On State Estimation for Hybrid Abstractions of Legged Locomotion
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Motivating Examples

Legged locomotion is a fundamentally hybrid phenomenon. Low-order dynamical abstractions for locomotion can inform design and control of running robots [1,3,4] and generate hypotheses about neurobiomechanical control in legged animals [2]. To achieve these goals, we must identify parameters and estimate states in low-order hybrid models using data collected from a physical system.

Hybrid Dynamical Systems

Hybrid Dynamical Systems $H := (Q,D,F,R)$:

- $Q \subset \mathbb{R}$ is a finite set indexing the discrete modes;
- $D := \cup_{q \in Q} D_q$ where each $D_q$ is an $n$-dimensional differentiable manifold with boundary $\partial D_q$;
- $F := (F_q)_{q \in Q}$ is a set of vector fields which generate the flow in each discrete mode, i.e. $x := F_q(x), \ x \in D_q$;
- $R := (R_{pq})_{p,q \in Q}$ is a set of diffeomorphisms which move the state between domains, i.e. $R_{pq} : D_p \rightarrow D_q$.

Standard results ensure existence and uniqueness of the hybrid flow $\Phi : \mathbb{R} \times D \rightarrow D$.

Stochastic Hybrid Dynamical Systems

Injecting uncertainty into a hybrid abstraction enables Bayesian inference for state estimation:

$x_{n+1} = \Phi(x_n), \ y_n = h(x_n) \rightarrow x_{n+1} = \Phi(x_{n+1}), \ y_n = h(x_{n+1})$, $x_n \sim \mathcal{N}(0,W), \ y_n \sim \mathcal{N}(0,V)$

Note that additive noise is not generally applicable due to possible nonlinearity of hybrid domains.

State Estimation for Hybrid Dynamical Systems

Popular approaches to state estimation for hybrid dynamical systems can be unsuitable or perform poorly when applied to the family of models for intermittent contact described above.

Kalman Filter [5]

- Requires linear dynamics
- Lags during hybrid transitions

$x_{n+1} = F_{x_n}x_n + w_n, \ w_n \sim \mathcal{N}(0,W)$
$y_n = H_{x_n}x_n, \ x_n \sim \mathcal{N}(0,V)$

Piecewise Affine Identification [6]

- Requires piecewise affine state space

$y_n = p\begin{bmatrix} x_n \end{bmatrix} + w_n, \ x_n \in D_p, \ w_n \sim \mathcal{N}(0,W)$

Interacting Multiple Model Filter [7]

- Typically requires linear dynamics and state-independent hybrid transitions

$x_{n+1} = F_{x_n}x_n + w_n, \ w_n \sim \mathcal{N}(0,W)$
$y_n = H_{x_n}x_n, \ x_n \sim \mathcal{N}(0,V)$
$q_n \sim $ a Markov chain

Particle Filter [8]

- Sample impoverishment is problematic for transient hybrid modes (e.g. bounce)

$x_{n+1} = f(x_n, q_n, w_n), \ w_n \sim \mathcal{N}(0,W)$
$y_n = g(q_n, y_n, w_n), \ y_n \sim \mathcal{N}(0,U)$
$q_n \sim \mathcal{L}(q_n, y_n), \ y_n \sim \mathcal{N}(0,V)$

Future Directions

Much work remains to create an efficient state estimation framework applicable to hybrid systems possessing non-linear dynamics and deterministic mode transitions. In particular, we advocate developing methods which are agnostic to hybrid transitions.

References and further reading


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