Reduction and Identification for Hybrid Dynamical Models of Terrestrial Locomotion

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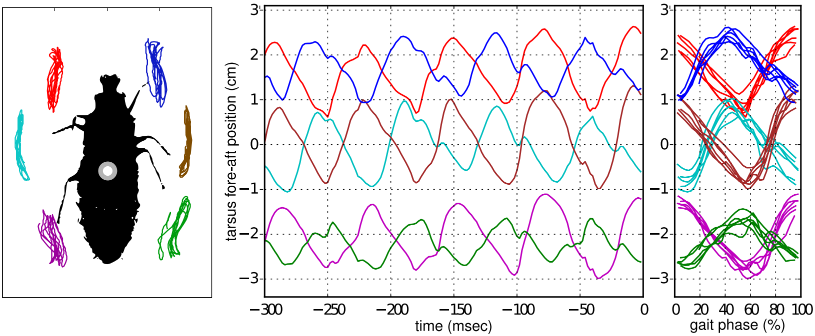
Dynamics of terrestrial locomotion

americana

Periplaneta americana

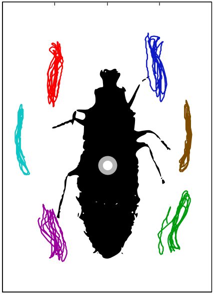
video courtesy of Poly-PEDAL Lab, UC Berkeley

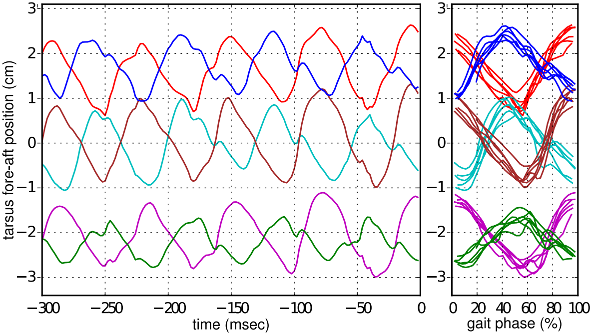
Empirically, animals use few degrees-of-freedom



Cockroach dynamics ∼ 7 dimensional (Revzen & Guckenheimer 2011)

Empirically, animals use few degrees-of-freedom





Cockroach dynamics ∼ 7 dimensional (Revzen & Guckenheimer 2011)

Mechanisms:

Neural synchronization
Cohen et al. 1982

Muscle activation synergy
Ting & Macpherson 2005

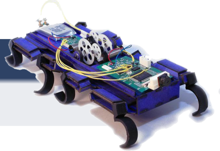
Physiological symmetry
Golubitsky et al. 1999

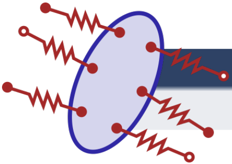
Granular media solidification
Li et al. 2009

Reduced-order model describes dynamic locomotion

physical system

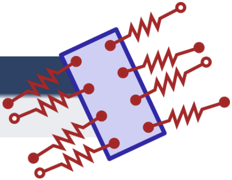
animal, robot

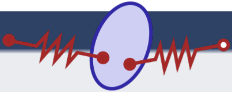




detailed model

10–100 DOF





reduced-order model

< 10 DOF

Full & Koditschek 1999

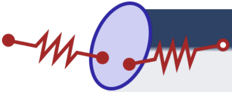
Mechanical self-stabilization in animals

physical system

Blaberus discoidalis

Jindrich & Full 2002

↓



reduced model

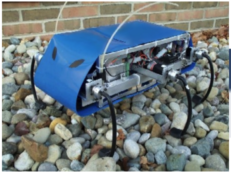
Lateral Leg-Spring

Schmitt & Holmes 2000

blaberus

video courtesy of Poly-PEDAL Lab

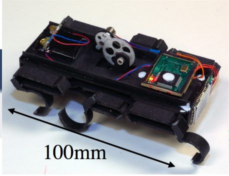
Fast & maneuverable dynamic robots



Saranli et al. 2001

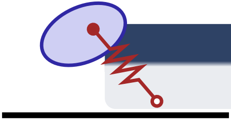
physical system

RHex, DynaRoACH



Hoover, Burden et al. 2010

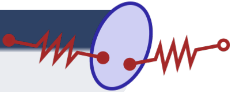
↓ ↓



Ghigliazza et al. 2003

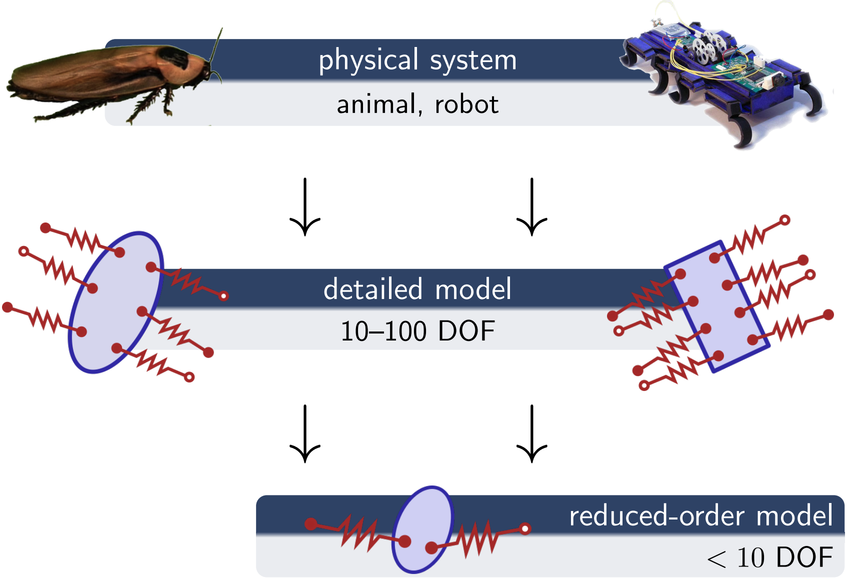
reduced-order model

SLIP, LLS



Proctor & Holmes 2008

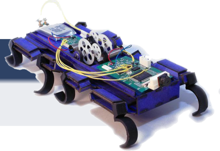
Obstacles to using reduced-order models



Obstacles to using reduced-order models

physical system

animal, robot



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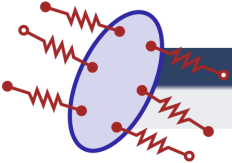
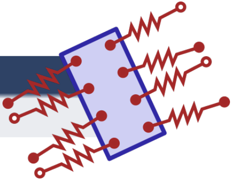


detailed model

10–100 DOF



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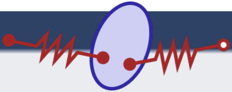


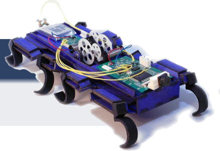
reduced-order model

< 10 DOF

Obstacles to using reduced-order models

physical system

animal, robot



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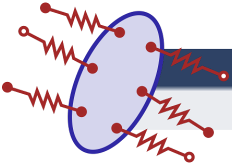

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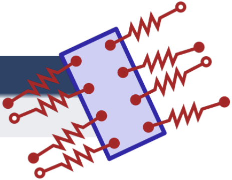
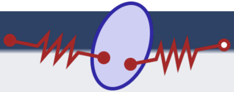
10–100 DOF



reduction



reduced-order model

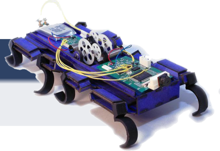
< 10 DOF

reduction
Grizzle et al. 2002
Ames et al. 2006
Proctor et al. 2010

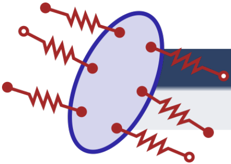
Obstacles to using reduced-order models

physical system

animal, robot

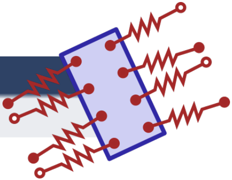


identification

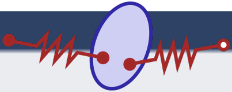


detailed model

10–100 DOF



reduction



reduced-order model

< 10 DOF

identification

Mazor et al. 1998

Ferrari-Trecate et al. 2003

Vidal 2008

reduction

Grizzle et al. 2002

Ames et al. 2006

Proctor et al. 2010

Overview

Motivation

reduced-order models describe dynamic locomotion

Reduction

hybrid dynamics reduce dimensionality near periodic orbits

Identification

reduction enables scalable algorithm for parameter estimation

Conclusion

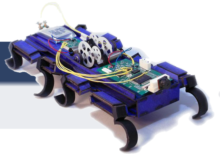
novel quantitative predictions for biomechanics

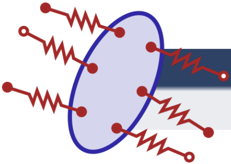
model-based design and control of dynamic robots

Reduction

physical system

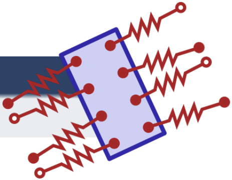
animal, robot



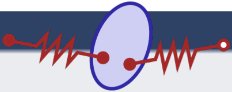


detailed model

10–100 DOF



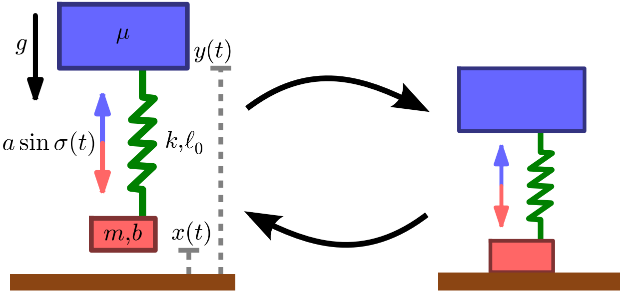
reduction



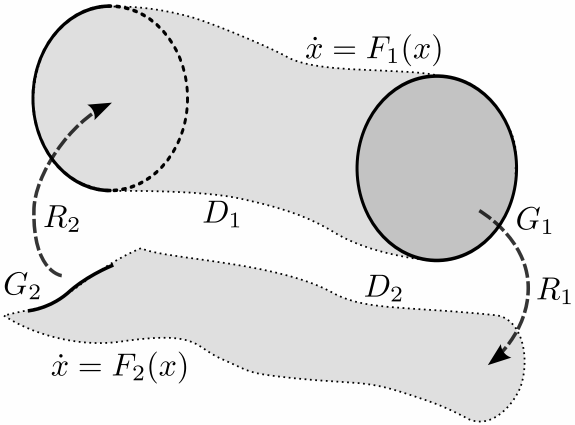
reduced-order model

< 10 DOF

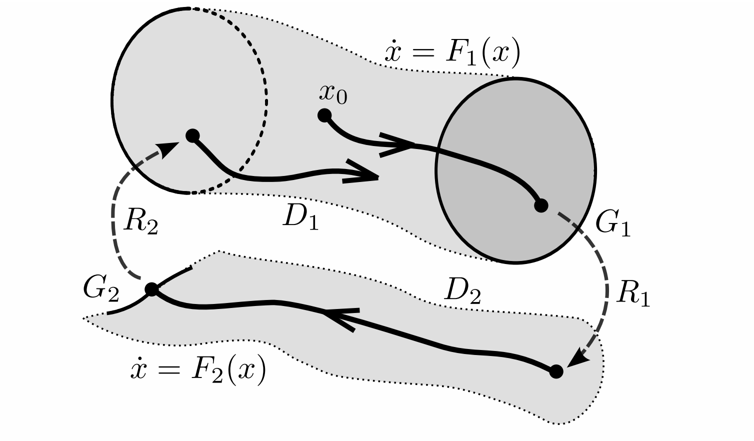
Example (vertical hopper)



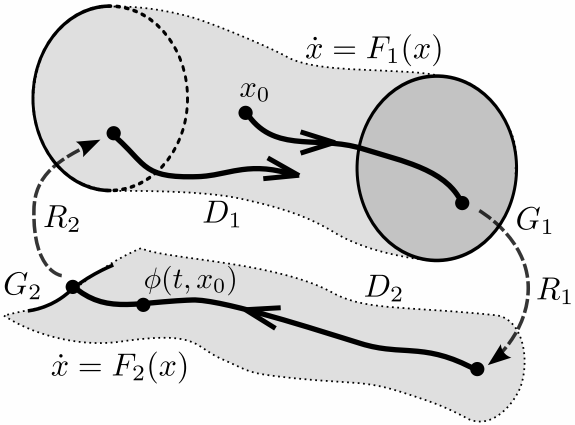
Hybrid dynamical system



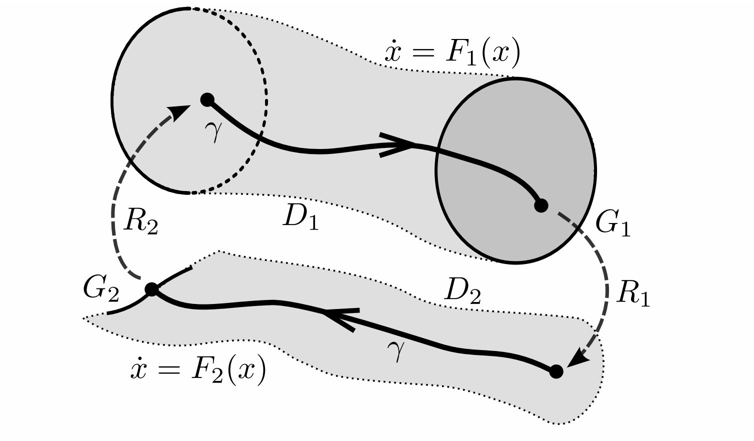
Trajectory for a hybrid dynamical system



Trajectory for a hybrid dynamical system



Periodic orbit γ for a hybrid dynamical system



Assumptions on hybrid periodic orbit γ

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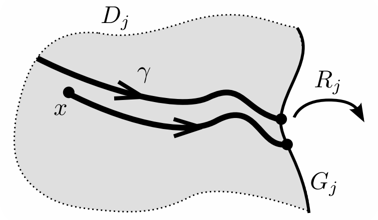
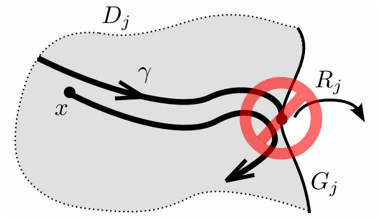
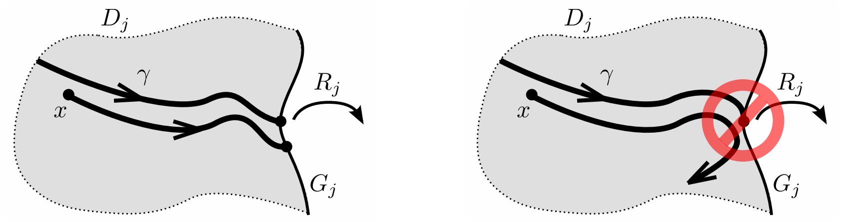




Assumption (transversality)

periodic orbit γ *passes transversely through each guard* G_j

Assumptions on hybrid periodic orbit γ



Assumption (transversality)

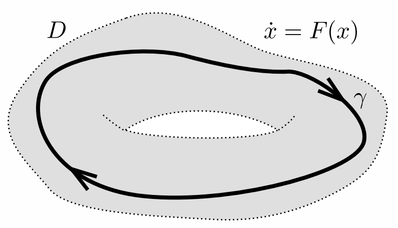
periodic orbit γ passes transversely through each guard G_j

Assumption (dwell time)

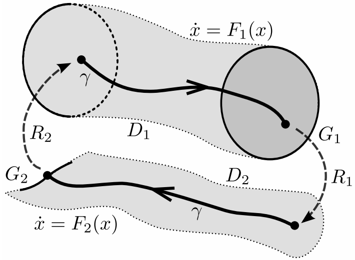
$\exists \varepsilon > 0$: *periodic orbit γ spends at least ε time units in each domain D_j*

Poincaré map for periodic orbit γ

smooth dynamical system



hybrid dynamical system



Poincaré map for periodic orbit γ

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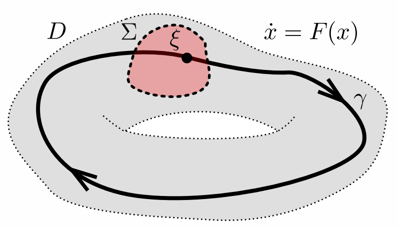
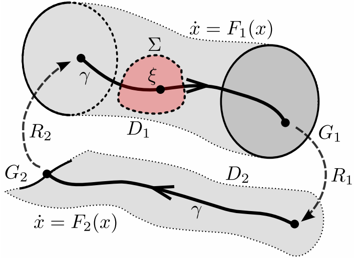


hybrid dynamical system



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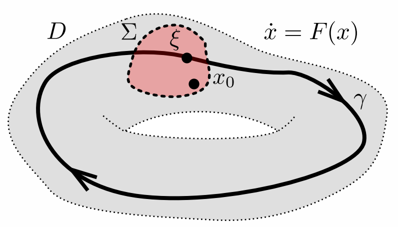

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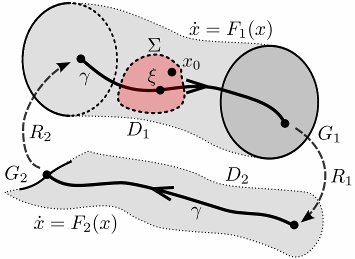

Poincaré map for periodic orbit γ

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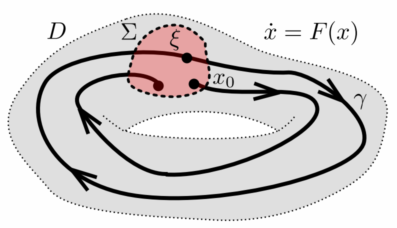

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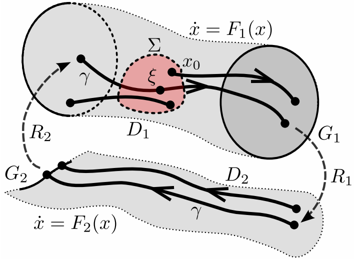

Poincaré map for periodic orbit γ

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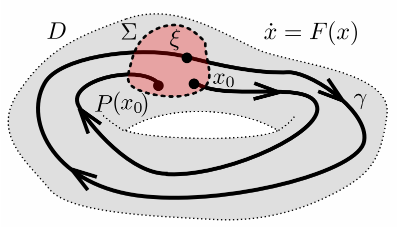
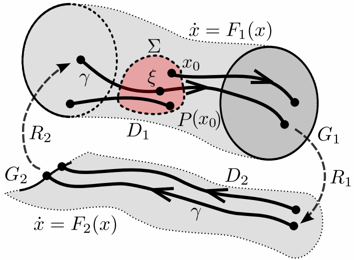


hybrid dynamical system



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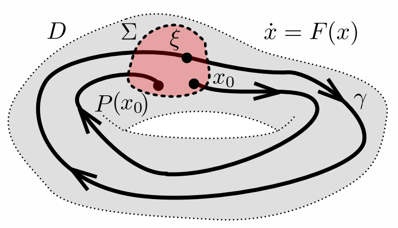
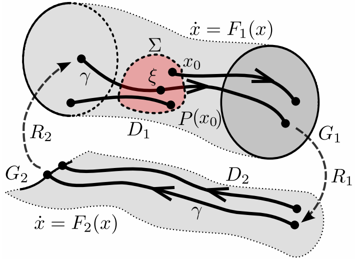


hybrid dynamical system

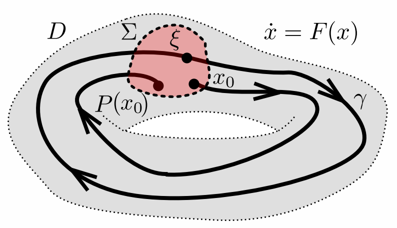


Theorem (Hirsch and Smale 1974, Grizzle et al. 2002)

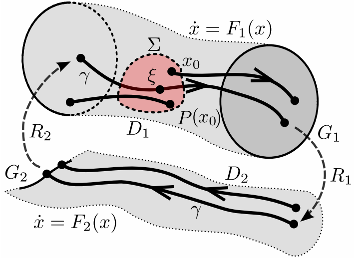
The Poincaré map P is smooth in a neighborhood of ξ.

Rank of Poincaré map P with fixed point $P(\xi) = \xi$

smooth dynamical system



hybrid dynamical system



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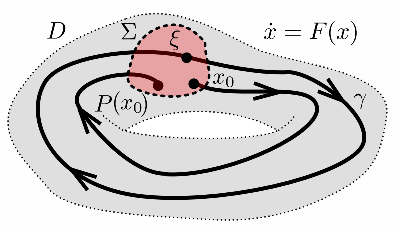

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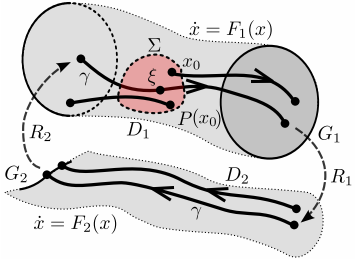
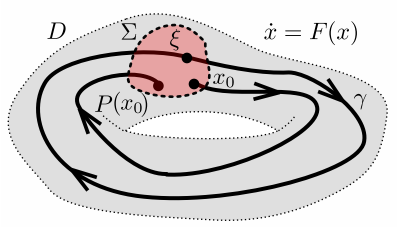


$$\operatorname{rank} DP(\xi) = \dim D - 1$$

Hirsch and Smale 1974

Rank of Poincaré map P with fixed point $P(\xi) = \xi$

smooth dynamical system



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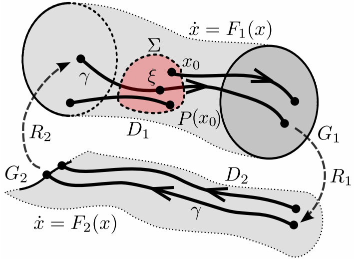
Hirsch and Smale 1974

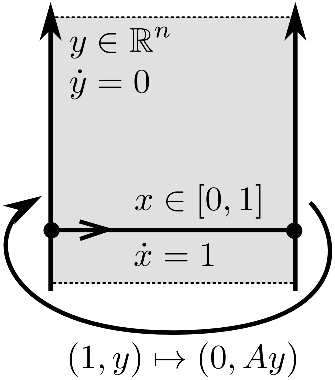
hybrid dynamical system



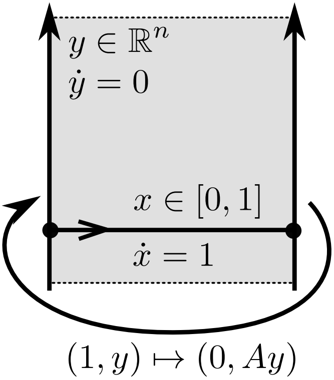
$\operatorname{rank} DP(\xi) \leq \min_j \dim D_j - 1$

Wendel and Ames 2010

Example (rank-deficient Poincaré map)

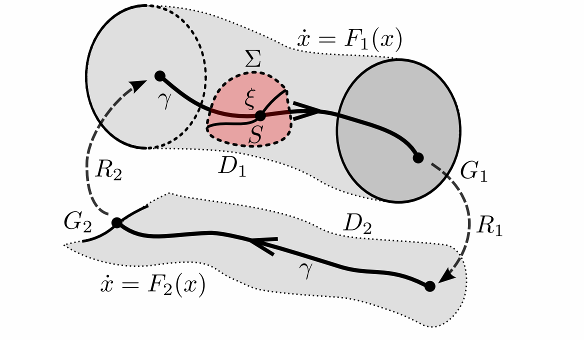


Example (rank-deficient Poincaré map)



If $A \in \mathbb{R}^{n \times n}$ is nilpotent (i.e. $A^n = 0_{n \times n}$), then $\operatorname{rank} DP^n = 0$.

Model reduction near hybrid periodic orbit γ



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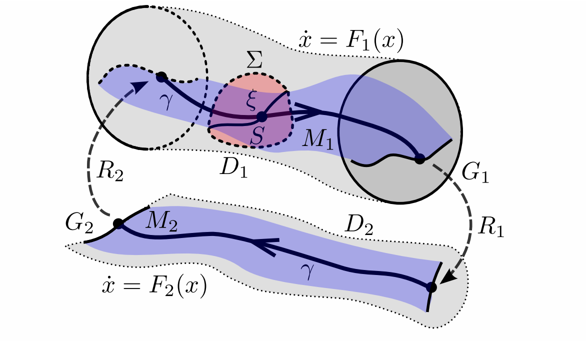
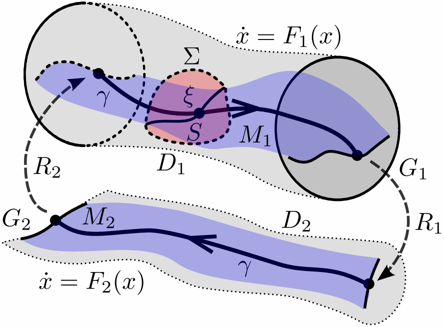


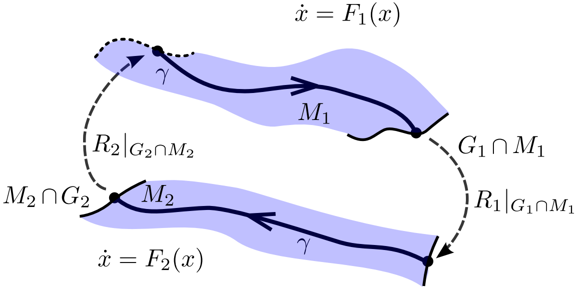
Model reduction near hybrid periodic orbit γ



Theorem (Burden, Revzen, Sastry CDC 2011)

Let $n = \min_j \dim D_j - 1$. *If* $\operatorname{rank} DP^n = r$ *near* ξ, *then trajectories starting near* γ *contract to a collection of hybrid-invariant* $(r+1)-$*dimensional submanifolds* $M_j \subset D_j$ *in finite time.*

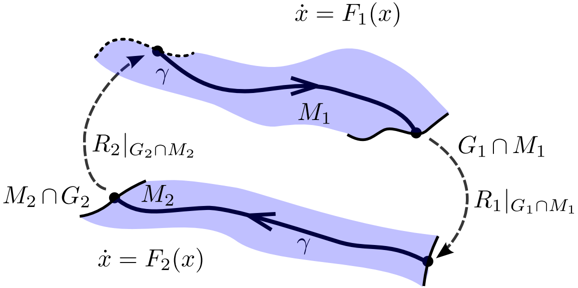
Model reduction near hybrid periodic orbit γ



Corollary (Burden, Revzen, Sastry CDC 2011)

The submanifolds M_j determine a hybrid system with periodic orbit γ.

Model reduction near hybrid periodic orbit γ



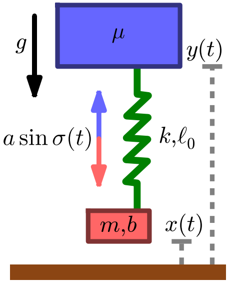
Corollary (Burden, Revzen, Sastry CDC 2011)

The submanifolds M_j determine a hybrid system with periodic orbit γ.

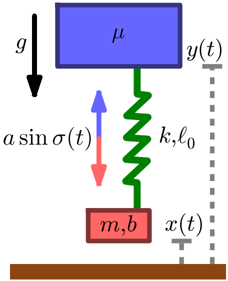
γ *is asymptotically stable in the original hybrid system*

$\iff$ γ *is asymptotically stable in the reduced hybrid system.*

Example (exact model reduction in vertical hopper)



Example (exact model reduction in vertical hopper)



Numerically linearizing Poincaré map P on ground,
we find $DP(\xi)$ has eigenvalues $\simeq -0.25 \pm 0.70j$,
therefore DP^2 is constant rank near ξ.

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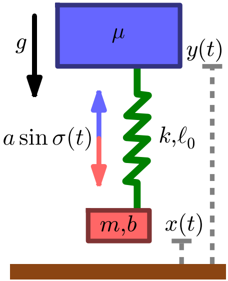

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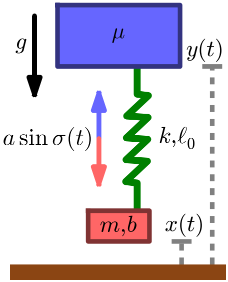

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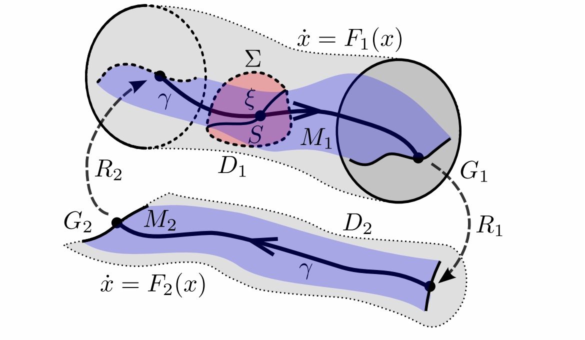
Interpretation: unilateral (Lagrangian) constraint appears after one "hop"

Approximate model reduction near hybrid periodic orbit γ



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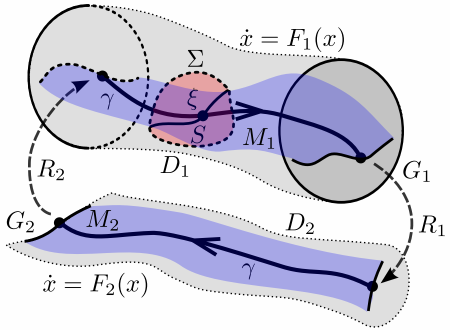
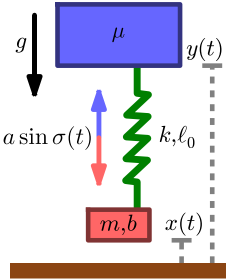


Theorem (Burden, Revzen, Sastry *(in preparation)*)

If ξ is exponentially stable and $\operatorname{rank} DP^n(\xi) = r$, then trajectories starting near γ contract super-exponentially to a collection of hybrid-invariant $(r+1)-$dimensional submanifolds $M_j \subset D_j$.

Example (structural stability in vertical hopper)



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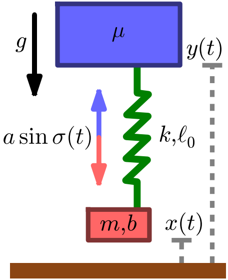

There exists a deadbeat control for vertical hopper,

i.e. smooth actuator feedback law $a(x, y, \dot{x}, \dot{y})$ such that hopper exactly tracks periodic orbit after one "hop"

Carver, Cowen, & Guckenheimer 2009

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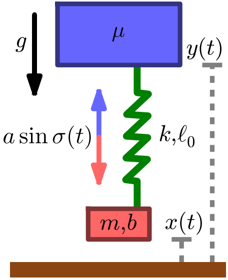

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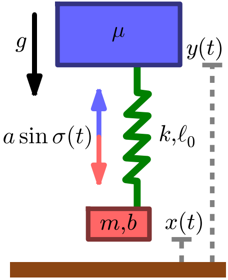

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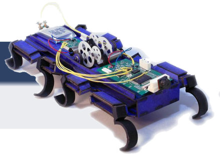
Theorem $\implies$ hopper contracts to periodic orbit at rate bounded by magnitude of perturbation

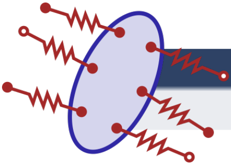
Identification

physical system

animal, robot

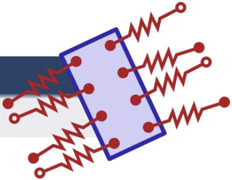


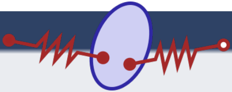
identification



detailed model

10–100 DOF

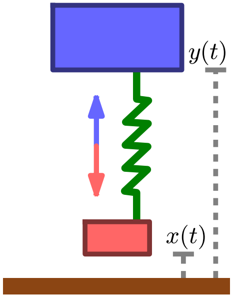




reduced-order model

< 10 DOF

Identification of initial conditions



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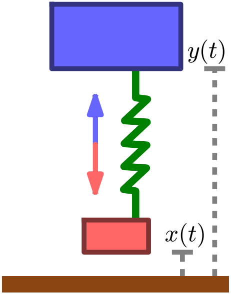

$$Y(\phi(t, z)) = y(t)$$

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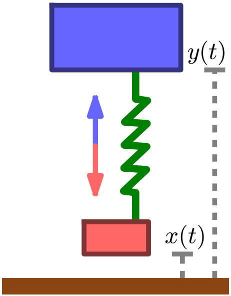

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$\eta_i = Y(\phi(iT, z^*)) + w_i$,
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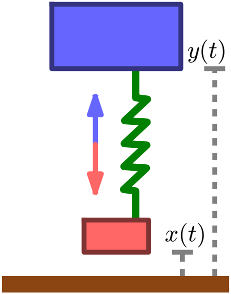

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Identification problem

Solve $\arg\min_{z \in D_j} \varepsilon(z, \{\eta_i\})$, where $\varepsilon(z, \{\eta_i\}) := \sum_i \|Y(\phi(iT, z)) - \eta_i\|^2$.

Identification on reduced hybrid model

Assumption (smooth observations)

Y is smooth along trajectories, i.e. $Y(\phi(t,z))$ is a smooth function of t.

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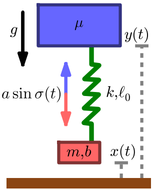
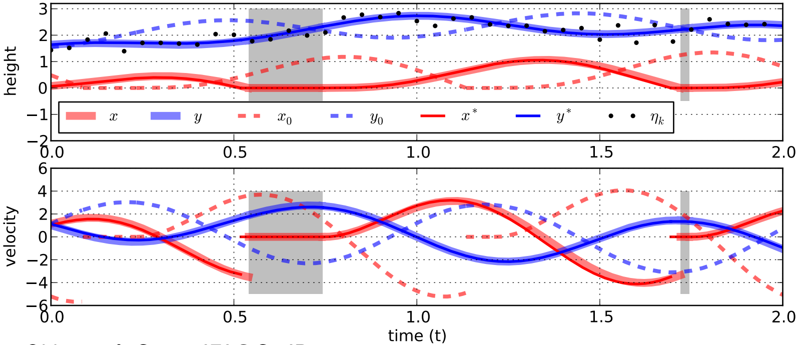
first-order algorithms applicable

Example (initial condition for vertical hopper)



Observe position of upper mass at 20Hz, additive noise with variance 0.2.

$$(\sigma_0, y_0, \dot{y}_0) \approx (8.0, 1.5, 1.1) : \text{initial}$$
$$(\sigma, y, \dot{y}) \approx (4.7, 1.6, 1.0) : \text{actual}$$
$$(\sigma^*, y^*, \dot{y}^*) \approx (4.6, 1.6, 1.1) : \text{estimated}$$

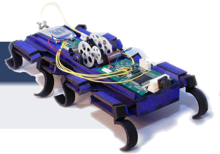


Burden, Ohlsson, & Sastry IFAC SysID 2012

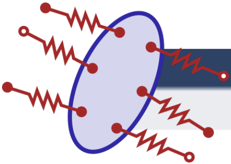
Reduction & Identification

physical system

animal, robot

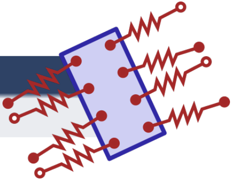


identification

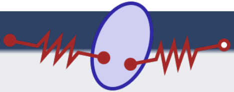


detailed model

10–100 DOF



reduction



reduced-order model

< 10 DOF

Novel quantitative predictions for biomechanics

observation

neural feedback
appears at a delay

Revzen, Burden et al. 2013

↕ identification

prediction

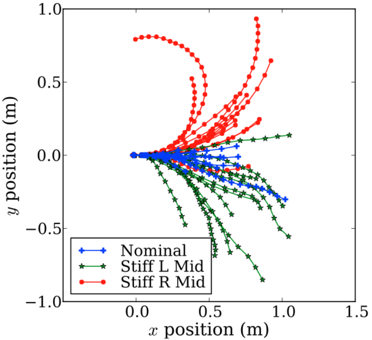
passive mechanics
sensitive to inertia

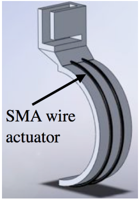
Full et al. 2002

Burden, Revzen, Moore, Sastry, & Full SICB 2013

lateral perturbation

Model-based design and control of dynamic robots

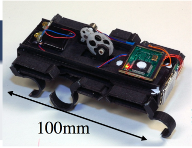




design

minimal use of actuators

Hoover et al. 2010



identification

control

asymmetric leg stiffness change

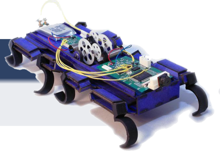
Proctor & Holmes 2008

Hoover, Burden, Fu, Sastry, & Fearing IEEE BIOROB 2010

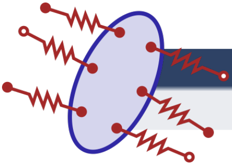
Model enables translation across morphology, scale

physical system

animal, robot

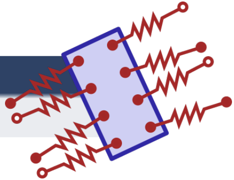


↓ identification ↓

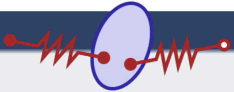


detailed model

10–100 DOF



↓ reduction ↓



reduced-order model

< 10 DOF

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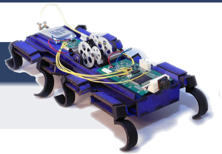

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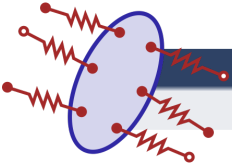

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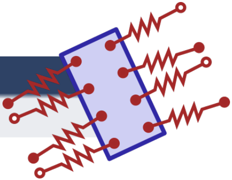
10–100 DOF



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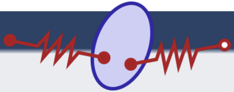


reduced-order model

< 10 DOF

Discussion & Questions — Thanks for your time!

Reduction

Hybrid dynamics reduce dimensionality near periodic orbits.

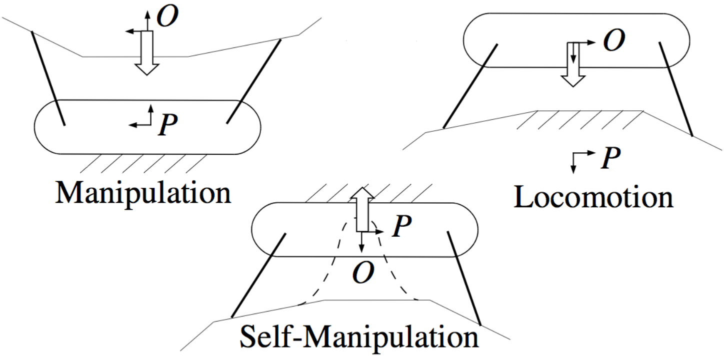
Identification

Reduction enables scalable algorithm for parameter estimation.

Collaborators

- Prof. Shankar Sastry
- Prof. Henrik Ohlsson
- Prof. Robert Full
- Prof. Aaron Hoover
- Prof. Shai Revzen
- Talia Moore

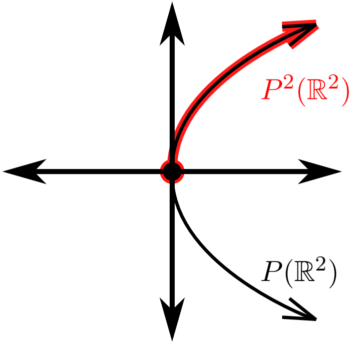
Locomotion is self-manipulation



Johnson, Haynes, & Koditschek, IROS 2012

Example ($\operatorname{rank} DP^n$ generically non-constant)

$$P(x, y) = (x^2, x)$$



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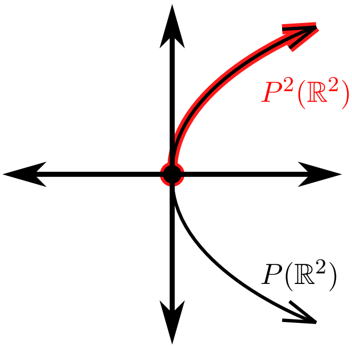

$$DP(x,y) = \begin{pmatrix} 2x & 0 \\ 1 & 0 \end{pmatrix}$$

$$\implies \operatorname{rank} DP = 1$$

$$DP^2(x,y) = \begin{pmatrix} 4x^3 & 0 \\ 2x & 0 \end{pmatrix}$$

$$\implies \operatorname{rank} DP^2(x,y) = \begin{cases} 0, & x = y = 0 \\ 1, & else \end{cases}$$

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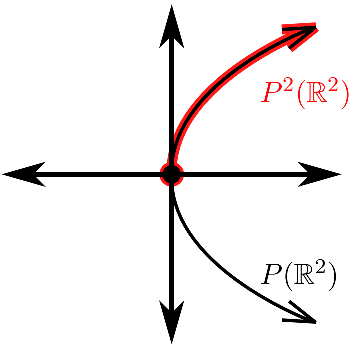

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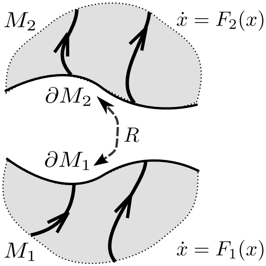
Note that P contracts superexponentially since $DP(0,0)$ is nilpotent: for all $\varepsilon > 0$ there exists $\delta > 0$ and $\|\cdot\|_\varepsilon$ such that

$$\|(x,y)\| < \delta \implies \|P(x,y)\| < \varepsilon \|(x,y)\|_\varepsilon$$

Gluing smooth dynamical systems



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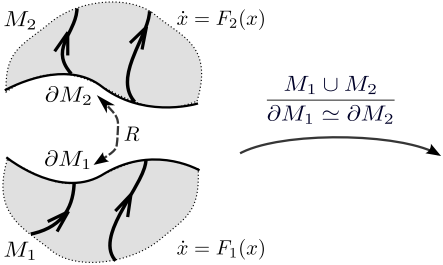

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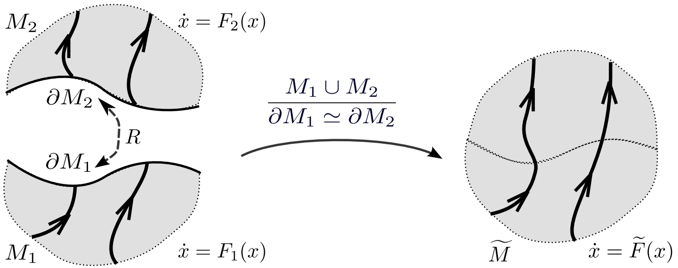

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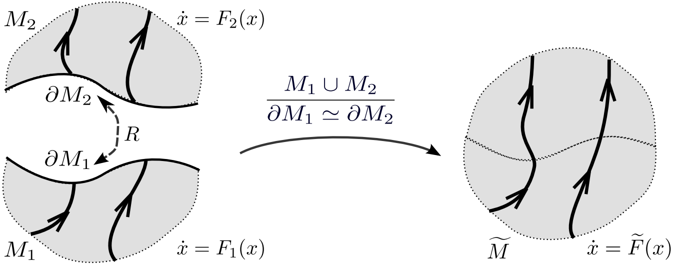


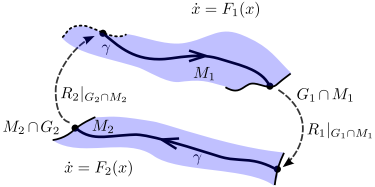
Lemma (Hirsch 1976)

Let F_j be a smooth vector field on n-dimensional manifold M_j, $j \in \{1, 2\}$. If $R : \partial M_1 \to \partial M_2$ is a diffeomorphism, F_1 points outward on ∂M_1, and F_2 points inward on ∂M_2, then the quotient $\widetilde{M} = \frac{M_1 \cup M_2}{\partial M_1 \simeq \partial M_2}$ is a smooth manifold, $M_j \subset \widetilde{M}$ is a smooth submanifold, and the vector field

$$\widetilde{F}(x) = \begin{cases} F_1(x), & x \in M_1; \\ F_2(x), & x \in M_2; \end{cases}$$

is smooth on $\widetilde{M}$.

Smoothing reduced-order hybrid system



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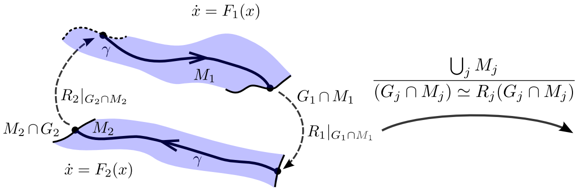

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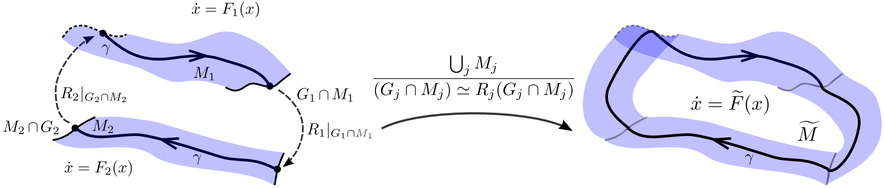

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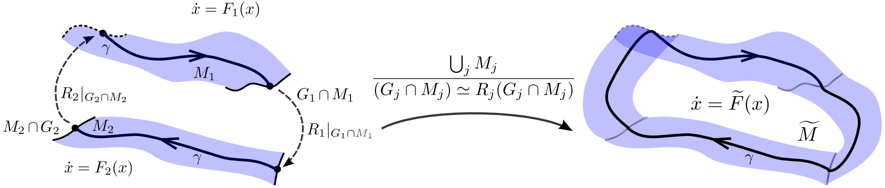

Corollary (Burden, Revzen, Sastry CDC 2011)

The topological quotient $\widetilde{M} = \frac{\bigcup_j M_j}{(G_j \cap M_j) \simeq R_j(G_j \cap M_j)}$ *is a smooth manifold,* $M_j \subset \widetilde{M}$ *is a smooth submanifold, and the vector field*

$$\widetilde{F}(x) = \begin{cases} F_1(x), & x \in M_1; \\ \vdots & \vdots \\ F_j(x), & x \in M_j; \\ \vdots & \vdots \end{cases}$$

is smooth on $\widetilde{M}$*.*