

## History and current status of a physiologically based system of photometry and colorimetry

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The CIE chromaticity diagram, which has been in common use for more than 60 years, disguises essential relations among cone excitations that become transparent in a system developed with D. I. A. MacLeod and initially proposed by the author to the CIE in 1979. This proposal led to the formation of a CIE committee to consider an ideal version of the system, to be employed either as a supplement to, or an alternative for, the 1931 “standard observer.” After 15 years, the task remains unfinished. The history of debate within the original committee and that of its successor (which is still active today) is briefly reviewed. Among cone fundamentals that might be chosen, a set derived and published by Stockman, MacLeod, and Johnson [*J. Opt. Soc. Am. A* **10**, 2491 (1993)] is favored here, and some of the advantages for displaying visual data in a system based on these fundamentals are illustrated. (The paper is based on the 1995 OSA Frederick Ives Medal address.) © 1996 Optical Society of America.

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### 1. INTRODUCTION

In 1924 the International Commission on Illumination (CIE) quantified the meaning of light by establishing a luminous-efficiency function, and in 1931 it sanctioned a system of colorimetry that includes the familiar  $x$ ,  $y$  CIE chromaticity diagram.<sup>1,2</sup> Light and color are to this day defined by these ancient prescriptions. Although they have served well enough for many purposes, the system based on the CIE standard observer nevertheless leaves something to be desired.

#### A. Photopic Luminous Efficiency Function

The photopic luminous-efficiency function,  $V$ -lambda, was pieced together by use of data from four studies, some based on brightness matching, others on flicker photometry (see Fig. 1). Brightness matching may seem to be the more obvious approach, but the procedure yields data that are unsuitable as a basis for photometry, partly because brightness matches for colors that differ are difficult and variable, but more importantly because they lead to serious additivity failures.<sup>3-6</sup>

Imagine that you are asked to match a white field ( $W$ ) on the left with a red field ( $R$ ) on the right by varying the radiance of the red one. Although the fields look so different that the match is difficult, you do it a few times, and we take an average. Then the red field is replaced with a green one ( $G$ ), and another match with  $W$  is made. Now we optically superimpose the two, which yields  $2W$  on the left and  $R + G$  on the right, and you can see immediately that the white field looks much brighter. It turns out that the cancellation of hue and saturation that occurs when red and green are mixed, to give a yellow, carries with it a cancellation of brightness. You will have to decrease the radiance of the white field at the left by a factor of 2 or 3 to restore a brightness match—this is no small effect!

The flicker method of photometry, which alternates a field of one color with that of another at roughly 10–15 Hz, establishes the relative radiances of fields of various wavelengths that are needed to minimize the sensation of flicker when they are alternated in turn with an unchanging reference. Such settings are both easy and reliable. Moreover, the data of flicker photometry, unlike those of brightness matching, exhibit nearly additive, transitive, and associative properties. The flicker method therefore provides an experimental basis that legitimizes the computation of luminance by integration of the spectral distribution of a light stimulus with the flicker-photometric spectral sensitivity of the eye. Herbert Ives (who endowed the Ives medal in 1928 to honor his father Frederick) was one of the pioneers in the field of flicker photometry, and his 1912 data from a dozen subjects,<sup>7</sup> along with values from many observers from other studies, contributed to the function that was adopted.<sup>7-10</sup> However, as Fig. 1 shows, strange weights were assigned to the data from the contributing studies. The adopted curve follows the Coblentz–Emerson<sup>8</sup> flicker data quite closely down to approximately 480 nm; but at the shortest wavelengths the Hyde–Forsythe–Cady data,<sup>10</sup> obtained by step-by-step brightness matching, were accepted instead. Ives had previously shown that, for specific conditions including a 2-deg field of view, step-by-step and flicker data were in good agreement for the range of wavelengths that he tested, but these did not include the shortest wavelengths. (Since then, a 2-deg field has become standard for photometry and colorimetry.)

It became evident long ago that  $V$ -lambda was seriously in error in the short wavelengths. An amended version proposed in 1951 by Judd<sup>11</sup> has since been used extensively by vision scientists without CIE sanction until recently when it was given an “advisory status” by the CIE (CIE Report 86-1990) and labeled  $V_M(\lambda)$ . Judd’s correction is similar to what the Coblentz–Emerson flicker data suggest.

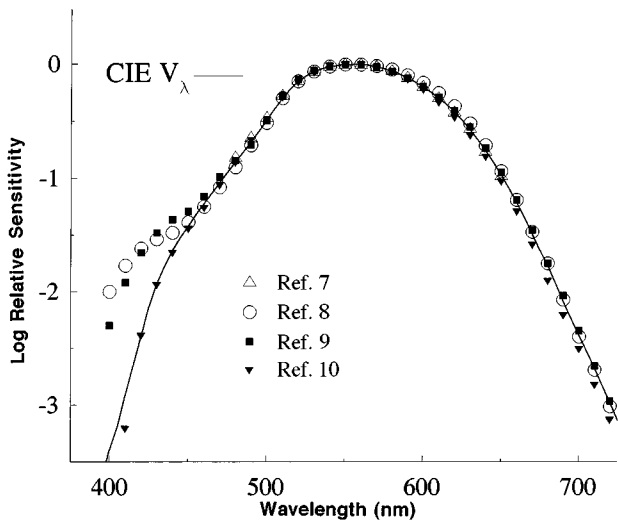


Fig. 1. Comparison of CIE luminous-efficiency function  $V_\lambda$  with the data of four studies upon which it was based. (Adapted from a figure provided by Andrew Stockman.) Open symbols, flicker photometry; filled symbols, step-by-step brightness matching.

**B. Color-Matching Experiment**

Because color-matching experiments provide the fundamental data for the CIE system of colorimetry, or any system of colorimetry for that matter, let us quickly review what is involved (see Fig. 2):

Monochromatic test lights to be matched are introduced, one at a time, into the upper half of the field—in this example, 480 nm, a greenish blue light. Three so-called primaries are chosen for the lower field; these are quite arbitrary, except that no two of them can match the third one. In this example they are 450 nm (blue, B), 550 nm (green, G), and 650 nm (red, R). The subject’s task is to adjust the amounts of these primaries by manipulating three knobs, each of which controls the intensity of a primary. The idea is, as the first equation indicates, given  $t$  units of the test light  $T$ —in this case 480 nm—to find the numbers of units  $r$ ,  $g$ , and  $b$  of the primaries required to make a match. In doing so, the subject is causing equal excitations to occur for each of the three classes of cones in his eye in response to the test and the matching stimuli. He acts, in effect, as an analog computer, using the feedback provided by his sensations to iterate toward a solution. In this example, the subject will find that no amount of R, the red primary, is helpful and will turn it off. Although a mixture of the remaining pair can be found that matches the test field in hue, the mixture field is less saturated than the test. In cases of this kind, a full match can be achieved by adding an amount of one of the primaries (in this case R) to the upper (test) half of the field. The second equation in the figure shows this. (This operation is legitimate because, for a substantial range of conditions, there is an isomorphic relation between the rules of algebra and the experimental procedures, where “=” translates to “matches with,” “+” means optical superimposition, and “-” means “add it to the test field.”) The equation can then be rewritten as shown at the bottom of the figure, to indicate that a negative amount of the R primary was used in making the match.

**C. History of the Standard Observer**

The 1931 CIE system of colorimetry was based on color matching experiments conducted separately in England in the 1920’s by W. David Wright<sup>12</sup> and J. Guild.<sup>13</sup> As an appendix to my book *Human Color Vision*,<sup>14</sup> published in 1979, Wright (a current OSA Fellow) authorized a reprinting of remarks he had made in 1969 to the Colour Group of Great Britain concerning the origins of the 1931 CIE system. This very personal account reveals quite a bit about how things got done in the CIE. Here is a sample, one that relates to the adoption of the 1931 chromaticity diagram:

Guild . . . presented a paper to the Royal Society . . . in April 1931 giving his own [color-matching] curves which he had measured a year or two before I had even got cracking but he had not published them. He then went on to compare his and my results . . . He then made plans to submit our mean data for adoption by the C.I.E. at the meeting . . . in Cambridge, England in September 1931 . . .

In 1931, colorimetry really burst on the C.I.E. and in the 1931 *Proceedings* we have several pages of discussion reported and, of course, the resolutions which determined the 1931 system. Two people were particularly involved in these discussions: Priest from America and Guild from this country. Priest was the official American delegate and I think it was quite clear that he had come briefed to delay the adoption of any standard observer, since he thought we were rushing things. He in fact raised a succession of objections . . . Then overnight, T. Smith, who was Head of the Light Division [at the National Physical Laboratory] . . . and Guild would recalculate a lot of data to meet Priest’s criticisms, and Priest would turn up next morning with something else to object to. In the end they wore Priest down and he accepted most of the proposals that Guild was going to put forward at the C.I.E. meeting . . . [at which] there was quite a bit of discussion but they eventually approved the standard observer data . . . [W]hilst approval was given at the meeting, subsequently France reversed all its decisions and opposed all the resolutions—shades of de Gaulle!—and Germany reversed their vote, but as long as Britain and America agreed that was really all that mattered.

As I have already noted, color matches are possible be-

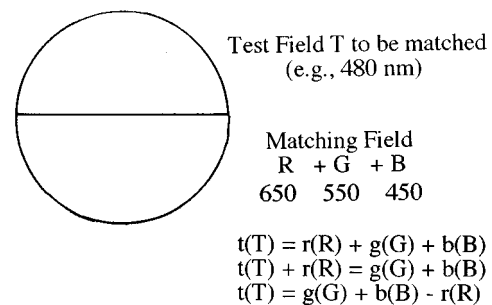


Fig. 2. Color-matching procedure using primaries of 450, 550, and 650 nm, and the equations that describe the match for a test field of 480 nm. See text for details.

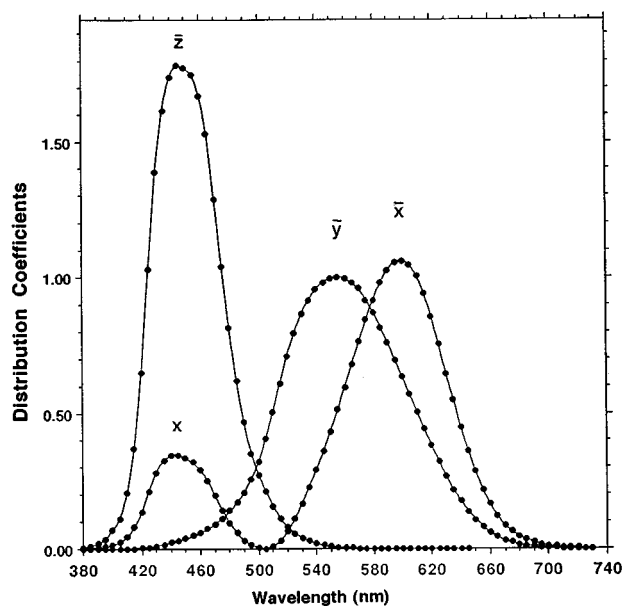


Fig. 3. Distribution functions of the CIE 1931 colorimetric system.

cause they depend upon the excitations of the three classes of cone photoreceptors in the retina. Mathematically, the action spectra of these cones must be linear transformations of the color-matching data, but there is only one transformation, among an infinite set of possibilities that depend upon choice of real or imaginary primaries, that will yield these action spectra. In neither Wright's color-matching experiments nor those of Guild were absolute radiometric measurements made of their light stimuli, although relative values were precisely determined. The new color system was tied into the existing photometric one by use of a mathematical transformation of the experimental data that caused  $\bar{y}$ —the middle one of the three color-matching functions of Fig. 3—to be directly proportional to  $V_\lambda$ . This is a trick that could not have worked unless the different classes of human cones were to contribute additively to luminance as defined by  $V_\lambda$ , so that  $V_\lambda$  represents the sum of their sensitivities if properly weighted.

Tristimulus values, as they are called, are calculated by integrating spectral radiance with the  $\bar{x}$ ,  $\bar{y}$ ,  $\bar{z}$  color-matching functions, and these calculations predict color matches of typical observers to the extent that these functions represent linear transformations of the action spectra of  $S$ - (short-wave-sensitive-),  $M$ - (middle-wave-sensitive-), and  $L$ - (long-wave-sensitive-) cone photoreceptors as modified by absorption in the eye media. When all three tristimulus values match for each of a pair of lights, we have the prediction of a color match for the so-called standard observer of the CIE system. Implicit in all of this is the idea, which is very well documented, that the cone photoreceptors generate univariant responses that individually carry no information about the wavelength of absorbed light. Only by comparing signals from three classes of cones can the visual system extract the information needed for color perception. This idea is often attributed to Thomas Young, who wrote

about it almost 200 years ago,<sup>15</sup> although the notion was in the air, and he was not the first to think of it.<sup>16</sup>

#### D. Relation of Color-Matching Functions to Cone Action Spectra

The  $\bar{x}$ ,  $\bar{y}$ , and  $\bar{z}$  functions are not the action spectra of the cones themselves but are linear transformations of them. For this reason they relate only indirectly to the underlying physiology, leading to a system of specification that, though useful, is not all that easy for the novice to grasp. The CIE chromaticity diagram is in turn derived from these functions by calculation of the values of  $y$  and  $x$  as fractions of the  $X+Y+Z$  sum. Textbook attempts to explain all this in just a page or two are certain to leave the student confused. It would be much better to start with the spectral sensitivities of the three classes of cones and to build a system of photometry and colorimetry upon this foundation. In 1931 this was not possible because these cone sensitivity functions, known as fundamentals, were not yet known.

How can these cone fundamentals be determined? One approach would be to isolate cones, extract their photopigments, and establish their absorptions in solution directly by spectrophotometry. Curiously, although rod photopigments were successfully dealt with in this manner more than a century ago, extraction of human cone photopigments has not proved possible. But such measurements have been made with the technique of microspectrophotometry.<sup>17</sup> Cone receptors, lying sideways on a microscope slide, are identified. To measure the absorption of light by a single cone, a tiny beam of light is projected through the cone's outer segment, where its photopigment resides. Physiological measures have also proved possible in recent years, with the use of suction electrodes into which individual cones can be drawn and their electrical responses recorded, and a few human cones have been tested.<sup>18</sup> (More recently, Merbs and Nathans<sup>19</sup> have actually made cone photopigments from DNA.)

#### E. Contributions of "Color-Blind" Individuals

Without denying the confirming importance of direct measurements, it is psychophysical data, obtained from intact human subjects with entirely noninvasive procedures (including but not limited to color matching) that have given us our best estimates of cone spectral sensitivities. In this regard, the contributions of color-blind individuals have been especially helpful. Actually, with only very rare exceptions, such people are not actually color *blind*; they are instead merely color *deficient*. Two of the relatively common kinds of color-deficient individuals, comprising one or two percent of the male population, are called protanopes and deuteranopes. Where color discrimination is concerned, they behave as if they lack either the  $L$  or the  $M$  class of cone photoreceptors. The methods of molecular biology have confirmed the genetic basis of their problem. As had long been predicted from population studies, including the tracing of family histories and noting the paucity of female colorblinds and the skipping of generations, it was found that the genes for both the  $L$  and  $M$  photopigments are on the X chromosome and that the gene for the  $S$  pigment is located on an autosome (chromosome 7).<sup>20,21</sup>

Because the vision of protanopes and deuteranopes is otherwise normal, the most plausible explanation for their condition is that all of their  $L$  and  $M$  cones contain only one of the two kinds of visual pigments that are housed separately in the  $L$  and  $M$  cones of normal observers. This would give the protanopes and deuteranopes normal spatial vision (which they have) and normal yellow–blue discrimination (which they have), but it eliminates the possibility of red–green discrimination because there is no basis for comparing the outputs of  $L$  and  $M$  cones, whose responses never differ from one another. Because the  $S$  cones are very insensitive to wavelengths longer than approximately 520 nm, normal color discrimination for the longer wavelengths of the spectrum depends almost entirely on the  $L/M$ -cone excitation ratio. When small fields are used, especially if observers are allowed to adapt to them for a few seconds, protanopes and deuteranopes can match all of the longer wavelengths merely by adjusting their relative radiances. (Actually, for large stimuli, most of them do show some residual red–green discrimination. Although I am one of several investigators<sup>22</sup> who have studied this residual ability, I will resist the temptation to say more about it here.)

#### F. Smith–Pokorny Cone Fundamentals

On the basis of small-field experiments with protanopes and deuteranopes under conditions where  $S$  cones were unable to contribute significantly to the measurements, in 1975 Vivianne Smith and Joel Pokorny at the University of Chicago developed estimates of cone spectral sensitivities that have been widely used by basic vision researchers.<sup>23</sup> These are shown in Fig. 4. The Smith–Pokorny curves are specifiable as linear transformations of the CIE distribution functions  $\bar{x}$ ,  $\bar{y}$ , and  $\bar{z}$  as modified by Judd with later improvements suggested by Hans Vos of the Netherlands.<sup>24</sup> In addition, the Smith–Pokorny  $L$ - and  $M$ -cone sensitivity functions, if

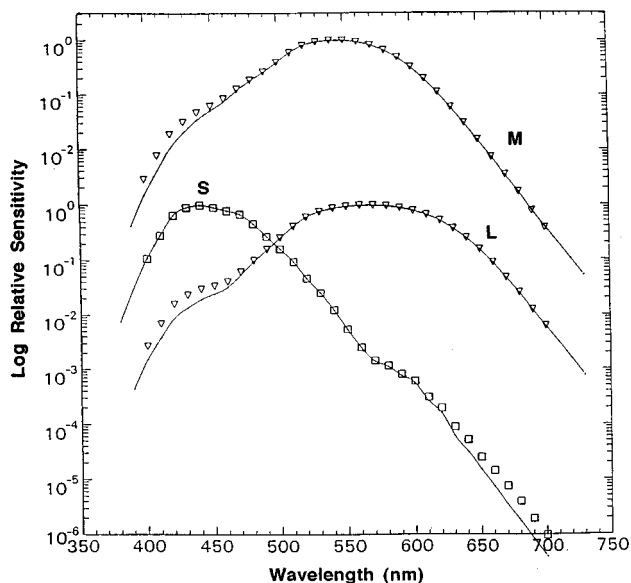


Fig. 4. Estimates of human cone spectral sensitivities by Smith and Pokorny<sup>23</sup> (open symbols) and Vos and Walraven<sup>27</sup> (solid curves). (Figure provided by Andrew Stockman.)

suitably scaled relative to one another (an operation that does not affect predictions of color matches), sum neatly to yield the Judd–Vos modified luminous-efficiency function. In this system, therefore, the  $S$  cones make no contribution to luminance.

## 2. EARLY COMMITTEE WORK

### A. Formation of the Committee

In 1979, at the 19th quadrennial meeting of the CIE in Kyoto, I proposed the formation of a CIE committee that would be concerned with the development of a new system of photometry and colorimetry.<sup>25</sup> I was not thinking at that time about replacing the existing CIE system, which seemed much too firmly entrenched and widely used for this to be possible or even desirable. The idea was to develop an alternative system for use by anyone who might find it preferable for his or her purposes and to seek its sanction by the CIE. With the enthusiastic backing of the late, great Gunter Wyszecki (who ascended to the presidency of the CIE by the time he died prematurely only a few years later), the committee was formed. I chaired it for a while, and at the next quadrennial meeting of the CIE, held in Amsterdam in 1983, I was invited to describe the system I had in mind to a plenary session of the convention. It was based on the Smith–Pokorny functions.<sup>26</sup>

### B. Points of Contention

Although my talk in Amsterdam seemed to be well received, my ideas were not accepted by members of my committee as easily as I had hoped. There was substantial disagreement on various points, and because the committee did not seem to be getting anywhere, I eventually bowed out. Later the committee was almost terminated, but today a reorganized version of it is hard at work, with Françoise Viénot of France as its energetic chair. My long-time colleague at the University of California, San Diego (UCSD), Donald MacLeod, who has been with this project from the start and is a member of the Viénot committee, has been joined in recent years by another UCSD colleague, Andrew Stockman. (Both have contributed in important ways to this paper.)

Early on, a major sticking point concerned the choice of the cone sensitivity curves that would lie at the foundation of any new system. The Dutch investigators Pieter Walraven and the aforementioned Hans Vos had a set of their own,<sup>27</sup> which they naturally favored. Actually, the differences between the Vos–Walraven functions and those of Smith and Pokorny are modest (see Fig. 4), but they are large enough to be visually significant. More important, the Vos–Walraven functions had the very undesirable property—at least from my point of view—of being based on the assumption that  $S$  cones make a tiny, yet significant, contribution to luminance.

At this point, because it relates to the dispute concerning whether or not they contribute to luminance, let us quickly review a few facts about  $S$  cones:

- They are sparsely represented in the retina and are nearly absent in the center of the critical foveal region. Direct anatomical evidence about this now exists.<sup>28</sup>

- They contribute only minimally to spatial vision. My work with minimally distinct borders with Peter Kaiser at the University of Rochester, and later with Brian Tansley at UCSD, showed this. For small fields, contour perception initially depends almost entirely upon differences in *L*- and *M*-cone excitation, and if *S*-cone excitation differs as well for fields on the two sides of a border, its influence is trivial, although Rhea Eskew, Conrad Olson, and I eventually were able to measure it as ~0.3% that of an *L*-*M* difference.<sup>29,30</sup>

- *S* cones contribute very importantly to color vision by providing the short-wavelength input to the axis of color discrimination<sup>31</sup> usually called yellow-blue but that actually varies, from the standpoint of color appearance, from yellowish green to violet.

### C. MacLeod-Boynton (Luther) Chromaticity Diagram

I return now to the issue of why an assumed contribution of *S* cones to luminance, even if a very small one, struck me as undesirable. I can explain this by describing a chromaticity diagram (Fig. 5) that Donald MacLeod and I published 17 years ago in *JOSA*.<sup>32</sup> But let us consider first the features that provide the more familiar CIE chart (Fig. 6) with a number of desirable properties that have led to its widespread use.

- By plotting fractional tristimulus values, it represents, in two dimensions that are more easily visualized than three, what is actually a three-dimensional proposition.

- A given location on the diagram represents the chromaticity of all possible spectral distributions that yield that color and that will match at equal luminance.

- Mixtures of any two colors plot along straight lines connecting the chromaticities of their components, and the location of a mixture along the connecting line can be determined by a center-of-gravity principle, according to the relative amounts of these components.

- One can visualize at a glance the triangular domain of colors encompassed by three additive primaries, for example, those provided by the phosphors used in color television.

- It is also clear at a glance that the only physical stimuli that cannot be produced by mixtures of at least two others are monochromatic lights whose chromaticities plot on the curved portion of the spectrum locus, and one can infer from this that the linear long-wavelength portion of the locus implies a negligible contribution of the *z* component.

### 3. LUTHER DIAGRAM VERSUS THE CIE CHROMATICITY CHART

All of the properties just enumerated are retained in the constant-luminance diagram that Donald MacLeod and I developed<sup>32</sup> (Fig. 5), which is based upon the Smith-Pokorny fundamentals of Fig. 4. Because the underlying concept is similar to that of Robert Luther, a German investigator in the 1920's,<sup>33</sup> we have called it a Luther diagram. Our chart has the important advantage, compared with the CIE diagram, that cone excitations are directly represented. Those of both the *L* and the *M*

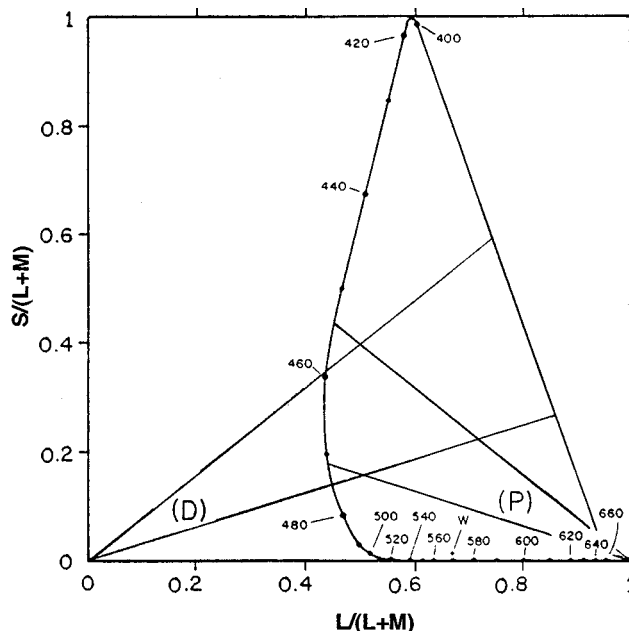


Fig. 5. Constant-luminance chromaticity diagram of MacLeod and the author<sup>32</sup> based on the Smith-Pokorny fundamentals (Luther diagram).

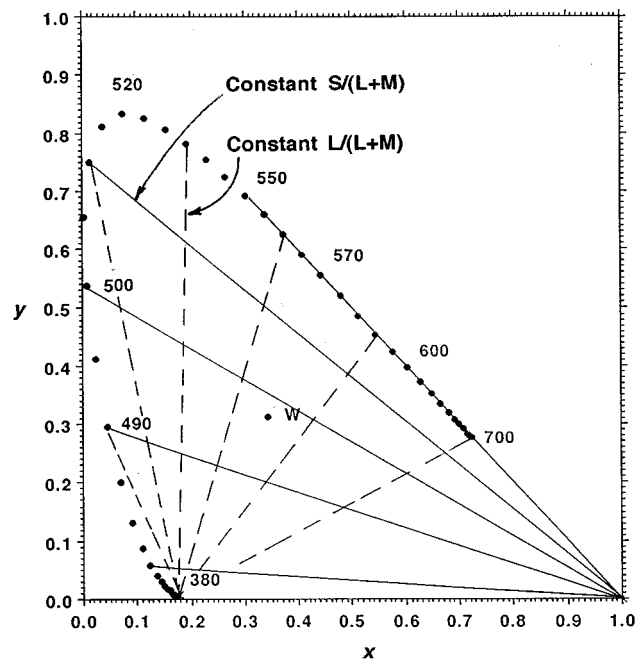


Fig. 6. CIE chromaticity diagram. For a fixed luminance, solid lines describe loci of constant *S*-cone excitation, dashed lines of constant *L*-cone excitation. In the diagram of Fig. 5, the solid and dashed lines all would intersect at right angles.

cones are plotted along the abscissa, where the one trades off for the other (the fraction of *L* is arbitrarily represented here), something that is possible only if luminance depends upon the sum of *L*- and *M*-cone excitations with no contribution allowed from *S* cones. *S*-cone excitation at constant luminance is explicitly represented along the ordinate, orthogonal to the *L*-*M* dimension. Because *S* cones are assumed *not* to contribute to luminance, the

vertical scaling is completely arbitrary relative to the horizontal one. Most important, no matter where you are in this chart, a given horizontal distance implies the same amount of trade-off between *L*- and *M*-cone excitation, and a given vertical distance implies a fixed change in the excitation of *S* cones. Naotake Kambe and I suggested<sup>34</sup> that 1 unit of *S* be defined as that amount of *S*-cone excitation equivalent to what would be produced by an equal-energy white, which in turn is equivalent to that produced by a monochromatic light of approximately 500 nm.

Specifying the excitation levels of the three kinds of cones provides an ideal way to characterize the impact of radiant energy on vision in terms of its initial effects. Our system, like that of the CIE, reduces the visual effectiveness of spectral radiance distributions to only three numbers, but unlike the CIE values, ours are intuitively meaningful. Cone excitation levels are badly obscured by the CIE chromaticity diagram (Fig. 6), upon which lines of equal *S*-cone excitation radiate from the point 1,0 at angles that vary depending upon the ratio of *L* to *M* excitation. Lines representing fixed ratios of *L*- and *M*-cone excitation radiate from the so-called *S*-cone copunctal point at the lower left in various directions depending on the *L/M* ratio. These angular relations, which make it extremely difficult to visualize changes in cone excitations in the CIE chromaticity diagram, become transparently obvious in ours (Fig. 5), where all of these intersections would be at right angles. Moreover in our diagram the amounts of stimulus components upon which the center-of-gravity calculations are made are based on luminance units, which is not the case in the CIE system, where special units having no other function must be employed.

#### 4. STOCKMAN-MACLEOD-JOHNSON FUNDAMENTALS

##### A. Choice of Matching Data

In December 1993 Andrew Stockman, Donald MacLeod, and a UCSD undergraduate, Jeff Vivien, published a 20-page paper<sup>35</sup> in *JOSA A* that provided background for a monumental 30-page paper<sup>36</sup> that immediately followed, authored by Stockman, MacLeod, and another undergraduate, Nancy Johnson. (Without wishing to downgrade the contributions of these talented undergraduates, which Stockman tells me were very substantial, I will, for simplicity, sometimes refer in what follows to the work of "Stockman and MacLeod." It is not unusual for undergraduates to participate in vision research, and some go on to careers in physiological optics.) Toward the end of their 50 pages, Stockman and MacLeod reached the conclusion that the best set of human cone fundamentals should be based on color-matching data obtained by W. S. Stiles and associates in the Light Division of the National Physical Laboratory in Teddington, England.<sup>37</sup> The cone fundamentals depend most importantly upon the absorbance spectra of the photopigments located in their outer segments. All three classes of photopigments are much alike and, as already noted, their genetic basis has been determined.<sup>38</sup> Genes for the *L* and *M* cones, in the X chromosome, have DNA sequences that are 98% homolo-

gous. This value for the *S* pigment, which derives from chromosome 7, is less than 50% in comparison with either *L* or *M*. But in an eye with only one class of cones, sensitivity measured psychophysically, in terms of energy being delivered to the eye, differs from the absorbance spectrum of the photopigment contained in those cones, for a variety of reasons which Stockman and colleagues take into account in a highly quantitative fashion. Here I will merely mention what they are.

##### B. Factors to Consider

- *Absorption in the ocular media.* This is, by far, the most important factor. Not all of the light incident upon the cornea reaches the photoreceptors. Most of the absorption takes place in two places, the macular pigment and the lens (Fig. 7). The lens is nearly transparent to long wavelengths but absorbs substantially in the shorter ones, as shown in the bottom panel of this figure. There is a systematic increase in lens absorption with age, causing the lens to appear yellow if extracted from an elderly

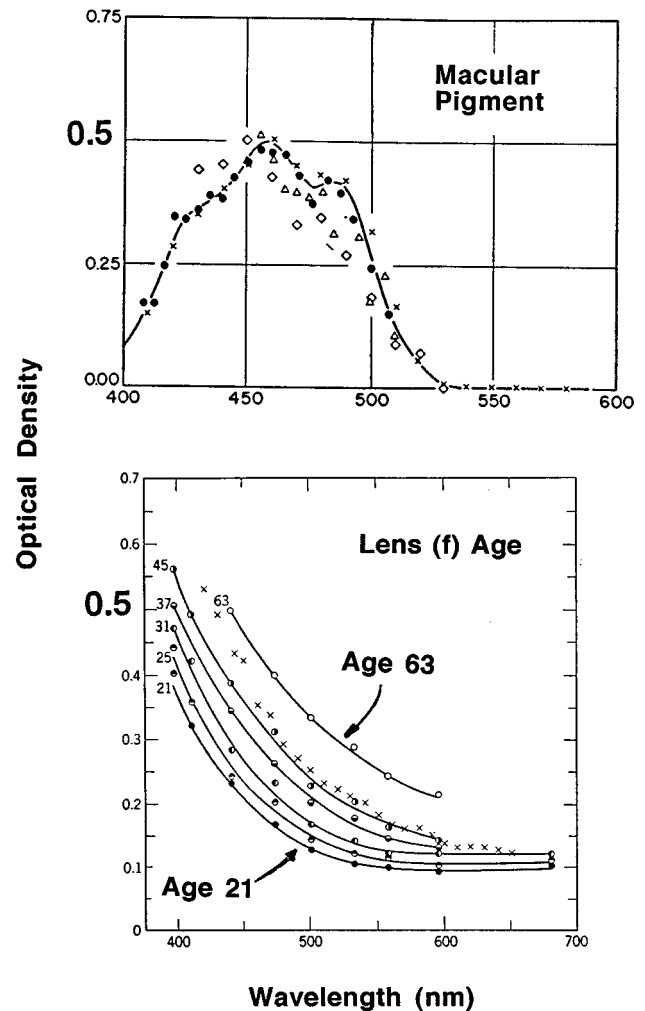


Fig. 7. Spectral sensitivity measured at the cornea differs from that measured at the retina for a variety of reasons, but absorption in the eye media, particularly in the macula and lens, is the major cause. Adapted from Wyszecki and Stiles,<sup>2</sup> pp. 110 and 112.

eye. The macular pigment also selectively absorbs in the short wavelengths, as shown at the top of the figure, but does so with an absorption spectrum peaking at  $\sim 460$  nm, very different from that of the lens. Macular pigment exhibits its highest density in the fovea, shades off with eccentricity, and is nearly absent at an eccentricity of 10 deg. The density of macular pigment varies enormously among observers. For persons of the same age with healthy eyes, lens absorption is much less variable, but the age effects are large. It should be obvious from these considerations alone that no standard observer can represent anything better than an average.

- *Waveguide complications.* As Brian O'Brien<sup>39</sup> and Jay Enoch<sup>40</sup> showed many years ago, receptors act as waveguides as the radiant energy is funneled toward the cone inner segments where the photopigments are located. Waveguide effects in the retina, which may slightly alter spectral sensitivity, are difficult to model quantitatively because of uncertainties about refractive indices and variations of receptor dimensions. At a 1964 symposium<sup>41</sup> I expressed the opinion that these effects are probably not systematic and merely add noise to the absorption spectrum of each receptor. Since many thousands of receptors enter into color perception, these effects would average out. At the University of Rochester, David Williams is currently examining waveguide influences in photoreceptors.

- *Polymorphism.* There are two common alleles of the *L*-cone photopigment gene, distributed roughly 60–40 in the population, leading to slightly different absorbances, and there are many types of hybrid photopigment genes. (A given individual may have as many as four extra *M*-pigment genes, but only the principal one seems to be expressed.) In the “News and Views” section of *Nature* in 1992, John Mollon wrote,<sup>42</sup> concerning these matters, “The significance of these discoveries for psychologists cannot be exaggerated,” and in his lead sentence he says “You and I may pass through our lives in different perceptual worlds.” This, I think, somewhat exaggerates the case. The perceptual consequences, which are small enough to be difficult to measure psychophysically, are also small for practical purposes. Otherwise there could be no color television, color photography, or color printing that would satisfy those of us with normal color vision, and the CIE concept of the standard observer would make no sense.

- *Self-screening.* The spectral absorption spectrum of a concentrated solution is broader than that of a dilute one. As a result, absorption in a receptor will differ a little from its absorbance, that is, from its absorption in an infinitely dilute solution. The *L* and *M* cones probably do not differ in this respect, although this is not certain. In any event, the self-screening effect will be less for the squat cones in the peripheral retina than for the skinny ones in the fovea.<sup>43</sup>

- *Angle of incidence.* Rays passing the margins of the pupil strike the receptors at an angle relative to the axial rays through the center. This affects self-screening and is sufficiently selective among the receptor types to produce slight mismatches of monochromatic lights when these are simultaneously compared for the two conditions. (This is known as the Stiles–Crawford Effect,

Type II, and was initially quantified by Jay Enoch and W. S. Stiles<sup>44</sup> while Enoch was on leave in Stiles’s lab in Teddington, England in 1958–1959. Shortly after Enoch went home, Mituso Ikeda, then a graduate student in the Institute of Optics at Rochester, joined Stiles and me for my sabbatical year.)

Taking all these factors into account, as well as data from color-deficient observers, Stockman and MacLeod also published spectral sensitivity data of their own (Nancy Johnson collected most of the data), using a clever adaptational technique that convincingly is able to isolate individual cone types. Armed with all of this information, they examined candidates for cone fundamentals, with special emphasis on the Smith–Pokorny set—the basis of the system that I had proposed in 1983—and two other sets derived from extensive color-matching data from Stiles’s laboratory.

### C. Stiles–Burch Color-Matching Study

The Stiles investigation had been recommended by the CIE in the 1950’s to address some of the problems of the 1931 system, especially when fields larger than 2 deg were involved. The work was carried out with the NPL tricolorimeter, which Stiles had commissioned—a massive instrument housed in a temperature- and humidity-controlled room at the National Physical Laboratory (NPL). James Burch, who states that “my main interest in optics has always been on the metrological side,” was brought on board to strengthen the effort, replacing Stiles’s assistant, R. W. Donaldson, who had died unexpectedly.<sup>45</sup> Calibration was meticulous, and Stiles’s assistants, Pam Fowler and Jean Vigil, were highly trained and dedicated workers. I have some personal knowledge about these matters based on the sabbatical year that I spent in Stiles’s lab. It is commonly accepted among vision researchers that the data of Stiles are as trustworthy as is humanly possible.

The NPL color-matching work was carried out in two phases. The first phase, which was regarded by Stiles as preliminary, was actually a rather extensive study in its own right. It yielded data from ten subjects tested with a 2-deg field, allowing a useful comparison with the original Wright–Guild data.<sup>46</sup> In the second phase, a 10-deg field was used, approximately 50 subjects were tested.<sup>37</sup> Unlike the Wright–Guild experiments, absolute radiometry was carried out routinely, obviously based on NPL standards.

### D. Choosing between the 2-degree and the 10-degree Data

Let us return now to Stockman and MacLeod. After considering all of the factors relating cone absorption to psychophysically measured spectral sensitivity as derived mathematically from color-matching data as well as from the data from dichromats and their own measurements, Stockman and MacLeod derived two sets of cone fundamentals, one for each of the two phases of the NPL work. At present, they are recommending that the committee use the set based on the Stiles–Burch large-field work for the following reasons:

- In 1964 the CIE approved a set of 10-deg color-matching functions based largely on the NPL data.<sup>2</sup> Cone fundamentals derived from these data would therefore relate to an already existing and approved system.

- Many more subjects were tested in the large-field study, allowing a better estimate of average data and individual differences.

- To relate the 10-deg data to what would be predicted for a 2-deg field, reasonable assumptions about the distribution of macular pigment and self-screening can be made (based partly also on the Stiles–Burch 2-deg data), and it should be possible to develop a scheme to interpolate for intermediate field sizes.

When all of this is said, however, there is not all that much difference between fundamentals derived from the Stiles–Burch data and those of Smith and Pokorny derived from the CIE functions. I hope the committee doesn't debate this issue for another 16 years!

Toward the end of the second *JOSA A* paper,<sup>36</sup> Stockman, MacLeod, and Johnson provide some calculations back and forth between cone absorbance spectra and cone fundamentals, with some adjustments being made to the cone fundamentals based on what was required to produce ideal versions of absorption spectra. There is empirical evidence that all photopigment absorbances are very smooth functions of wavelength, although the exact basis for this smoothness, at a molecular level, remains obscure. Also, as originally noted by H. J. A. Dartnall<sup>47</sup> more than 40 years ago, all photopigment absorbance curves have nearly the same shape if suitably plotted. If this were true for human cone photopigments, the shapes of their absorbance curves would differ only in their lateral positions, usually defined by the wavelengths of their peak sensitivities, known as  $\lambda$ -max.

The cone absorbance functions that Stockman and MacLeod believe are consistent with the 10-deg cone fundamentals are shown in the curves in Fig. 8, which I have borrowed from their paper.<sup>36</sup> The horizontal scale is logarithmic. The curves drawn through the data for the *L* and *M* cones are consistent with the long-wavelength portion of the *S*-cone curve, shifted laterally. The agree-

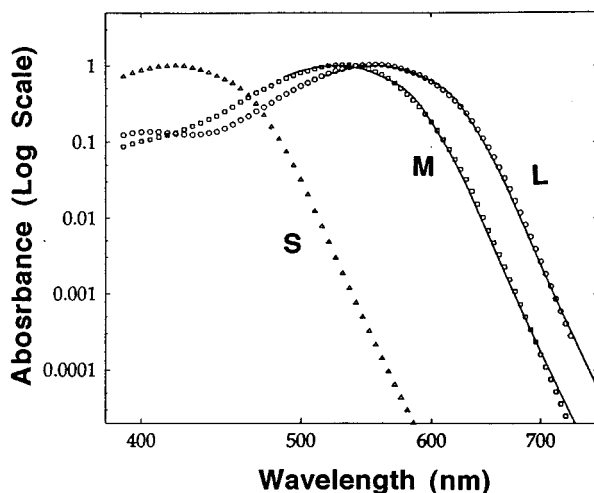


Fig. 8. Estimates of the absorbance spectra of human *L*, *M*, and *S* cones (from Ref. 36).

ment, though surely not perfect (it looks much worse on a linear scale), is surely good enough for practical purposes.

## 5. OTHER CURRENT COMMITTEE CONSIDERATIONS

The Viénot committee is currently struggling with the question of whether luminosity should be redefined to be precisely equal to the sum of the *L* and *M* functions, as it is here. To do so implies that the system is linear and that *S* cones contribute negligibly to luminance. My view is that this is precisely how luminous efficiency *should* be defined, after the best weights of the *L* and *M* functions are determined for optimal agreement with flicker photometry and minimally distinct border measurements, which are known to agree reasonably well with one another. I am happy to report that, after having debated the issue for 12 years, the committee seems ready to accept the null contribution of *S* cones to luminance.

I am still advocating the same kind of system that I proposed at Amsterdam in 1983, and although I continue to feel that the Smith–Pokorny functions are good enough for all practical purposes, I can see at least a political rationale for using the Stiles–Burch data. Whereas my original idea was to develop a supplemental system for the CIE, the committee may, in the end, recommend a complete replacement of the 1931 system, especially because the modern use of narrow-band sources has tended to expose some of its more serious predictive errors in the existing system that were not so obvious in the old days.

Well, the standard observer is a 65-year-old geezer, and perhaps it *is* time for him to retire!

## 6. PLOTS BASED ON STOCKMAN–MACLEOD–JOHNSON FUNDAMENTALS

I will finish by presenting a series of graphs that are based on calculations that I have made using the Stockman–MacLeod 2-deg fundamentals derived from the 10-deg Stiles–Burch data. Figure 9 shows the Stockman–MacLeod cone fundamentals plotted as log sensitivity versus wavelength in nanometers, with the three functions scaled to unit values at their peaks. Qualitatively, they exhibit the usual features: The *L* and *M* curves overlap extensively, with sensitivities on the order of 3 or 4 log units below their peaks at the extremes of the visible spectrum. The *S* function is much more separated laterally and has already dropped by 4 log units by the time it reaches the middle of the spectrum, beyond which values become rather uncertain, which doesn't matter much, because sensitivity there is negligible anyway for all practical visual purposes. It is easy to assess the upper part of the *S*-cone function by use of long-wavelength adapting lights to selectively depress the *L*- and *M*-cone sensitivities, and many investigators have done this.

### A. Relation of *L* and *M* Fundamentals to Luminance

Figure 10 shows the *L* and *M* fundamentals scaled so that, when added, they produce the function at the top,

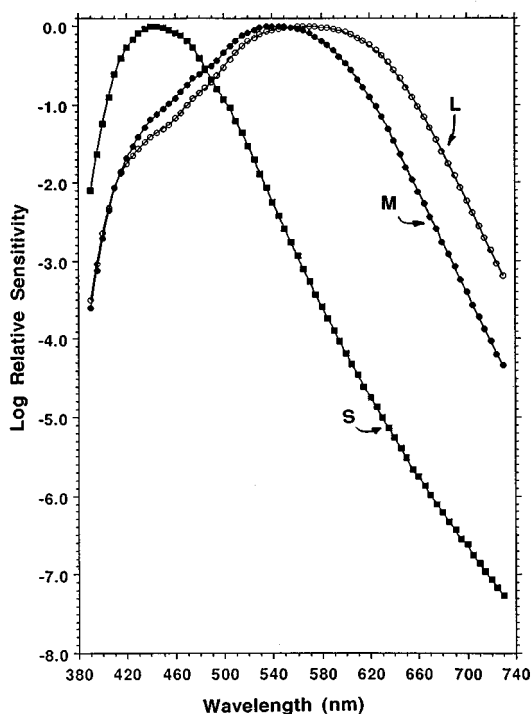


Fig. 9.  $L$ -,  $M$ -, and  $S$ -cone fundamentals derived by Stockman, MacLeod, and Johnson.<sup>36</sup> All subsequent figures are based on calculations using these functions.

labeled  $L + M$ , which is normalized to unit sensitivity and which agrees more or less with spectral luminous efficiency as defined by flicker photometry. Figure 11 illustrates exactly the same data but plotted on a linear ordinate. Linear summation is, of course, much easier to visualize on the linear plot, but all detail is lost in the tails of the curves, which have important implications for color perception.

Figure 12 shows how luminance is divided between the  $L$  and  $M$  cones as a function of wavelength for the particular weights of  $L$  and  $M$  that I have chosen. No matter what this choice, the upper curve is necessarily the mirror image of the lower one, since whatever the  $L$  cones don't contribute to luminance, the  $M$  cones must, in order to sum to unity. The graph shows a relatively high sensitivity of  $L$  cones at short wavelengths, reaching a minimum at approximately 460 nm, rising in a more or less ogival fashion to reach an asymptote at long wavelengths. If horizontal lines were drawn across the graph, they would intersect the curves twice in the range from 390 to ~570 nm. At equal luminance these wavelength pairs match for both  $L$  and  $M$  cones; we call these *tritan pairs*, because they would match for a tritanope lacking  $S$  cones. (There are such people, but they are very rare.) Tritan pairs differ in appearance for normal subjects only because  $S$ -cone sensitivity differs for all such pairs. Figure 13 shows this. The sensitivity for  $S$  cones at constant luminance is plotted here, on a logarithmic ordinate because of the millionfold range depicted. Because the function is descending throughout the range of tritanopic metamers just described, tritan pairs differ in appearance because the short-wavelength member of any such pair produces more  $S$ -cone excitation than the long-

wavelength one. The more separated the members of a tritan pair, the greater is this difference.

### B. Luther Diagram Based on Stockman–MacLeod–Johnson Cone Fundamentals

We are ready now, with the help of Fig. 14, to examine the Luther chromaticity diagram on the basis of the fundamentals advocated by Stockman and MacLeod. It is worth stressing again that a major advantage of this dia-

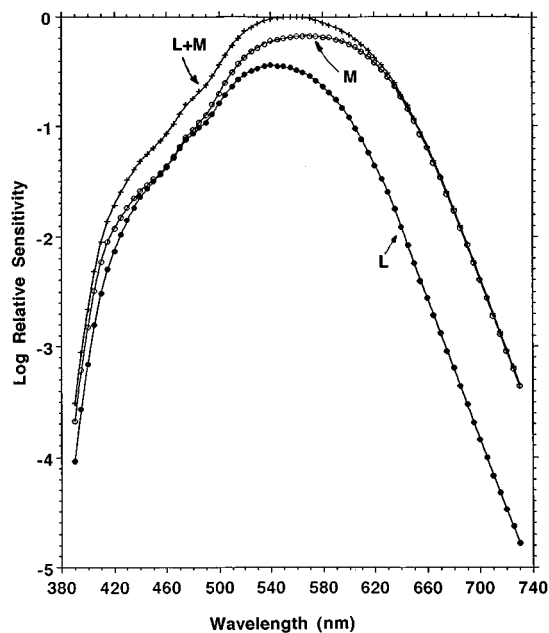


Fig. 10. The  $L$  and  $M$  curves of Fig. 9, if suitably weighted, sum to a function that is very similar to the luminous-efficiency function.

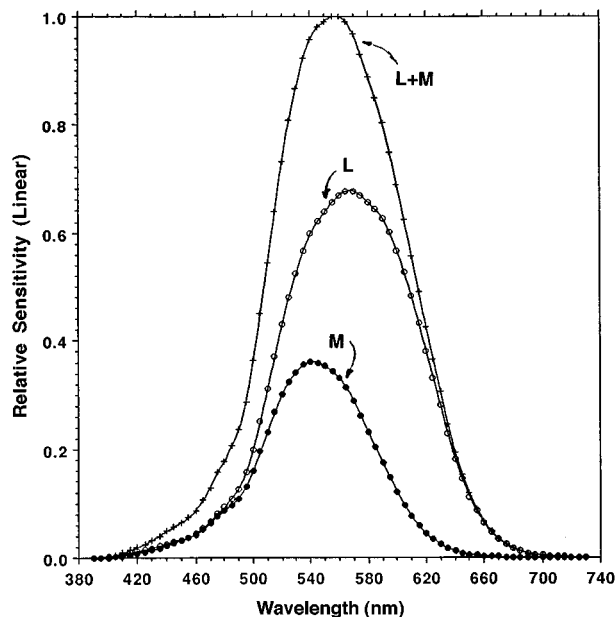


Fig. 11. Data of Fig. 10 plotted on a linear, rather than a logarithmic, ordinate. Summation is easier to visualize than in Fig. 10, but details are lost in the tails of the curves.

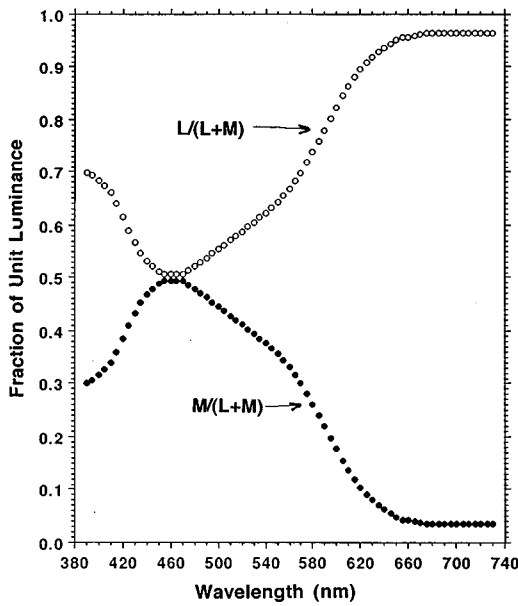


Fig. 12. Contributions of *L* and *M* cones to unit luminance.

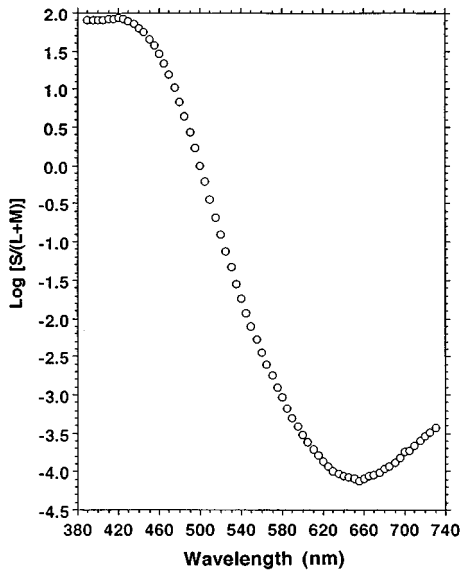


Fig. 13. Excitation of *S* cones at constant luminance.

gram pertains to the nature of what is represented on the two axes. To repeat: The scaling of the diagram is arbitrary, but no matter how it is done, up and down always refers to pure changes of *S*-cone excitation, whereas left and right implies a pure exchange of *L* for *M*, all of it at constant luminance. As also noted above, the diagram otherwise retains all of the properties of the familiar CIE chart or any other linear chromaticity diagram. A possible disadvantage, cited by some of the current committee members, is that the white point appears almost to fall on the *x* axis, at an ordinate value of 1 on a scale that runs from zero to more than 80. What this shows, however, is a physiological truth: The amount of *S*-cone excitation required to change yellow to white is less than one eightieth of what a 420-nm spectral light can produce at the same luminance! However, most of what we see in

the natural world of reflecting objects never comes close to this; the chromaticities of realizable surfaces lie almost at the bottom of the chart, mostly below an ordinate value of 3.

If, as in Fig. 15, wavelengths shorter than ~470 nm are not depicted, then the white point shows more clearly, still at the ordinate value of 1 but on a greatly expanded vertical scale, truncated at the top. In general, the ordinate of a Luther diagram can be scaled to serve whatever purpose is at hand.

**C. Tritan Pairs and Brindley Isochromes: Clearly Revealed**

I will now provide two illustrations to show how this type of chromaticity chart can clearly reveal properties that are either just barely discernible or not discernible at all on the conventional chromaticity diagram. In the CIE diagram (Fig. 6), one can see a very gentle curvature of the spectrum locus depicted at the left side. The intersections with the curve of lines drawn upward from the tritanopic copunctal point represent tritan pairs. In Fig. 16 the vertical scale of the Luther diagram is compressed a bit and the horizontal one is expanded greatly, covering only the leftmost 10% or so of the function of Fig. 14. Now the tritan pairs are clearly revealed, lying above and below one another on the horseshoe curve.

A second example (Fig. 17) expands the lower right-hand corner of the large diagram (Fig. 14) in the vicinity of 700 nm. This clearly reveals that the largest ratio of *L*- to *M*-cone excitation—the reddest red, if you will, is not at the spectral extreme but lies instead at ~700 nm,

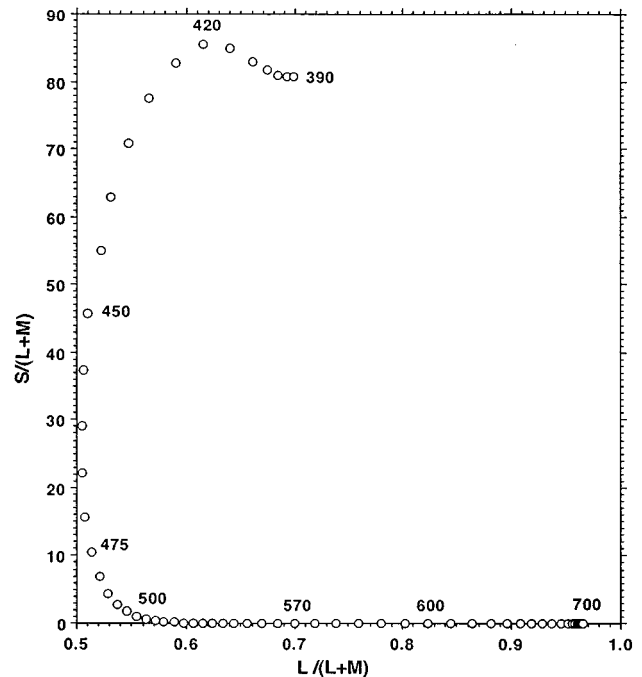


Fig. 14. Luther diagram, equivalent to Fig. 5 except that the Stockman–MacLeod fundamentals are used instead of those of Smith and Pokorny. The white point would appear to be nearly on the abscissa because so little *S*-cone excitation is needed to convert from yellow to white.

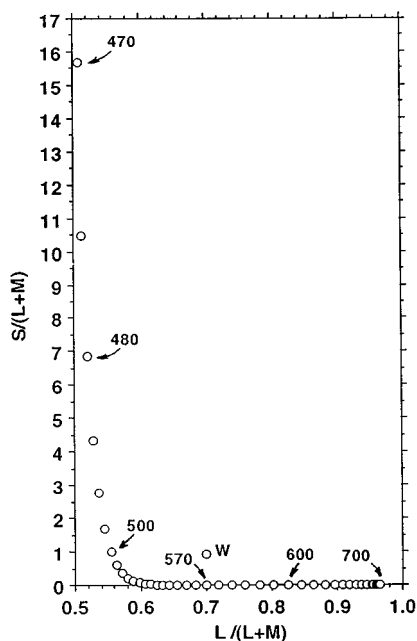


Fig. 15. Figure 14 is rescaled, and ordinate values greater than 17 are not shown. This makes the white point somewhat easier to visualize.

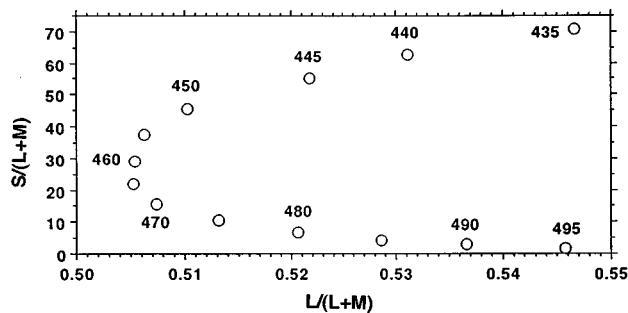


Fig. 16. Rescaling of the leftmost portion of Fig. 14. Tritanopic metamers fall along vertical lines that intersect the spectrum locus twice.

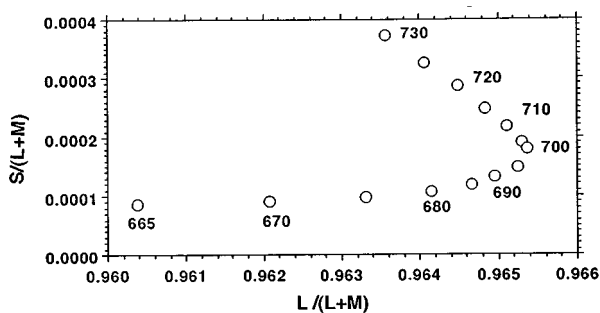


Fig. 17. Rescaling of the lower right-hand portion of Fig. 14 to show the Brindley isochromes. Vertical lines that intersect the spectrum locus twice would show wavelengths that produce the same  $L/M$  cone excitation ratios.

as Brindley showed experimentally some years ago.<sup>48</sup> The ranges covered here are very limited on both axes. The ordinate shows only tiny fractional values of  $S$ -cone

excitation, less than a thousandth of the already small amount needed to turn yellow into white. The Brindley isochromes are depicted as pairs of stimuli lying above and below one another. Because the tiny contribution of  $S$  cones is below threshold, these pairs match exactly. (Although the conventional CIE chromaticity diagram could also be truncated and expanded—it has never been customary to do so—directions corresponding to the axes of the Luther diagram would be impossible to discern.)

Figure 18 is a reprise of Fig. 14 but with the ordinate scaled logarithmically. This chart lacks the basic advantages of linear chromaticity diagrams, as can be seen most dramatically by the representation of the extra-spectral purples, the solid diamonds at the upper right, which would fall along a straight line in any linear dia-

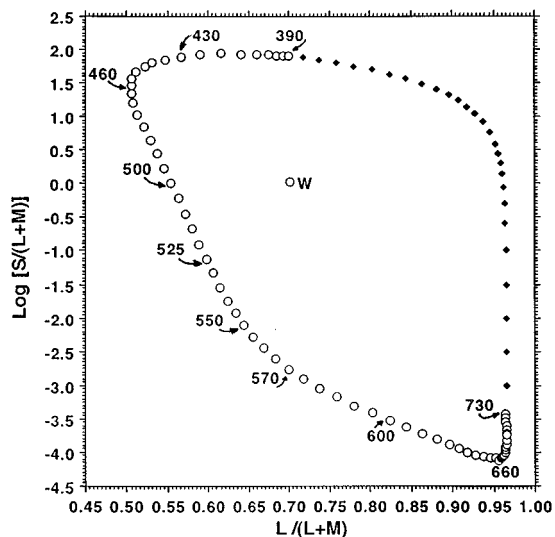


Fig. 18. Luther diagram modified by using a logarithmic ordinate. This moves the white point nearer to the center of the chart at the expense of introducing severe nonlinearities, as illustrated by the curve (solid diamonds) representing the extra-spectral purples.

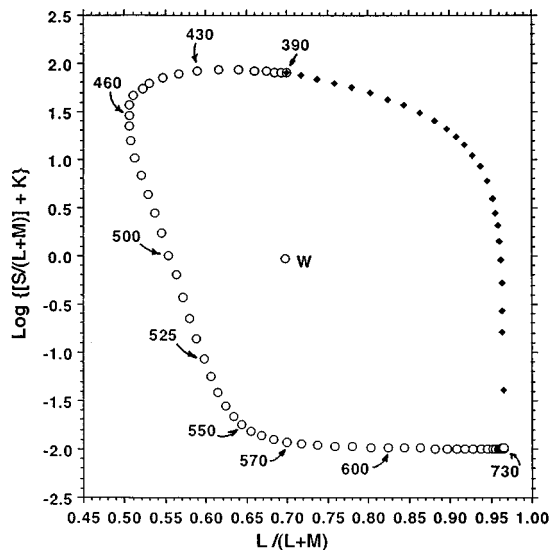


Fig. 19. Plot of Fig. 18, modified by adding a small constant before taking the logarithms of ordinate values.

gram. Complementary colors do not plot across from one another, either, for example, a line drawn between 480 nm and 580 nm would connect through the white point by a substantially curved line. However, plotting logarithmic values of  $S$  does move the white point toward the middle of the chart. Another disadvantage relates to the points in the lower right-hand corner. We saw the upturn of  $S$ -cone excitation at short wavelengths, which is visually inconsequential, in Fig. 16. The logarithmic scale exaggerates it outrageously. Adding a small constant to the  $S/(L + M)$  value before taking the logarithm (Fig. 19) results in much more reasonably depicted long wavelengths. And there is reason to believe that in a substantial region around white, this is a reasonably uniform diagram, one in which equally discriminable differences are represented by similar distances. The same scheme can be used to improve the spacing of the OSA colors.

#### D. OSA Uniform Color Scale Samples on Luther Diagram

About 20 years ago, after many years of effort, OSA produced its collection of Uniform Color Scale Samples, which are intended to fill three-dimensional color space as uniformly as possible.<sup>49</sup> Figure 20 shows their chromaticities for the middle luminance level  $L = 0$  (based on calculations made by MacLeod) as they appear in the Luther chromaticity diagram. Note that the middle gray at  $j = g = 0$  is at an ordinate value of  $\sim 1$ , as it should be, and that the most saturated purples at the top (point) do not even exceed an ordinate value of 2 by very much. It is remarkable, really, how little excitation of  $S$  cones is

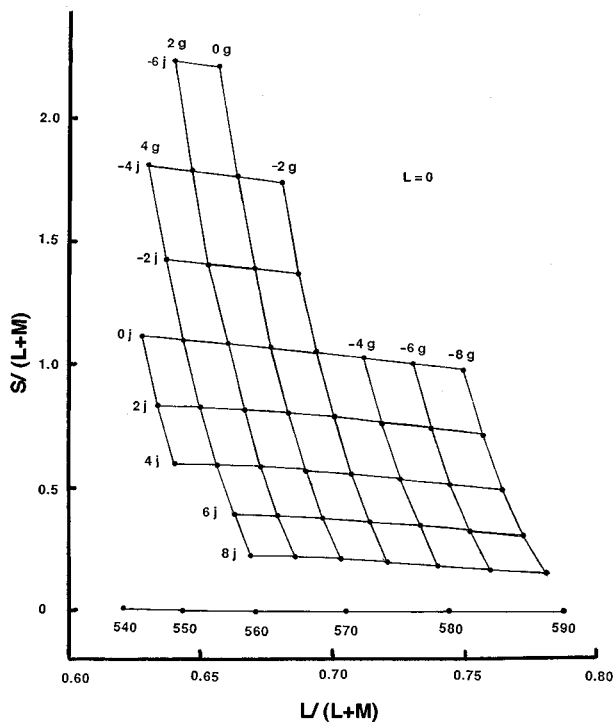


Fig. 20. Chromaticities of the samples of the CIE Uniform Color Scales at luminance  $L = 0$  plotted in the Luther diagram.

needed to produce these colors relative to what is physically possible with spectral lights.

## 7. CONCLUDING REMARKS

It was an honor and a pleasure to have had the unusual and unexpected opportunity to address a plenary session of the Optical Society of America. I hope that these concepts are useful for readers who haven't thought lately about the representation of color and that they are to some extent intelligible for those who may have thought very little about it, which I recognize as not unusual for people in the field of optics today, in which vision plays a relatively minor role in comparison with the early days when the eye was the only radiation detector available. Nevertheless, it is good to keep in mind that without human vision there would be no science of optics, and it is unlikely that the narrow band of the electromagnetic spectrum that is visible would have excited any special interest.

I am also very pleased that OSA has seen fit to present the Ives medal to a vision researcher, even one like me who closed his lab and retired five years ago from UCSD. There is, however, one relevant project with which I have recently been involved. In 1979 Holt, Rinehart & Winston published the first edition of my book *Human Color Vision*.<sup>14</sup> I was very flattered when, three years ago, I was approached by the Optical Society of America for permission to reprint a limited number of copies of that book. OSA has just published a substantially revised second edition with the same title, coauthored with my friend and colleague Peter Kaiser of York University.<sup>50</sup> (Kaiser, who provided the impetus for this revision and did most of the work, is first author.)

Finally, on behalf of all of us who are members of the vision research community and who have found a happy home in OSA, I am most pleased that the Ives committee and the Board of Directors have recognized in this way the continuing relevance of physiological optics within the Society.

## SPECIAL ACKNOWLEDGMENTS

I am very grateful to those who supported my nomination for the Ives Medal and Quinn Prize. This honor is the capstone of a long career to which former graduate students, postdoctoral fellows, and other colleagues in visual science (Donald MacLeod paramount among them) have greatly contributed. Both my biological father, Merrill Boynton, and my academic mentor, Lorrin Riggs, have provided splendid examples of human conduct, and to the extent that I have been able to emulate them, their principles have served me well. Finally, without the support of my wife Allie on the home front, no such honor could possibly have come my way.

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