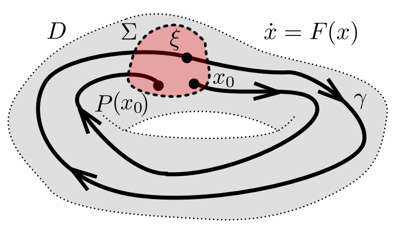

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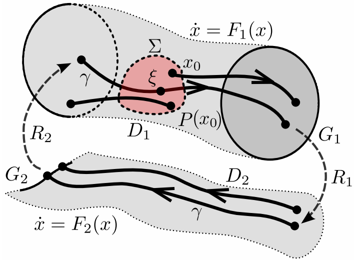
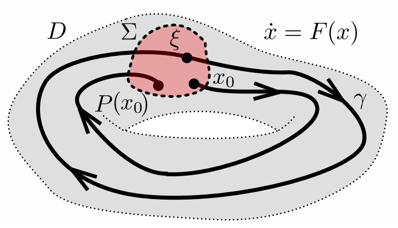


Theorem (Hirsch and Smale 1974, Grizzle et al. 2002)

The Poincaré map P is smooth in a neighborhood of ξ.

Rank of Poincaré map P with fixed point $P(\xi) = \xi$

smooth dynamical system



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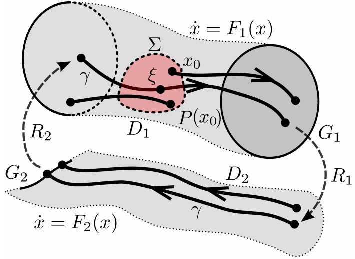

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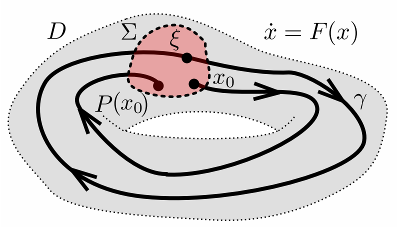
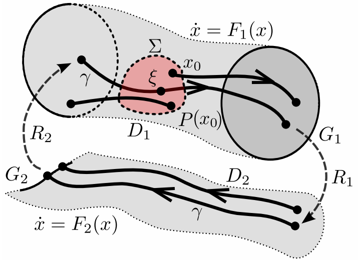


$\operatorname{rank} DP(\xi) = \dim D - 1$

Hirsch and Smale 1974

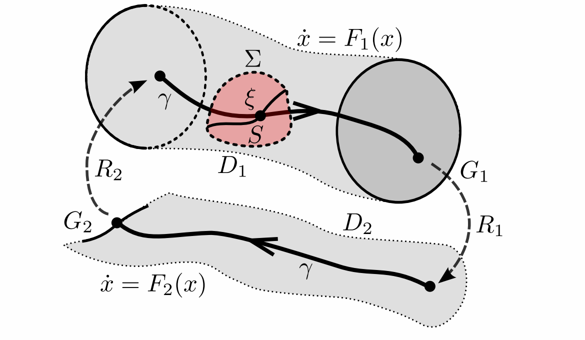
hybrid dynamical system



$\operatorname{rank} DP(\xi) \le \min_j \dim D_j - 1$

Wendel and Ames 2010

Model reduction near hybrid periodic orbit γ



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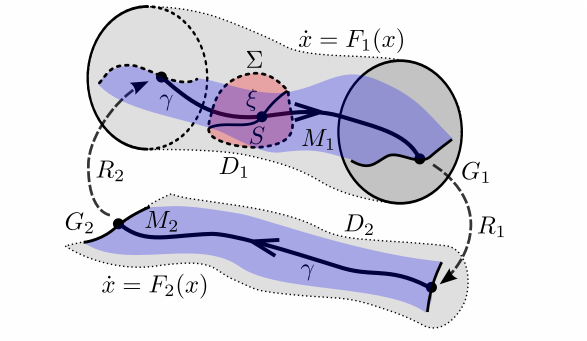

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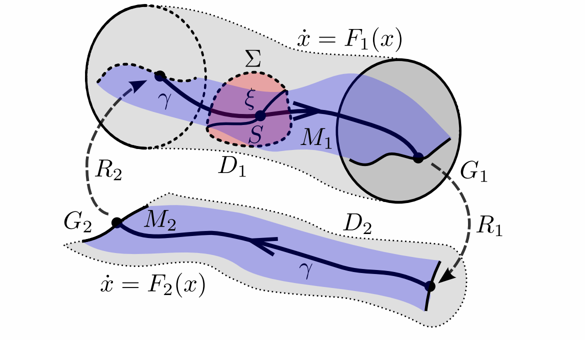
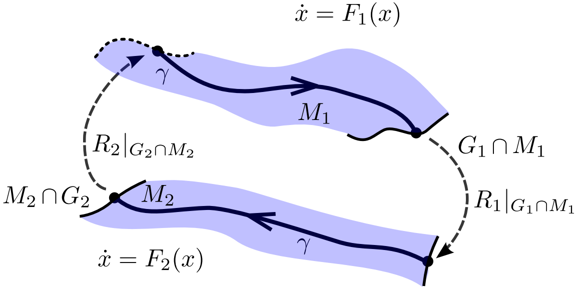


Theorem (Burden, Revzen, Sastry CDC 2011)

Let $n = \min_j \dim D_j - 1$*. If* $\operatorname{rank} DP^n = r$ *near* ξ*, then trajectories starting near* γ *contract to a collection of hybrid-invariant* $(r+1)-$*dimensional submanifolds* $M_j \subset D_j$ *in finite time.*

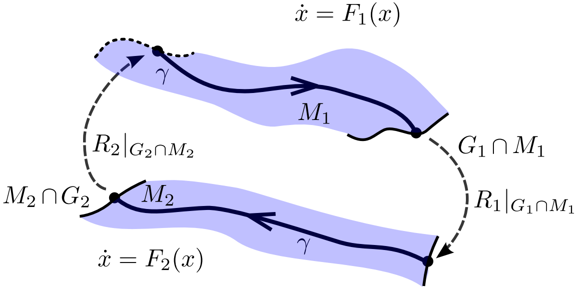
Model reduction near hybrid periodic orbit γ



Corollary

The submanifolds M_j determine a hybrid system with periodic orbit γ.

Model reduction near hybrid periodic orbit γ



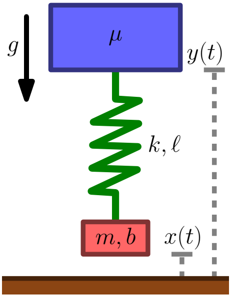
Corollary

The submanifolds M_j determine a hybrid system with periodic orbit γ.

γ *is stable in the original hybrid system*

$\iff$ γ *is stable in the reduced hybrid system.*

Example (exact model reduction in vertical hopper)



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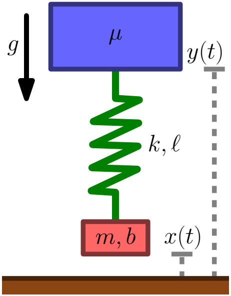

Numerically linearizing Poincaré map P on ground,
we find $DP(\xi)$ has eigenvalue $\simeq 0.57$,
therefore DP^2 is constant rank near ξ.

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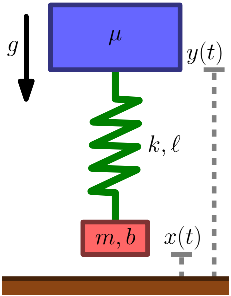

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Theorem $\implies$ dynamics collapse to 1-DOF hopper

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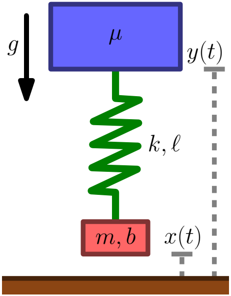

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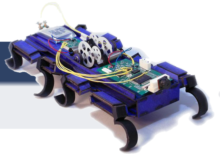
Theorem $\implies$ dynamics collapse to 1-DOF hopper

Interpretation: unilateral (Lagrangian) constraint appears after one "hop"

Identification

physical system

animal, robot



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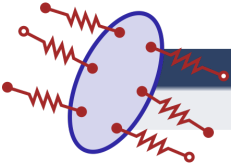
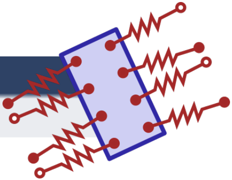
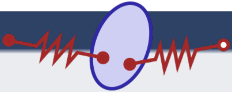


detailed model

10–100 DOF

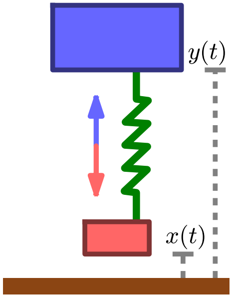




reduced-order model

< 10 DOF

Identification of initial conditions



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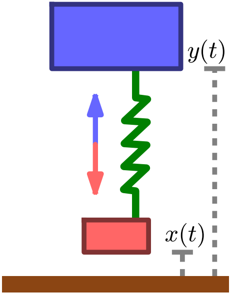

$$Y(\phi(t, z)) = y(t)$$

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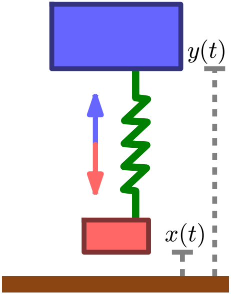

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$\eta_i = Y(\phi(iT, z^*)) + w_i$,

w_i iid random variables

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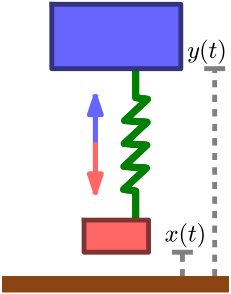

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Identification problem

Solve $\arg\min_{z \in D_j} \varepsilon(z, \{\eta_i\})$, where $\varepsilon(z, \{\eta_i\}) := \sum_i \|Y(\phi(iT, z)) - \eta_i\|^2$.

Identification on reduced hybrid model

Assumption (smooth observations)

Y is smooth along trajectories, i.e. $Y(\phi(t,z))$ is a smooth function of t.

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Identification on $\bigcup_j D_j$

$$\arg\min_{z \in D_j} \varepsilon\left(z, \{\eta_i\}\right)$$

- $\nabla\varepsilon$ undefined on $G_j \subset D_j$
- R_j not generally invertible

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Identification on $\bigcup_j M_j$

$$\arg\min_{z \in M_j} \varepsilon\left(z, \{\eta_i\}\right)$$

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- $R_j|_{M_j}$ invertible

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first-order algorithms applicable

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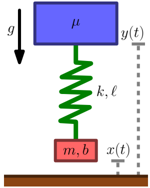
first-order algorithms applicable

Identification on $\bigcup_j M_j$ in practice

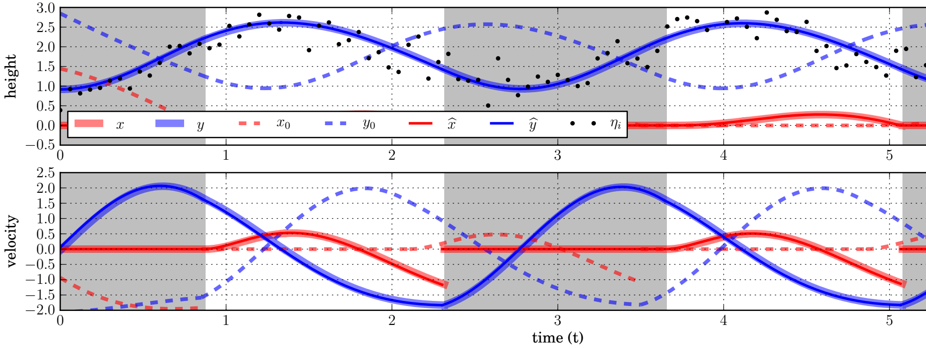
For any $z \in M$ there exists $(t,u) \in \mathbb{R}_{\geq 0} \times \Sigma \cap M$ such that $\phi(t,u) = z$.

Example (initial condition for vertical hopper)



Observe position of upper mass at 20Hz, additive noise with variance 0.3.

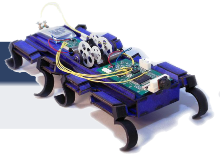
$$(t_0, y_0) \approx (2.29, 0.03) : \text{initial}$$
$$(t, y) \approx (3.41, 0.14) : \text{actual}$$
$$(\hat{t}, \hat{y}) \approx (3.42, 0.16) : \text{estimated}$$



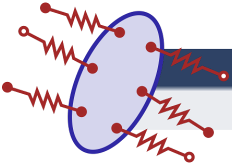
Reduction & Identification

physical system

animal, robot

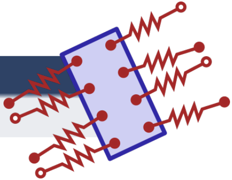


identification

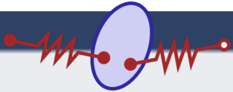


detailed model

10–100 DOF



reduction



reduced-order model

< 10 DOF

Novel quantitative predictions for biomechanics

observation

neural feedback
appears at a delay

Revzen, Burden et al. 2013

identification

prediction

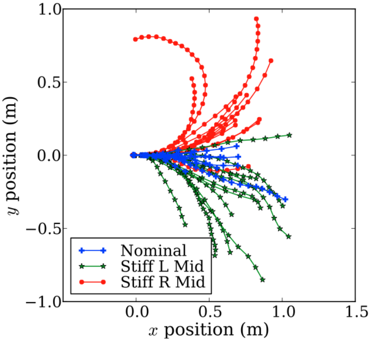
passive mechanics
sensitive to inertia

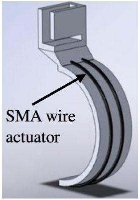
Full et al. 2002

Burden, Revzen, Moore, Sastry, & Full SICB 2013

lateral perturbation

Model-based design and control of dynamic robots

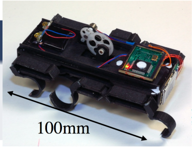




design

minimal use of actuators

Hoover et al. 2010



identification

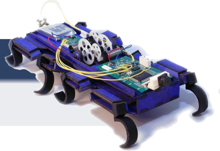
control

asymmetric leg stiffness change

Proctor & Holmes 2008

Hoover, Burden, Fu, Sastry, & Fearing IEEE BIOROB 2010

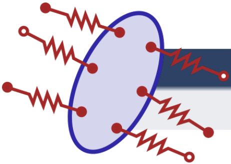
Model enables translation across morphology, scale

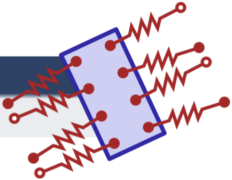


physical system

animal, robot

identification

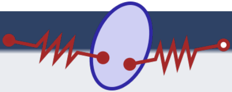




detailed model

10–100 DOF

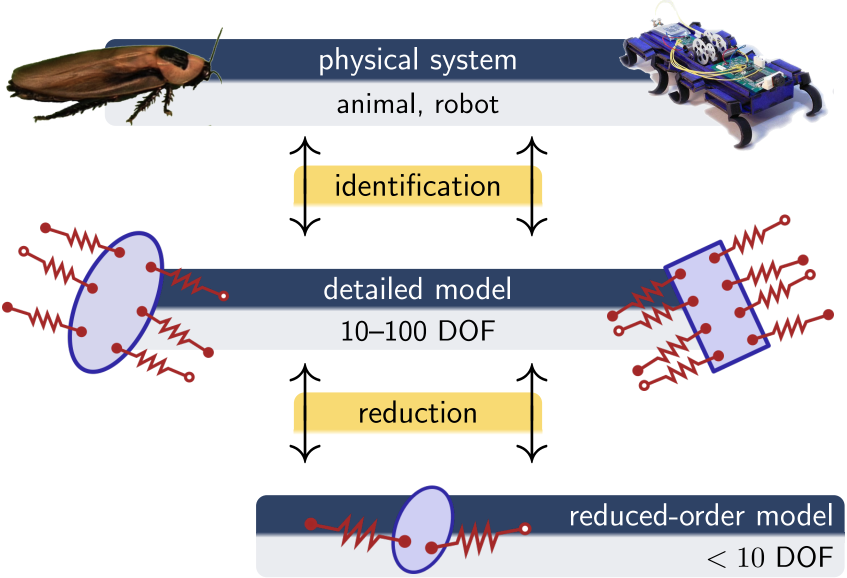
reduction



reduced-order model

< 10 DOF

Model enables translation across morphology, scale



Discussion & Questions — Thanks for your time!

Reduction

Hybrid dynamics reduce dimensionality near periodic orbits.

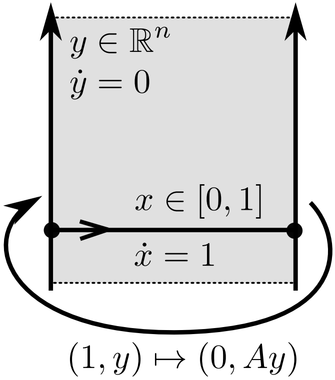
Identification

Reduction enables scalable algorithm for parameter estimation.

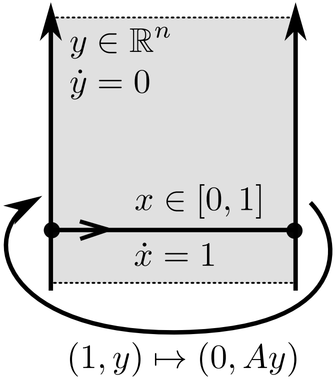
Collaborators

- Prof. Shankar Sastry
- Prof. Robert Full
- Prof. Ron Fearing
- Prof. Shai Revzen
- Prof. Aaron Hoover
- Talia Moore

Example (rank-deficient Poincaré map)



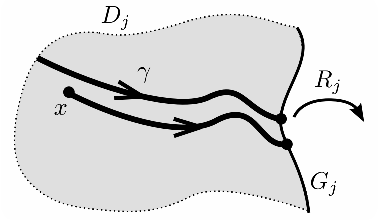
Example (rank-deficient Poincaré map)

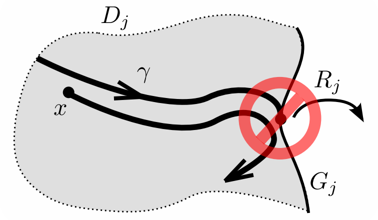


If $A \in \mathbb{R}^{n \times n}$ is nilpotent (i.e. $A^n = 0_{n \times n}$), then $\operatorname{rank} DP^n = 0$.

Assumptions on hybrid periodic orbit γ

Assumptions on hybrid periodic orbit γ

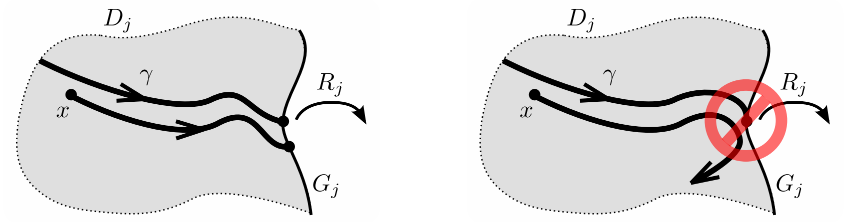




Assumption (transversality)

periodic orbit γ passes transversely through each guard G_j

Assumptions on hybrid periodic orbit γ



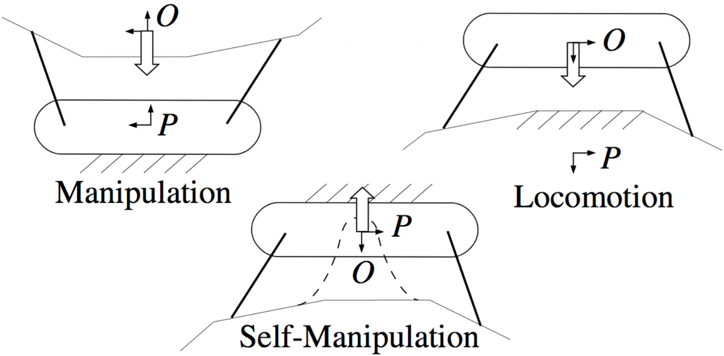
Assumption (transversality)

periodic orbit γ passes transversely through each guard G_j

Assumption (dwell time)

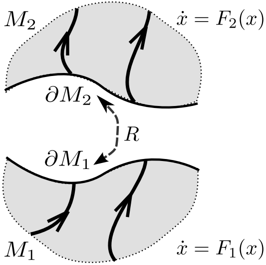
$\exists\varepsilon > 0$: *periodic orbit γ spends at least ε time units in each domain D_j*

Locomotion is self-manipulation

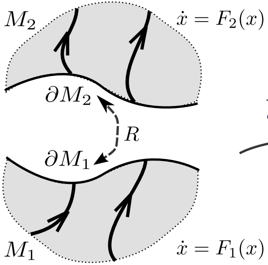


Johnson, Haynes, & Koditschek, IROS 2012

Gluing smooth dynamical systems

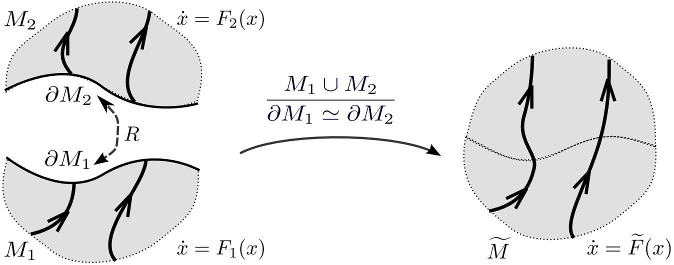


Gluing smooth dynamical systems

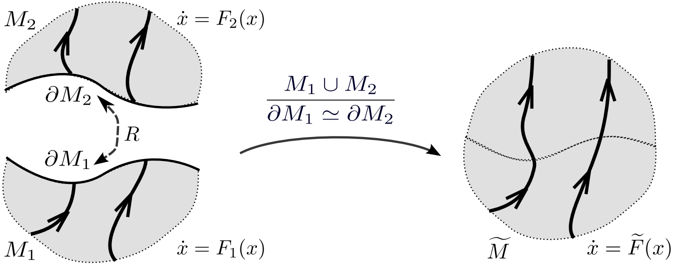


$$\frac{M_1 \cup M_2}{\partial M_1 \simeq \partial M_2}$$

Gluing smooth dynamical systems



Gluing smooth dynamical systems



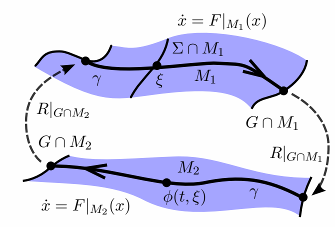
Lemma (Hirsch 1976)

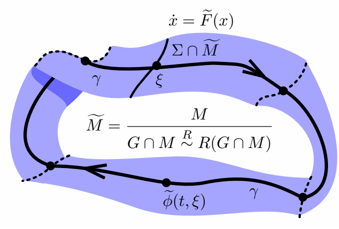
Let F_j be a smooth vector field on n-dimensional manifold M_j, $j \in \{1, 2\}$. If $R : \partial M_1 \to \partial M_2$ is a diffeomorphism, F_1 points outward on ∂M_1, and F_2 points inward on ∂M_2, then the quotient $\widetilde{M} = \frac{M_1 \cup M_2}{\partial M_1 \simeq \partial M_2}$ is a smooth manifold, $M_j \subset \widetilde{M}$ is a smooth submanifold, and the vector field

$$\widetilde{F}(x) = \begin{cases} F_1(x), & x \in M_1; \\ F_2(x), & x \in M_2; \end{cases}$$

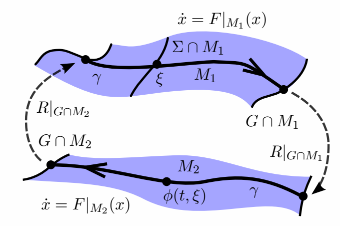
is smooth on $\widetilde{M}$.

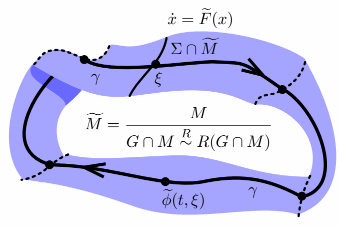
Smoothing reduced-order hybrid system





Smoothing reduced-order hybrid system





Corollary (Burden, Revzen, Sastry CDC 2011)

The topological quotient $\widetilde{M} = \frac{\bigcup_j M_j}{(G_j \cap M_j) \simeq R_j(G_j \cap M_j)}$ *is a smooth manifold,* $M_j \subset \widetilde{M}$ *is a smooth submanifold, and the vector field*

$$\widetilde{F}(x) = \begin{cases} F_1(x), & x \in M_1; \\ \vdots & \vdots \\ F_j(x), & x \in M_j; \\ \vdots & \vdots \end{cases}$$

is smooth on $\widetilde{M}$.