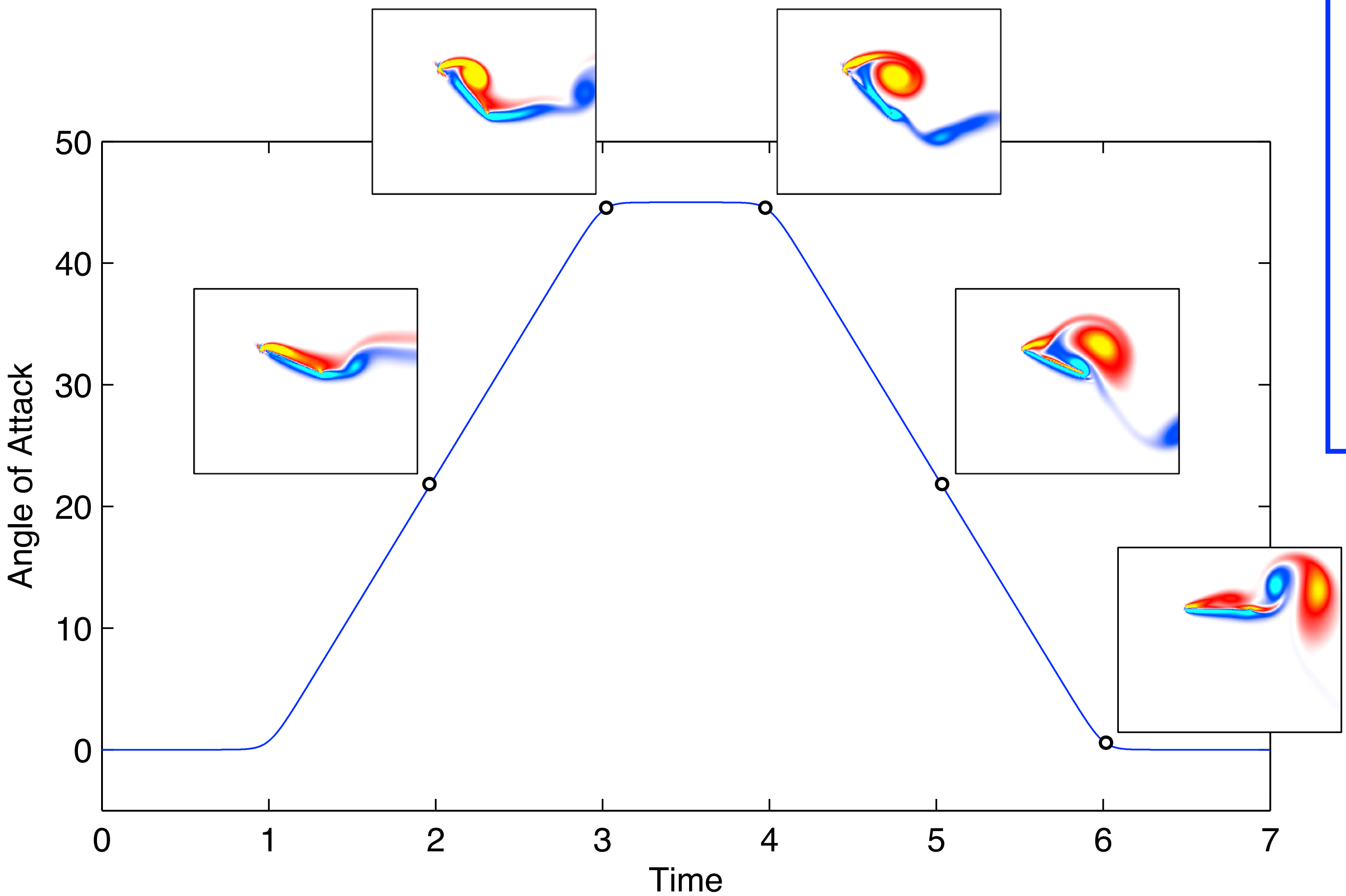
State-Space Representation of Unsteady Aerodynamic Models



$$\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k+1} = \begin{bmatrix}A_{\text{ERA}} & 0 & 0\\ 0 & 1 & \Delta t\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k} + \begin{bmatrix}B_{\text{ERA}}\\ 0\\ \Delta t\end{bmatrix}\ddot\alpha_k$$

input

$$C_L(k\Delta t) = \begin{bmatrix}C_{\text{ERA}} & C_{L_\alpha} & C_{L_{\dot\alpha}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k} + D_{\text{ERA}}\ \ddot\alpha_k$$

ERA Model

quasi-steady and added-mass contributions

fast dynamics

Steve Brunton & Clancy Rowley
Princeton University

63rd APS DFD November 21, 2010

Applications of Unsteady Models

- Conventional UAVs (performance/robustness)
- Micro air vehicles (MAVs)
- Flow control, flight dynamic control
- Autopilots
- Flight simulators

FLYIT Simulators, Inc.

Predator (General Atomics)

Safety Concerns

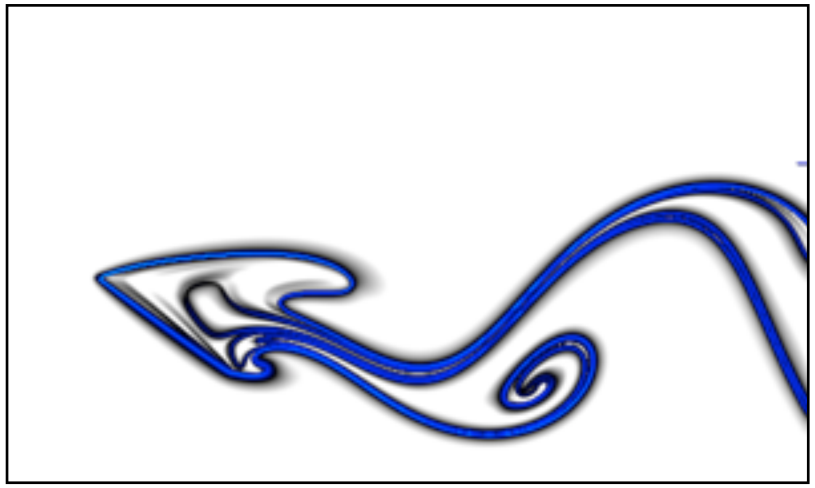
- severe weather
- wake vorticity
- gust disturbances

Wake Vorticity

Flexible Wing
(University of Florida)

3 Types of Unsteadiness

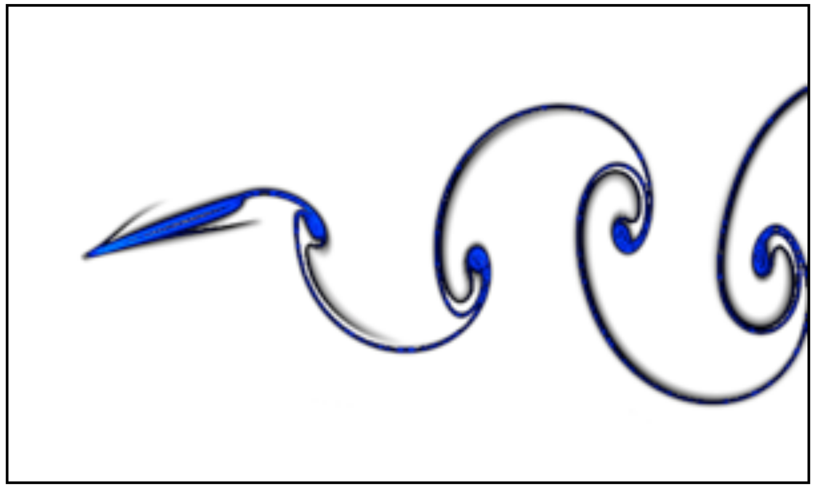
1. High angle-of-attack



$$\alpha > \alpha_{\text{stall}}$$

Large amplitude, slow

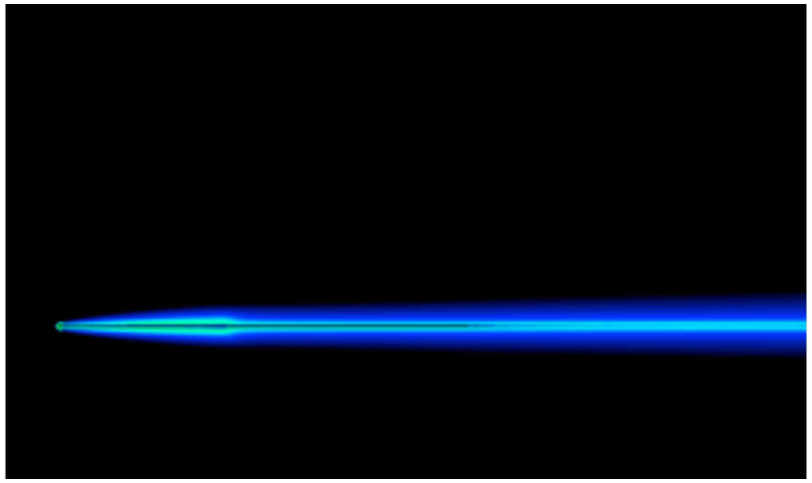
2. Strouhal number



$$St = \frac{Af}{U_\infty}$$

Moderate amplitude, fast

3. Reduced frequency



$$k = \frac{\pi f c}{U_\infty}$$

Small amplitude, very fast

Closely related

$$\alpha_{\text{eff}} = \tan^{-1}(\pi St)$$

Brunton and Rowley, AIAA ASM 2009

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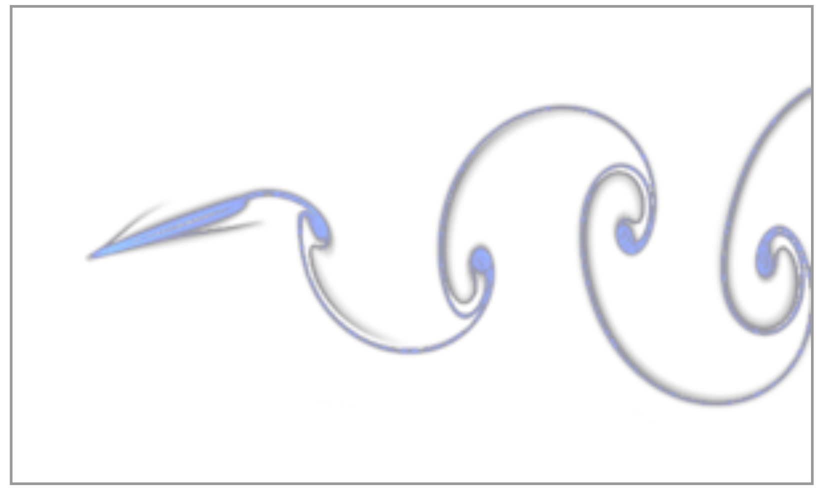

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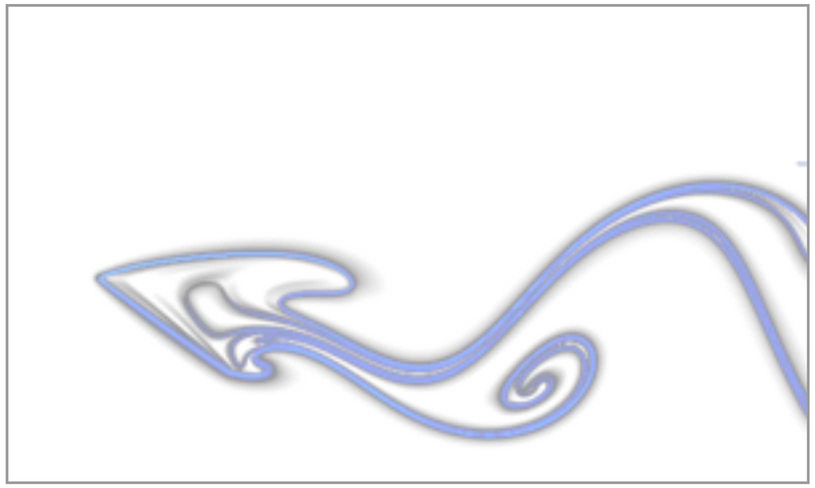

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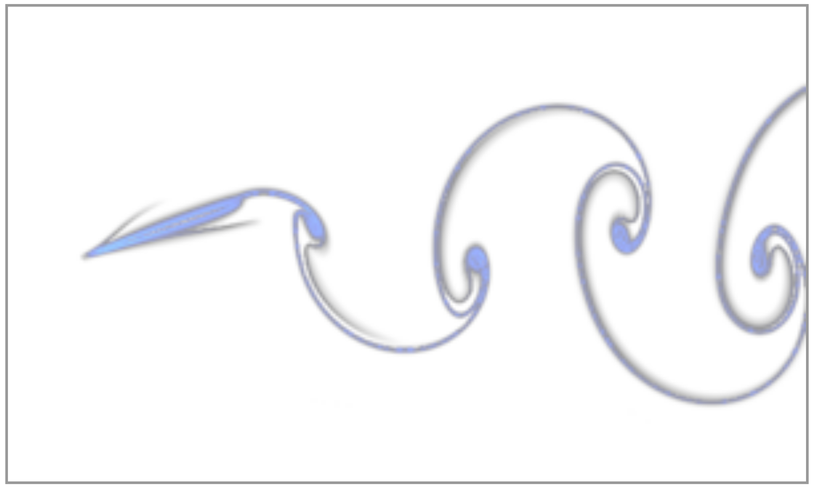

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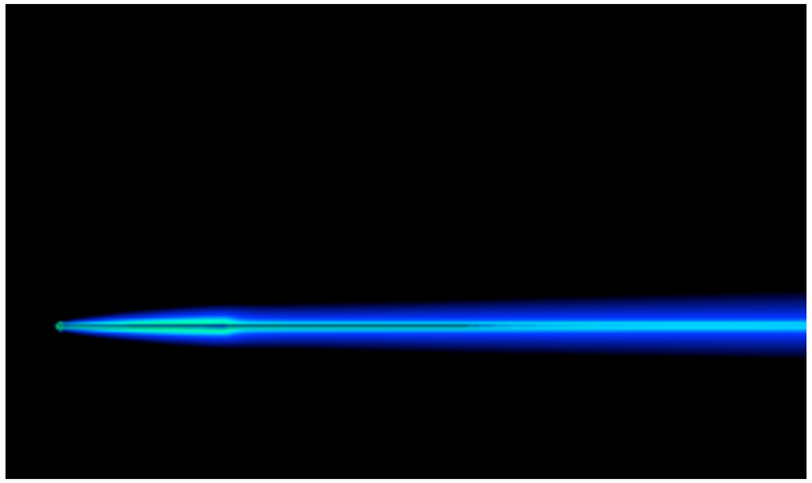

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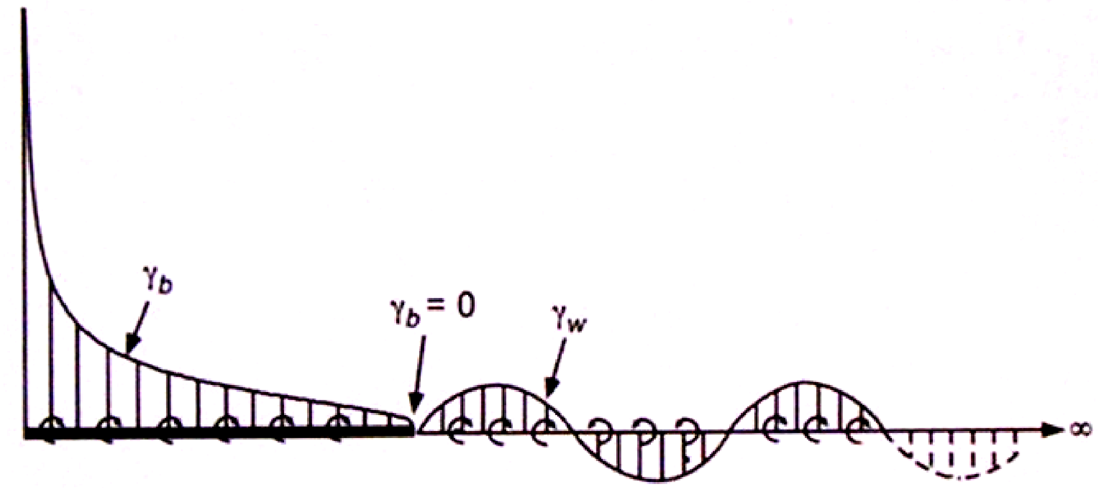
Candidate Lift Models

$$C_L = 2\pi\alpha$$

$$C_L = C_{L_\alpha}\alpha$$

$$C_L = C_L(\alpha)$$



$$C_L(t) = C_L^\delta(t)\alpha(0) + \int_0^t C_L^\delta(t-\tau)\dot{\alpha}(\tau)d\tau$$

Wagner's Indicial Response

$$C_L = \underbrace{\frac{\pi}{2}\left[\ddot{h} + \dot{\alpha} - \frac{a}{2}\ddot{\alpha}\right]}_{\text{Added-Mass}} + \underbrace{2\pi\left[\alpha + \dot{h} + \frac{1}{2}\dot{\alpha}\left(\frac{1}{2} - a\right)\right]}_{\text{Circulatory}} C(k)$$

Theodorsen's Model

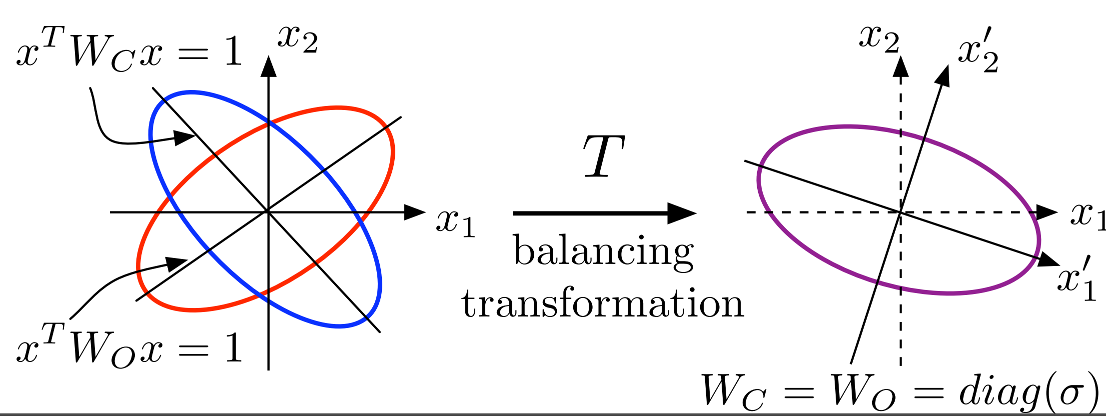
New models!

Model Criteria

Captures input output dynamics accurately

Computationally tractable

fits into control framework



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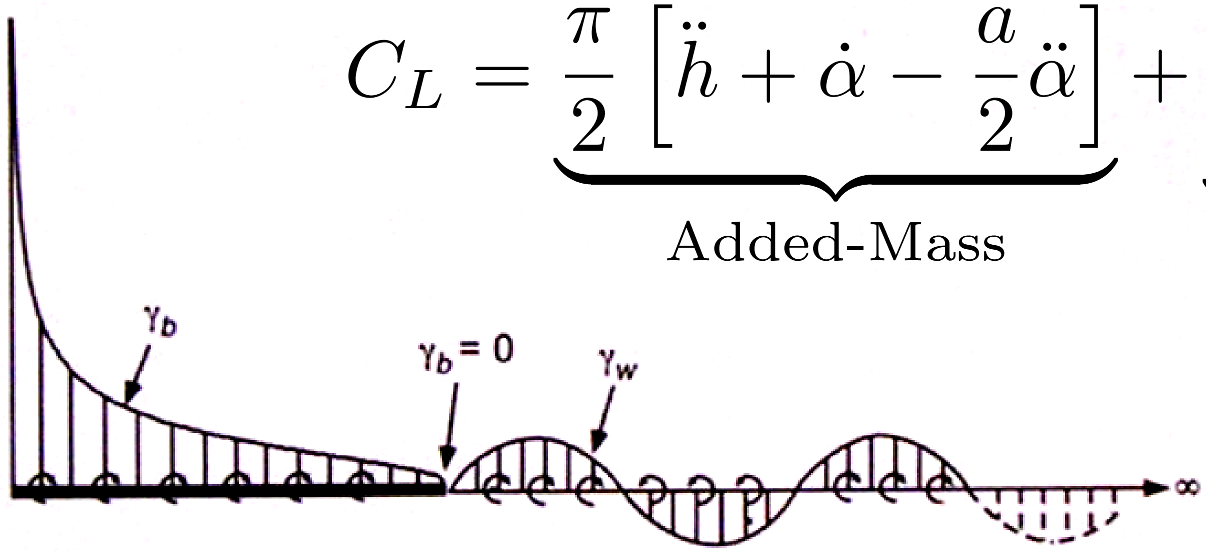


$$k = \frac{\pi f c}{U_\infty}$$

2D Incompressible, inviscid model

Unsteady potential flow (w/ Kutta condition)

Linearized about zero angle of attack

Apparent Mass

Increasingly important for lighter aircraft

Not trivial to compute, but essentially solved

force needed to move air as plate accelerates

Circulatory Lift

Captures separation effects

Need improved models here

source of all lift in steady flight

Theodorsen, 1935.

Leishman, 2006.

Bode Plot of Theodorsen

$$C_L = \underbrace{\frac{\pi}{2}\left[\ddot{h} + \dot{\alpha} - \frac{a}{2}\ddot{\alpha}\right]}_{\text{Added-Mass}} + \underbrace{2\pi\left[\alpha + \dot{h} + \frac{1}{2}\dot{\alpha}\left(\frac{1}{2} - a\right)\right]}_{\text{Circulatory}} C(k)$$

$$k = \frac{\pi f c}{U_\infty}$$

Frequency response

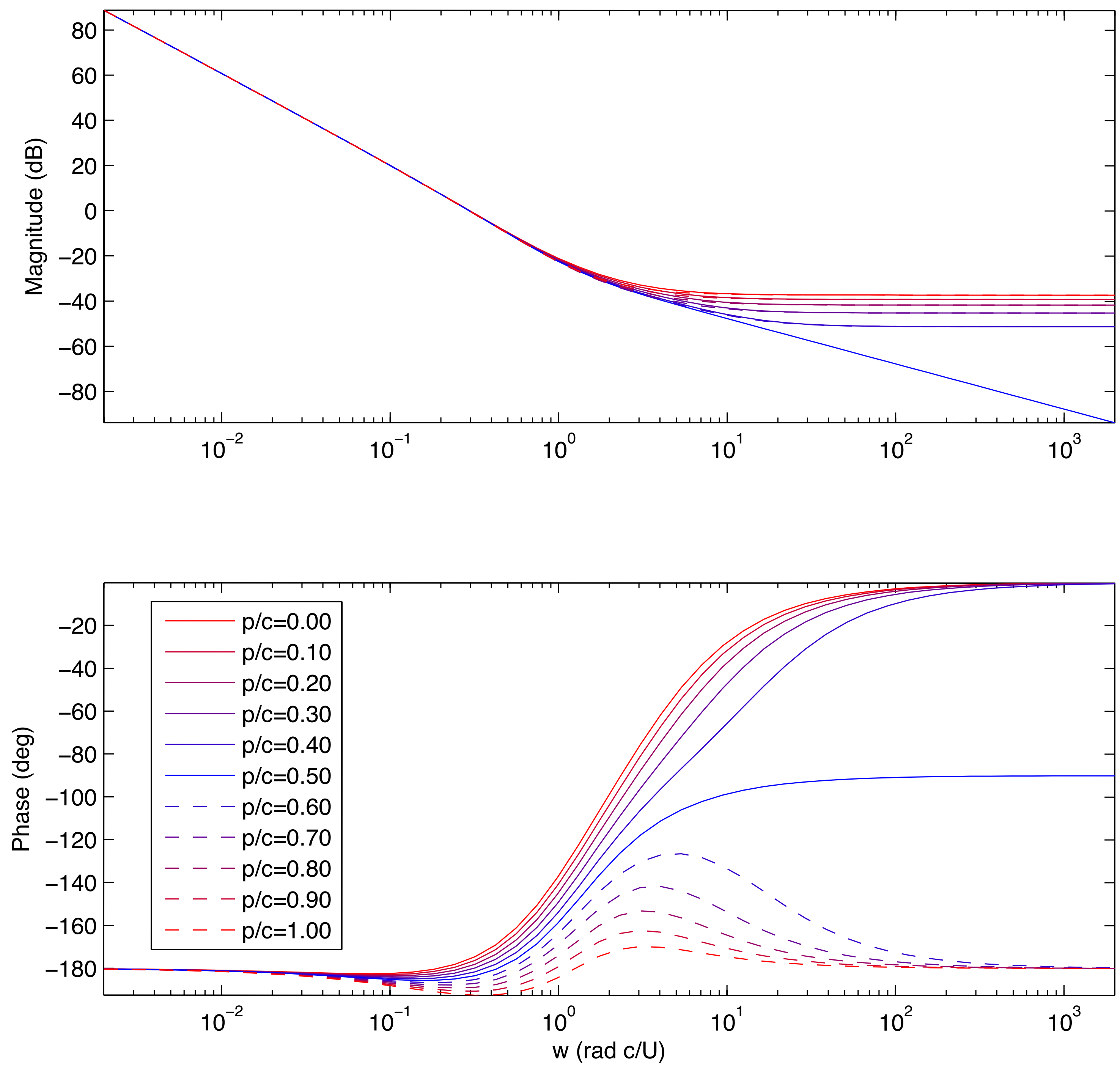
input is $\ddot{\alpha}$ (α is angle of attack)

output is lift coefficient $\mathrm{C_L}$

Low frequencies dominated by quasi-steady forces

High frequencies dominated by added-mass forces

Crossover region determined by Theodorsen's function $\mathrm{C(k)}$

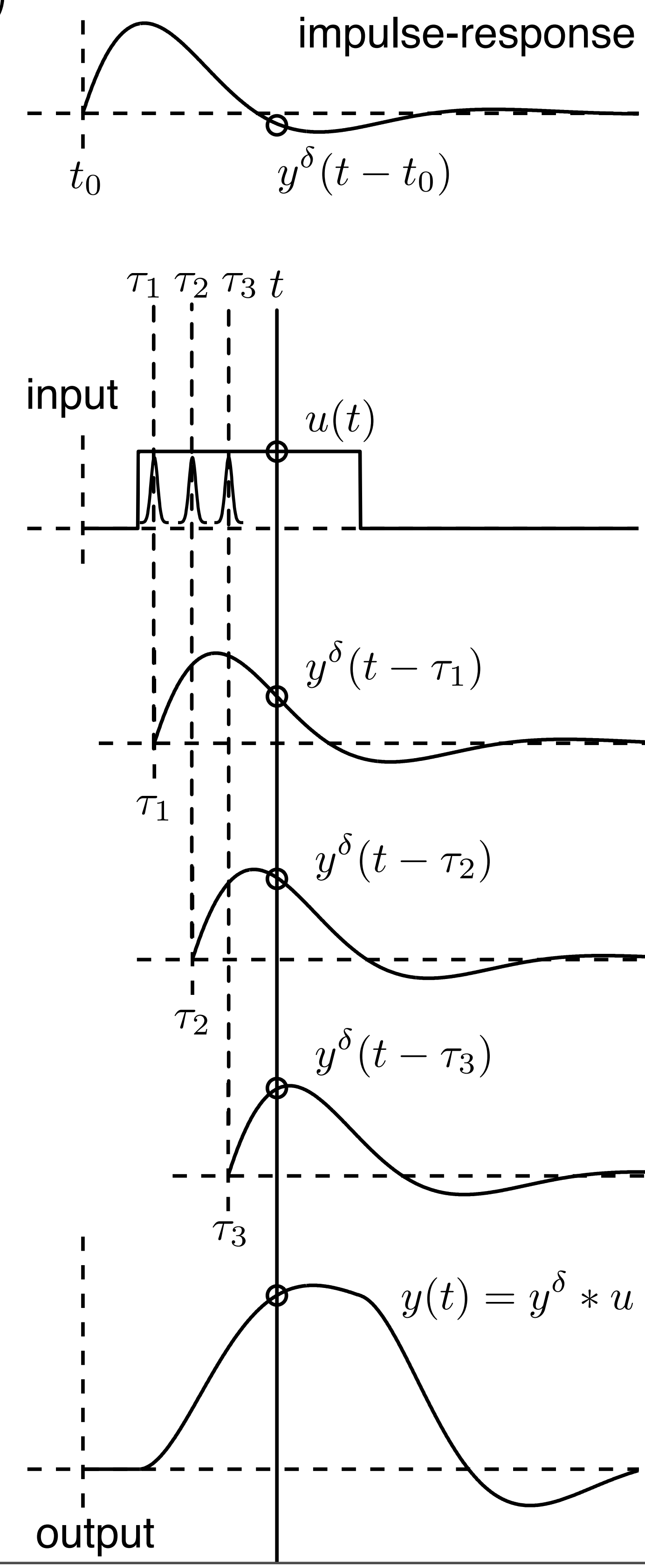


Wagner's Indicial Response

Given an impulse in angle of attack, $\alpha = \delta(t)$, the time history of Lift is $C_L^\delta(t)$

The response to an arbitrary input $\alpha(t)$ is given by linear superposition:

$$C_L(t) = \int_0^t C_L^\delta(t-\tau)\alpha(\tau)d\tau = \left(C_L^\delta * \alpha\right)(t)$$



Given a step in angle of attack, $\dot{\alpha} = \delta(t)$, the time history of Lift is $C_L^S(t)$

and the response to an arbitrary input $\alpha(t)$ is given by:

$$C_L(t) = C_L^S(t)\alpha(0) + \int_0^t C_L^S(t-\tau)\dot{\alpha}(\tau)d\tau$$

Model Summary

Reconstructs Lift for arbitrary input

Linearized about $\alpha = 0$

Based on experiment, simulation or theory

convolution integral inconvenient for feedback control design

Wagner, 1925.

Leishman, 2006.

Reduced Order Wagner

fast dynamics

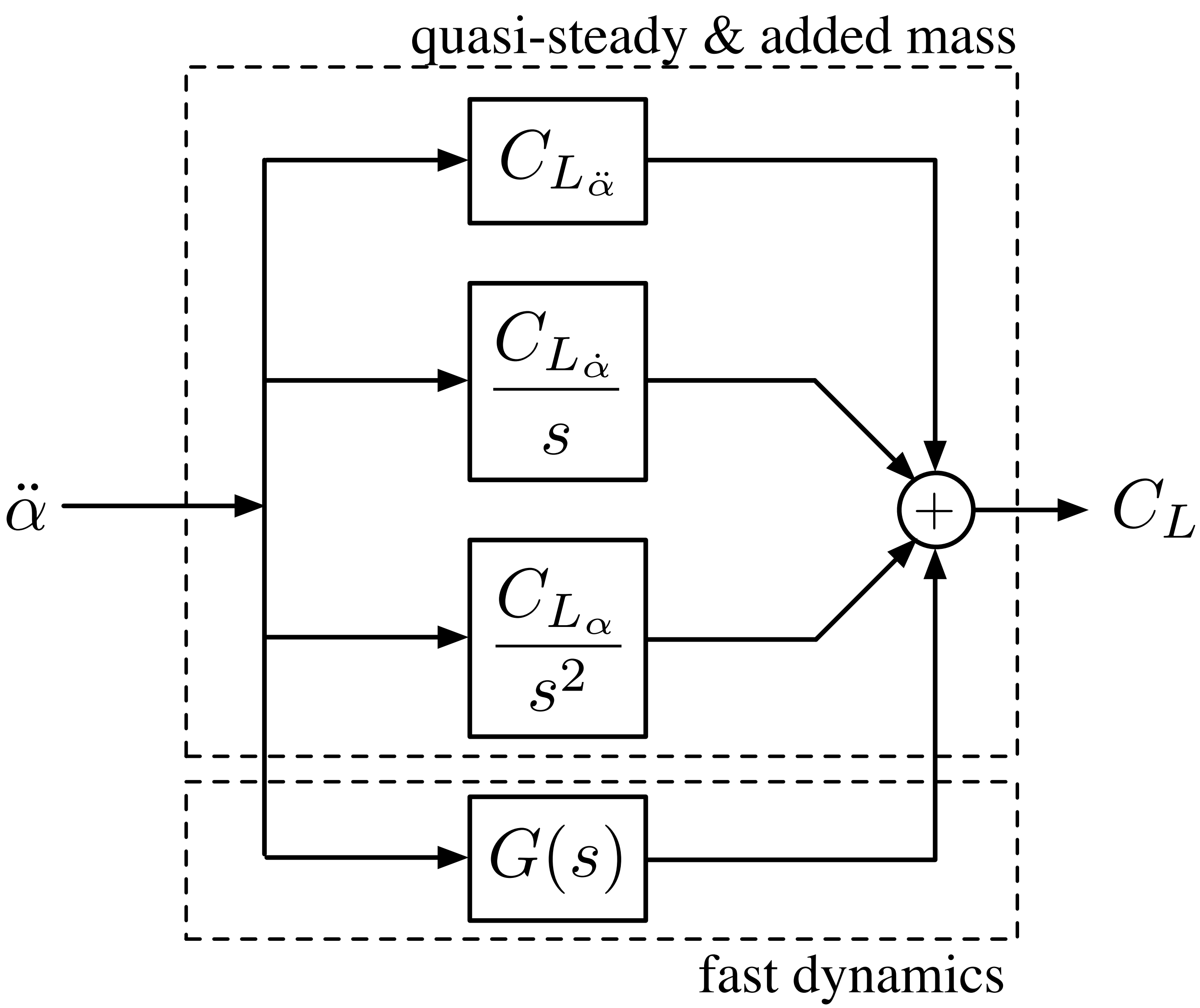
$$\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k+1} = \begin{bmatrix}A_{\text{ERA}} & 0 & 0\\ 0 & 1 & \Delta t\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k} + \begin{bmatrix}B_{\text{ERA}}\\ 0\\ \Delta t\end{bmatrix}\ddot\alpha_k$$

input

$$C_L(k\Delta t) = \begin{bmatrix}C_{\text{ERA}} & C_{L_\alpha} & C_{L_{\dot\alpha}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix}_{k} + D_{\text{ERA}}\ \ddot\alpha_k$$

ERA Model

quasi-steady and added-mass



Model Summary

Linearized about $\alpha = 0$

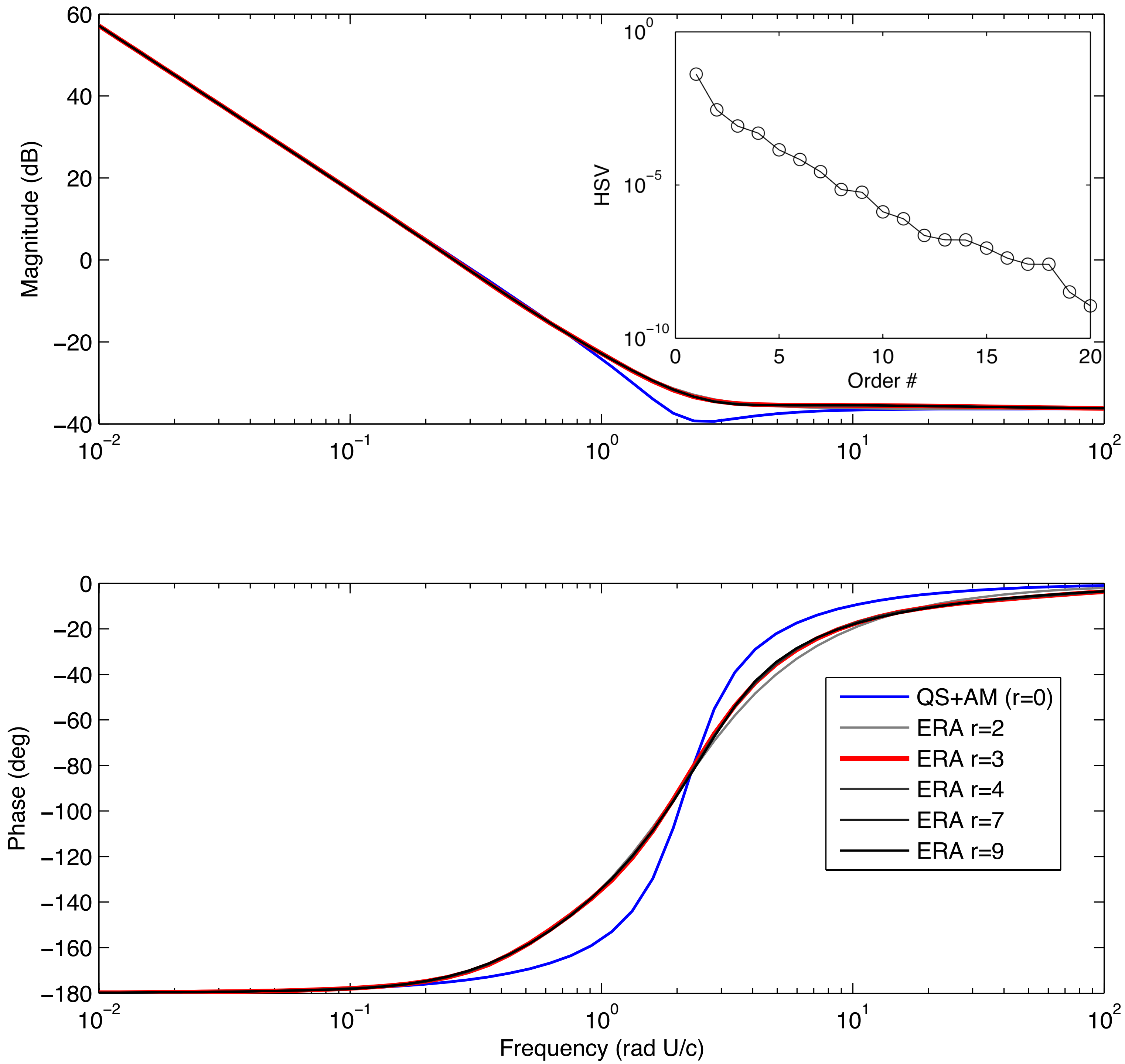
Based on experiment, simulation or theory

Recovers stability derivatives $C_{L_\alpha}, C_{L_{\dot\alpha}}, C_{L_{\ddot\alpha}}$ associated with quasi-steady and added mass

ODE model ideal for control design

Brunton and Rowley, *in preparation*.

Bode Plot - Pitch (LE)



Frequency response

input is $\ddot{\alpha}$ (α is angle of attack)

output is lift coefficient C_L

Pitching at leading edge

Model without additional fast dynamics [QS+AM (r=0)] is inaccurate in crossover region

Models with fast dynamics of ERA model order >3 are converged

Punchline: additional fast dynamics (ERA model) are essential

Brunton and Rowley, *in preparation*.

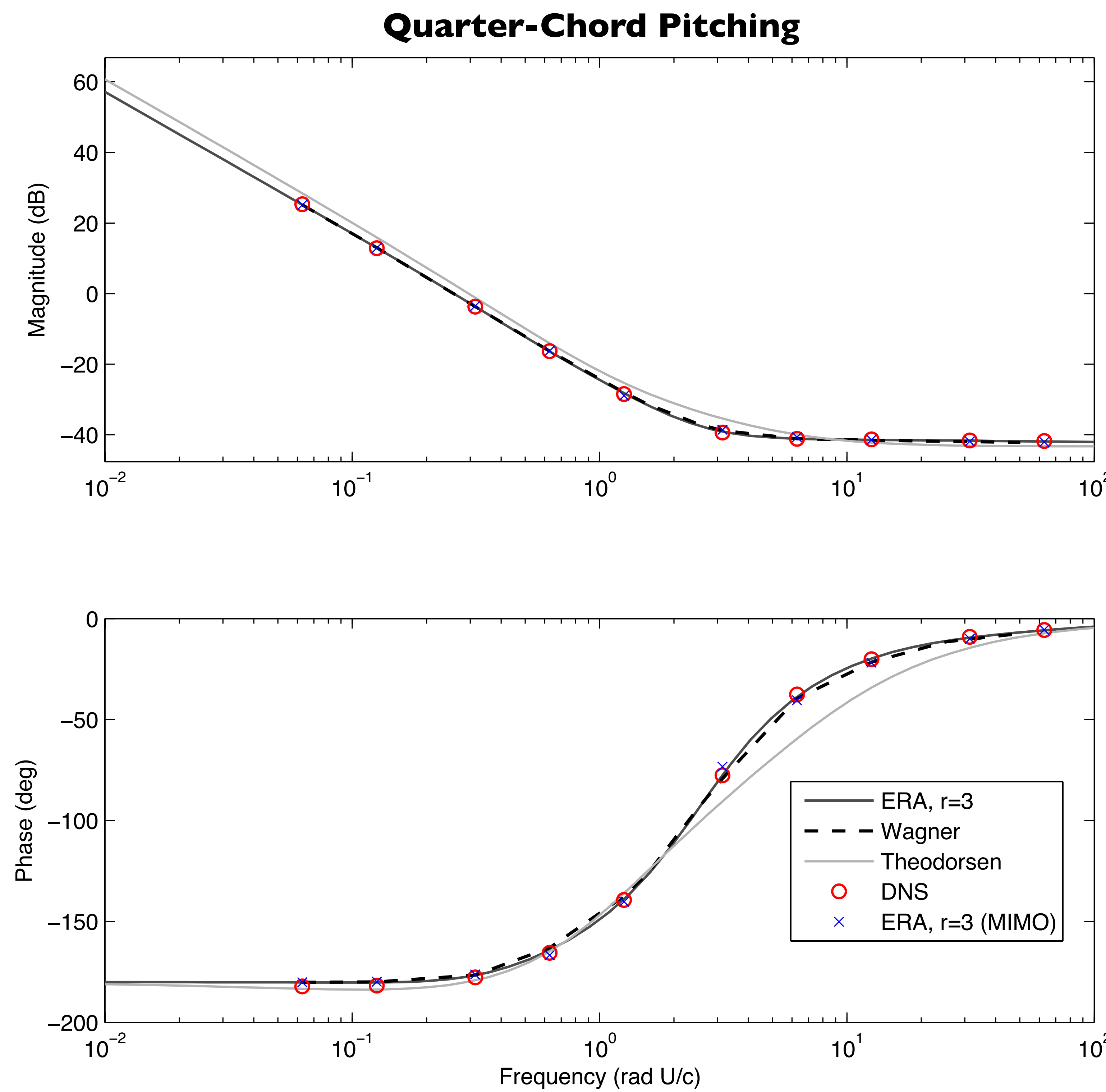
Bode Plot - Pitch (QC)

Frequency response

input is $\ddot{\alpha}$ (α is angle of attack)

output is lift coefficient C_L

Pitching at quarter chord



Reduced order model with ERA r=3 accurately reproduces Wagner

Wagner and ROM agree better with DNS than Theodorsen's model.

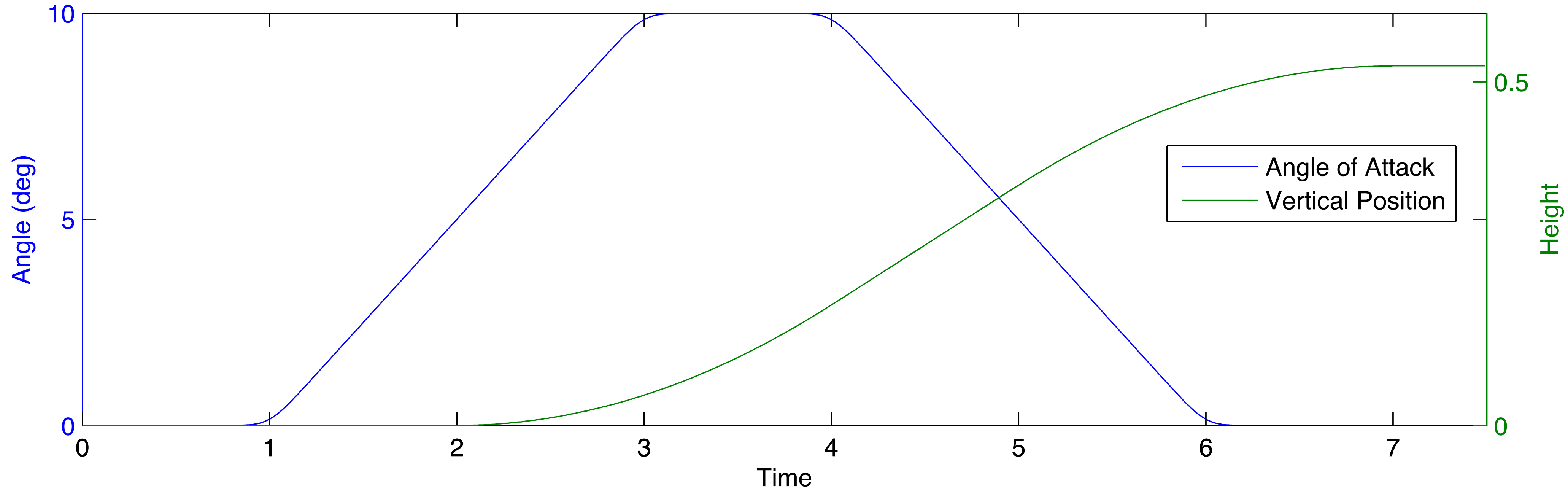
Asymptotes are correct for Wagner because it is based on experiment

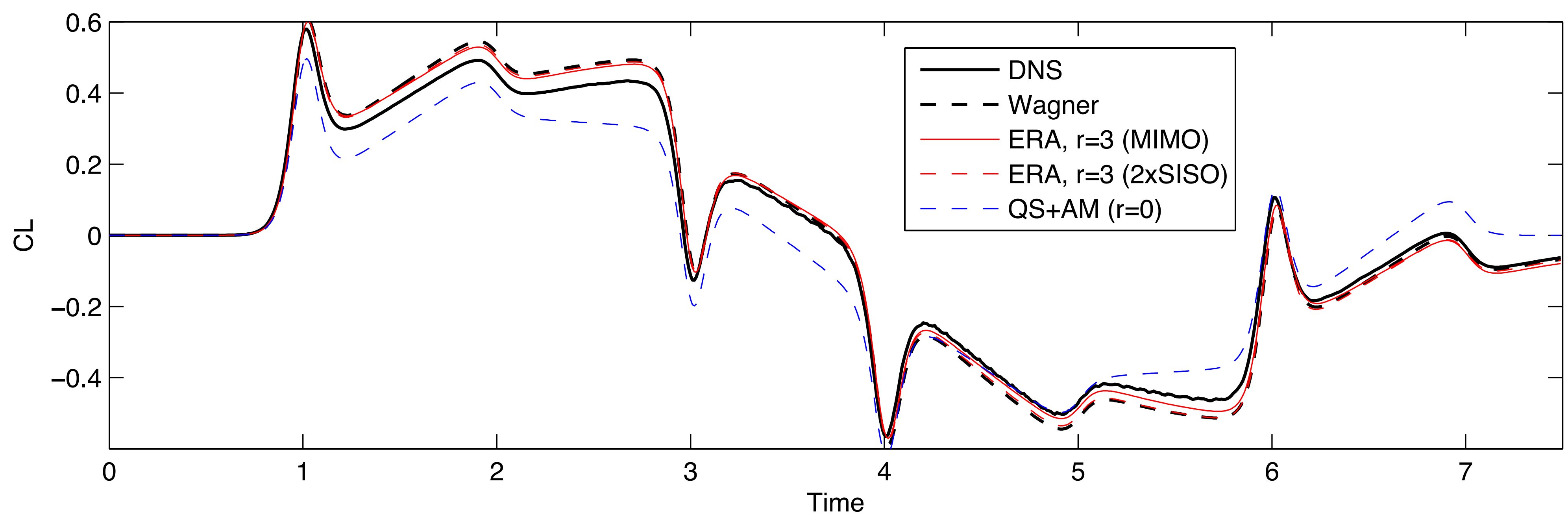
Model for pitch/plunge dynamics [ERA, r=3 (MIMO)] works as well, for the same order model

Brunton and Rowley, *in preparation*.

Pitch/Plunge Maneuver

Canonical pitch-up, hold, pitch-down maneuver, followed by step-up in vertical position





OL, Altman, Eldredge, Garmann, and Lian, 2010

Brunton and Rowley, *in preparation.*

Reduced order model for Wagner's indicial response accurately captures lift coefficient history from DNS

Summary

Reduced order model for Wagner's indicial response

- Based on eigensystem realization algorithm (ERA)
- Systematic reduced order models based on step-response
- Linear input-output system ideal for flight dynamic framework

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Future Directions

Combine ERA models with nonlinear heuristic and POD models

- Capture unsteady forces due to vortex shedding and stall

Generalize theory to large angle of attack

Develop H2 optimal controller to minimize gust disturbance

References:

Haller, 2002

Shadden *et al.*, 2005

Leishman, 2006.

Brunton and Rowley, 2010

Wagner, 1925.

Theodorsen, 1935.

Brunton and Rowley, AIAA ASM 2009

Brunton and Rowley, *in preparation*.

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