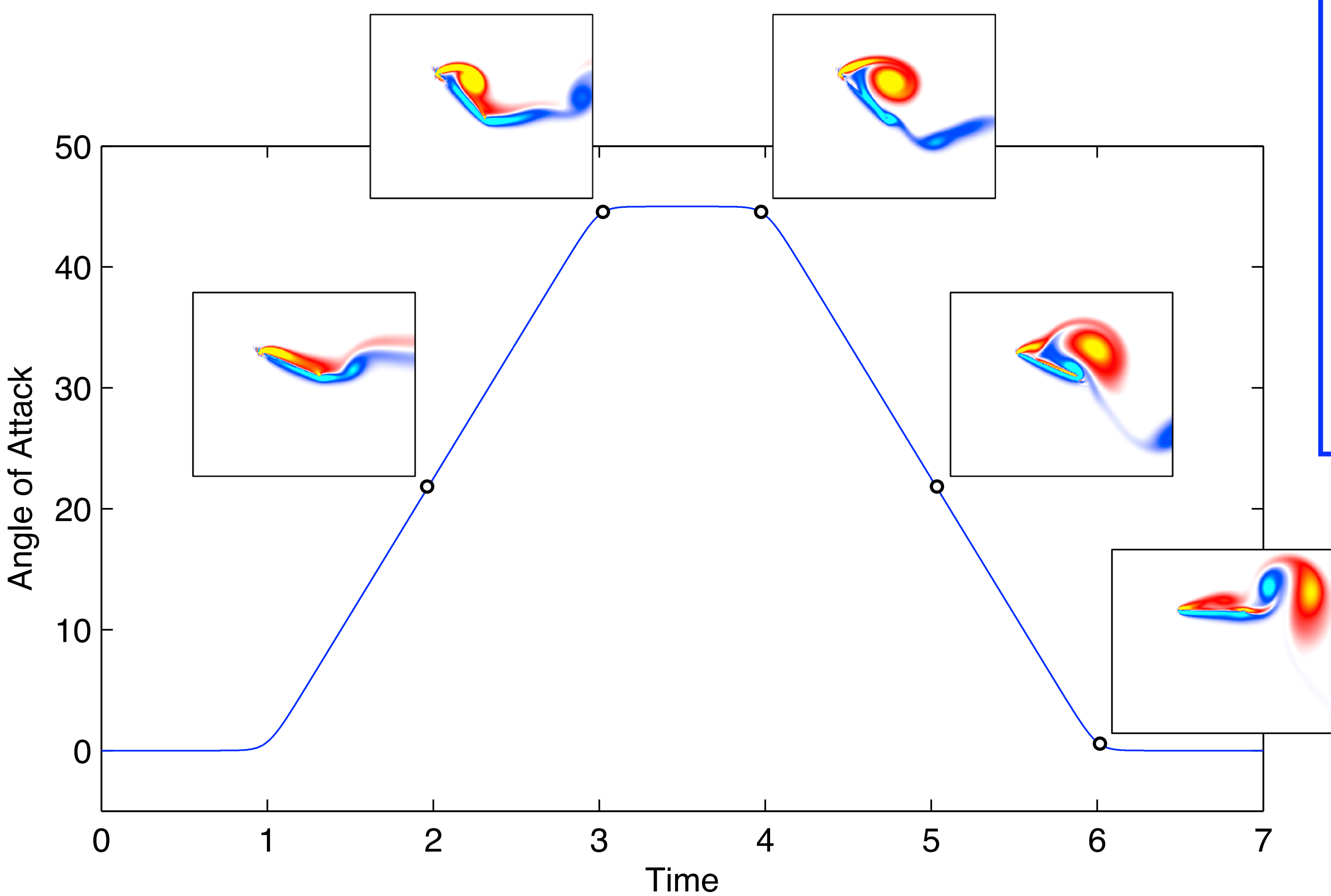
System Identification and Models for Flight Control



$$\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot{\alpha}\end{bmatrix}_{k+1} = \begin{bmatrix}A_{\mathrm{ERA}} & 0 & 0\\ 0 & 1 & \Delta t\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot{\alpha}\end{bmatrix}_{k} + \begin{bmatrix}B_{\mathrm{ERA}}\\ 0\\ \Delta t\end{bmatrix}\ddot{\alpha}_k$$

input

$$C_L(k\Delta t) = \begin{bmatrix}C_{\mathrm{ERA}} & C_{L_\alpha} & C_{L_{\dot{\alpha}}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot{\alpha}\end{bmatrix}_{k} + D_{\mathrm{ERA}}\,\ddot{\alpha}_k$$

ERA Model

quasi-steady contribution

Steve Brunton and Clancy Rowley
Princeton University

FAA/JUP Meeting October 7, 2010

Major Applications

1. Autopilot

2. Flight Simulators

a. small, agile UAV

b. severe weather

c. wake vorticity

d. cheaper than full CFD to compute aerodynamic response of airframe

e. necessary when using for onboard control

Goals

1. *System Identification*

a. *Stability derivatives*

b. *Additional fast dynamics*

c. *Markov parameters and ERA algorithm*

$$\begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k+1} = \begin{bmatrix} A_{\mathrm{ERA}} & 0 & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k} + \begin{bmatrix} B_{\mathrm{ERA}} \\ 0 \\ \Delta t \end{bmatrix} \underbrace{\ddot{\alpha}_k}_{\text{input}}$$

$$C_L(k\Delta t) = \begin{bmatrix} C_{\mathrm{ERA}} & C_{L_\alpha} & C_{L_{\dot{\alpha}}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k} + D_{\mathrm{ERA}}\, \ddot{\alpha}_k$$

ERA Model

quasi-steady contribution

---

2. *Flight Control*

a. *Stability augmentation*

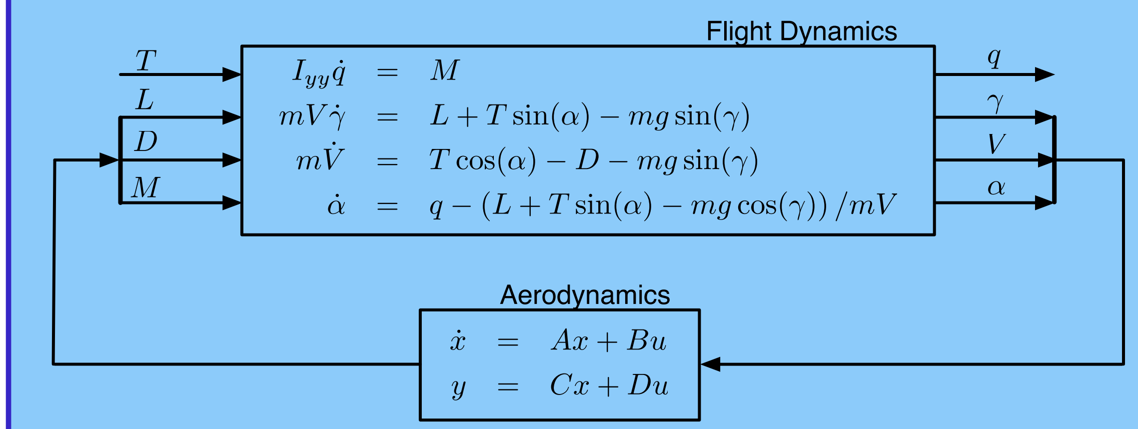
b. *Control augmentation*

c. *6DOF inertial model*

d. *Coupled aerodynamic model*

e. *Interesting control problem when inertial/aerodynamic timescales are close*

coupled model



NextGen ConOps V2.0: UAVs

2.7.2.2 Unmanned Aircraft Systems

UAS operations are some of the most demanding operations in NextGen. UAS operations include scheduled and on-demand flights for a variety of civil, military, and state missions.

Because of the range of operational uses, UAS operators may require access to all NextGen airspace. ...

2.7.2.3 Vertical Flight

... Rotorcraft are also used for UAS applications for commercial, police, and security operations. These operations add to the density and complexity of operations, particularly in and around urban areas.

3.3.1.2.3 Integrated Environmental Operations

UAS performing security functions and the airport perimeter security intrusion detection system may have the capability to assist with wildlife management programs.

5.3.3 Weather Information Enterprise Services

- **Enterprise Service 3:** UASs Are Used for Weather Reconnaissance. [R-169]

En route weather reconnaissance UASs are equipped to collect and report in-flight weather data. Specialized weather reconnaissance UASs are used to scout potential flight routes and trajectories to identify available “weather-favorable” airspace...

UAV Challenges

Shadow (Aerocam)

UAVs and NextGen:

May require access to all NextGen airspace

Civil, military and state missions

Mobile communications relays

Security/Policing

Weather reconnaissance

and much more ...

Safety Hazards:

Extremely light, very difficult to control in high crosswinds

No human failsafes

Especially dangerous in takeoff and landing

"... 65 of the 195 Predators the Air Force has acquired since 1994 had been lost because of Class A mishaps."

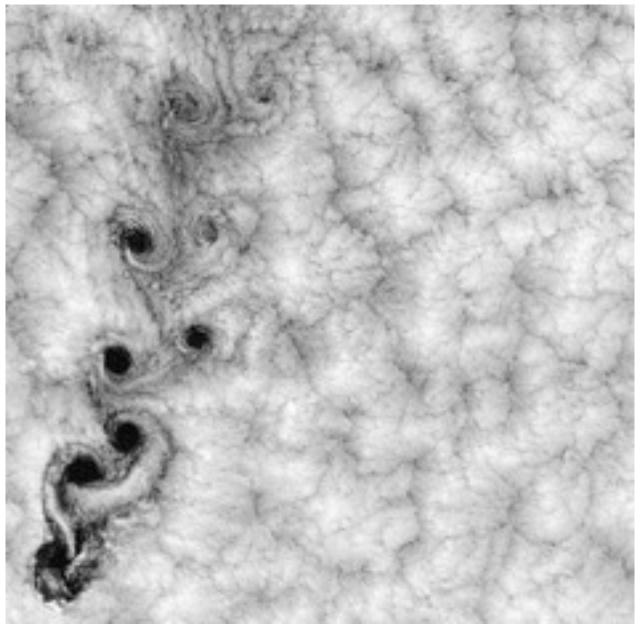
"... 36% were attributed to human error. And 15% of the accidents occurred during landing"

Government Computer News (Oct. 9, 2009)

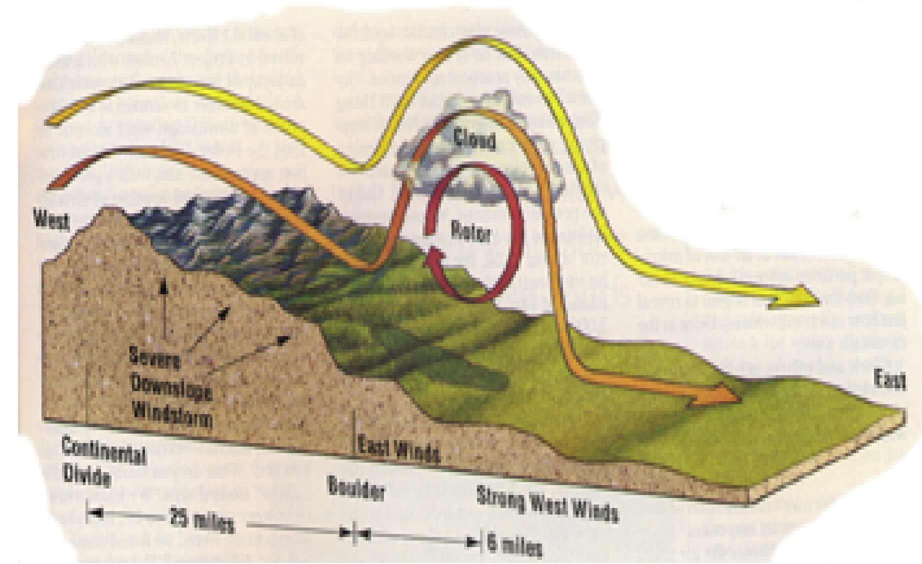
Predator (General Atomics)

Wind Disturbances

Free air turbulence



Wind rotors



Wake vorticity

(Slides and history courtesy of Rob Stengel)

Microburst wind shear

Safety Hazards: **Landing and takeoff (congestion during storms, takeoff waiting lines)**

Especially problematic for lightweight UAVs

Flight Simulators

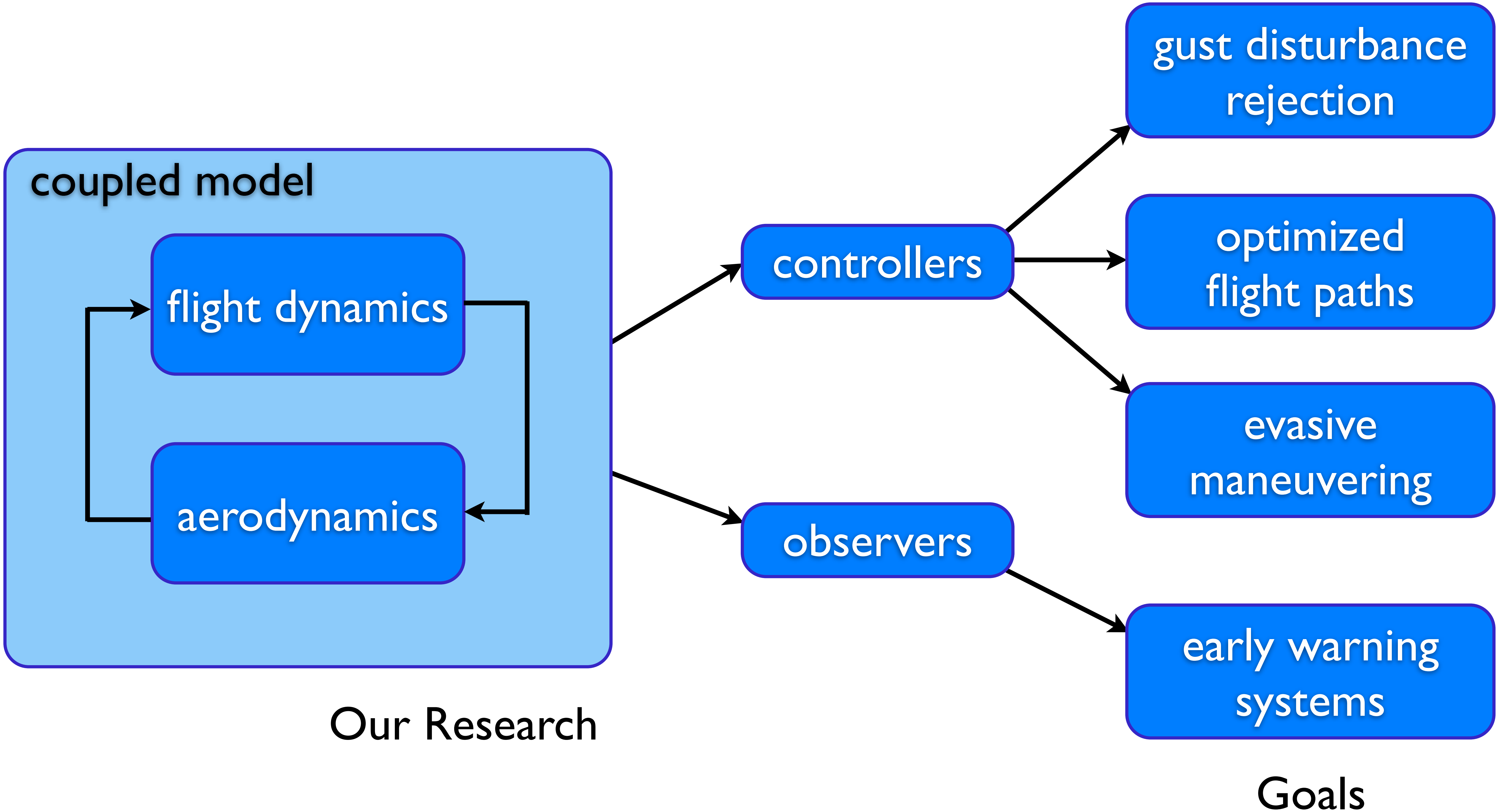
Goal: Pilot flies real aircraft for 5-10 minutes, and reduced order aerodynamic model is automatically generated.

Not specific to unsteady aerodynamics

Physics based, generalizable to nonlinear affects, such as wake vorticity, turbulence, etc.

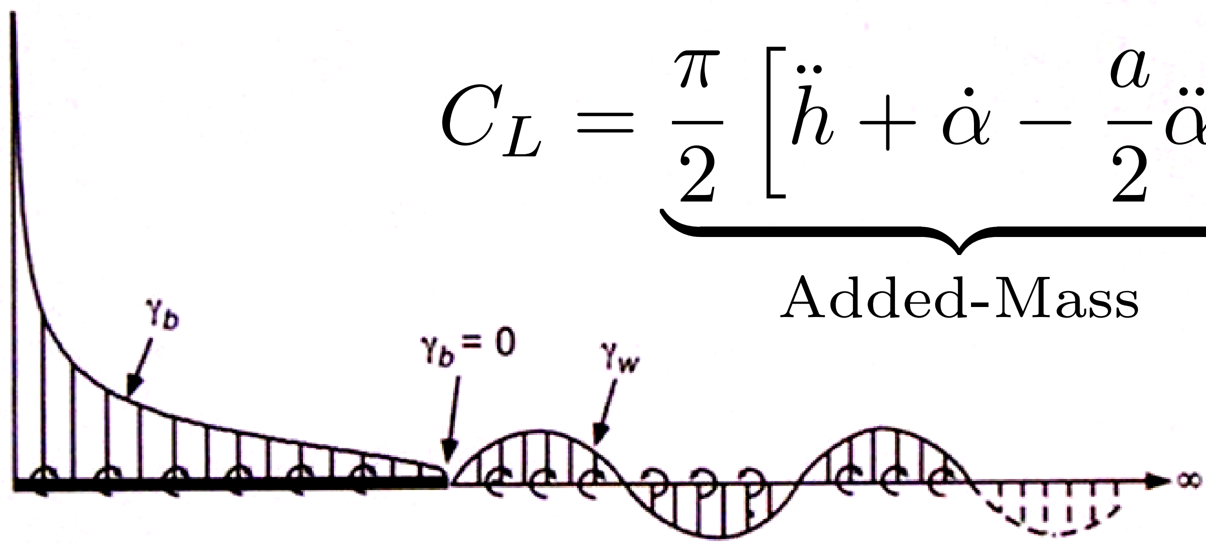
FLYIT Simulators, Inc.
The New Standard in Aviation Training

Flight Dynamic Model



Theodorsen's Model

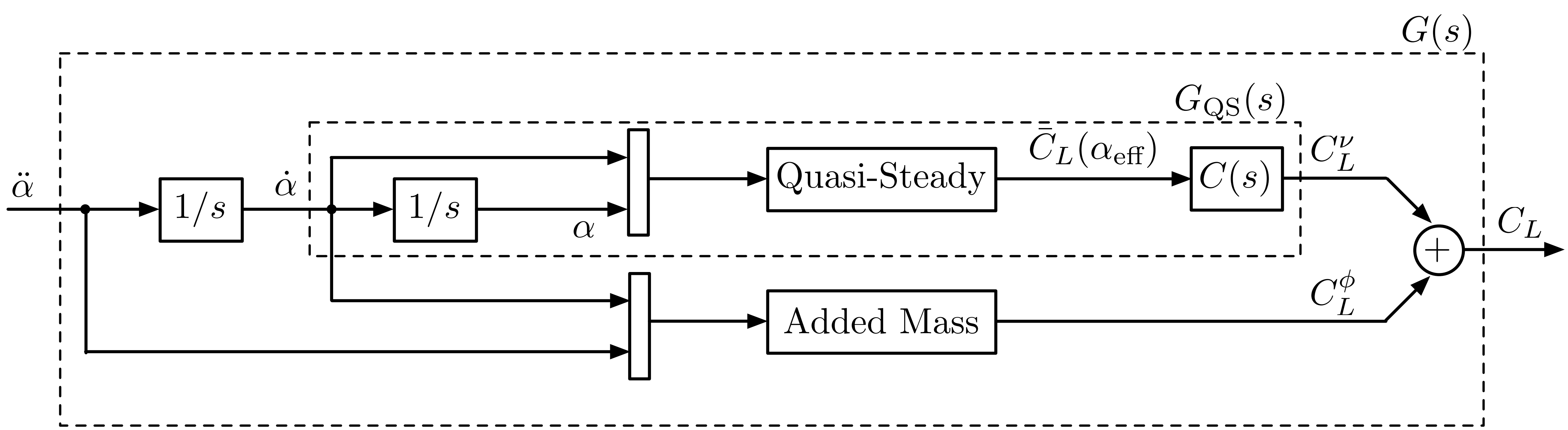
$$C_L = \underbrace{\frac{\pi}{2}\left[\ddot{h} + \dot{\alpha} - \frac{a}{2}\ddot{\alpha}\right]}_{\text{Added-Mass}} + \underbrace{2\pi\left[\alpha + \dot{h} + \frac{1}{2}\dot{\alpha}\left(\frac{1}{2} - a\right)\right]}_{\text{Circulatory}} C(k)$$



Theodorsen, *NACA-496*, 1935.

Leishman, 2006.

$$\boxed{k = \frac{\pi f c}{U_\infty}}$$

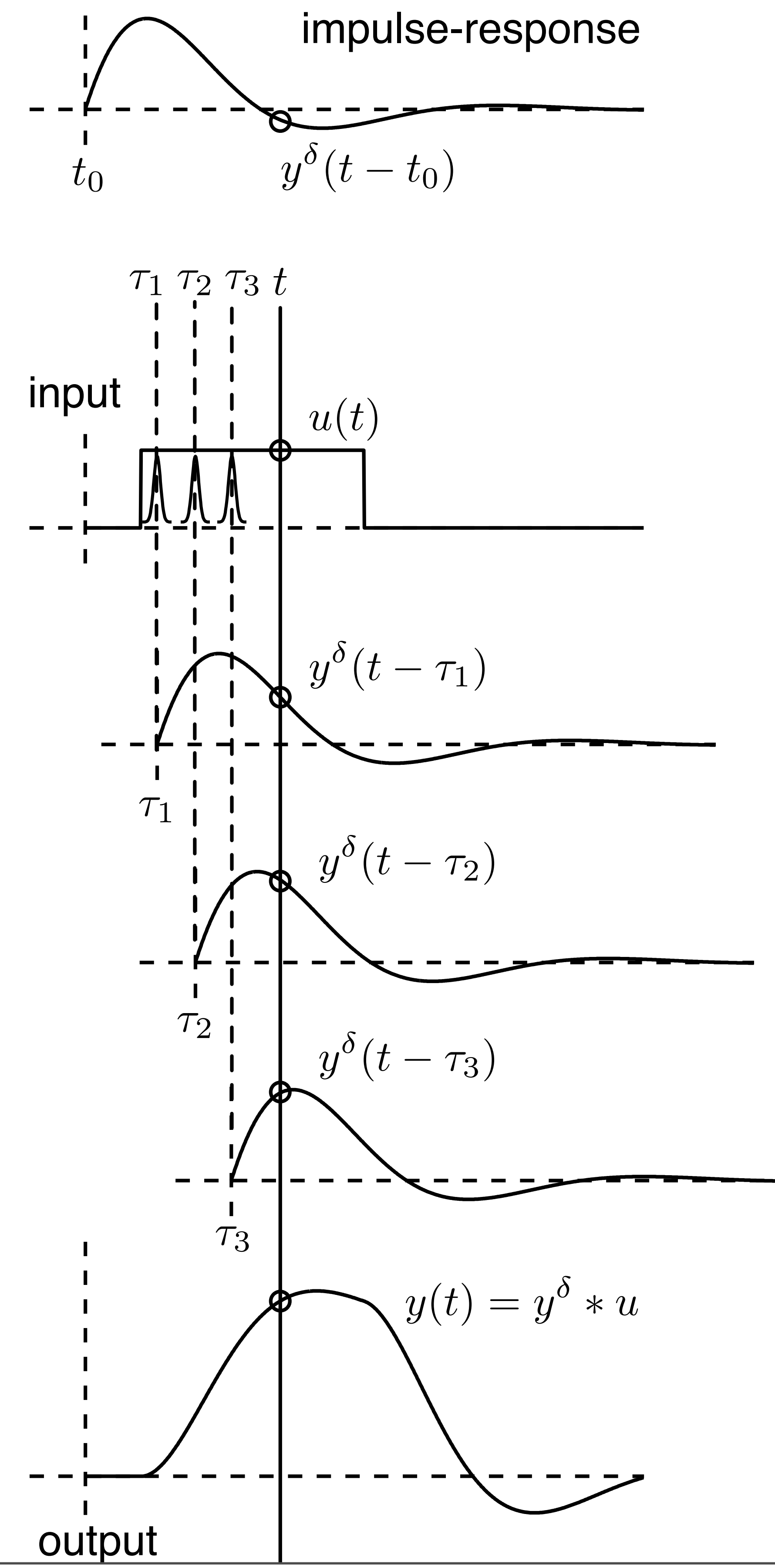


Wagner's Indicial Response

Given $y^\delta(t)$ for an impulse response $u = \delta(t)$,

The response to an arbitrary input $u(t)$

is given by linear superposition



$$y(t) = y^\delta(t)u(0) + \int_0^t y^\delta(t - \tau)u(\tau)d\tau$$

In particular, input is pitch rate,
and output is lift coefficient:

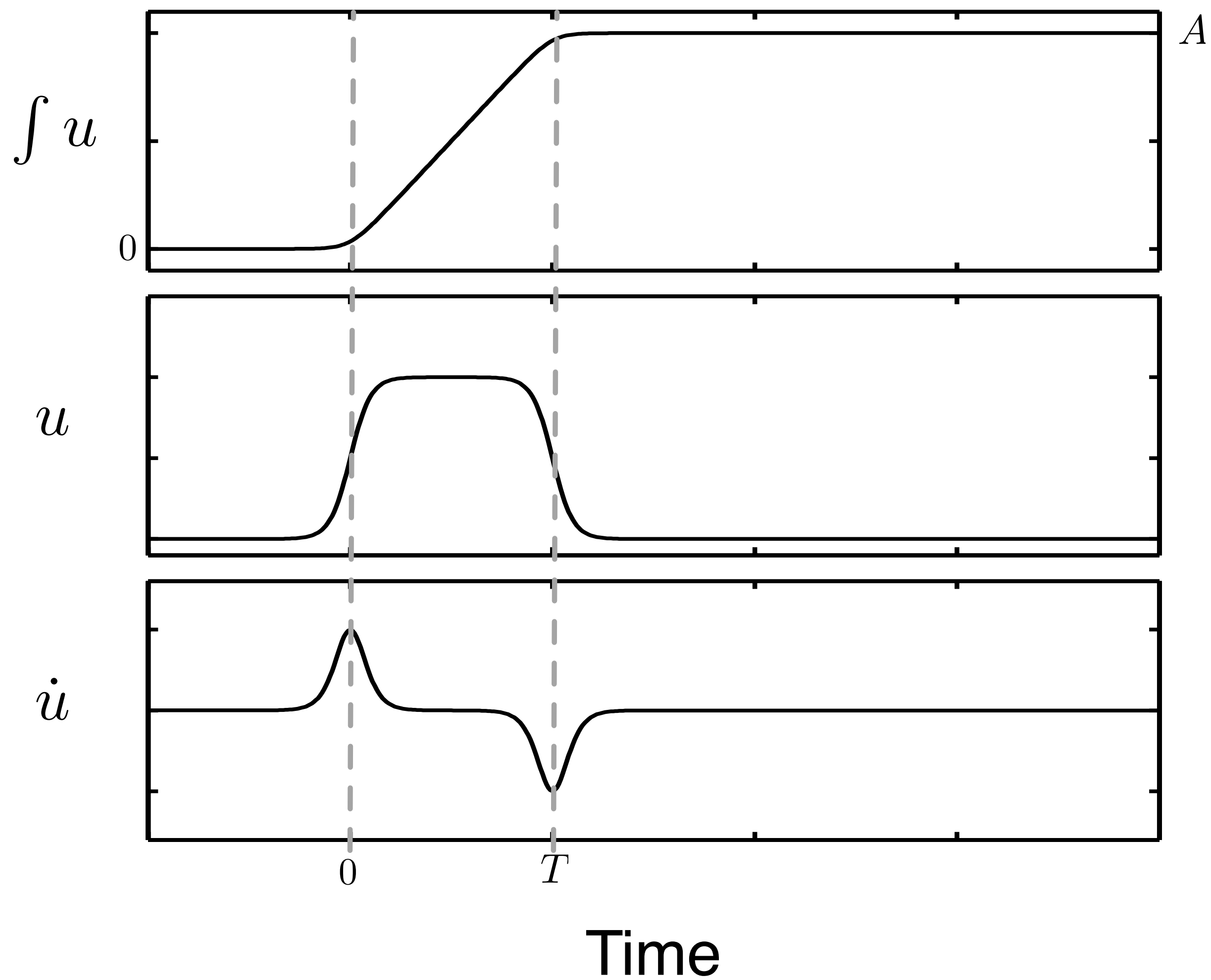
$$u = \dot{\alpha}$$

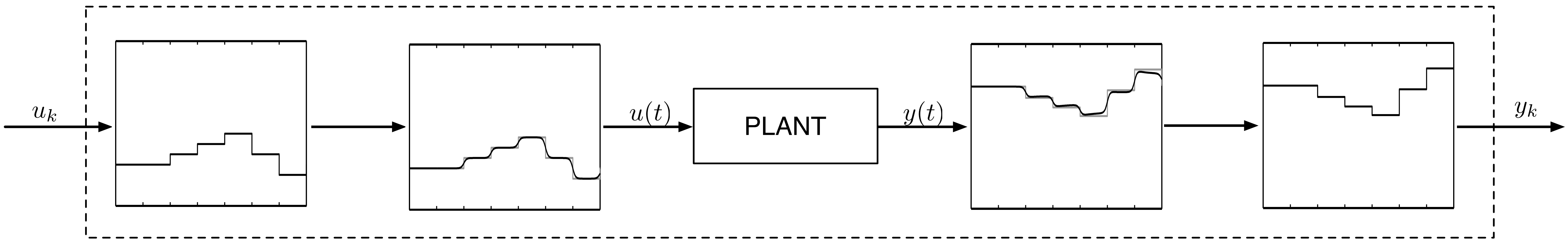
$$y = C_L$$

Wagner, 1925.

Leishman, 2006.

(Indicial) Step Response





Eigensystem Realization Algorithm

$$\left.\begin{aligned} \mathbf{x}(k+1) &= A\mathbf{x}(k) + B\mathbf{u}(k) \\ \mathbf{y}(k) &= C\mathbf{x}(k) \end{aligned}\right\} \xrightarrow{\text{Reduction}} \begin{aligned} \mathbf{x}_r(k+1) &= A_r\mathbf{x}_r(k) + B_r\mathbf{u}(k) \\ \mathbf{y}(k) &= C_r\mathbf{x}_r(k) \end{aligned}$$

$\mathbf{x} \in \mathbb{R}^n$ $\quad n$ large

$\mathbf{x}_r \in \mathbb{R}^r$ $\quad r$ small

1. Gather outputs $\mathbf{y}(k) = CA^kB$ from an impulse-response experiment, and arrange into Hankel matrices:

$$H = \begin{bmatrix} CB & CAB & \cdots & CA^{m_c}B \\ CAB & CA^2B & \cdots & CA^{m_c+1}B \\ \vdots & \vdots & \ddots & \vdots \\ CA^{m_o}B & CA^{m_o+1}B & \cdots & CA^{m_c+m_o}B \end{bmatrix} \qquad H' = \begin{bmatrix} CAB & CA^2B & \cdots & CA^{m_c+1}B \\ CA^2B & CA^3B & \cdots & CA^{m_c+2}B \\ \vdots & \vdots & \ddots & \vdots \\ CA^{m_o+1}B & CA^{m_o+2}B & \cdots & CA^{m_c+m_o+1}B \end{bmatrix}$$

2. Compute the singular value decomposition of H:

$$H = U\Sigma V^* = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} = U_1\Sigma_1V_1^*$$

3. Let Σ_r be the first $r \times r$ block of Σ_1 and U_r, V_r the first r columns of U_1, V_1 so that the reduced order model A_r, B_r, C_r is given by:

$$\begin{aligned} A_r &= \Sigma_r^{-1/2} U_r^* H' V_r \Sigma_r^{-1/2} \\ B_r &= \text{ first } p \text{ columns of } \Sigma_r^{1/2} V_1^* \\ C_r &= \text{ first } q \text{ rows of } U_r \Sigma_r^{1/2} \end{aligned}$$

Juang and Pappa, *J. Guid. Contr. Dyn.*, **8**:5, 1985.

Ma, Z., Ahuja, S., and C. Rowley, *Theor. Comput. Fluid. Dyn.*, to appear.

Recently shown to yield reduced order models equivalent to those obtained through Balanced Proper Orthogonal Decomposition

ERA Model

$$\begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k+1} = \begin{bmatrix} A_{\mathrm{ERA}} & 0 & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k} + \begin{bmatrix} B_{\mathrm{ERA}} \\ 0 \\ \Delta t \end{bmatrix} \underbrace{\ddot{\alpha}_k}_{\text{input}}$$

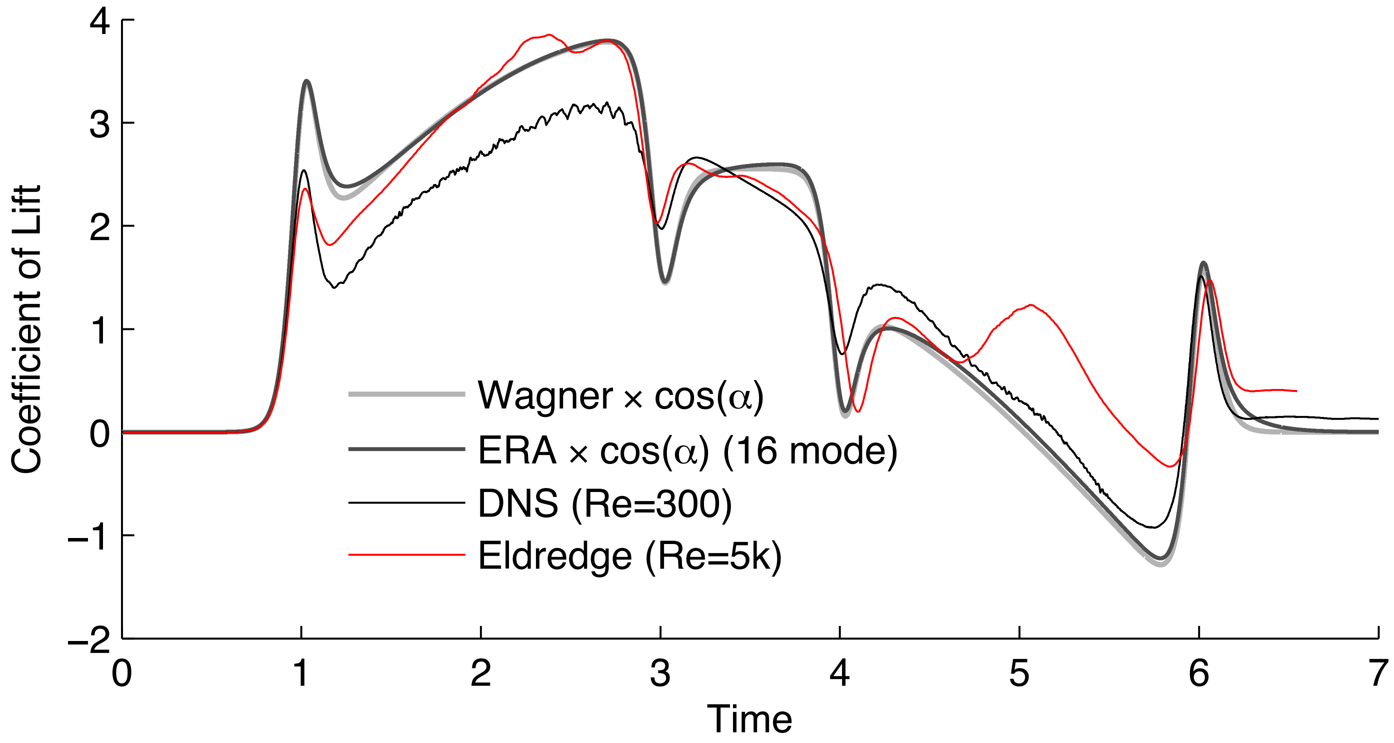
$$C_L(k\Delta t) = \begin{bmatrix} C_{\mathrm{ERA}} & C_{L_\alpha} & C_{L_{\dot{\alpha}}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot{\alpha} \end{bmatrix}_{k} + D_{\mathrm{ERA}}\, \ddot{\alpha}_k$$

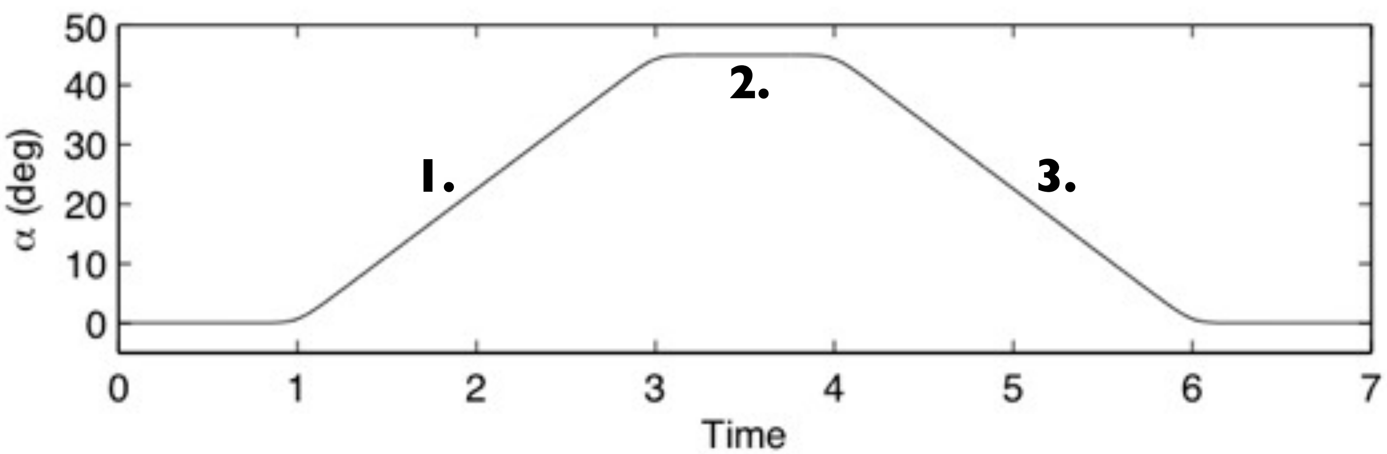
ERA Model

quasi-steady plus added-mass contribution

additional fast dynamics

Canonical Pitch-ramp Maneuver





Leading-edge pitch-ramp maneuver

Large added-mass forces appear as spikes

Reduced order ERA model captures unsteady lift

Canonical Maneuver

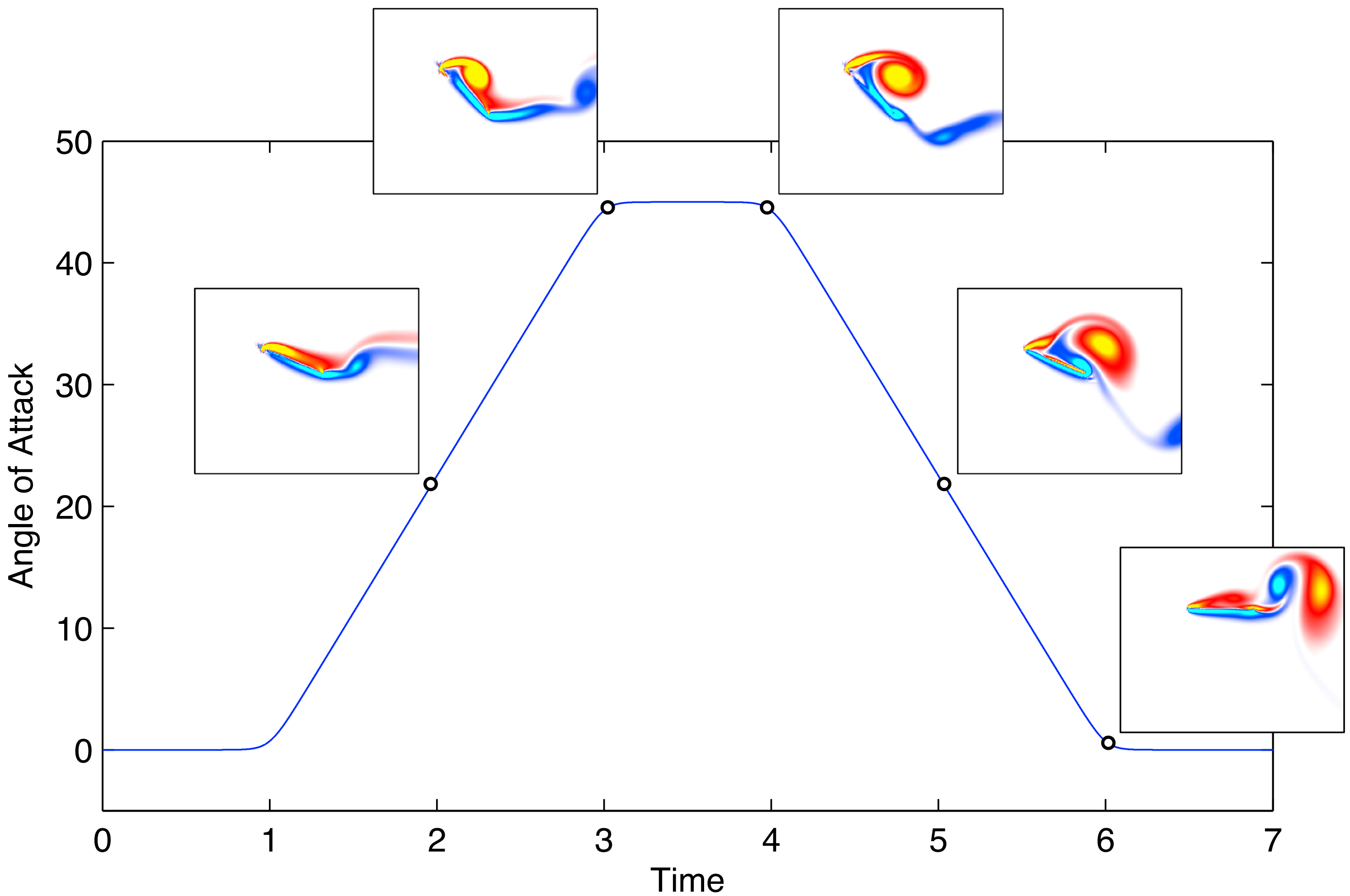
M. Ol, J. Eldredge *et al*, *48th AIAA ASM, 2010.*

Developed to compare models, simulations and experiments

Qualitatively similar for range of Reynolds numbers from 300 - 40k

1. **Pitch-up to 45°**
2. **Hold at 45°**
3. **Pitch-down to 0°**

Canonical Pitch-ramp Maneuver



Canonical Maneuver

M. Ol, J. Eldredge *et al*, *48th AIAA ASM, 2010.*

Developed to compare models, simulations and experiments

Qualitatively similar for range of Reynolds numbers from 300 - 40k

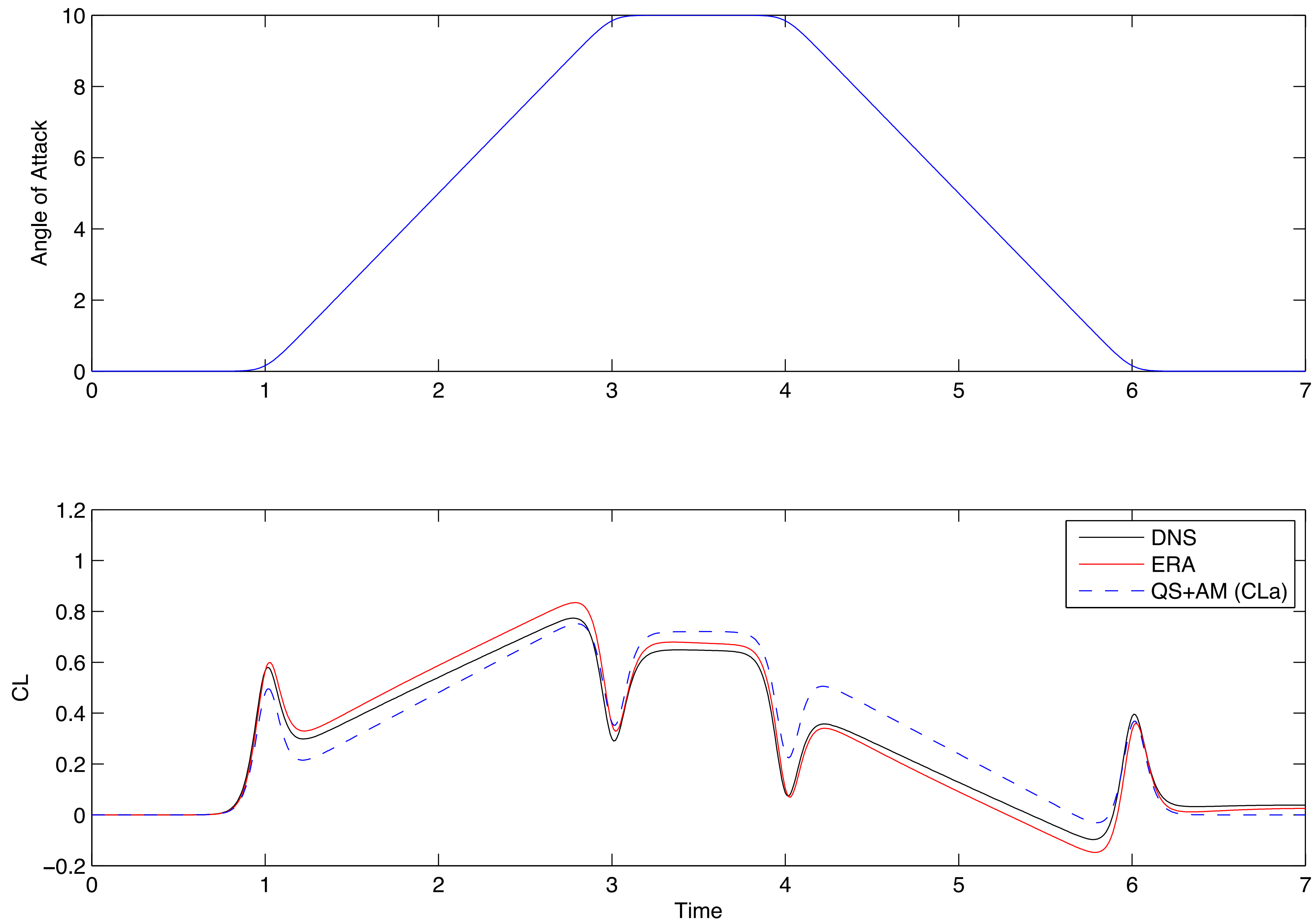
1. **Pitch-up to 45°**
2. **Hold at 45°**
3. **Pitch-down to 0°**

Leading-edge pitch-ramp maneuver

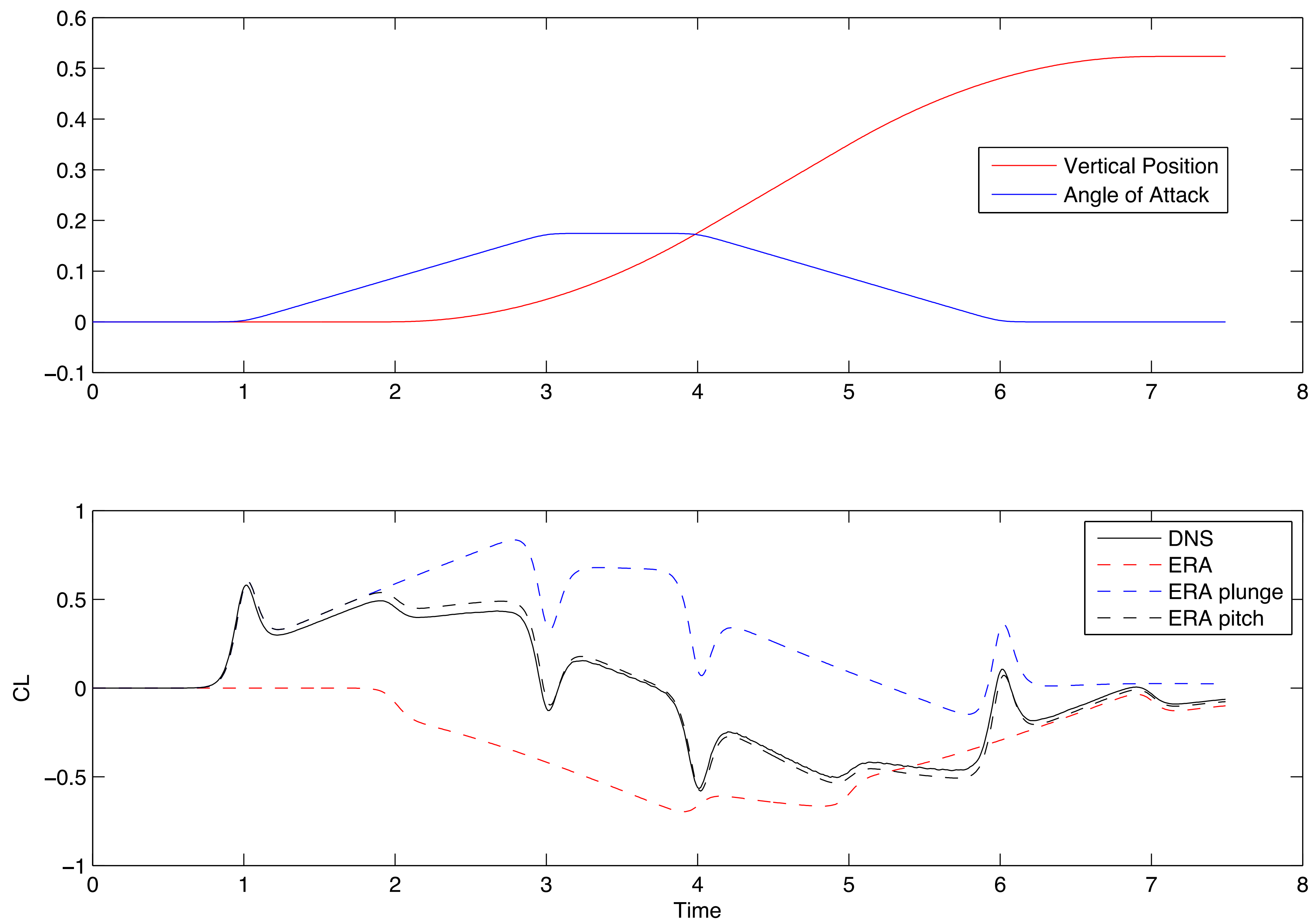
Large added-mass forces appear as spikes

Reduced order ERA model captures unsteady lift

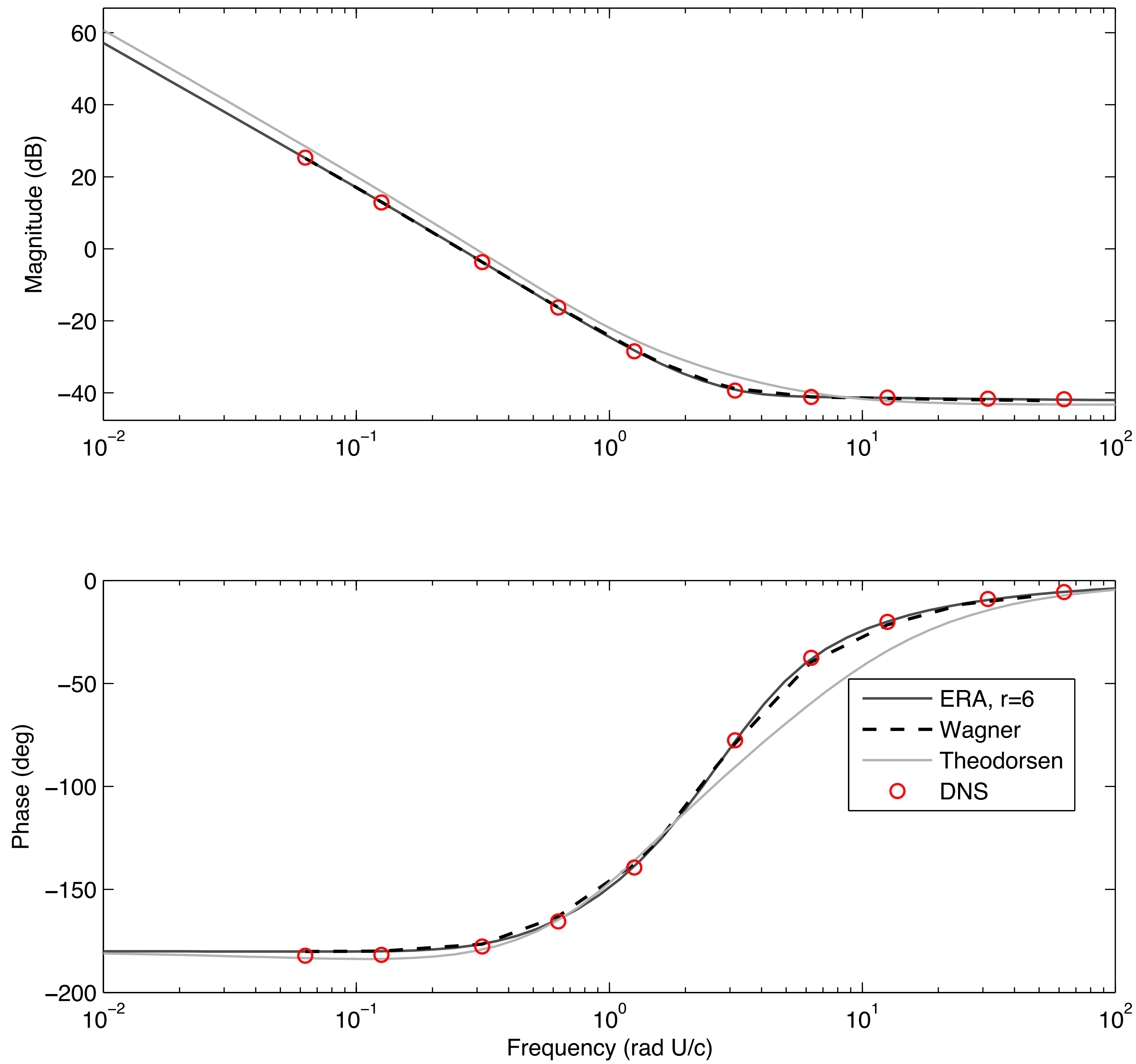
Canonical Pitch-ramp Maneuver



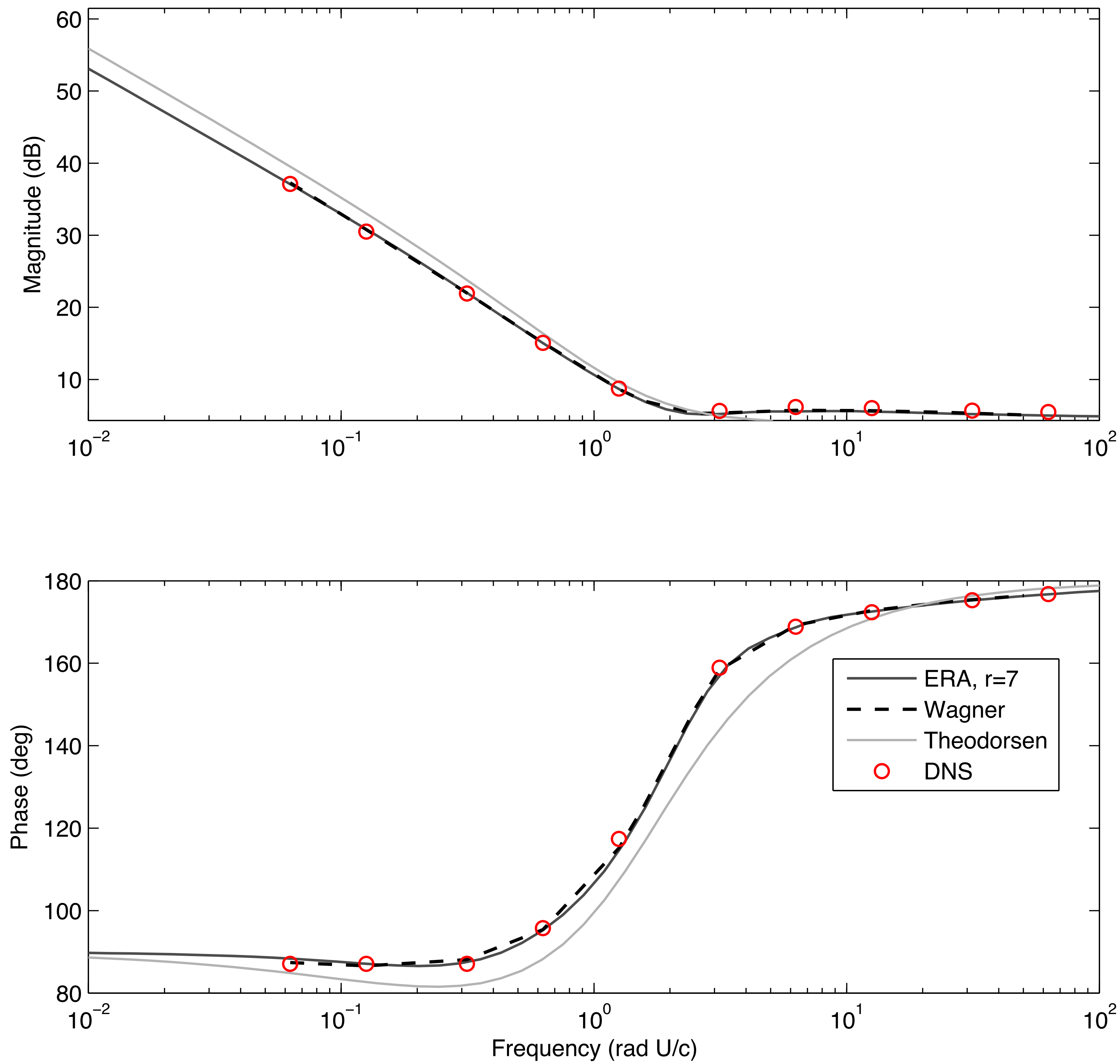
Combined Pitch/Plunge Maneuver



Pitching at Quarter Chord



Vertical Plunging



Summary

1. Reduced Order Model for Wagner

a. Stability derivatives

b. Additional fast dynamics

c. Markov parameters and ERA algorithm

$$\begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot\alpha \end{bmatrix}_{k+1} = \begin{bmatrix} A_{\mathrm{ERA}} & 0 & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot\alpha \end{bmatrix}_{k} + \begin{bmatrix} B_{\mathrm{ERA}} \\ 0 \\ \Delta t \end{bmatrix} \underbrace{\ddot\alpha_k}_{\text{input}}$$

$$C_L(k\Delta t) = \begin{bmatrix} C_{\mathrm{ERA}} & C_{L_\alpha} & C_{L_{\dot\alpha}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \alpha \\ \dot\alpha \end{bmatrix}_{k} + D_{\mathrm{ERA}}\,\ddot\alpha_k$$

ERA Model

quasi-steady contribution

---

2. Advantages

a. More accurate than Quasi-steady

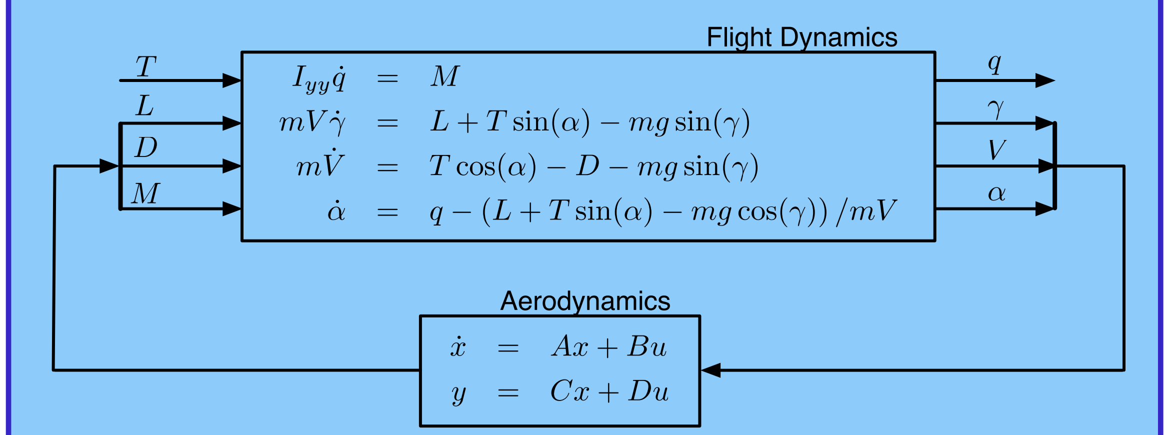
b. More accurate than Theodorsen

c. Efficient

d. ODE framework ideal for control

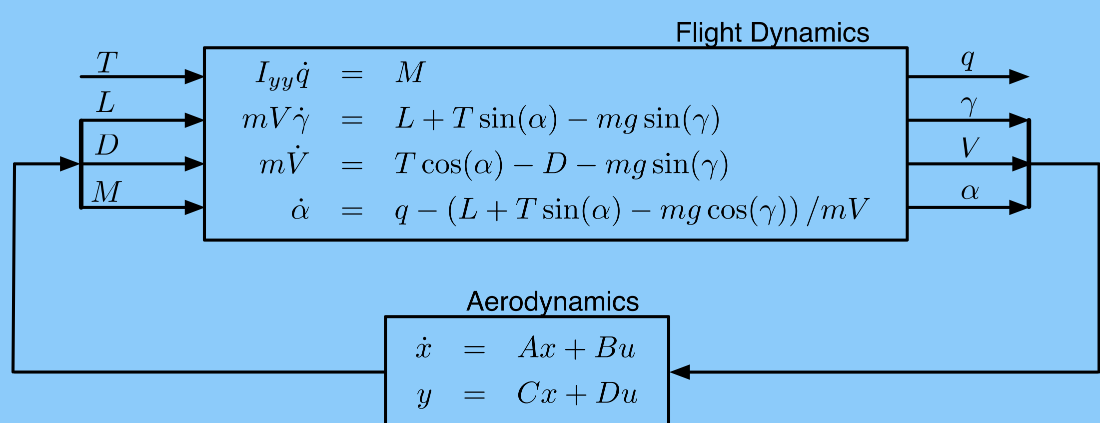
e. Fits naturally into fight dynamic framework

coupled model



Flight Dynamic Model

coupled model



controllers

observers

gust disturbance rejection

optimized flight paths

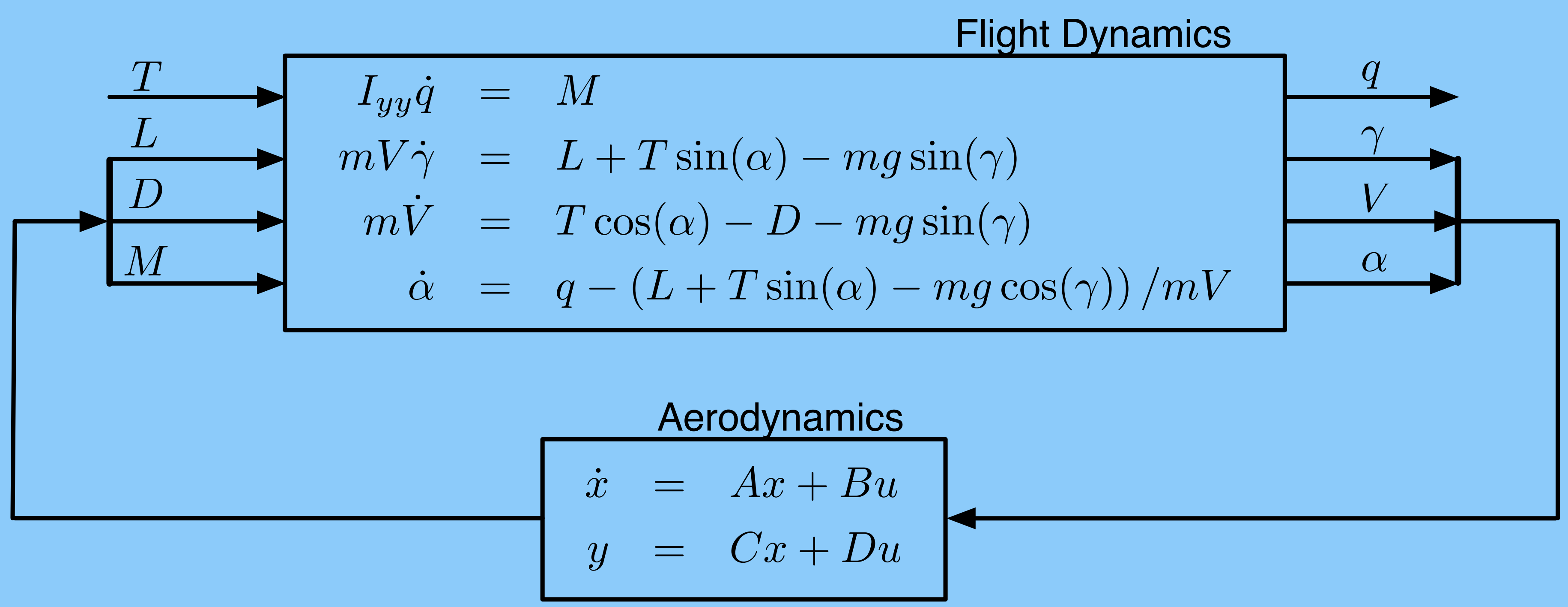
evasive maneuvering

early warning systems

Our Research

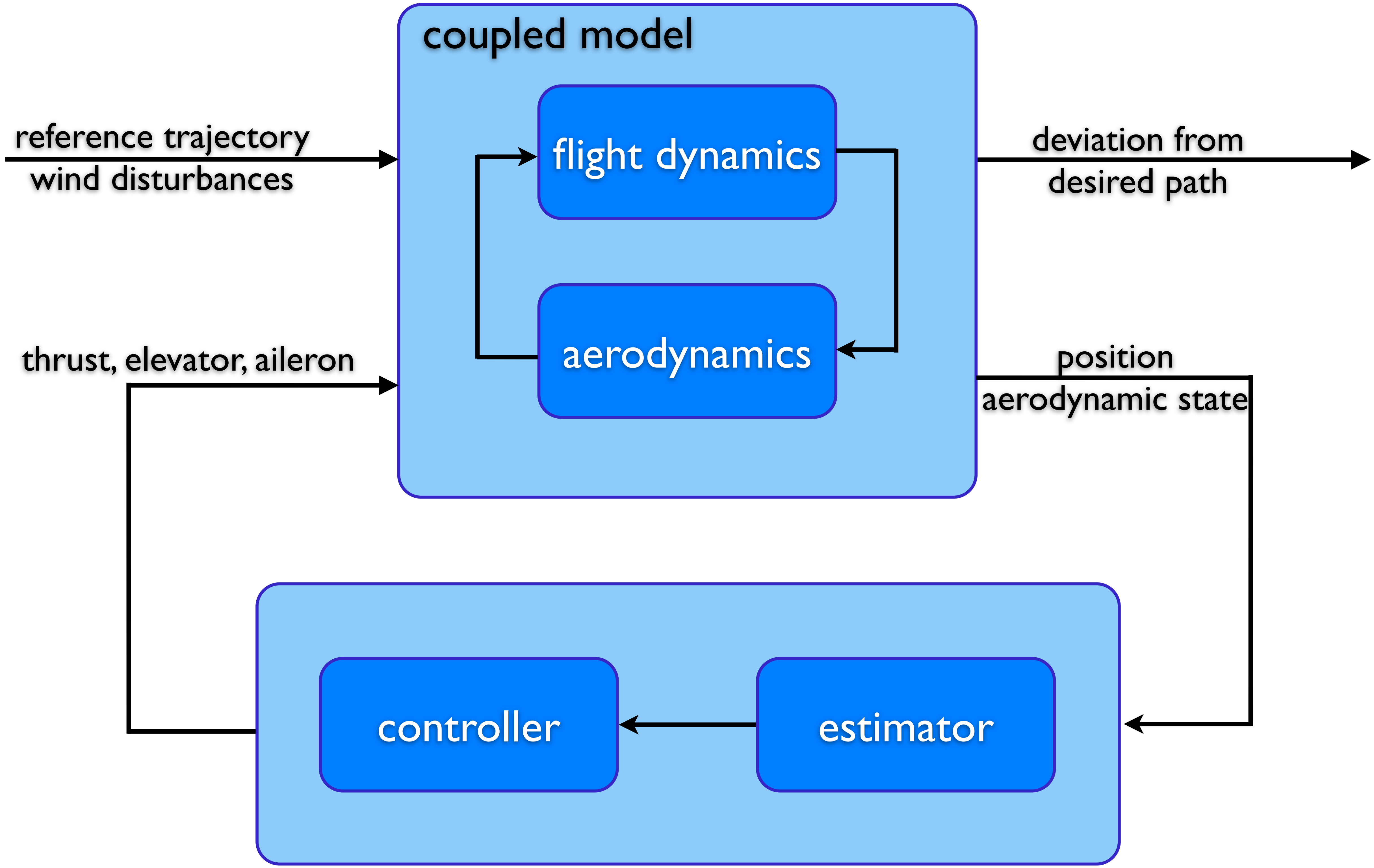
Goals

coupled model

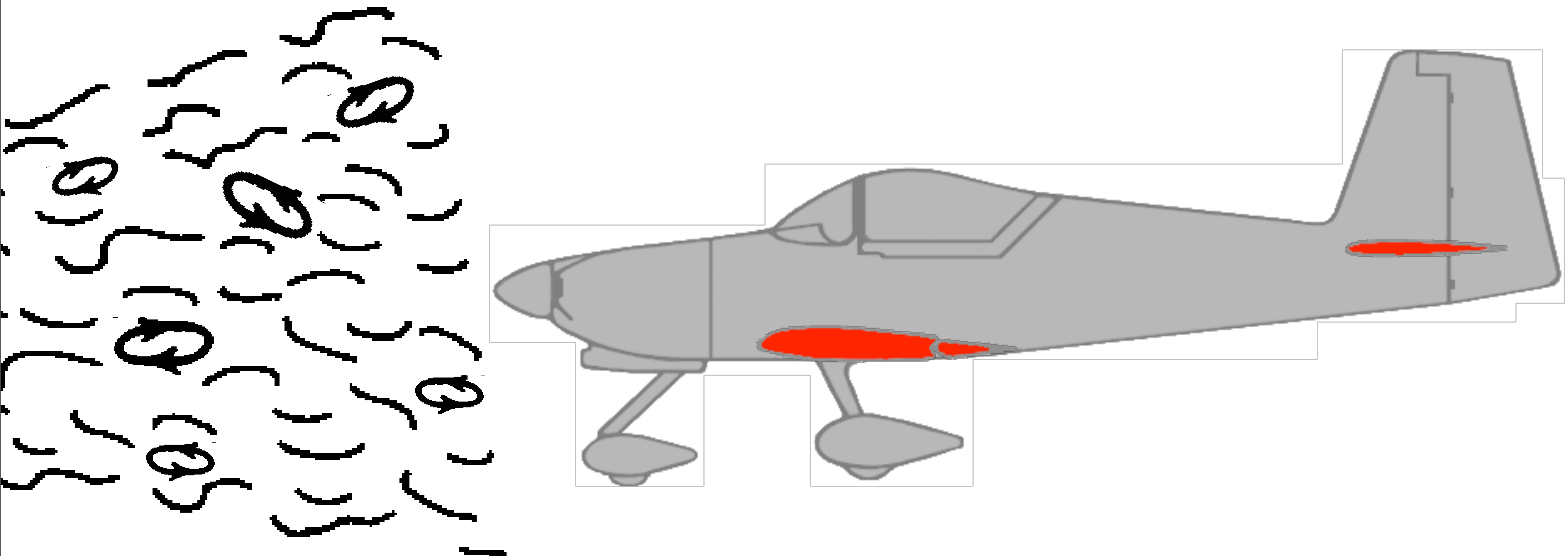


Interesting control scenario when time-scales of flight dynamics are close to time-scales of aerodynamics

Flight Dynamic Model



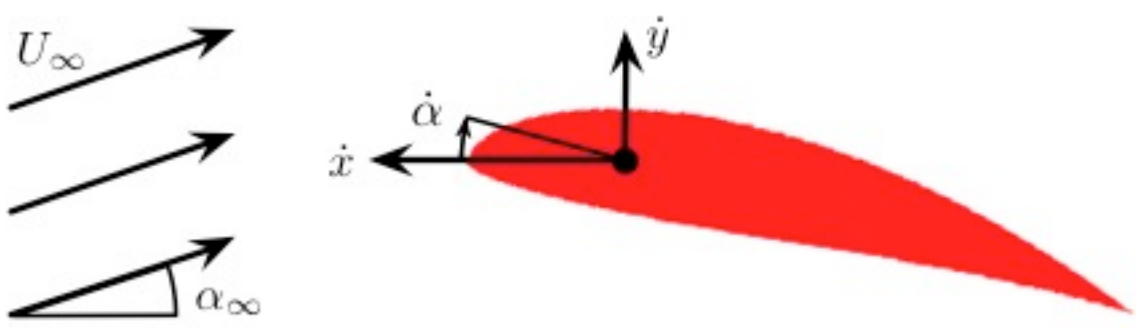
Next Step



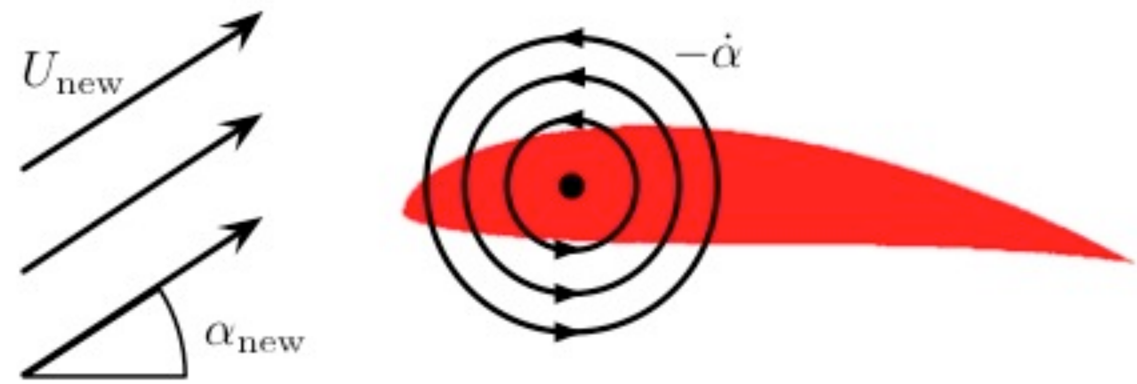
DISTURBANCE: Gust Field

INPUT: Flaperon

INPUT: Elevator



Moving Airfoil



Moving Base Flow

Base flow velocity:

$$u(x,y,t) = U_\infty \cos(\alpha + \alpha_\infty) - \dot{x} - \dot{\alpha}(y - y_C)$$
$$v(x,y,t) = U_\infty \sin(\alpha + \alpha_\infty) - \dot{y} + \dot{\alpha}(x - x_C)$$

Vorticity:

$$\nabla \times (u,v) = v_x - u_y = \dot{\alpha} + \dot{\alpha} = 2\dot{\alpha}$$

where (x_C, y_C) is the center of mass.

Moving Base Flow

Faster simulations (Cholesky decomposition)

allows more aggressive maneuvers and gusts

subject of current research

Immersed Boundary Method

T. Colonius and K. Taira, 2008
A fast immersed boundary method using a nullspace approach and multi-domain far-field boundary conditions.

Kornfeld, Hansman, and Deyst, *ICAT-99-5*, 1999.

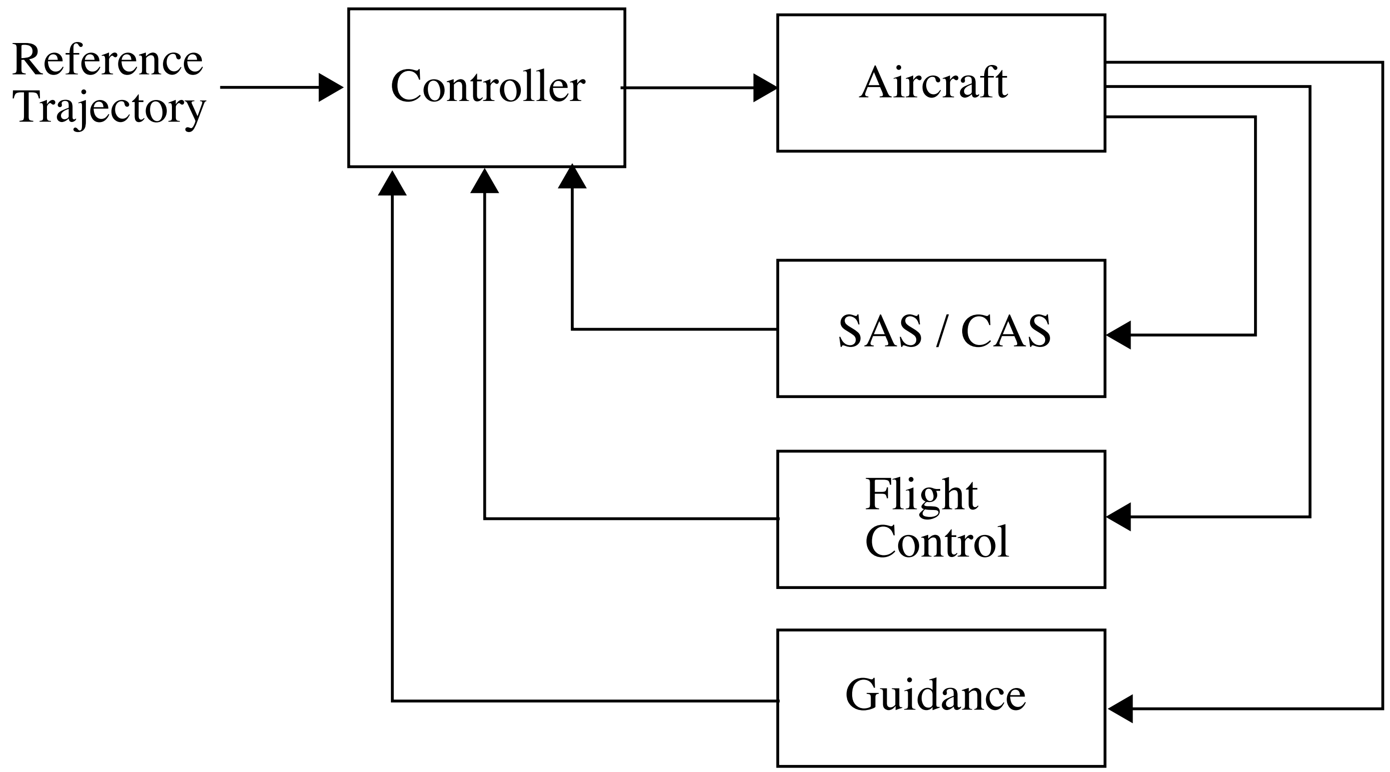


Figure 2.1: Classical Flight Control Loops

SAS: Stability Augmentation System

CAS: Control Augmentation System

Kornfeld, Hansman, and Deyst, *ICAT-99-5*, 1999.

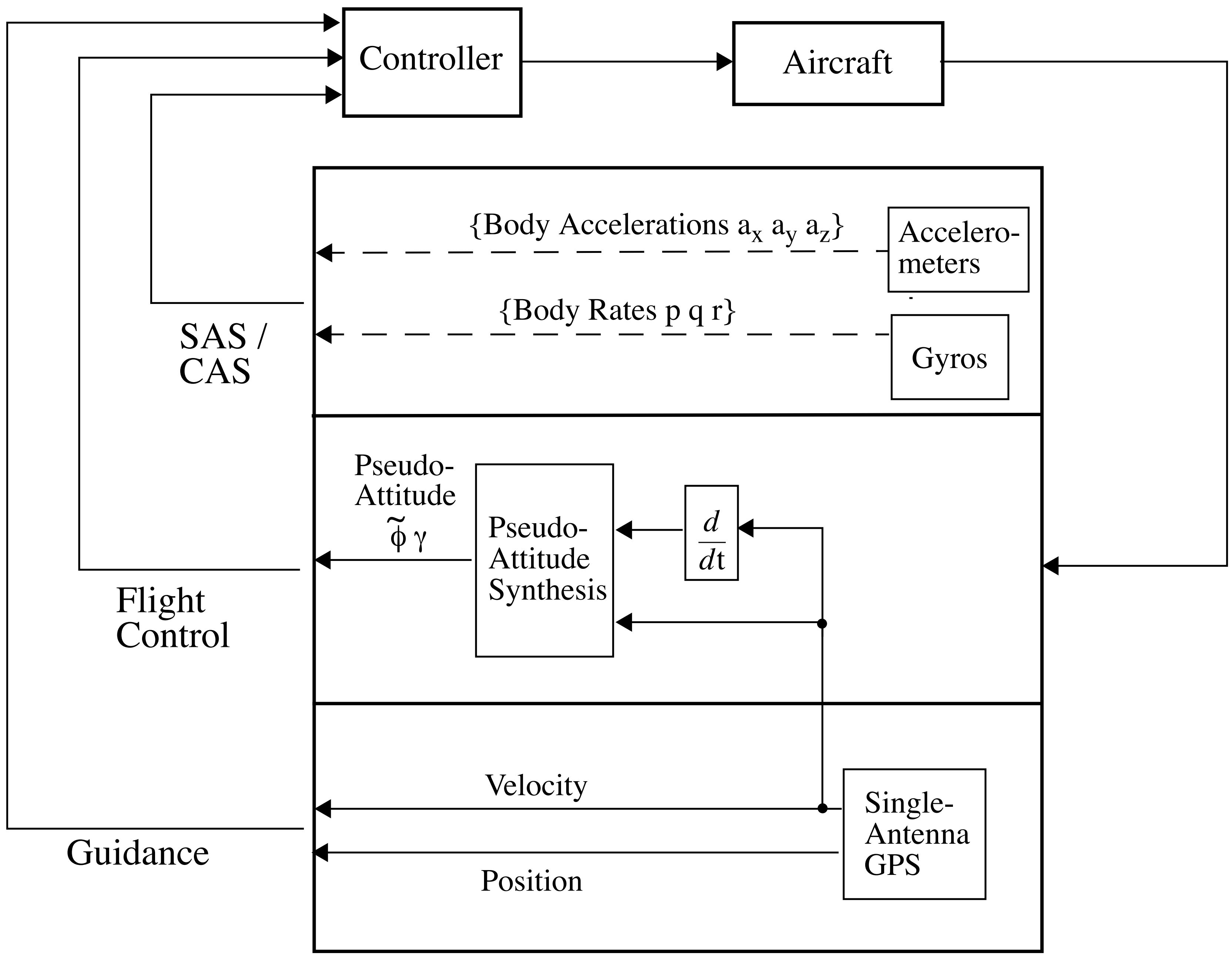


Figure 2.7: Single-Antenna GPS-Based Instrumentation Architecture