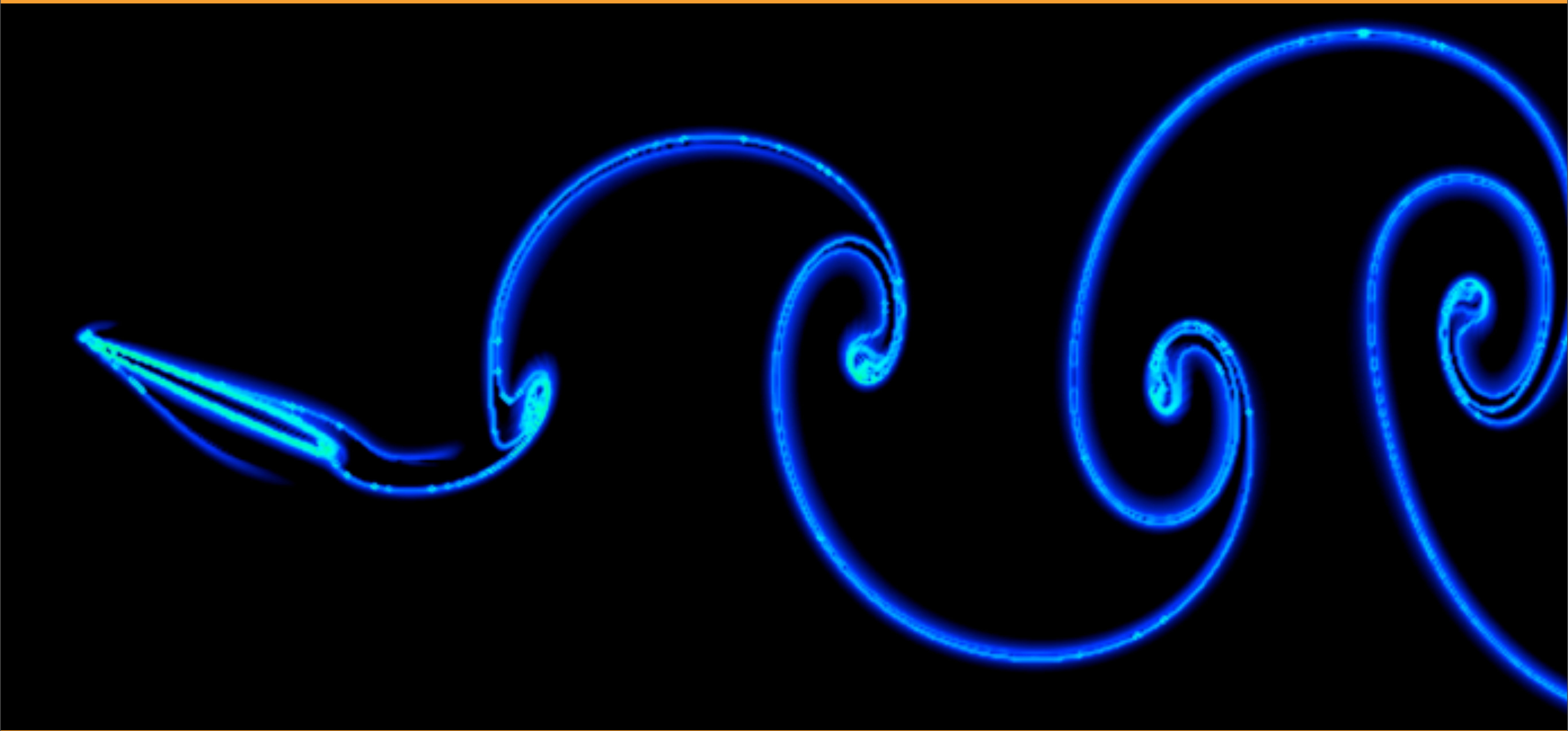
Unsteady aerodynamic models for separated flows past a flat plate at Re=100



Steve Brunton & Clancy Rowley
Princeton University
64th APS DFD November 21, 2011

Motivation

Need for State-Space Models

Need models suitable for control

Compatible with flight models

Bio Propulsion

High propulsive efficiency, maximum lift coefficient

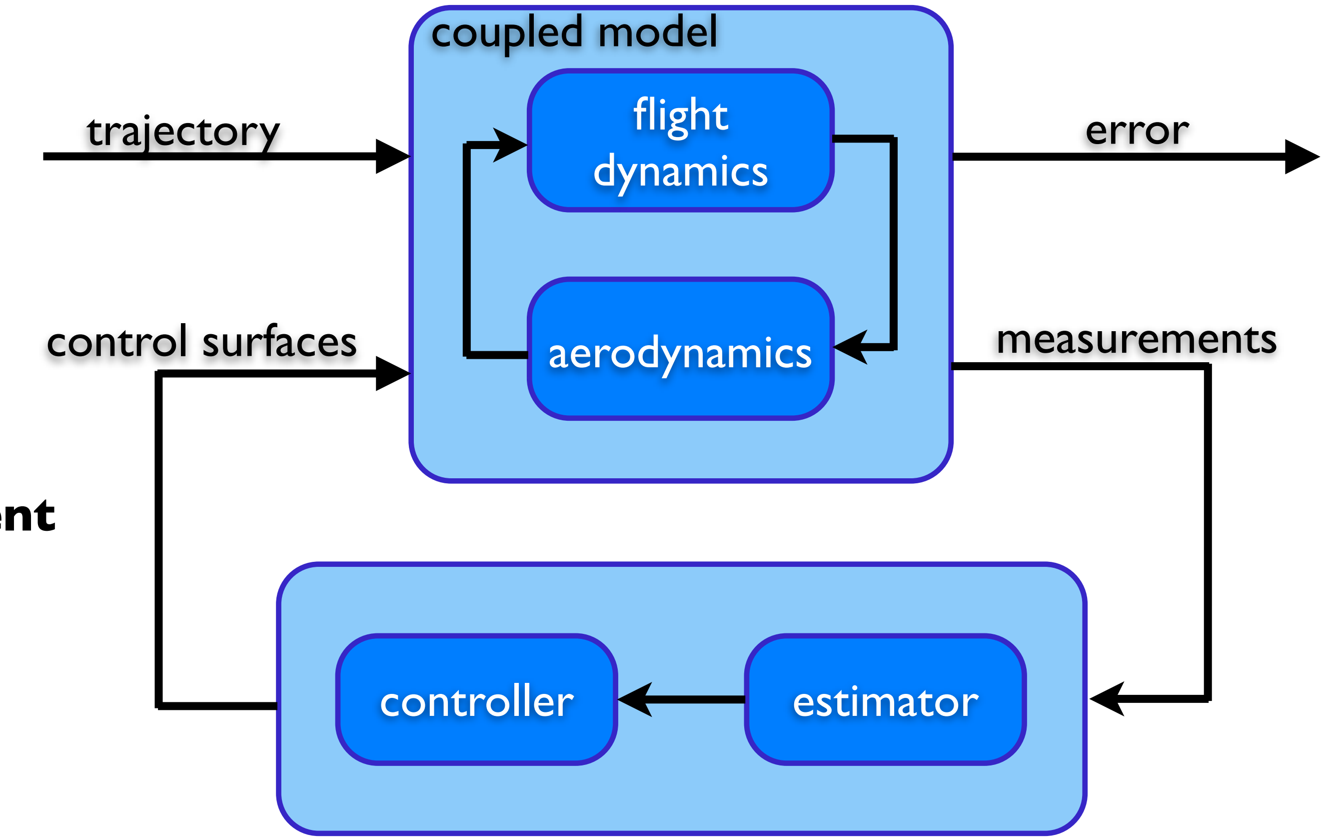
Efficient utilization of gusts and wake vorticity

Unmanned Aerial Vehicles

Flow control, flight dynamic control

Autopilots / Flight simulators

Gust disturbance mitigation



2D Model Problem

Added-Mass

Transient

Periodic Vortex Shedding

Lift

Drag

$\mathrm{Re} = 300$

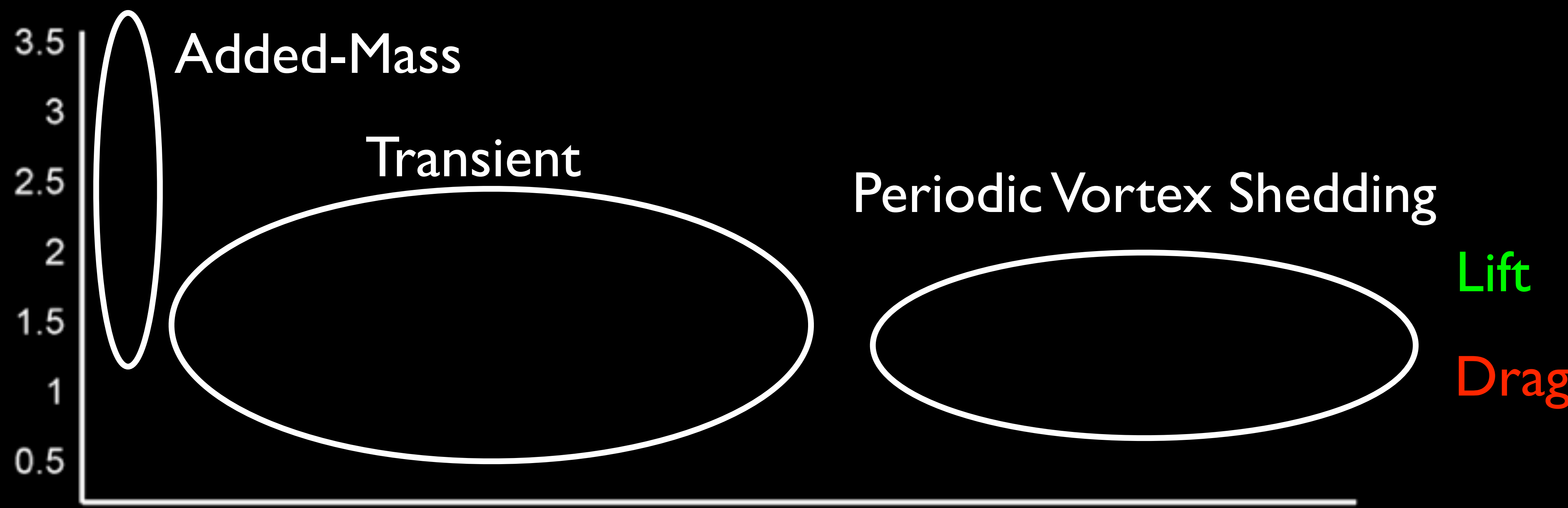
$\alpha = 32^\circ$

2D Incompressible Navier-Stokes

Immersed boundary method

Taira & Colonius, 2007.

2D Model Problem



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Unsteady Aerodynamic Forces

Added Mass

Increasingly important for small/light aircraft

Unsteady potential flow forces (F=ma)

force needed to move air as plate accelerates

Circulatory/Viscous

Captures separation effects

Need improved models here

source of all lift in steady flight... and more

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The mass of the body and surrounding fluid are being accelerated, to different extents.

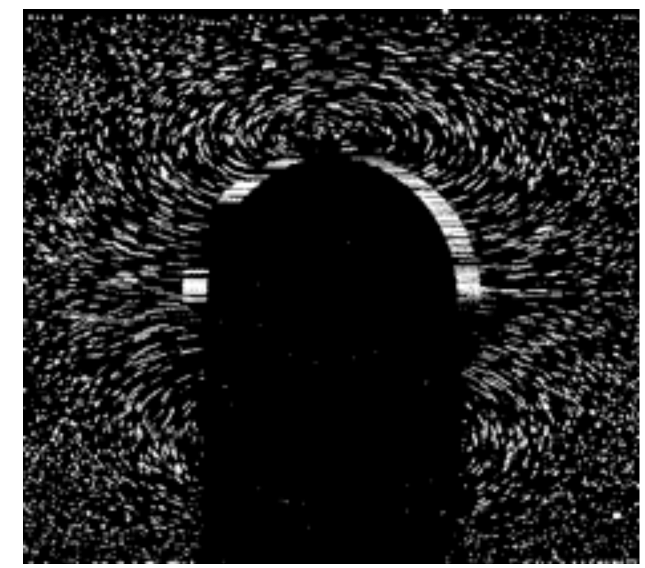
Kinetic energy T will be in some manner proportional to U (for potential and Stokes flows)

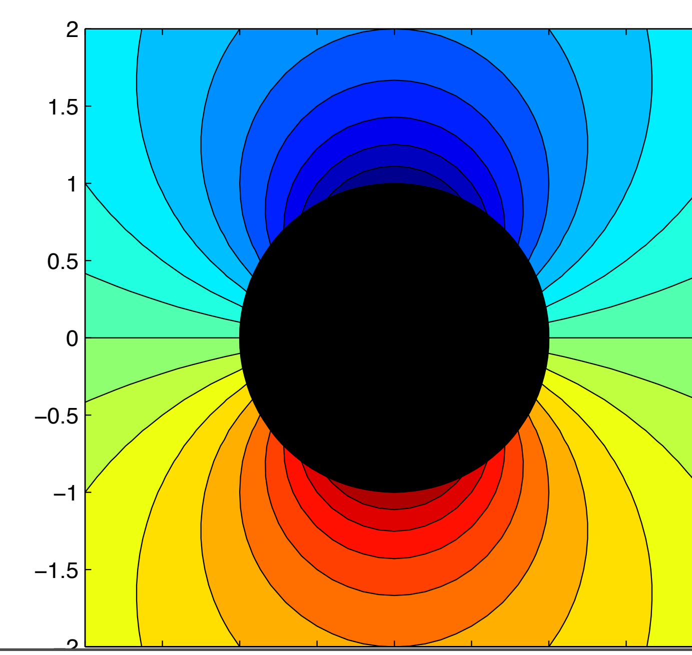
$$T = \rho \frac{I}{2} U^2 \qquad \text{where} \qquad I = \int_V \frac{u_i}{U} \cdot \frac{u_i}{U} dV$$

If body accelerates, T probably increases, and energy must be supplied:

$$\frac{dT}{dt} = -FU \quad \Longrightarrow \quad F_i = -\underbrace{\rho I_{ij}}_{\text{AM}} \dot{U}_j$$

cylinder moving in Lab frame





Lamb, 1945.

Milne-Thompson, 1962

Newman, 1977.

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Beer bubble acceleration

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Boundary layer

Laminar separation bubble

Leading edge vortex

Periodic Vortex Shedding

Milne-Thompson, 1973.

Stengel, 2004.

Theodorsen's Model - 1935

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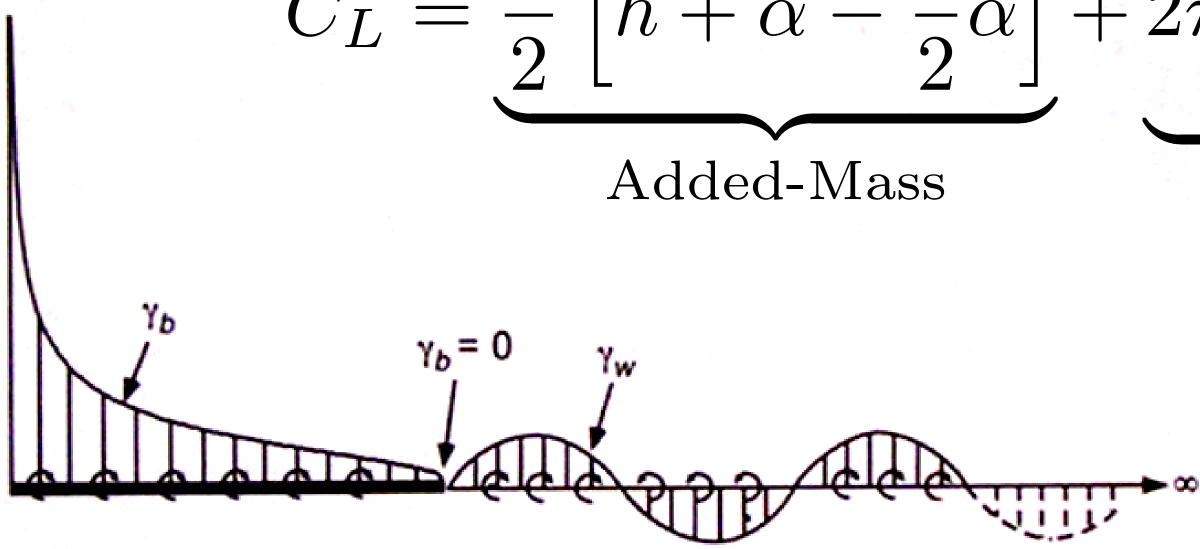
source of all lift in steady flight... and more

$$C_L = \underbrace{\frac{\pi}{2}\left[\ddot{h} + \dot{\alpha} - \frac{a}{2}\ddot{\alpha}\right]}_{\text{Added-Mass}} + \underbrace{2\pi\left[\alpha + \dot{h} + \frac{1}{2}\dot{\alpha}\left(\frac{1}{2} - a\right)\right]}_{\text{Circulatory}} C(k)$$



$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)}$$

2D Incompressible, inviscid model

Unsteady potential flow (w/ Kutta condition)

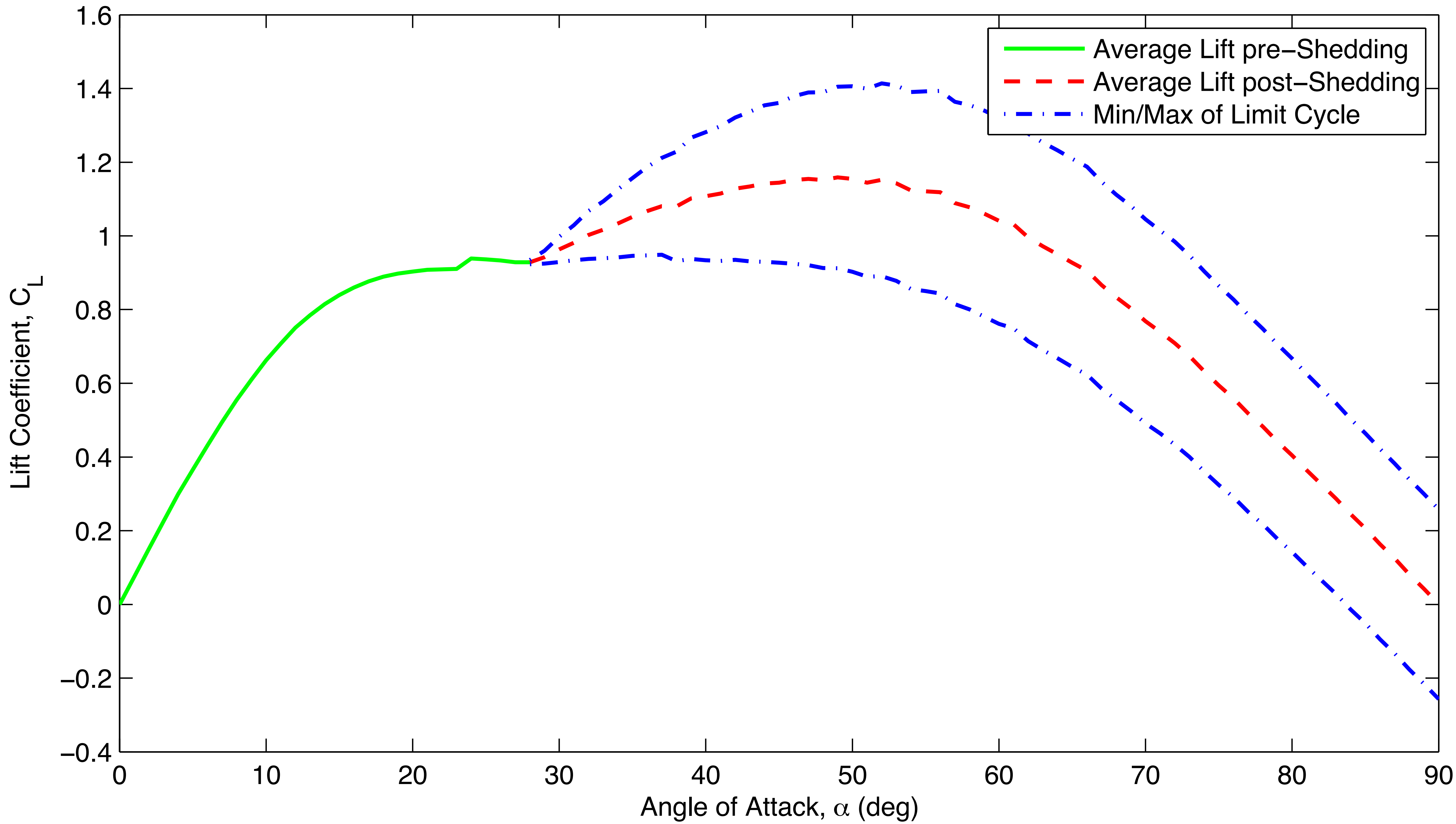
Linearized about zero angle of attack

$$\boxed{k = \frac{\pi f c}{U_\infty}}$$

Theodorsen, 1935.

Leishman, 2006.

Lift vs. Angle of Attack



Low Reynolds number, (Re=100)

Hopf bifurcation at $\alpha_{\text{crit}} \approx 28^\circ$ **(pair of imaginary eigenvalues pass into right half plane)**

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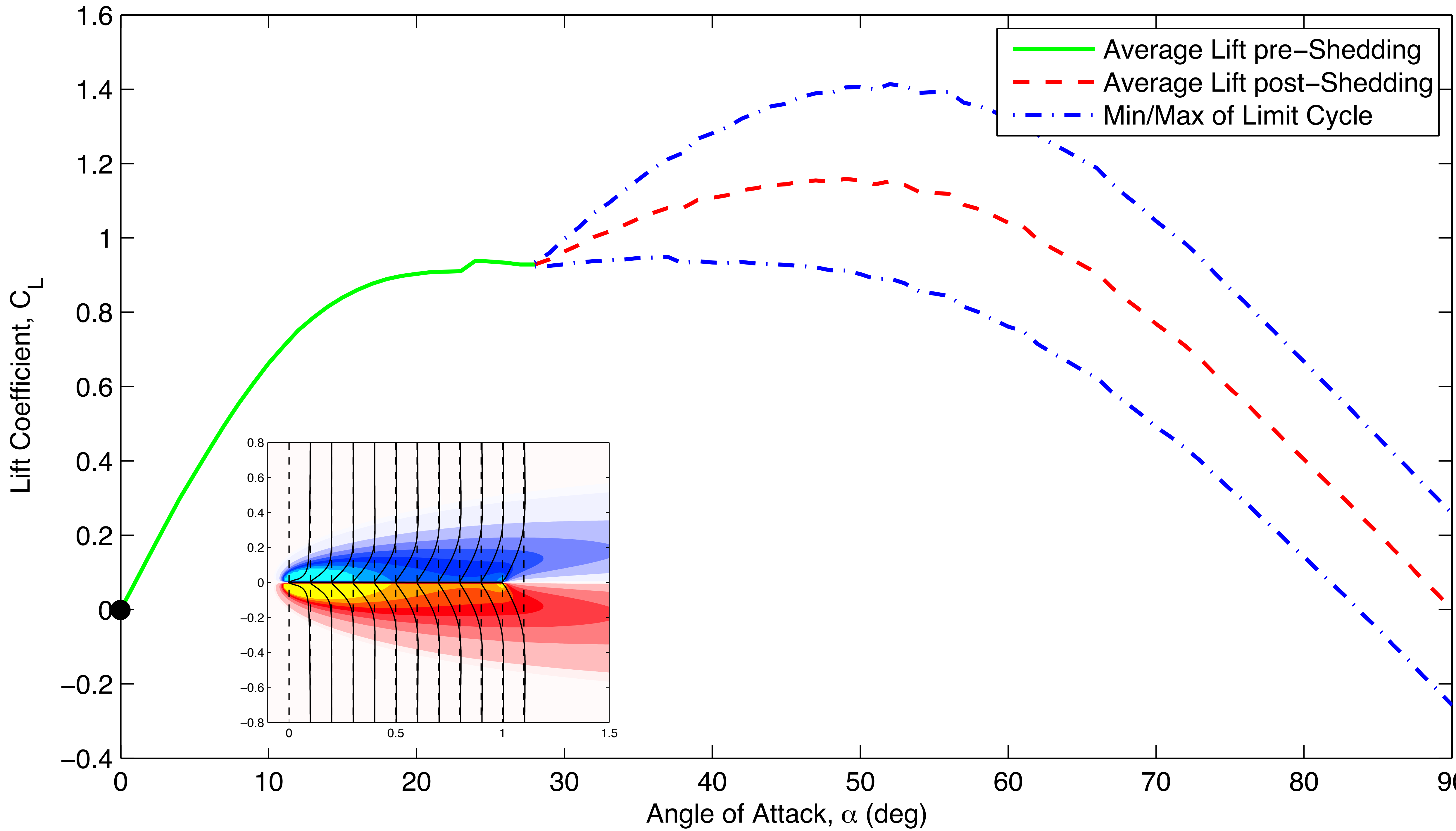

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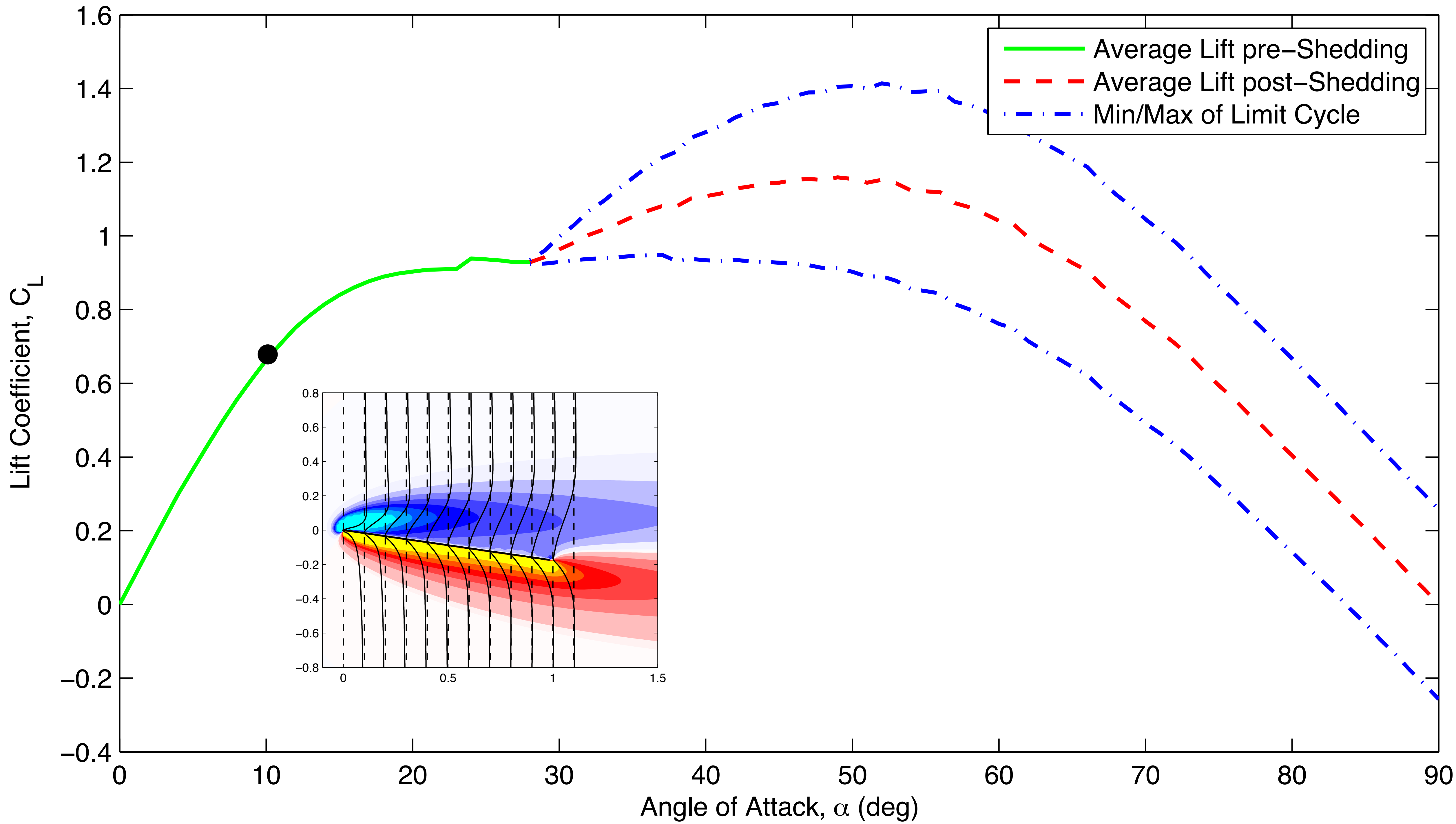

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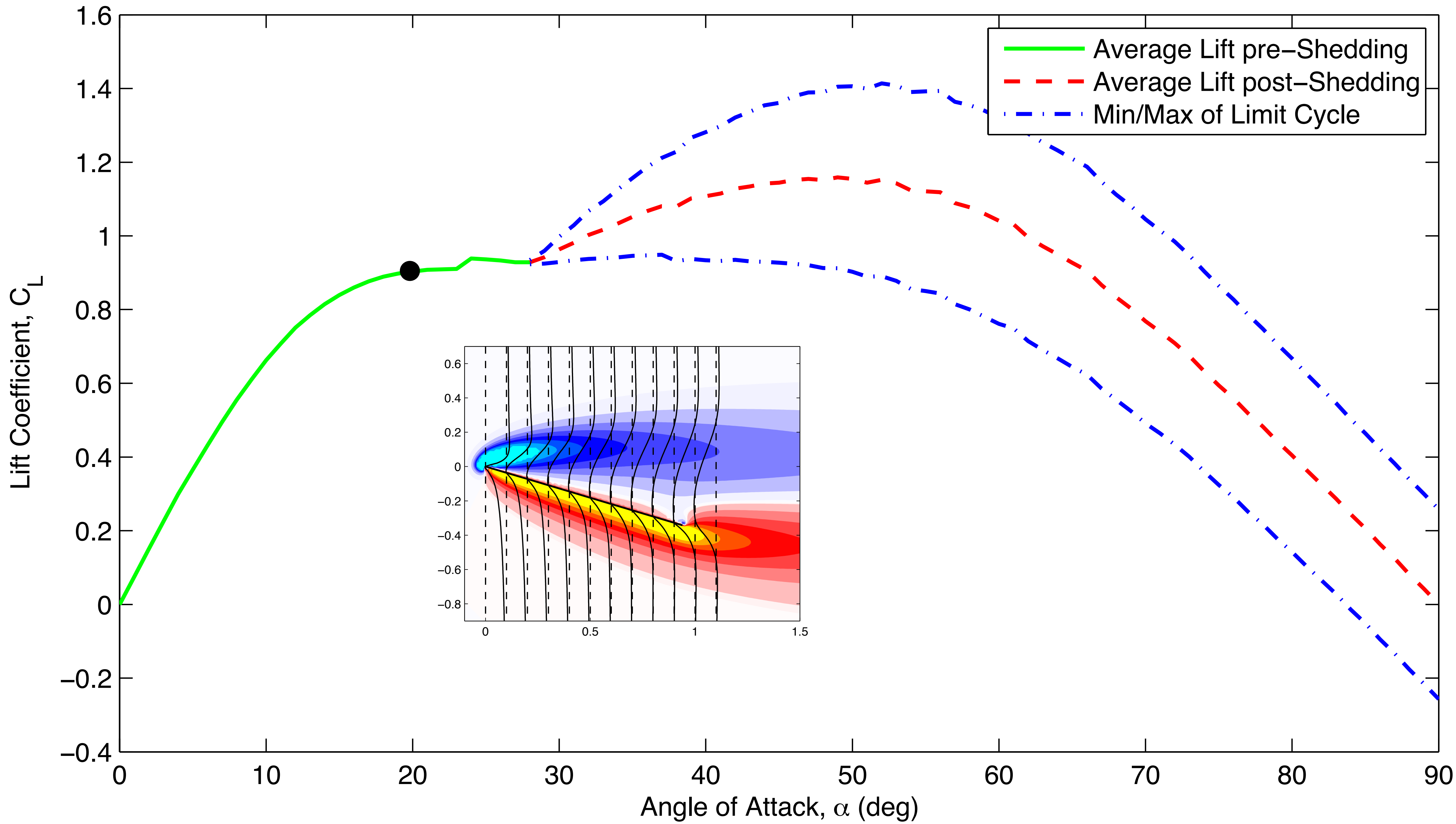

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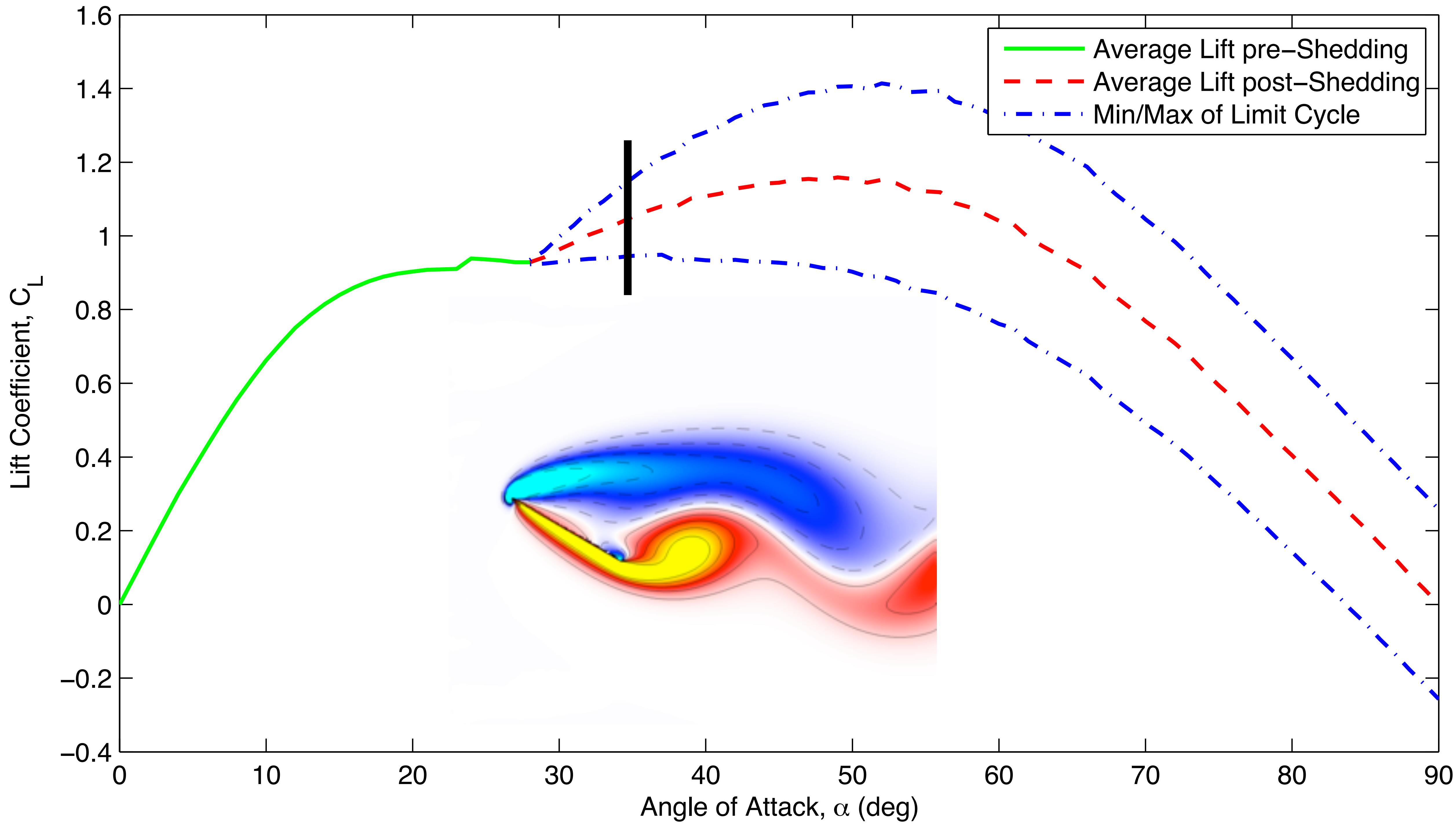

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Nonlinear Unsteady Models

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}; \mu)$$

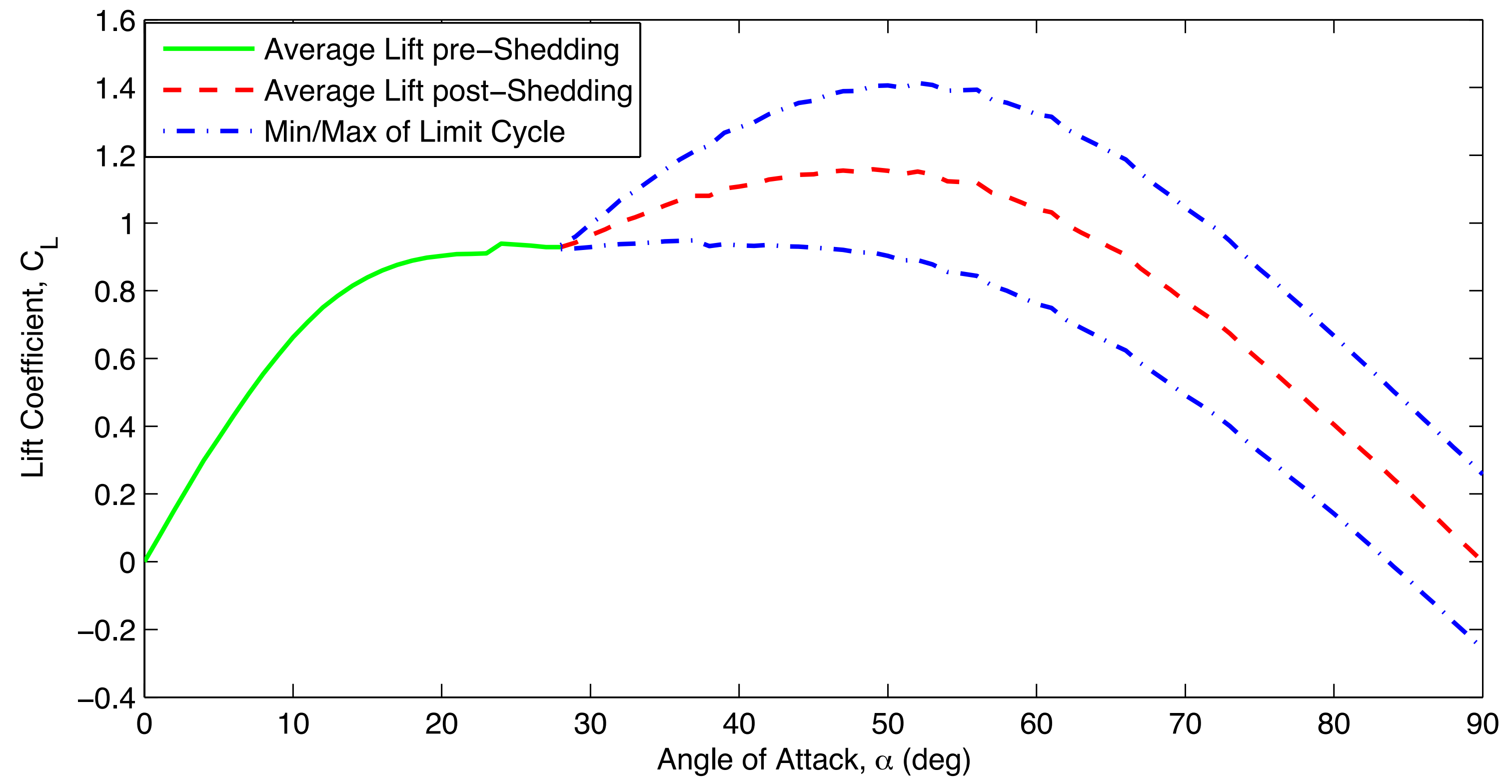
$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}; \mu)$$

u - input

y - output

x - state vector

μ - bifurcation parameter



For $\alpha_0 < \alpha_{\text{crit}}$, equilibrium x=0 is stable, with linear dynamics given by:

nonlinear lift model

$$\dot{\mathbf{x}} \triangleq \frac{d}{dt}\begin{bmatrix} x \\ \alpha \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} f_{\text{NS}}(x, \alpha, \dot{\alpha}, \ddot{\alpha}) \\ \dot{\alpha} \\ \ddot{\alpha} \end{bmatrix}$$

$$\begin{aligned} y &= g_{\text{lift}}(x, \alpha, \dot{\alpha}, \ddot{\alpha}) \\ &= g_\nu(x, \alpha, \dot{\alpha}) + g_\phi(\dot{\alpha}, \ddot{\alpha}) \end{aligned}$$

linearization at $\bar{\mathbf{x}}(\alpha_0)$

$$\frac{d}{dt}\begin{bmatrix} x \\ \alpha \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} B_3 \\ 0 \\ 1 \end{bmatrix}\ddot{\alpha}$$

$$C_L = \begin{bmatrix} C & C_\alpha & C_{\dot{\alpha}} \end{bmatrix}\begin{bmatrix} x \\ \alpha \\ \dot{\alpha} \end{bmatrix} + C_{\ddot{\alpha}}\ddot{\alpha}$$

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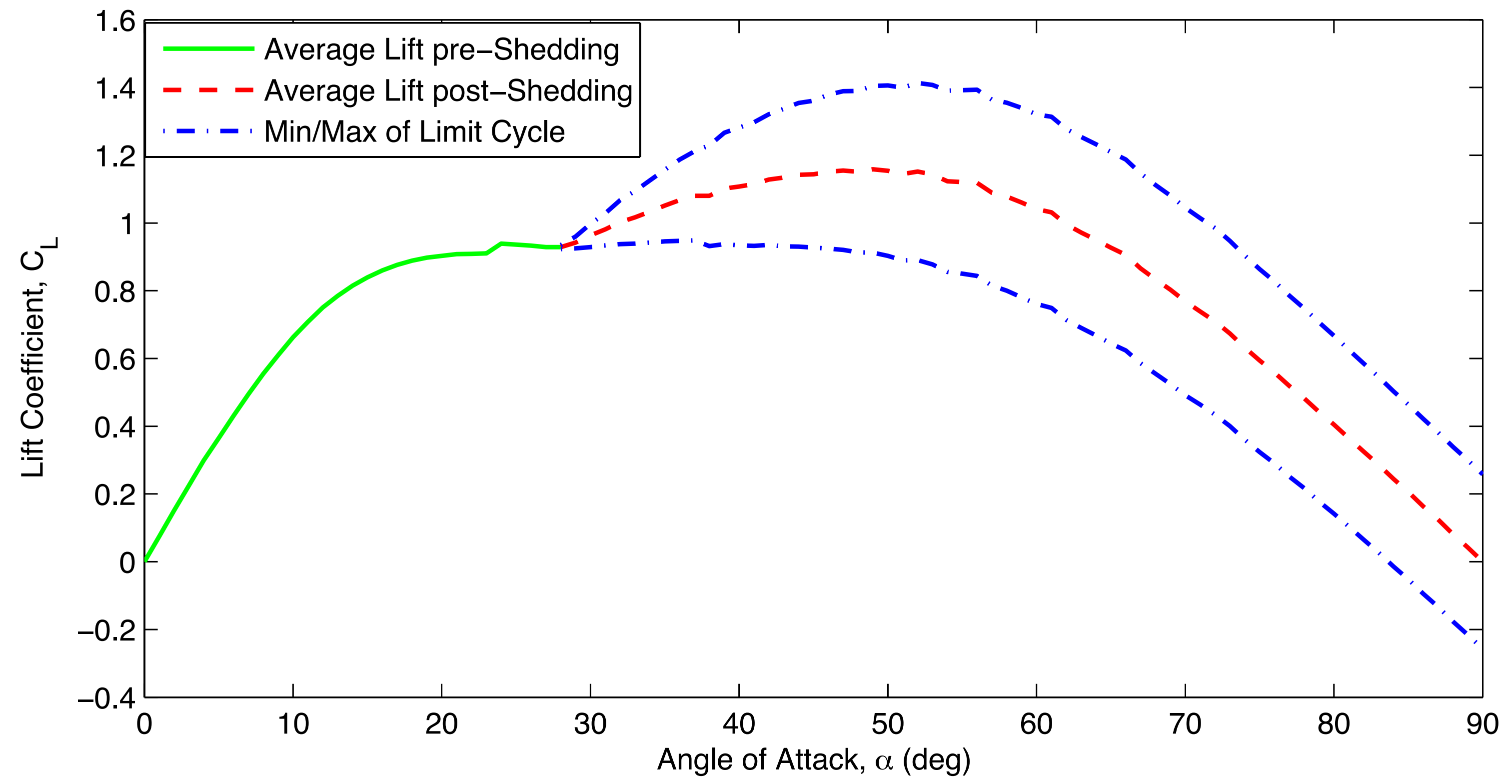

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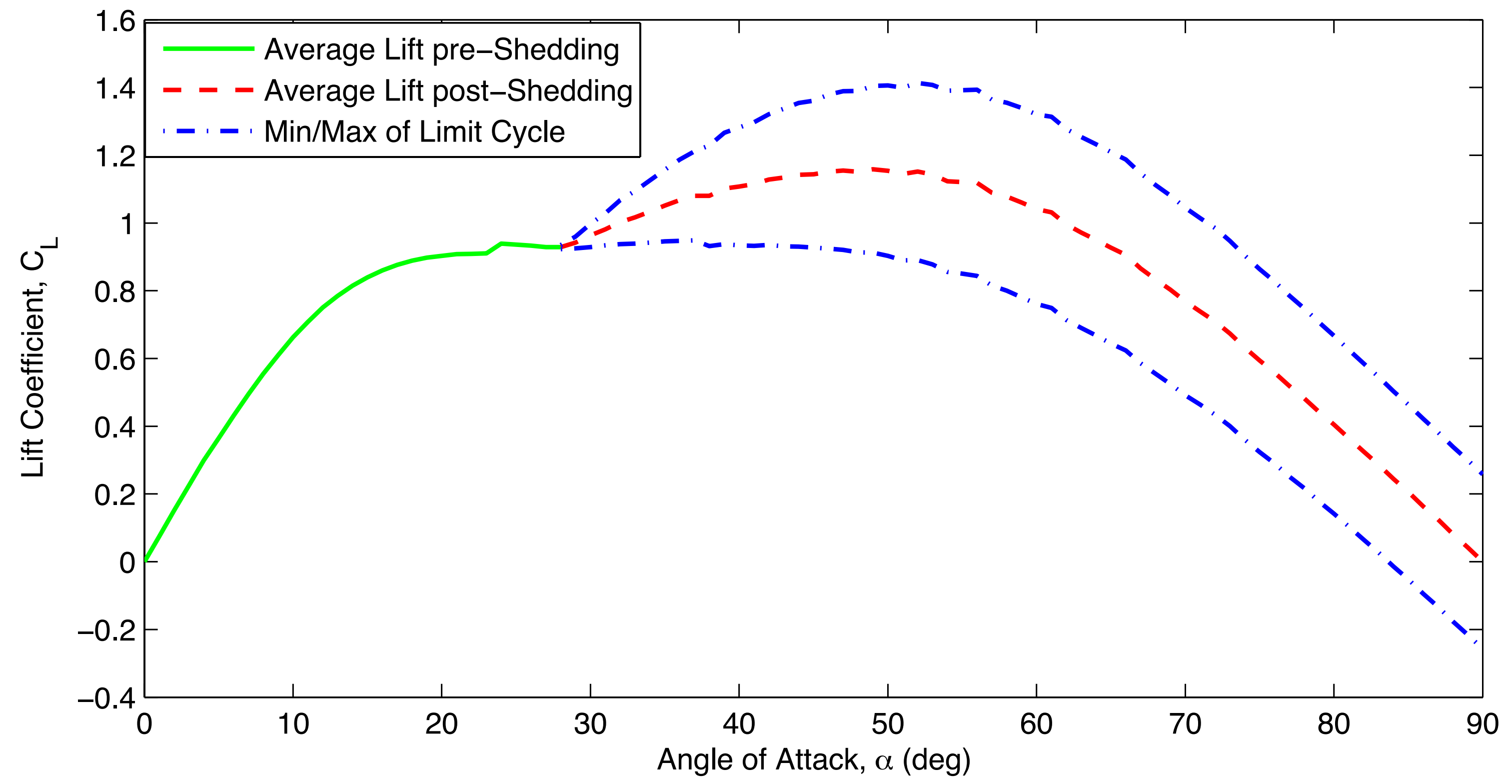

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$$\begin{aligned} \dot{x} &= Ax + B\ddot{\alpha} \\ \tilde{C}_L &= Cx \end{aligned}$$

$\Longleftarrow$

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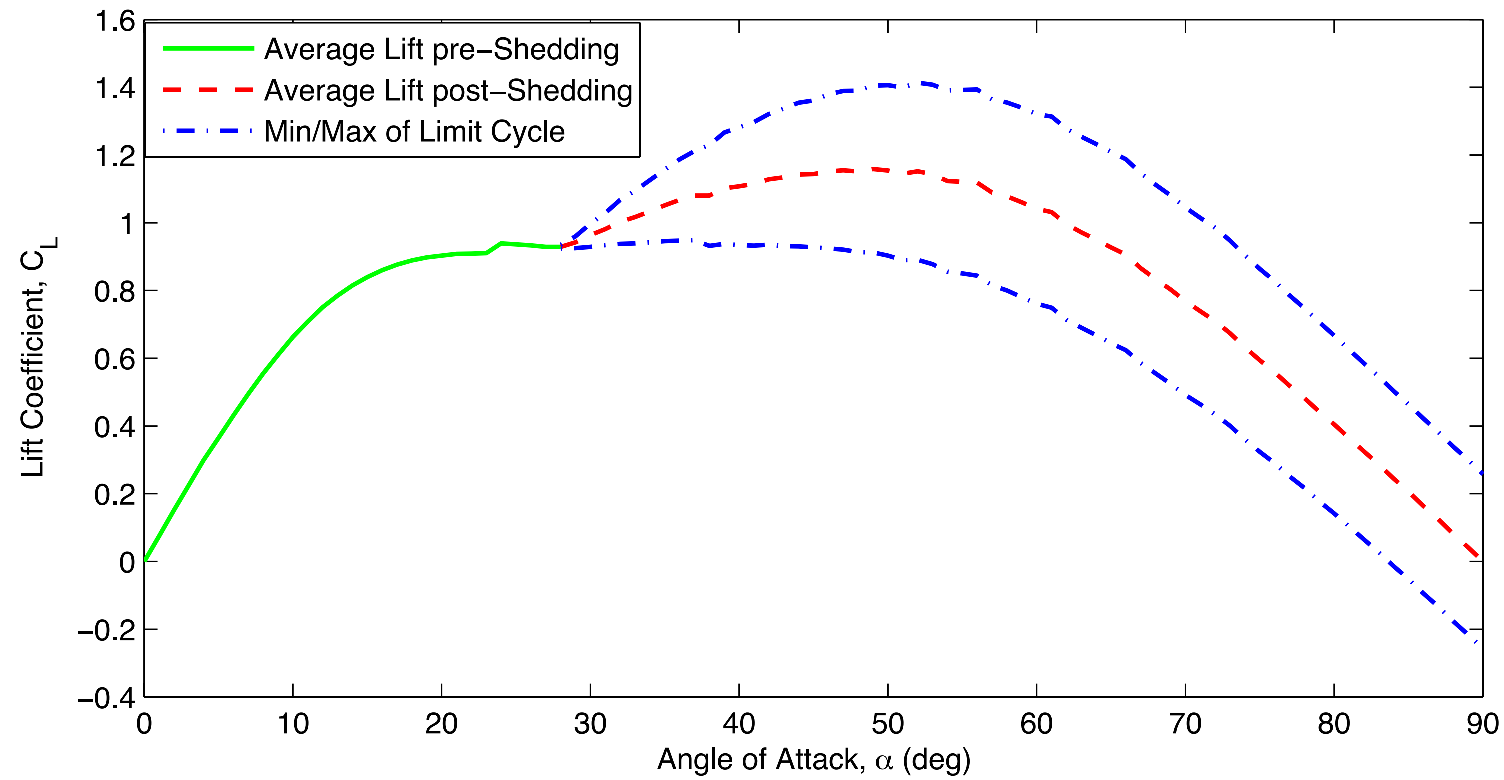

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$$A(\alpha_0) = \left[\begin{array}{cc|c} \mu & -\omega & \multirow{2}{*}{0} \\ \omega & \mu & \\ \hline \multicolumn{2}{c|}{0} & A_S \end{array}\right]$$

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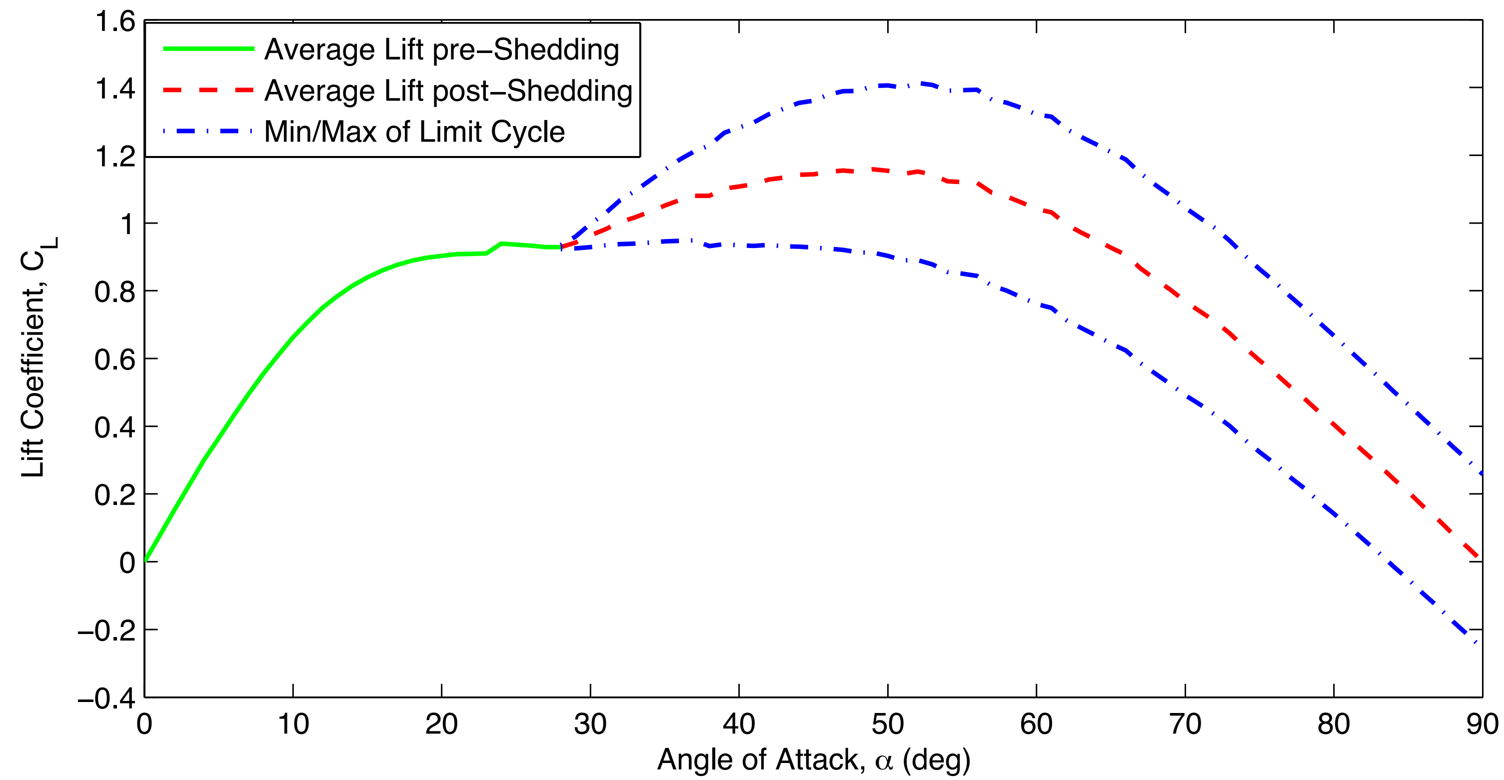

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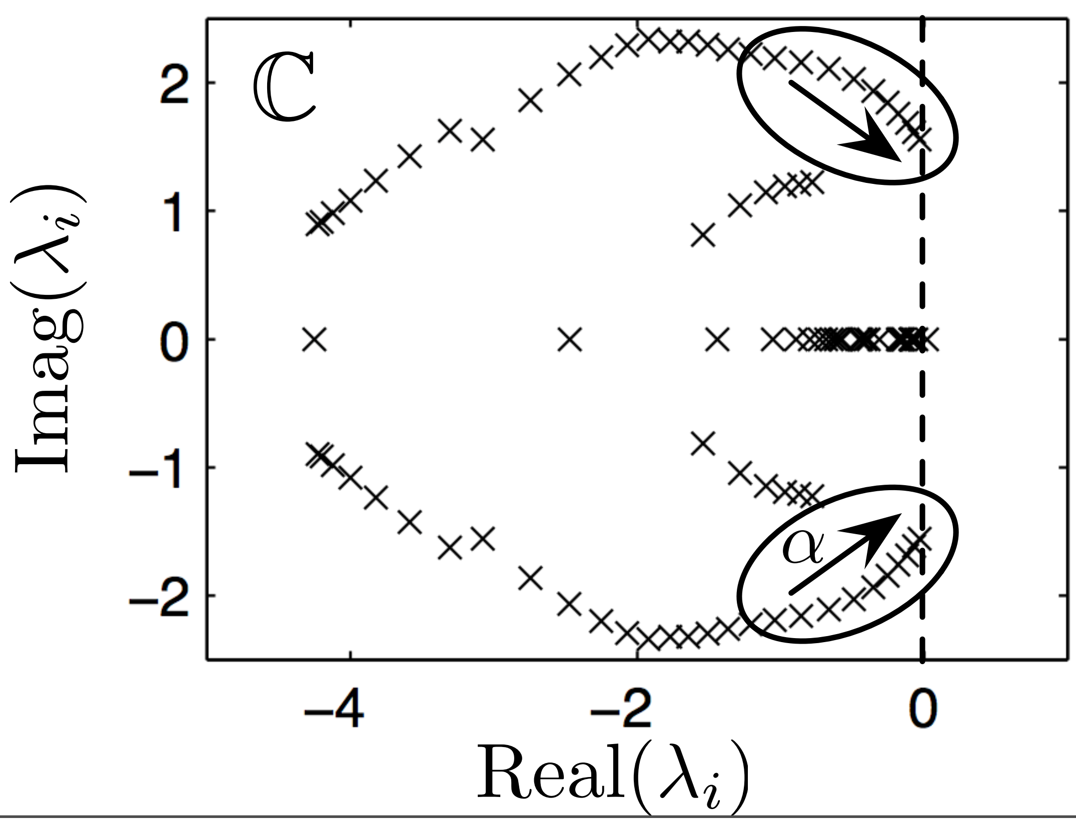

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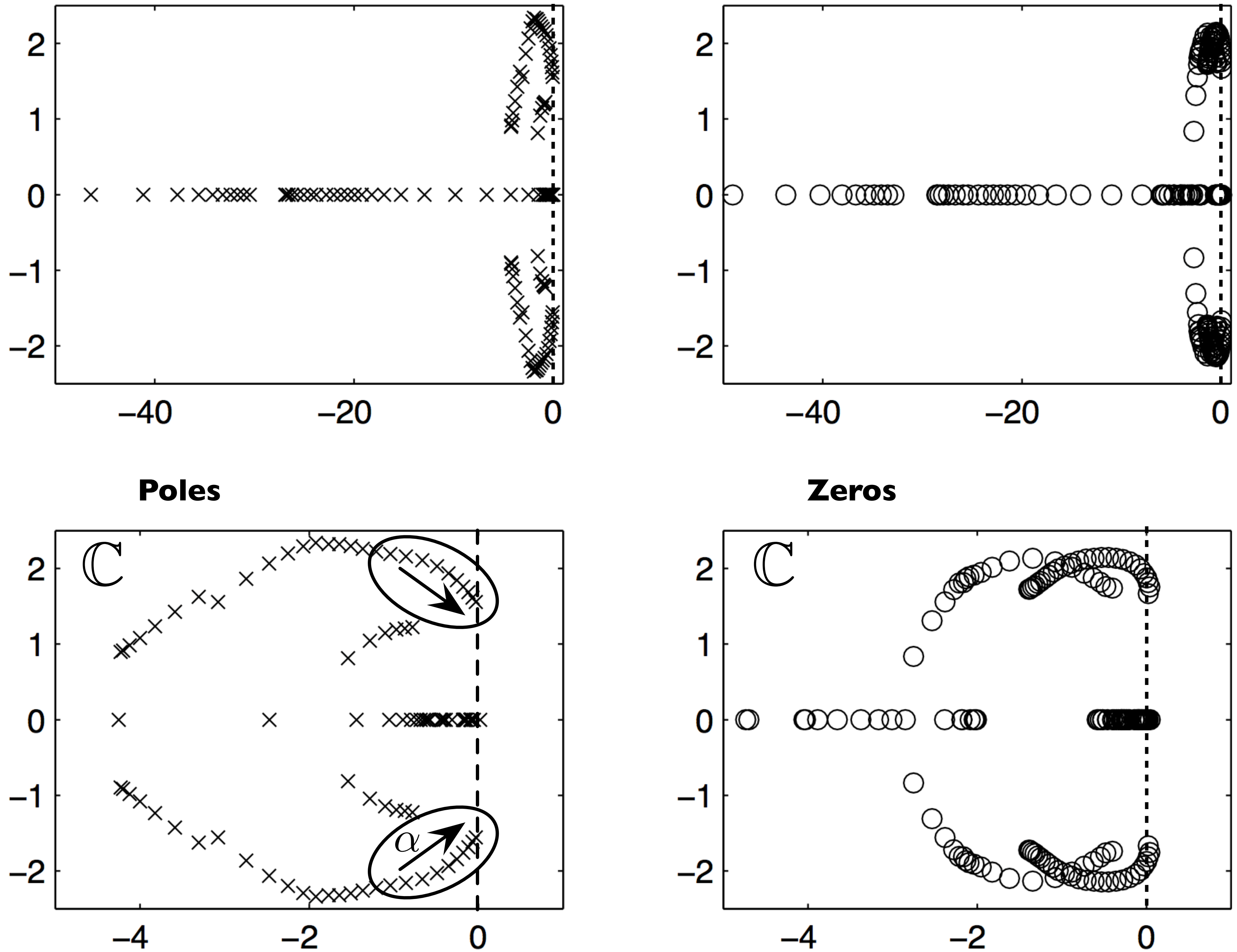
Poles and Zeros of ERA Models



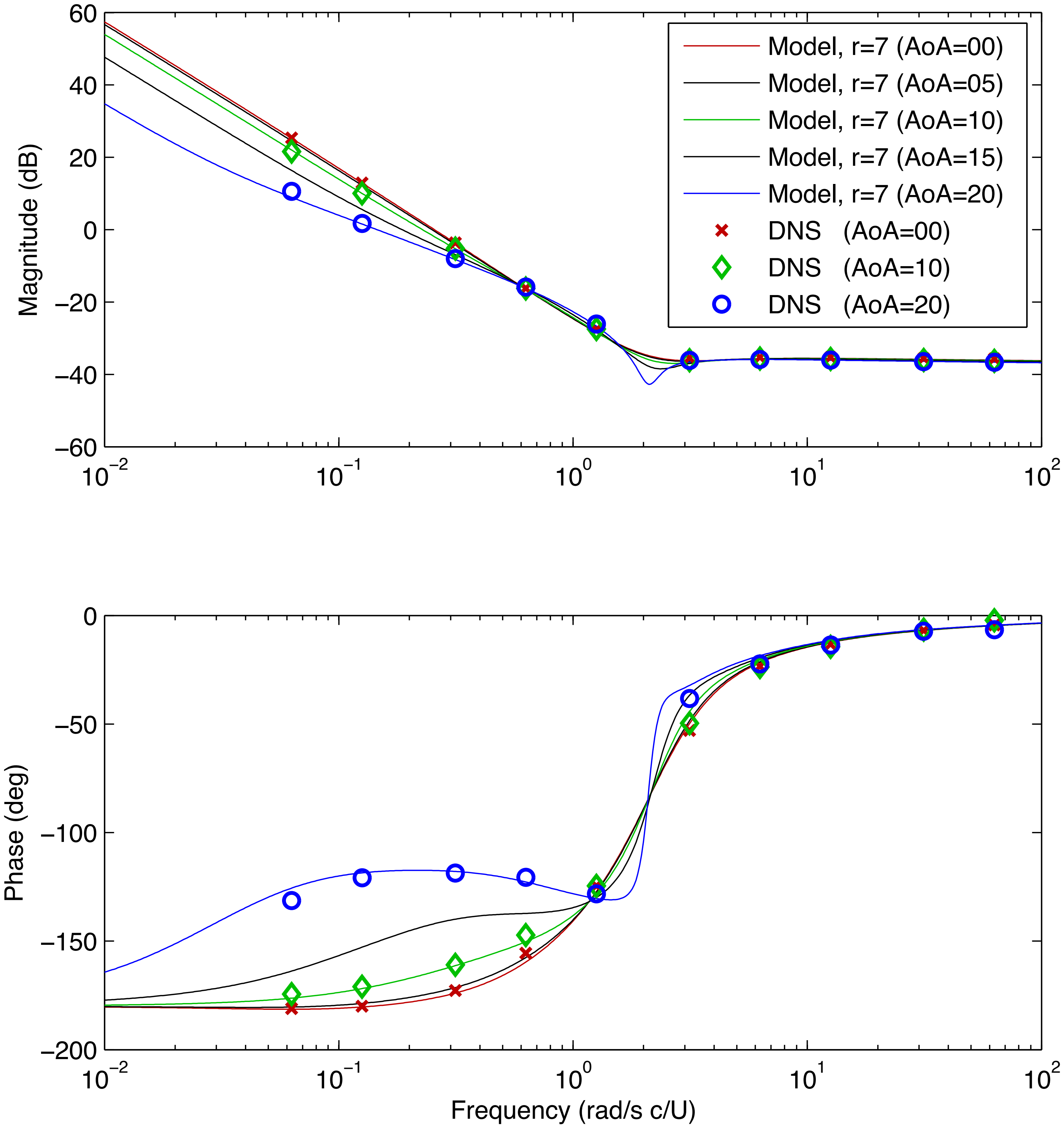
As angle of attack increases, pair of poles (and pair of zeros) march towards imaginary axis.

This is a good thing, because a Hopf bifurcation occurs at $\alpha_{\text{crit}} \approx 28^\circ$

Brunton and Rowley, AIAA ASM 2011

Bode Plot of Model (-) vs Data (x)

Frequency response for pitching about the leading edge



Direct numerical simulation confirms that local linearized models are accurate for small amplitude sinusoidal maneuvers

Results

Lift slope decreases for increasing angle of attack, so magnitude of low frequency motions decreases for increasing angle of attack.

At larger angle of attack, phase converges to -180 at much lower frequencies. I.e., solutions take longer to reach equilibrium in time domain.

Consistent with fact that for large angle of attack, system is closer to Hopf instability, and a pair of eigenvalues are moving closer to imaginary axis.

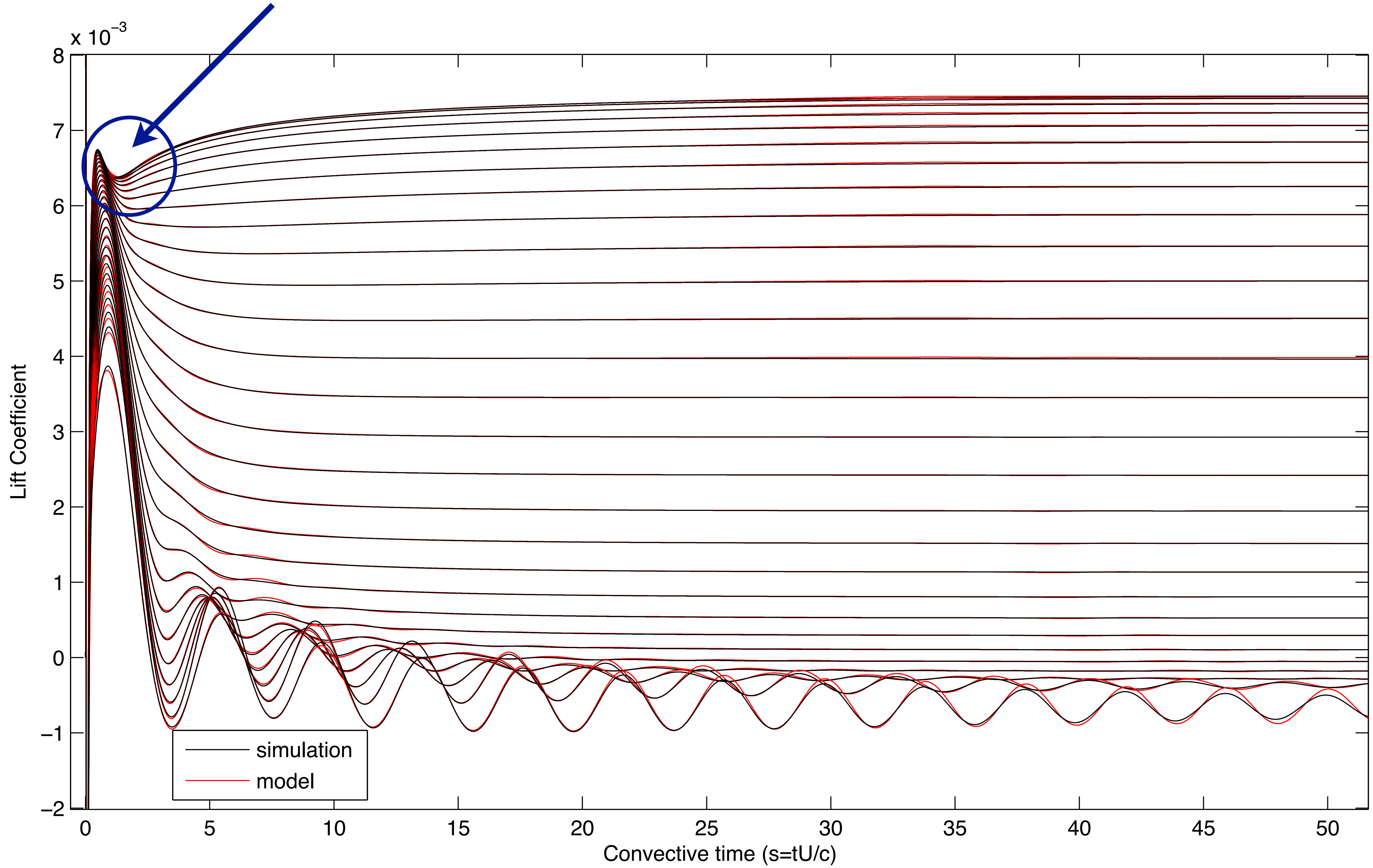
Brunton and Rowley, AIAA ASM 2011

Models at various angle of attack

Impulse response simulations after rapid step-up $\alpha \in [0^\circ, 27^\circ]$

Initial lift $C_L(\alpha_0)$ **subtracted off**

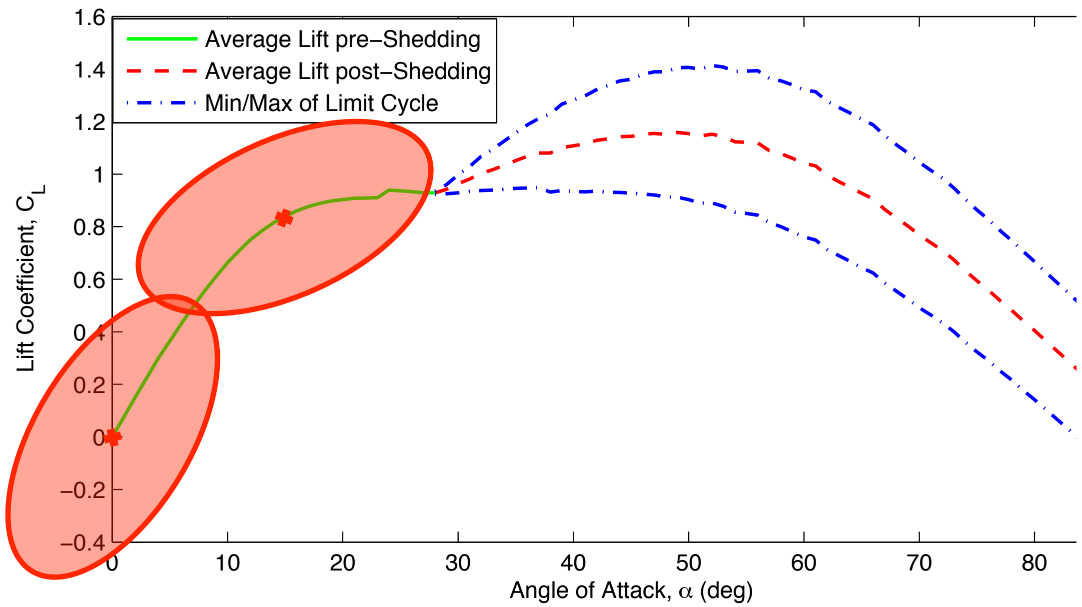
Model with order r=7 required to capture this flow feature, eventually develops into vortex shedding mode

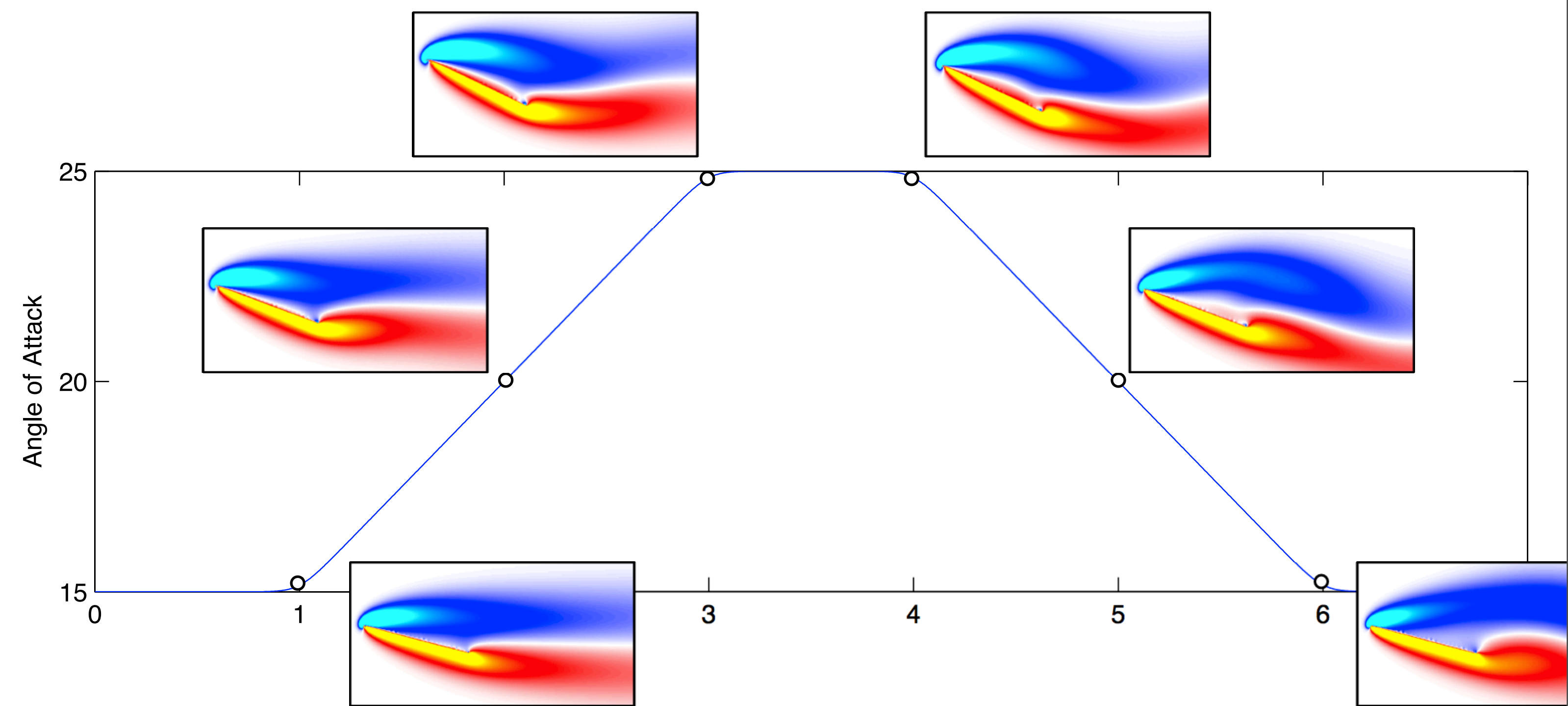


$\alpha = 0^\circ$

$\alpha = 27^\circ$

Large Amplitude Maneuver





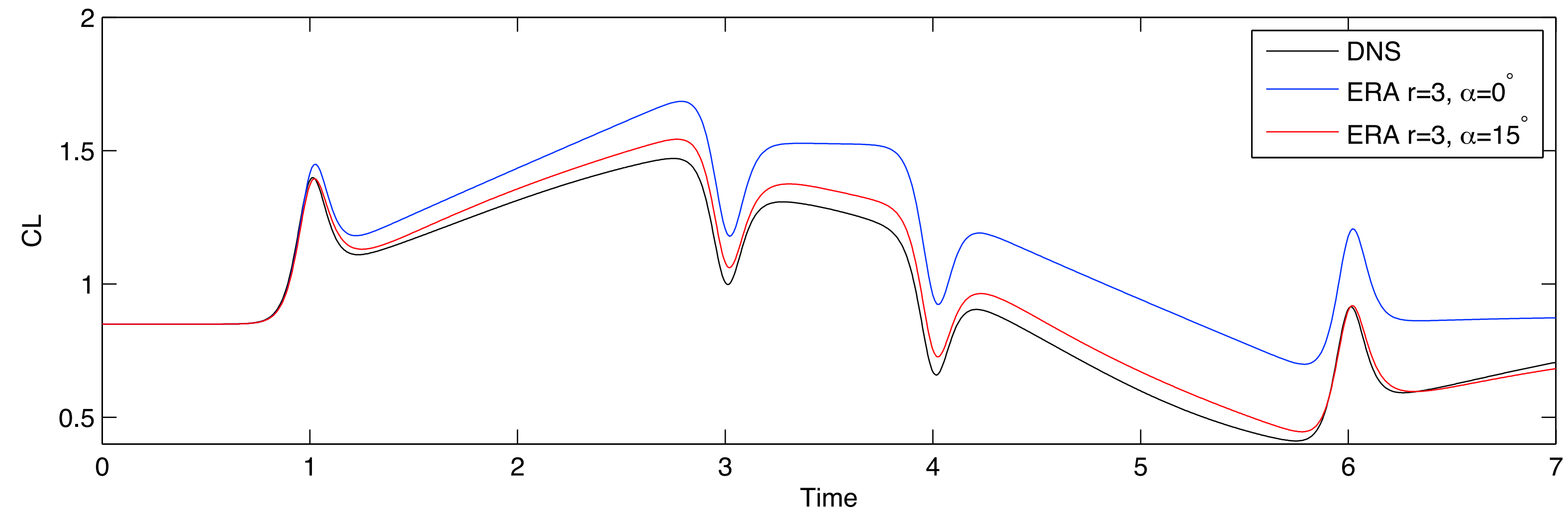
For pitching maneuver with

$$\alpha \in [\mathbf{15^\circ}, \mathbf{25^\circ}]$$

Model linearized at $\alpha = \mathbf{15^\circ}$

captures lift response more accurately

than model linearized at $\alpha = \mathbf{0^\circ}$



$$G(t) = \log\left[\frac{\cosh(a(t-t_1))\cosh(a(t-t_4))}{\cosh(a(t-t_2))\cosh(a(t-t_3))}\right] \qquad \alpha(t) = \alpha_0 + \alpha_{\max}\frac{G(t)}{\max(G(t))}$$

Brunton and Rowley, AIAA ASM 2011 **OL, Altman, Eldredge, Garmann, and Lian, 2010**

Summary

Aerodynamic models linearized from unsteady Navier-Stokes

- **Separate terms for added-mass, quasi-steady, and fluid transients**
- **Transient dynamics modeled with the eigensystem realization algorithm**
- **Accurate for separated flows up to the Hopf bifurcation**

Future Directions

Interpolate between models linearized at different angle of attack

- **Low-order model states are different at each angle of attack**

Include nonlinear terms based on Hopf normal form

Develop H2 optimal controller for partially stalled wings

References:

Theodorsen, 1935.

Lamb, 1945.

Milne-Thompson, 1962

Newman, 1977.

Leishman, 2006.

Taira & Colonius, 2007.

Brunton and Rowley, AIAA ASM 2009.

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