Pinned equilibria provide robustly stable multilegged locomotion

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I. Motivation

Models of terrestrial legged locomotion are almost invariably hybrid, containing a discontinuity in the equations of motion when feet land on the ground. While some animals are bipedal, the great majority of legged locomotion on land is done with 4, 6 or more legs. Poly-pedal gaits for moderate or high-speed locomotion usually include multiple legs striking the ground that once (or nearly at once, on average). Familiar examples include trotting and pacing in mammalian quadrupeds, and the ubiquitous alternating tripod gaits in the insect world. We will present several arguments for why such simultaneous touchdowns should not be observed in nature, leading to the conclusion that observing them requires that a unique form of robust stability is associated with such behaviors. We proceed to present a mathematical formulation of such a novel form of stability, demonstrate sufficient conditions for its existence, and show that it can appear in fairly pedestrian models of locomotion.

II. State of the Art

Conventional wisdom has it that intersecting guards in hybrid systems can cause substantial difficulties in the analysis of the dynamics, and thus most models of poly-pedal gates either assume that legs touch down at very different times or reduce legs to a single virtual leg, sidestepping the issues associated with guard intersection.

Several researchers have offered arguments for why we observed the gaits we do in the natural world [1], [2], [3]. Arguments based on collision angle tend to favor collisions to be equally distributed throughout the gait cycle – similar to rotary gallop – thereby minimizing the energy loss introduced by such collisions [4], [5]. From a "first principles" standpoint, each ground contact guard is a co-dimension 1 manifold, and thus the intersection of multiple guards is an object of co-dimension 2 or higher. Generic objects of co-dimension 2 would almost never intersect objects of dimension 1 such as a limit cycle.

For all these reasons the fact that we commonly observe multiple simultaneous touchdowns is both surprising and difficult to analyze.

III. Own Approach

We demonstrate that under plausible conditions for animal gaits, the intersecting hybrid transitions would not exhibit many of the problems that hindered a general mathematical theory of such systems. For the special case that remains, we demonstrate a simple theory which gives rise to what may well be a normal form for such systems. Generally speaking, the return map of such systems will be continuous but not differentiable, although in particularly simple cases differentiability can be shown.

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IV. Current Result

A fundamentally new, and potentially important kind of stability appears, which we have named *pinned equilibria*. Such equilibria are pinned to a neighborhood of the intersection of the guards, and resist changes to the system dynamics. Of particular interest to the study of locomotion is the observation that reflexes would place the guards in locations governed by sensory accuracy, and pinned equilibria would make the dynamics robust with respect to limb force production uncertainty – a seemingly ideal situation for locomotion.

V. Best Possible Outcome

We hope to develop a complete theory of pinned equilibria, including a thorough understanding of their behavior under uncertainties in the dynamics, the structure and arrangement of the guards, and changes in the number of guards participating in the intersection. We hope our current attempts at applying controllers based on pinned equilibria to multi-legged robot gait will achieve substantially improved performance compared with previously used controllers. Potentially, pinned equilibria might provide a fundamental new tool in the arsenal of controller designers.

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References

[1] R. Alexander and A. Jayes. A dynamic similarity hypothesis for the gaits of quadrupedal mammals. *Journal of zoology* 201(1):135–152, 1983.

[2] R. M. Alexander. Why mammals gallop. *Am Zool* 28(1):237–245, 1988.

[3] M. Golubitsky, I. Stewart, P. L. Buono, and J. J. Collins. Symmetry in locomotor central pattern generators and animal gaits. *Nature* 401(6754):693–695, 1999.

[4] J. Bertram, A. Ruina, C. Cannon, Y. Chang, and M. Coleman. A point-mass model of gibbon locomotion. *J Exp Biol* 202(19):2609–2617, OCT 1999.

[5] A. Ruina, J. Bertram, and M. Srinivasan. A collisional model of the energetic cost of support work qualitatively explains leg sequencing in walking and galloping, pseudo-elastic leg behavior in running and the walk-to-run transition. *J Theo Biol* 237(2):170–192, NOV 21 2005.