On State Estimation for Hybrid Abstractions of Legged Locomotion

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Motivating Examples

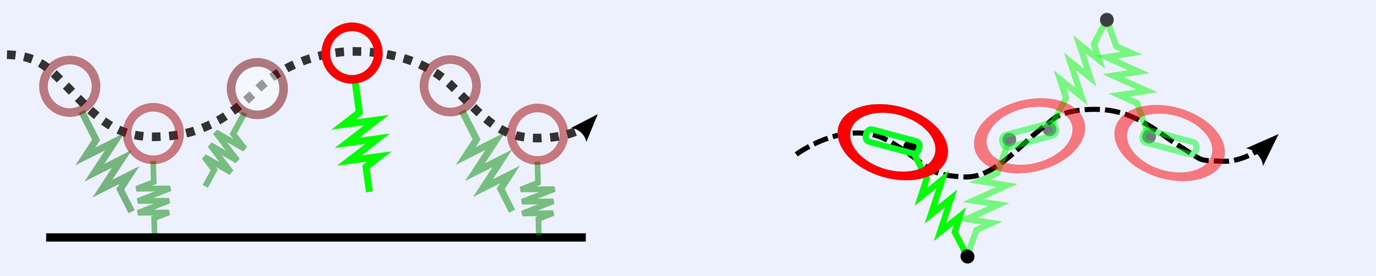
Legged locomotion is a fundamentally hybrid phenomenon. Low-order dynamical abstractions for locomotion can inform design and control of running robots [1,3,4] and generate hypotheses about neuromechanical control in legged animals [2]. To achieve these goals, we must identify parameters and estimate states in low-order hybrid models using data collected from a physical system.

Running Robot

Fig 1: **Milliscale hexapedal robot** [1]
10cm (L) x 6.5cm (W) x 3.5cm (H)
6mm DC motor; 90 mAHr LiPoly batt.
40 MIPS PIC; 230 Kbps Bluetooth radio

- Robot dynamics are poorly understood; need hybrid system ID tools to improve design and control

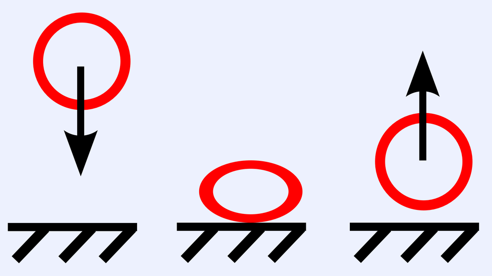
Hybrid Abstractions of Legged Locomotion



Vertical Plane: **SLIP** [3] Horizontal Plane: **LLS** [4]

Bouncing Ball $\ddot{h} = \begin{cases} -g & \text{flight} \\ -g + \partial_h V(h) + B(\dot{h}) & \text{bounce} \end{cases}$

V = elastic potential
B = damping



- Bounce time decreases as stiffness increases, complicating estimation

Abstraction for Intermittent Contact

Running Cockroach

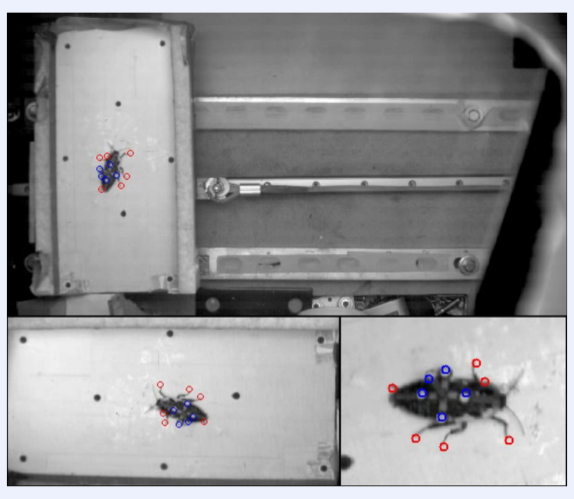
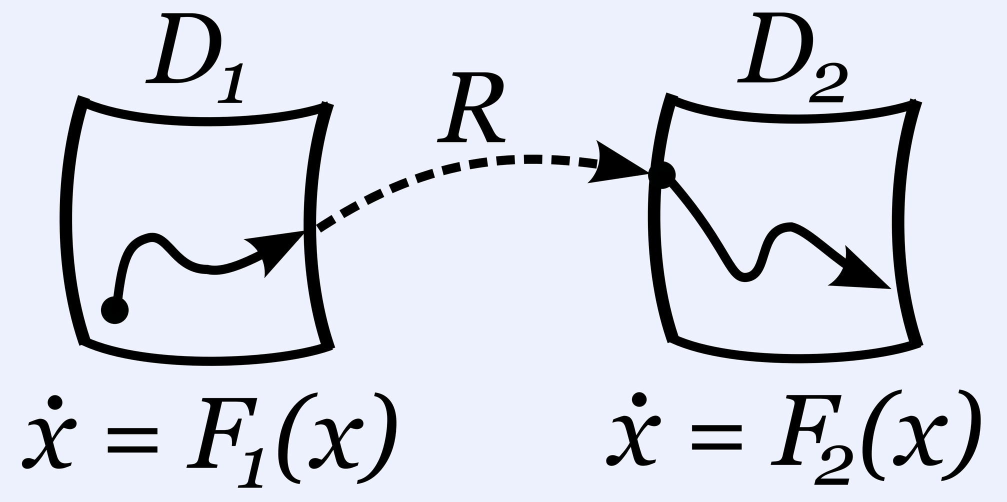


Fig 2: **Lateral Perturbation** of a running cockroach [2], tracking rigid body and foot state in moving frame

- Probing neuromechanical control architecture in legged animals is vastly simplified using predictions from low-order hybrid models of locomotion

Hybrid Dynamical Systems

Hybrid Dynamical System $H := (Q,D,F,R)$:



$Q \subset \mathbb{N}$ is a finite set indexing the discrete modes;
$D = \bigcup_{q \in Q} D_q$ where each D_q is an n-dimensional differentiable manifold with boundary ∂D_q;
$F = \{F_q\}_{q \in Q}$ is a set of vector fields which generate the flow in each discrete mode, i.e. $\dot{x} := F_q(x),\ x \in D_q$;
$R = \{R_{pq}\}_{p,q \in Q}$ is a set of diffeomorphisms which move the state between domains, i.e. $R_{pq} : \partial D_p \to \partial D_q$.

Standard results ensure existence and uniqueness of the hybrid flow $\Phi : \mathbb{R} \times D \to D$

Stochastic Hybrid Dynamical Systems

Injecting uncertainty into a hybrid abstraction enables Bayesian inference for state estimation:

$$x_{k+1} = \Phi_T(x_k),\ y_k = h(x_k) \longrightarrow x_{k+1} = \tilde{\Phi}_T(x_k, w_k),\ y_k = \tilde{h}(x_k, v_k),\ w_k \sim \mathcal{N}(0,W),\ v_k \sim \mathcal{N}(0,V)$$

Note that additive noise is not generally applicable due to possible nonlinearity of hybrid domains.

State Estimation for Hybrid Dynamical Systems

Popular approaches to state estimation for hybrid dynamical systems can be unsuitable or perform poorly when applied to the family of models for intermittent contact described above.

Kalman Filter [5]

- Requires linear dynamics
- Lags during hybrid transitions

$$\begin{aligned} x_{k+1} &= F x_k + w_k,\ w_k \sim \mathcal{N}(0,W) \\ y_k &= H x_k + v_k,\ v_k \sim \mathcal{N}(0,V) \end{aligned}$$

Piecewise Affine Identification [6]

- Requires piecewise affine state space

$$y_k = \theta_q^T \begin{pmatrix} x_k \\ 1 \end{pmatrix} + w_k,\ x_k \in D_q,\ w_k \sim \mathcal{N}(0,W)$$

Interacting Multiple Model Filter [7]

- Typically requires linear dynamics and state-independent hybrid transitions

$$\begin{aligned} x_{k+1} &= F_{q_k} x_k + w_k,\ w_k \sim \mathcal{N}(0,W) \\ y_k &= H_{q_k} x_k + v_k,\ v_k \sim \mathcal{N}(0,V) \\ q_k &\sim \text{a Markov chain} \end{aligned}$$

Particle Filter [8]

- Sample impoverishment is problematic for transient hybrid modes (e.g. *bounce*)

$$\begin{aligned} x_{k+1} &= f(x_k, q_k, w_k),\ w_k \sim \mathcal{N}(0,W) \\ q_{k+1} &= g(x_k, q_k, u_k),\ u_k \sim \mathcal{N}(0,U) \\ y_k &= h(x_k, q_k, v_k),\ v_k \sim \mathcal{N}(0,V) \end{aligned}$$

Future Directions

Much work remains to create an efficient state estimation framework applicable to hybrid systems possessing nonlinear dynamics and deterministic mode transitions. In particular, we advocate developing methods which are agnostic to hybrid transitions.

References and further reading

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