Near–Simultaneous Footfalls Lend Stability to Multi–Legged Gaits

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1 Motivation and State–of–the–Art

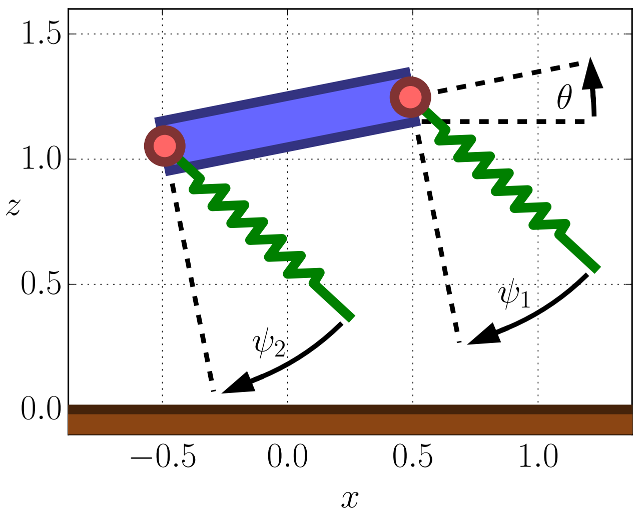
Parsimonious dynamical models for legged locomotion are piecewise–defined. The state flows in continuous–time according to an ordinary differential equation (ODE) until a *touchdown* or *liftoff* event occurs, triggering an instantaneous reset at a discrete time instant [1]. We've shown that models for periodic gaits with footfalls isolated in time (e.g. bipedal *walk* or *run*, quadrupedal *walk* or *gallop*) reduce to classical dynamical systems—smooth ODEs on smooth manifolds [2]. However, footfalls are not isolated for general behaviors.

During other typical gaits (*pace*, *trot*, *hop*, *pronk*, *alternating tripod*), transitions between gaits, or aperiodic maneuvers, footfall events can occur at arbitrary times with respect to one another. From a modeling perspective, this can introduce ambiguities or inconsistencies. In models with rigid limbs, for instance, others [3, 4] have observed that the dynamical system flow is generically discontinuous near simultaneous–footfall events. This implies that the model's behavior has unbounded sensitivity with respect to initial conditions. Though potentially relevant for rigidly–constructed artifacts, such singular model behavior provides a poor prediction for the dynamics of physical locomotors—biological or robotic—with viscoelastic limbs.

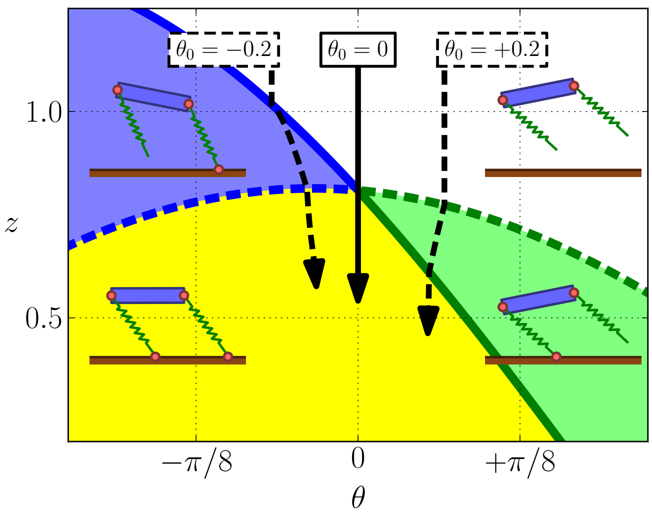
2 Our Approach and Results

We study models for locomotion with *viscoelastic* limbs as in Fig. 1a. The flow of such systems is generated by a vector field that is *event–selected* C^r [5, Def. 1], i.e. smooth except along a finite number of surfaces of discontinuity as in Fig. 1b. This flow is continuous but not classically differentiable; however, we show in [5] that it admits a strong first–order approximation (a *Bouligand* derivative [6]) that enables analytical characterization of contractive effects arising near simultaneous–footfall events.

We show that, in contrast to the case with rigid limbs, the flow of a multi–legged model with viscoelastic limbs is generally *contractive* near simultaneous–footfall events [5]. This implies that near–simultaneous touchdown lends stability to multi–legged gaits and maneuvers. For periodic gaits wherein limbs partition into groupings that touch down nearly simultaneously (*pace*, *trot*, *pronk*, *alternating tripod*), this provides a first–order synchronizing effect between limbs within a group. Furthermore, whenever a group of limbs undergoes near–simultaneous touchdown, they introduce a first–order stabilizing effect to one or more body posture degrees–of–freedom (DOF). In abstracted models of locomotion, these effects alone can stabilize a gait. In a more general setting, these effects can be exploited to lend stability to a behavior.



(a) Illustration of multi–legged model with three body degrees–of–freedom (position (x, z) and pitch θ) and two leg angle states ψ_1, ψ_2. Limbs attach to the body and ground through pin joints.



(b) Trajectories undergoing near–simultaneous footfall (initialized with pitch angle $\theta_0 = \pm 0.2$) contract toward simultaneous–footfall trajectory ($\theta_0 = 0$).

Figure 1: In a viscoelastic–limb model (a), discontinuous dynamics near a simultaneous–footfall event (b) lend stability to body pitch (θ) and synchronicity to limbs (ψ_1, ψ_2) .

Notes and Support

We presented an earlier version of these results at Dynamic Walking 2013 [7]. In the intervening years, we made substantial progress on both the underlying theory and the practical relevance for design and analysis of multi–legged locomotion.

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