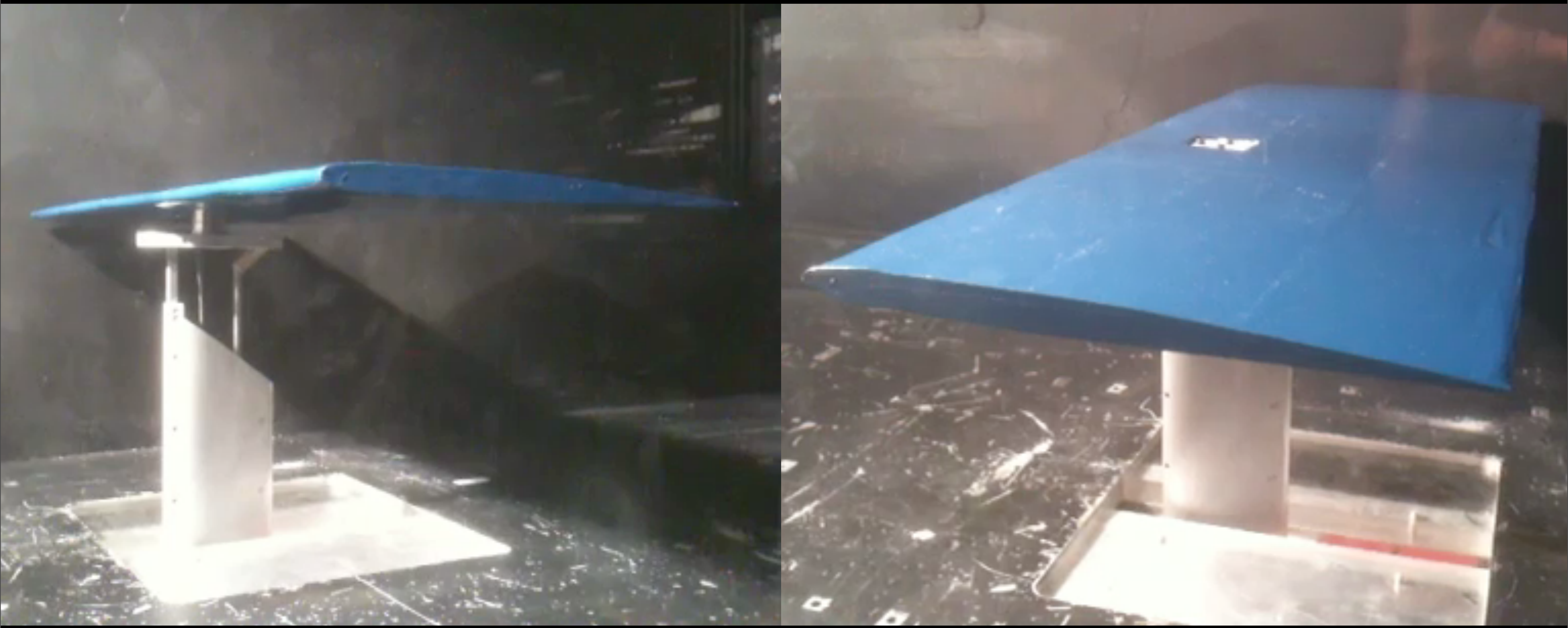
Linear Unsteady Aerodynamic Models from Wind Tunnel Measurements



Steven L. Brunton, Clarence W. Rowley
Princeton University

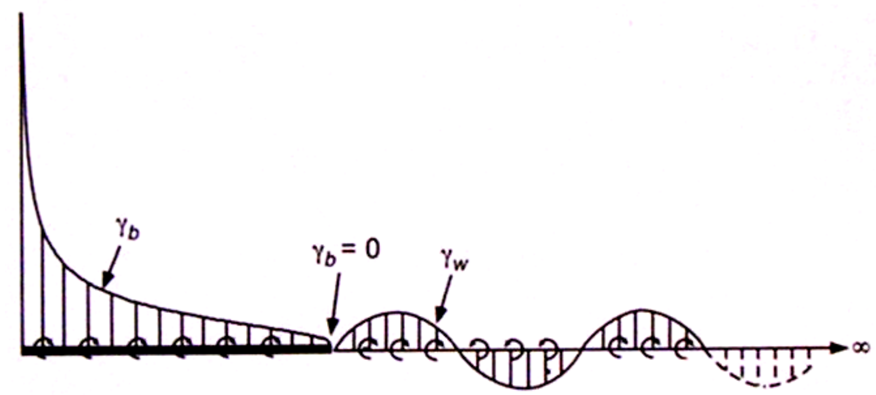
David R. Williams
Illinois Institute of Technology

AIAA Honolulu, June 28, 2011

Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



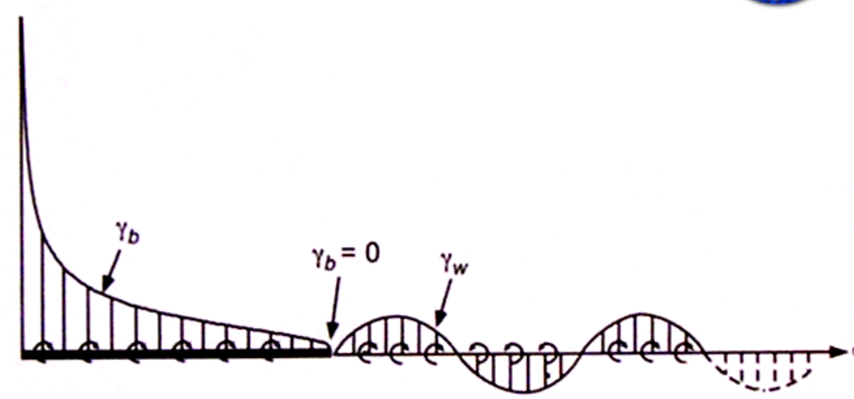
3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$

4. Conclusions and Future Work

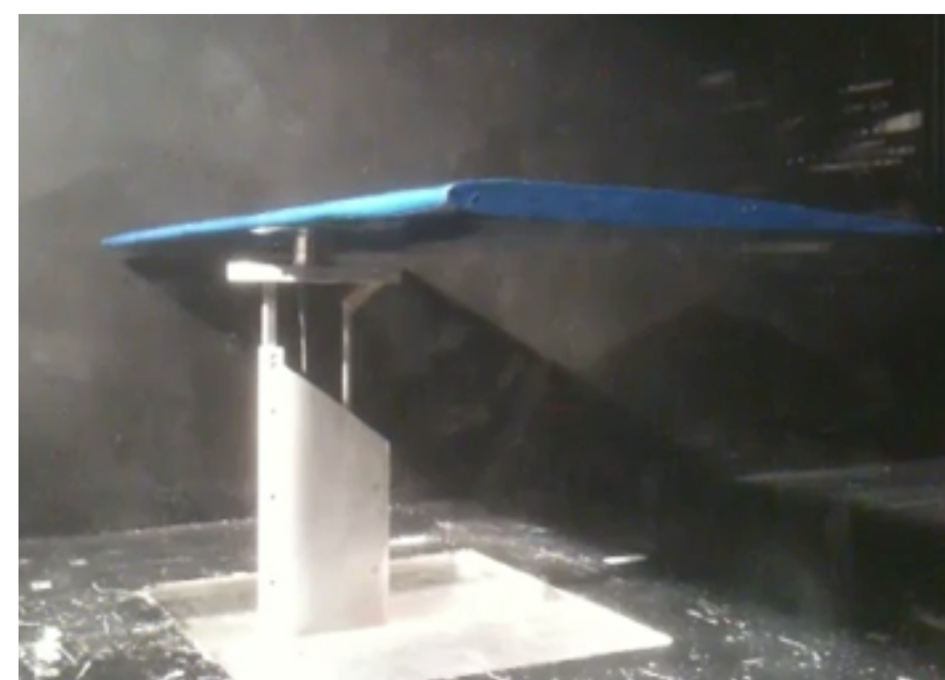
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID

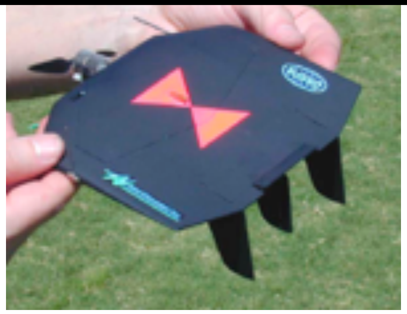


3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$



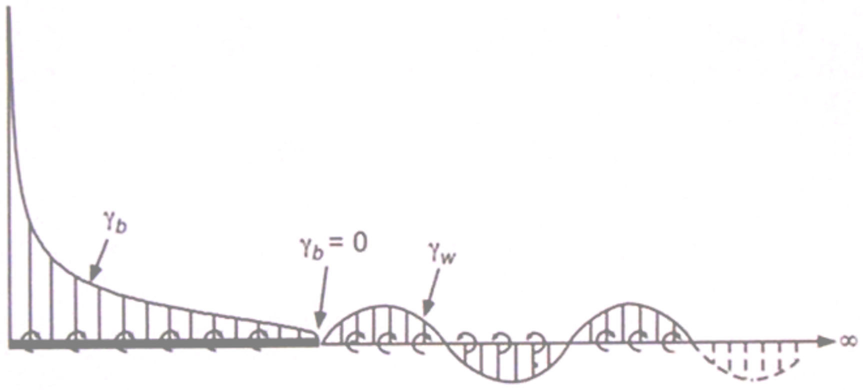
4. Conclusions and Future Work

Outline

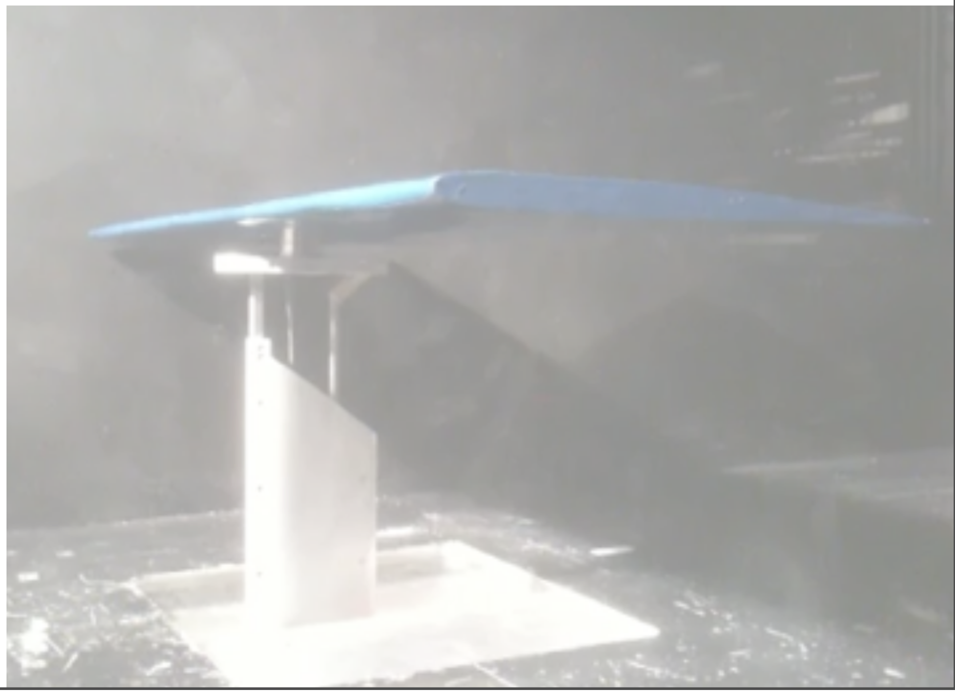


1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$



4. Conclusions and Future Work

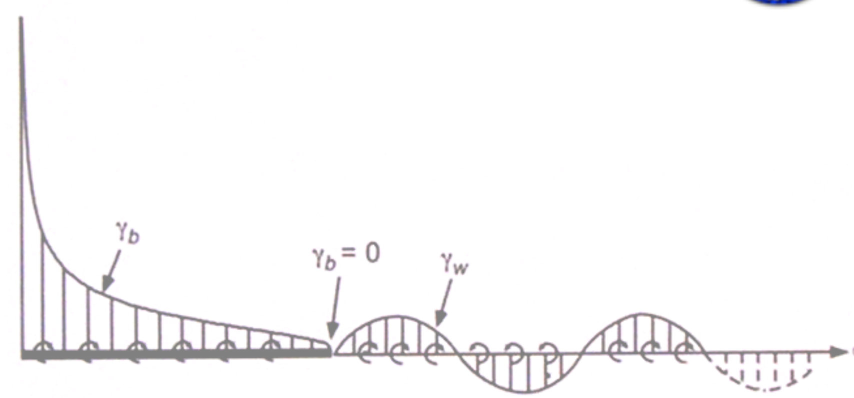
Outline

1. Motivation and Overview

   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

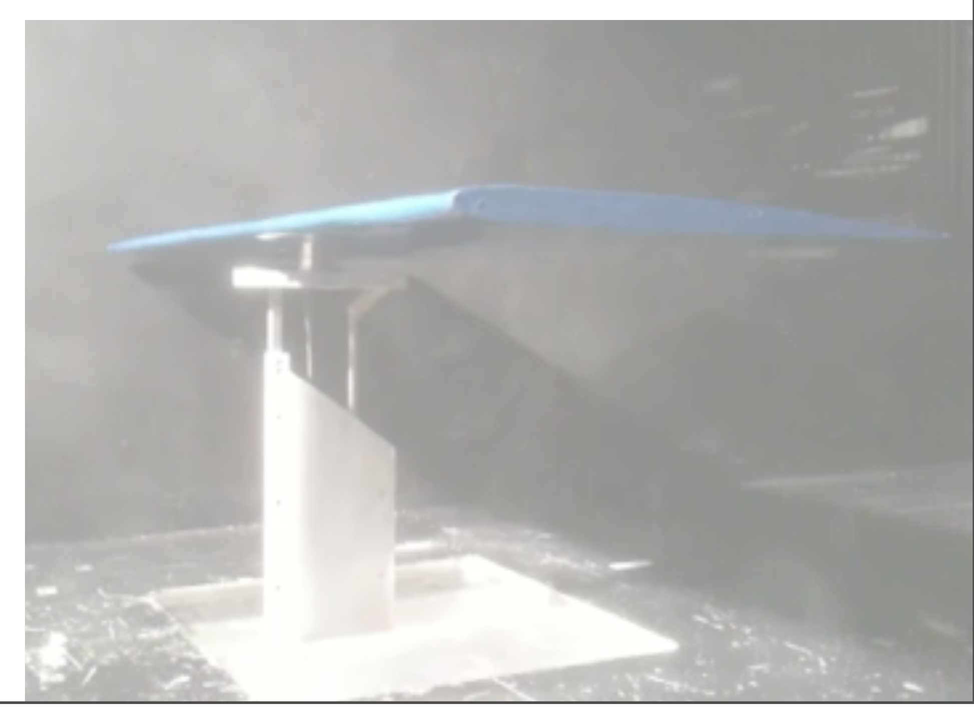
2. Review of Previous Work

   - State-space aerodynamic models
   - Indicial response and OKID



3. Wind Tunnel Experiments

   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$



4. Conclusions and Future Work

Motivation

Applications of Unsteady Models

- **Conventional UAVs (performance/robustness)**
- **Micro air vehicles (MAVs)**
- **Flow control, flight dynamic control**
- **Autopilots / Flight simulators**
- **Gust disturbance mitigation**
- **Understand bird/insect flight**

Predator (General Atomics)

Need for State-Space Models

- **Need models suitable for control**
- **Combining with flight models**

Bio-locomotion

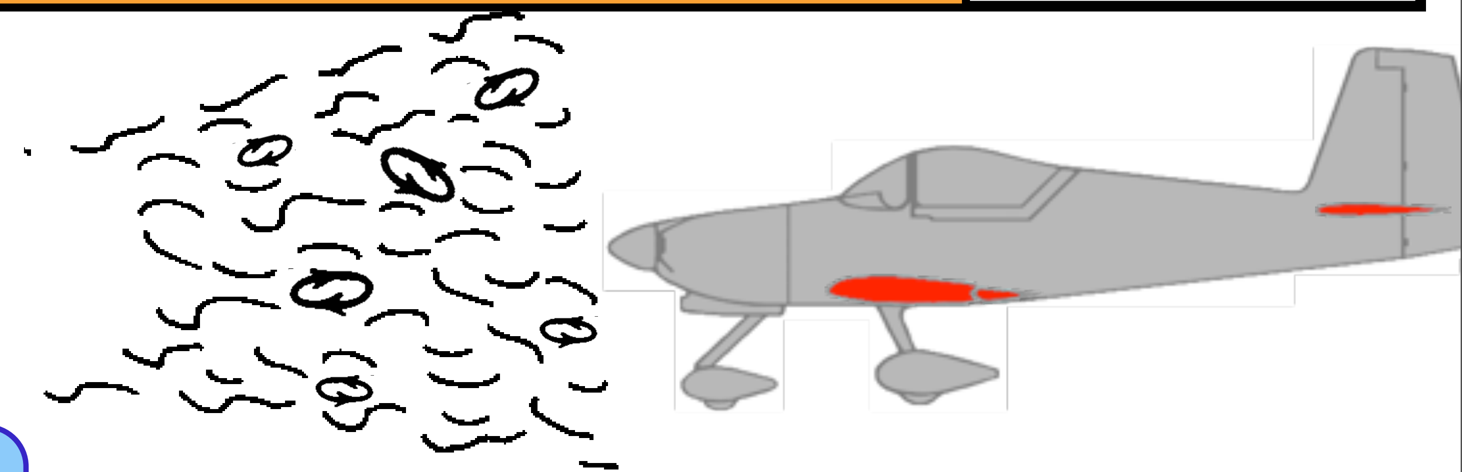
FLYIT Simulators, Inc.

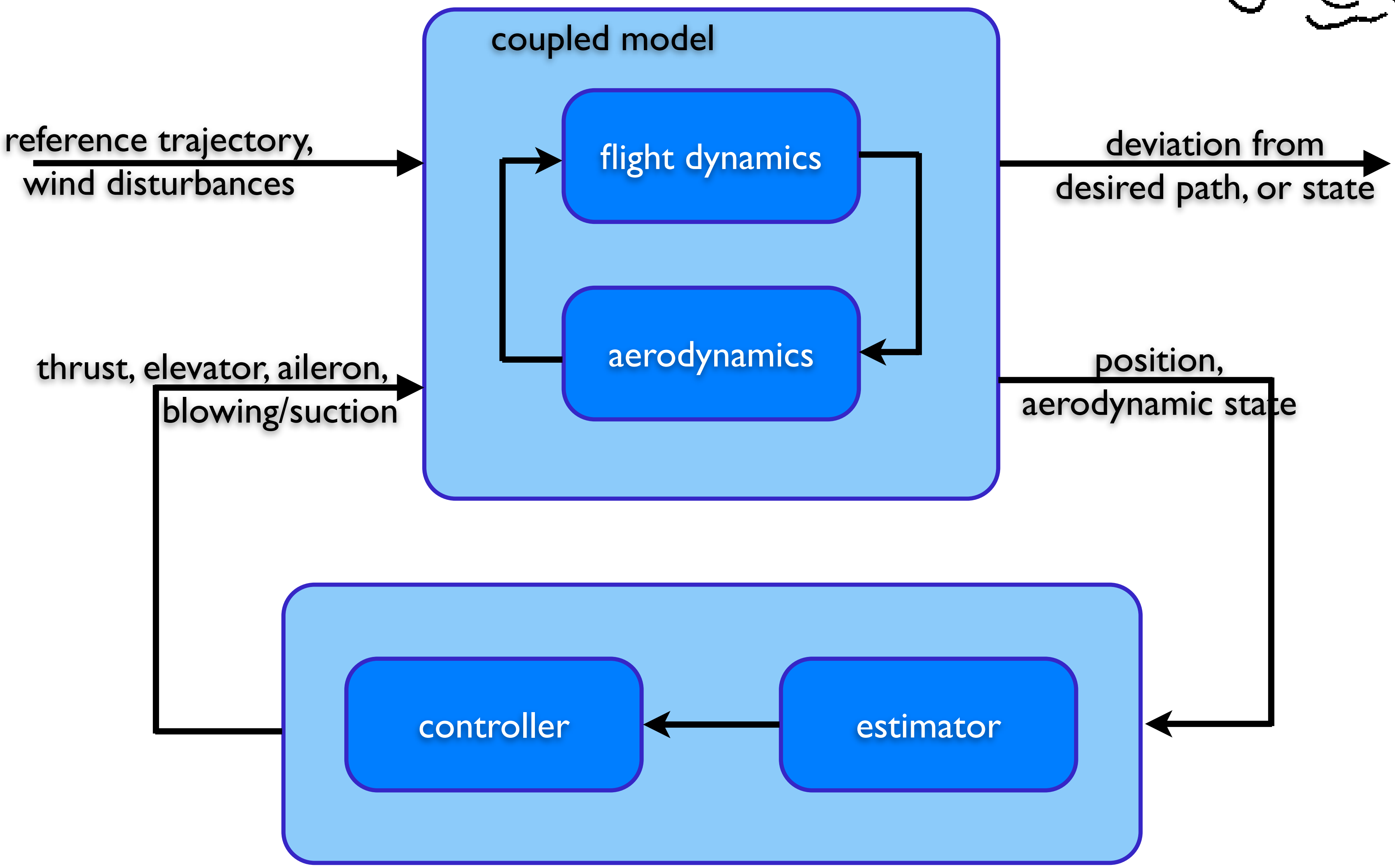
Flexible Wing (University of Florida)

www.gettyimages.com

www.gettyimages.com

Flight Dynamic Control





$\mathcal{H}_2$ – optimal control framework

Stall velocity and size

Smaller, lower stall velocity

RQ-1 Predator
(27 m/s stall)

Daedalus Dakota
(18m/s stall)

Puma AE
(10 m/s stall)

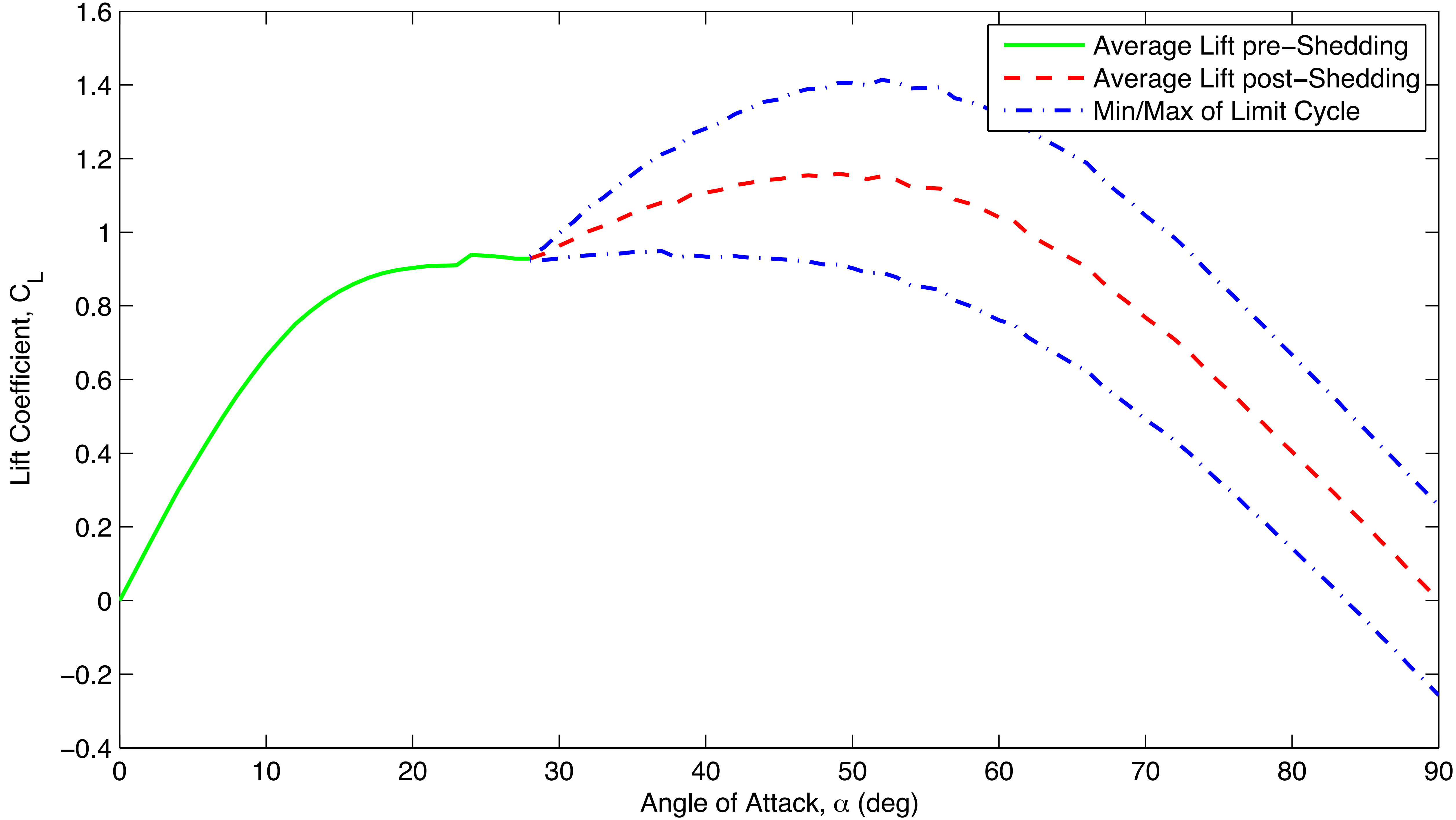
$$V_{\text{stall}} = \sqrt{\frac{2}{\rho}\left(C_{L_{\max}} S\right)^{-1} W}$$

S	Wing surface area
W	Aircraft weight
L	Lift force
C_L	Lift coefficient
V	Velocity of aircraft

Unsteady phenomena become more significant, easier to excite for smaller vehicles

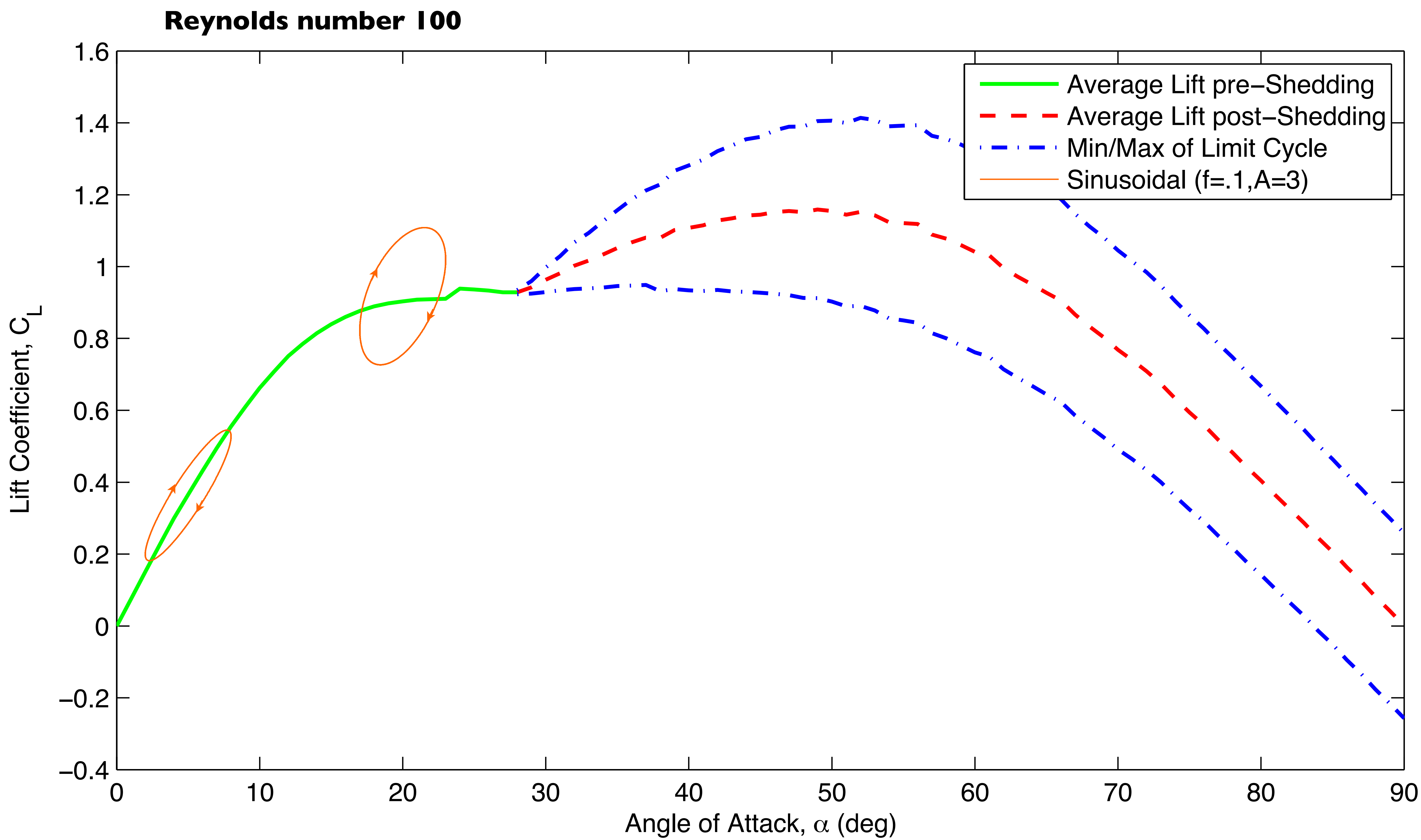
Lift vs. Angle of Attack

Reynolds number 100



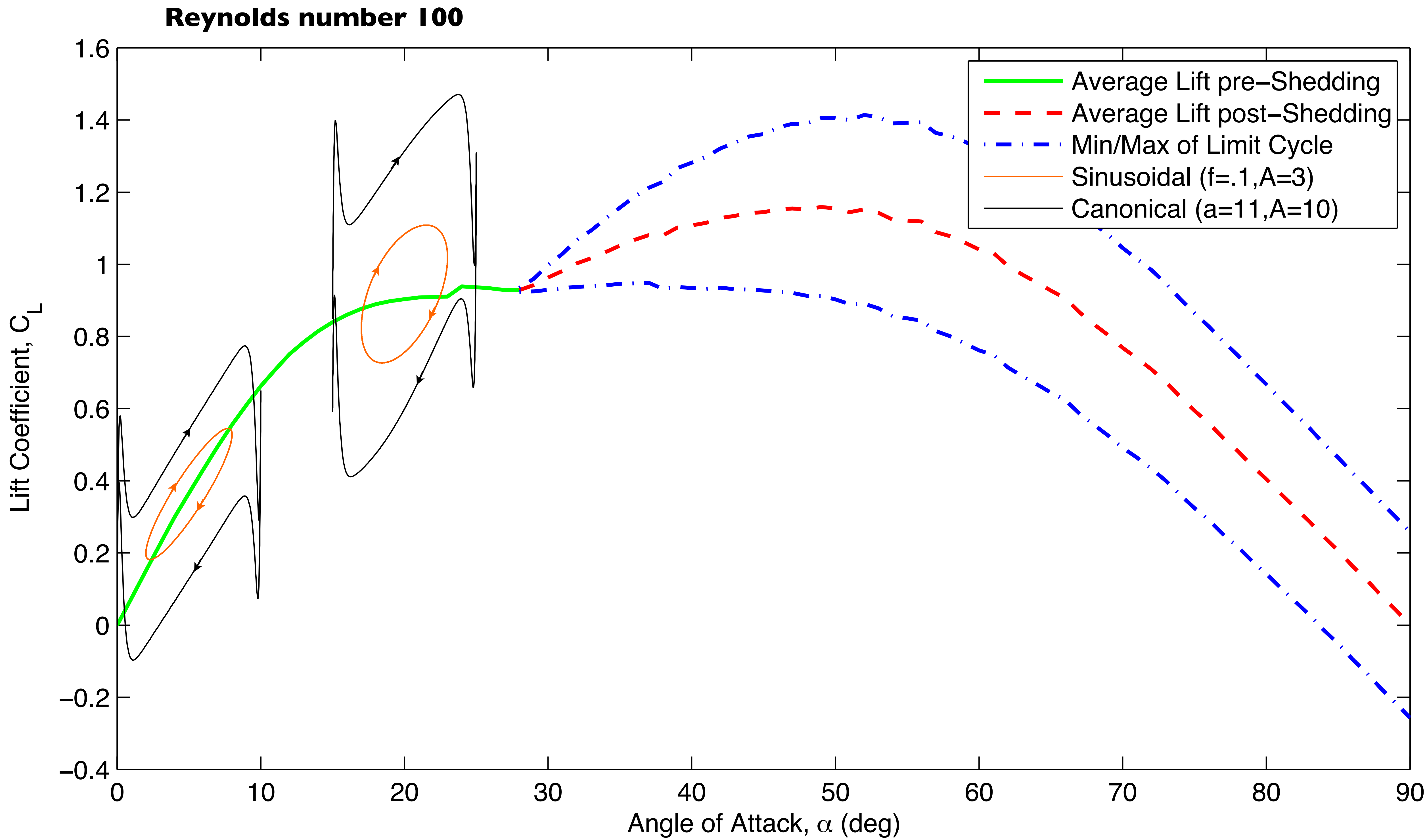
Need model that captures lift due to moving airfoil!

Lift vs. Angle of Attack



Need model that captures lift due to moving airfoil!

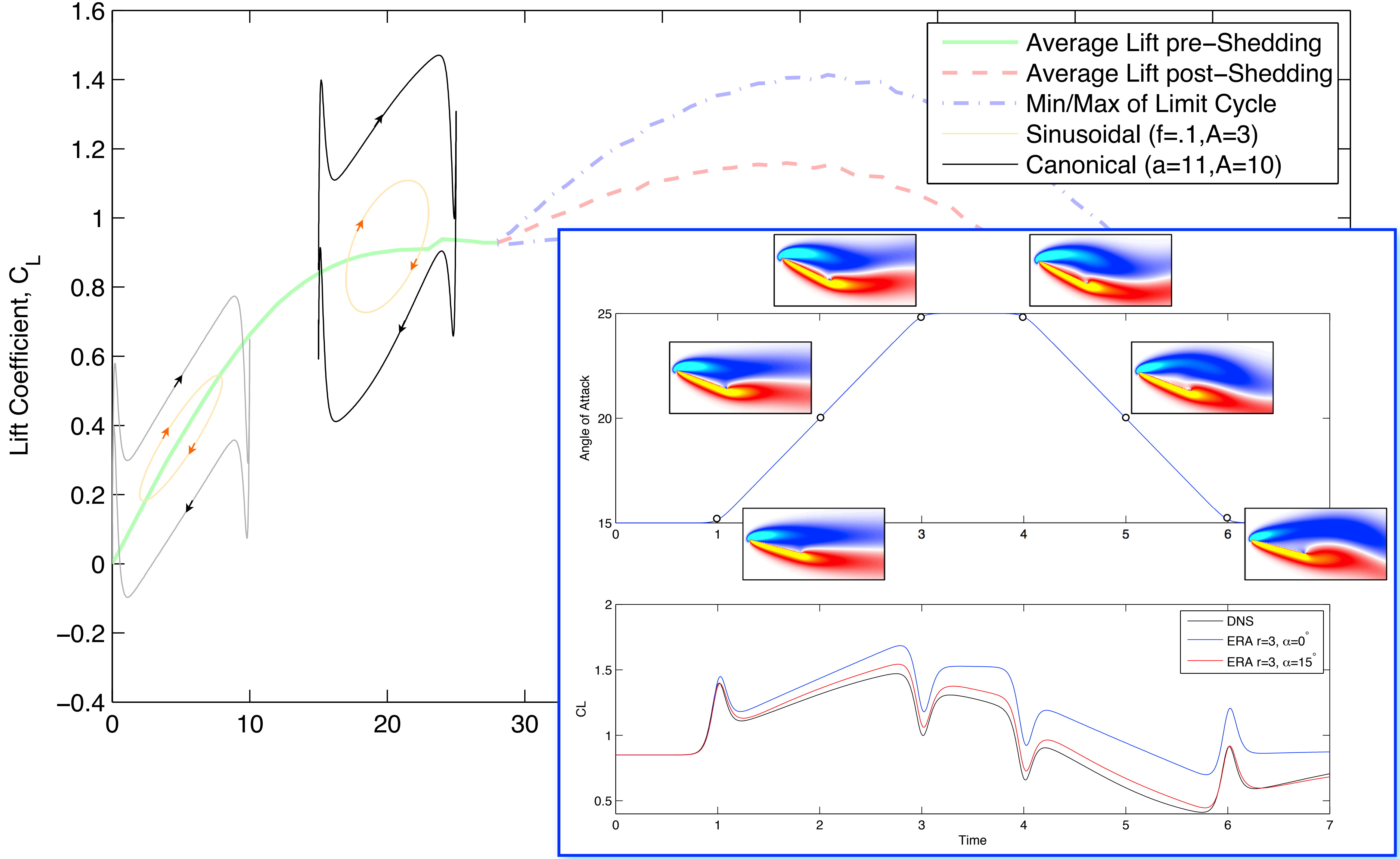
Lift vs. Angle of Attack



Need model that captures lift due to moving airfoil!

Lift vs. Angle of Attack

Reynolds number 100



Need model that captures lift due to moving airfoil!

2D Model Problem

Added-Mass

Transient

Periodic Vortex Shedding

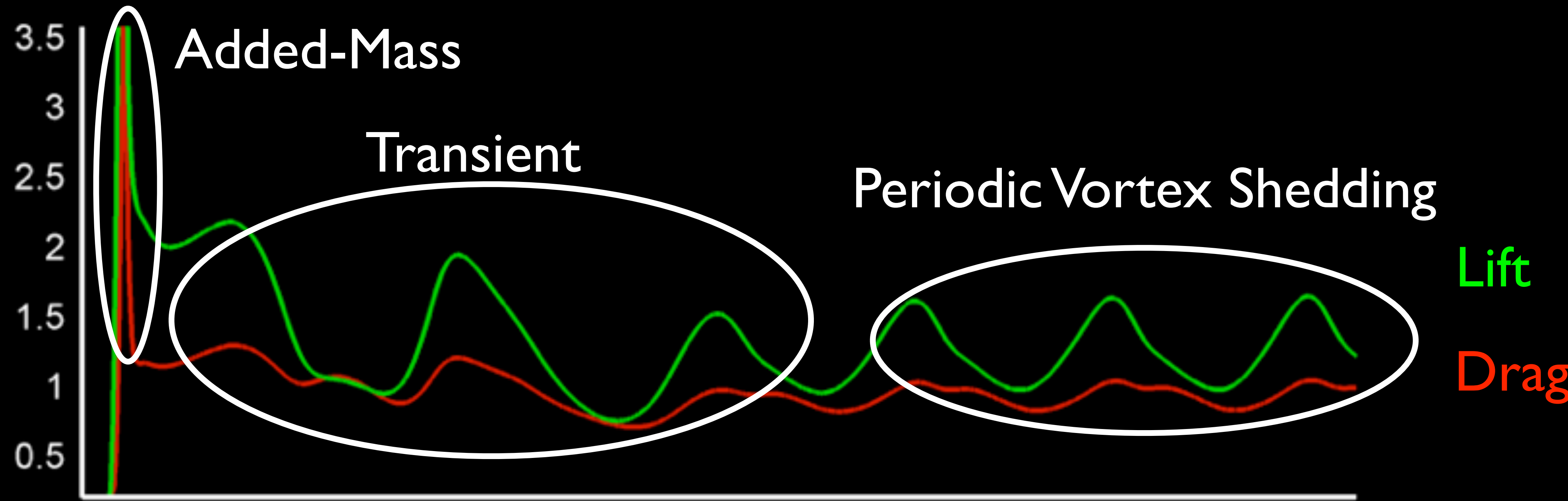
Lift

Drag

$\mathrm{Re} = 300$

$\alpha = 32^\circ$

2D Model Problem

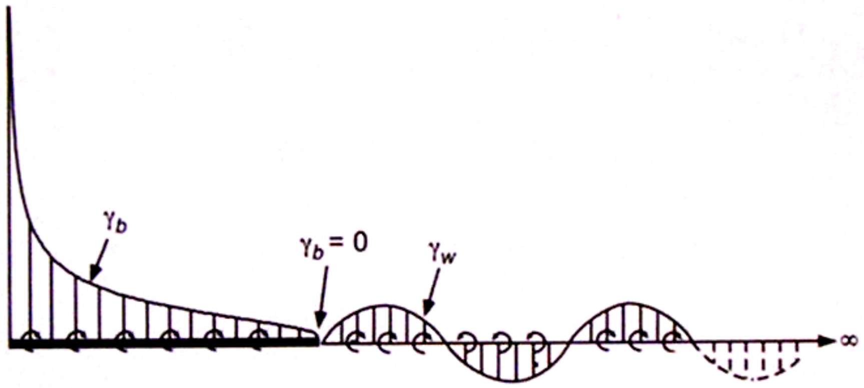


$\mathrm{Re} = 300$
$\alpha = 32^\circ$

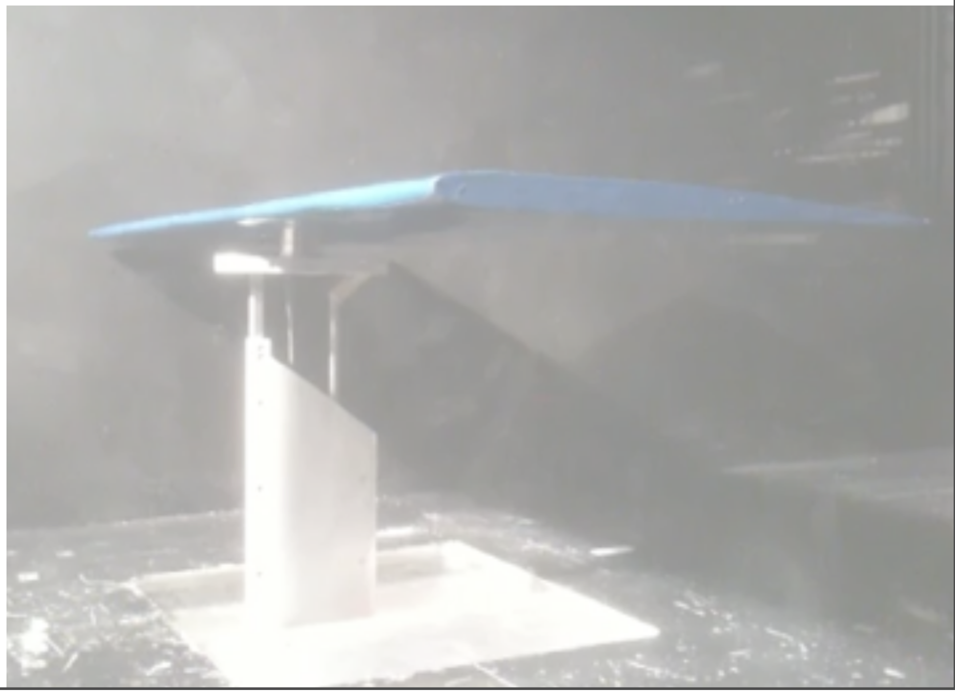
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$

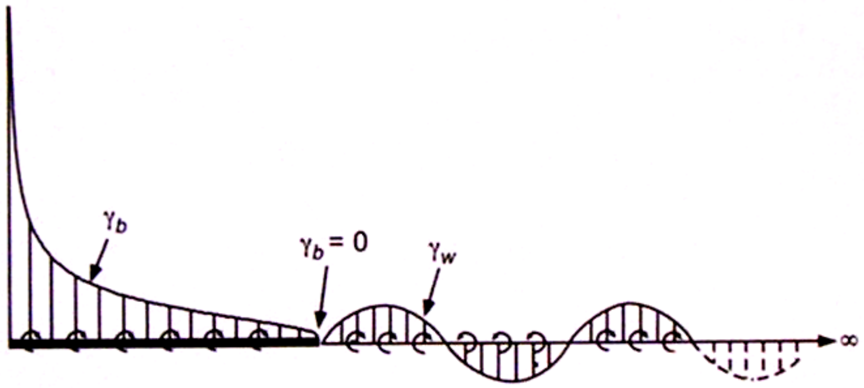


4. Conclusions and Future Work

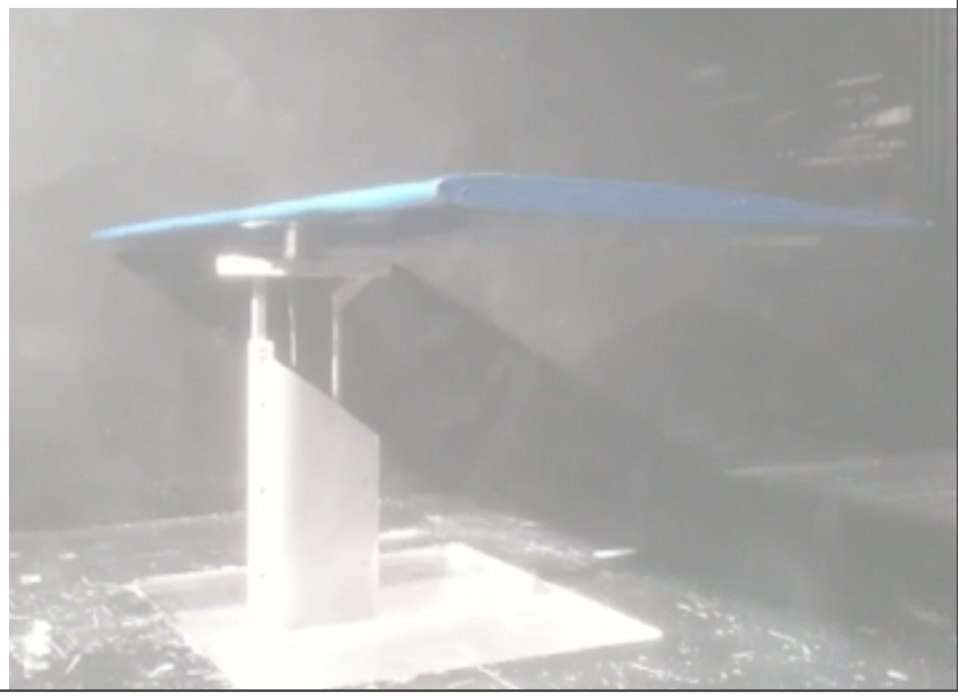
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. **Review of Previous Work**
   - **State-space aerodynamic models**
   - **Indicial response and OKID**

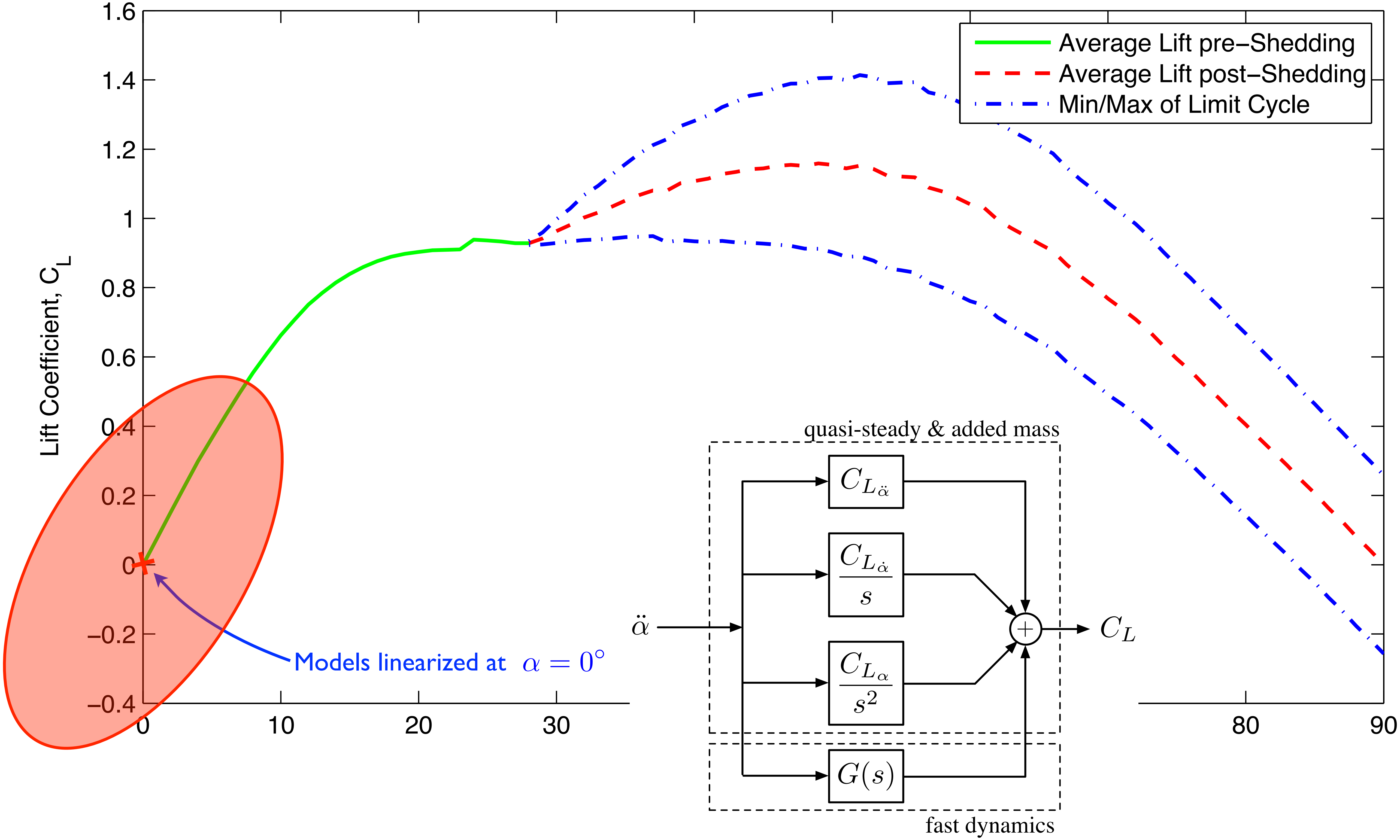


3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$



4. Conclusions and Future Work

Lift vs. Angle of Attack



State-Space Indicial Response

Indicial Response

Tuned to specific geometry, Re #

$$C_L(t) = C_L^\delta(t)\alpha(0) + \int_0^t C_L^\delta(t-\tau)\dot{\alpha}(\tau)d\tau$$

Theodorsen's Model

Physically motivated components

Parametrized by pitch point

Frequency domain, idealized assumptions

$$C_L = \underbrace{\frac{\pi}{2}\left[\ddot{h} + \dot{\alpha} - \frac{a}{2}\ddot{\alpha}\right]}_{\text{Added-Mass}} + \underbrace{2\pi\left[\alpha + \dot{h} + \frac{1}{2}\dot{\alpha}\left(\frac{1}{2} - a\right)\right]}_{\text{Circulatory}} C(k)$$

State-Space Model

Captures input output dynamics accurately

Computationally tractable

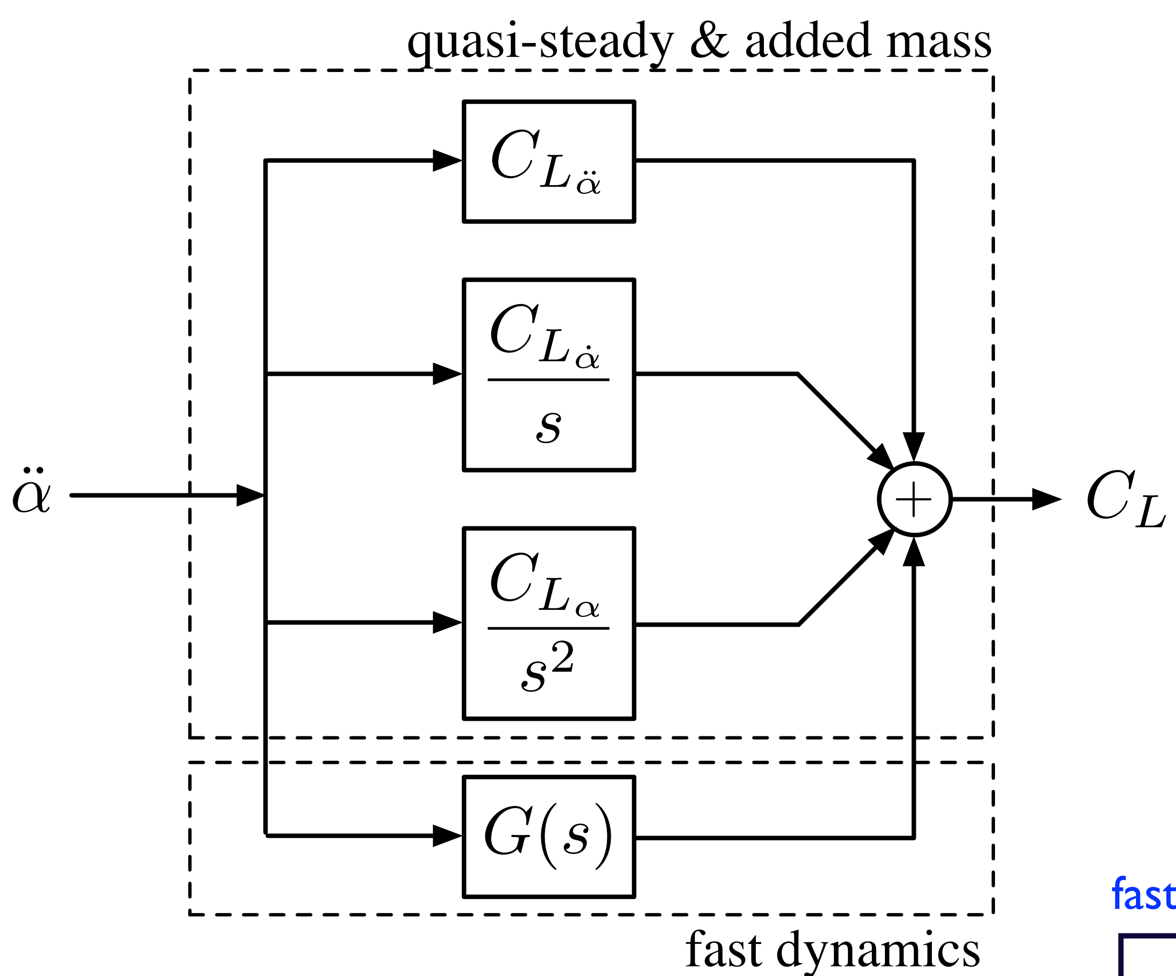
fits into control framework

fast dynamics

$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \alpha \\ \dot{\alpha}\end{bmatrix} = \begin{bmatrix} A_r & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha \\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix} B_r \\ 0 \\ 1\end{bmatrix}\ddot{\alpha}$$

$$C_L = \begin{bmatrix} C_r & C_{L_\alpha} & C_{L_{\dot{\alpha}}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha \\ \dot{\alpha}\end{bmatrix} + C_{L_{\ddot{\alpha}}}\ddot{\alpha}$$

quasi-steady and added-mass



Model Summary

Linearized about $\alpha = 0$

Based on experiment, simulation or theory

Recovers stability derivatives $C_{L_\alpha}, C_{L_{\dot\alpha}}, C_{L_{\ddot\alpha}}$ associated with quasi-steady and added-mass

ODE model ideal for control design

fast dynamics

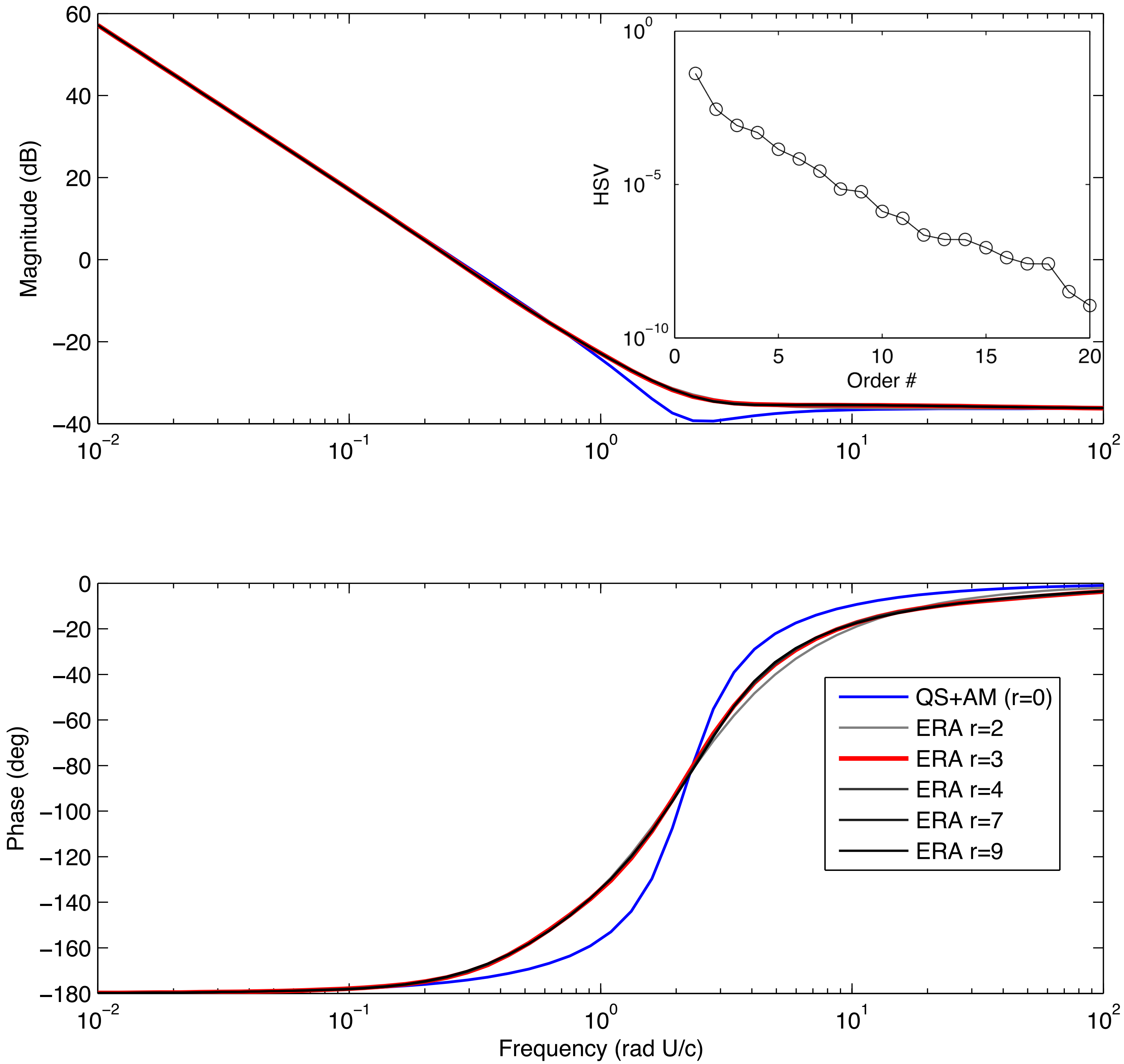
$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} = \begin{bmatrix}A_r & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}B_r\\ 0\\ 1\end{bmatrix}\ddot\alpha$$

$$C_L = \begin{bmatrix}C_r & C_{L_\alpha} & C_{L_{\dot\alpha}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + C_{L_{\ddot\alpha}}\ddot\alpha$$

quasi-steady and added-mass

Brunton and Rowley, *in preparation.*

Bode Plot - Pitch (LE)



Frequency response

input is $\ddot{\alpha}$ **(** α **is angle of attack)**

output is lift coefficient $\mathrm{C_L}$

Pitching at leading edge

Model without additional fast dynamics [QS+AM (r=0)] is inaccurate in crossover region

Models with fast dynamics of ERA model order >3 are converged

Punchline: additional fast dynamics (ERA model) are essential

Brunton and Rowley, *in preparation.*

Bode Plot - Pitch (QC)

Frequency response

input is $\ddot{\alpha}$ (α is angle of attack)

output is lift coefficient C_L

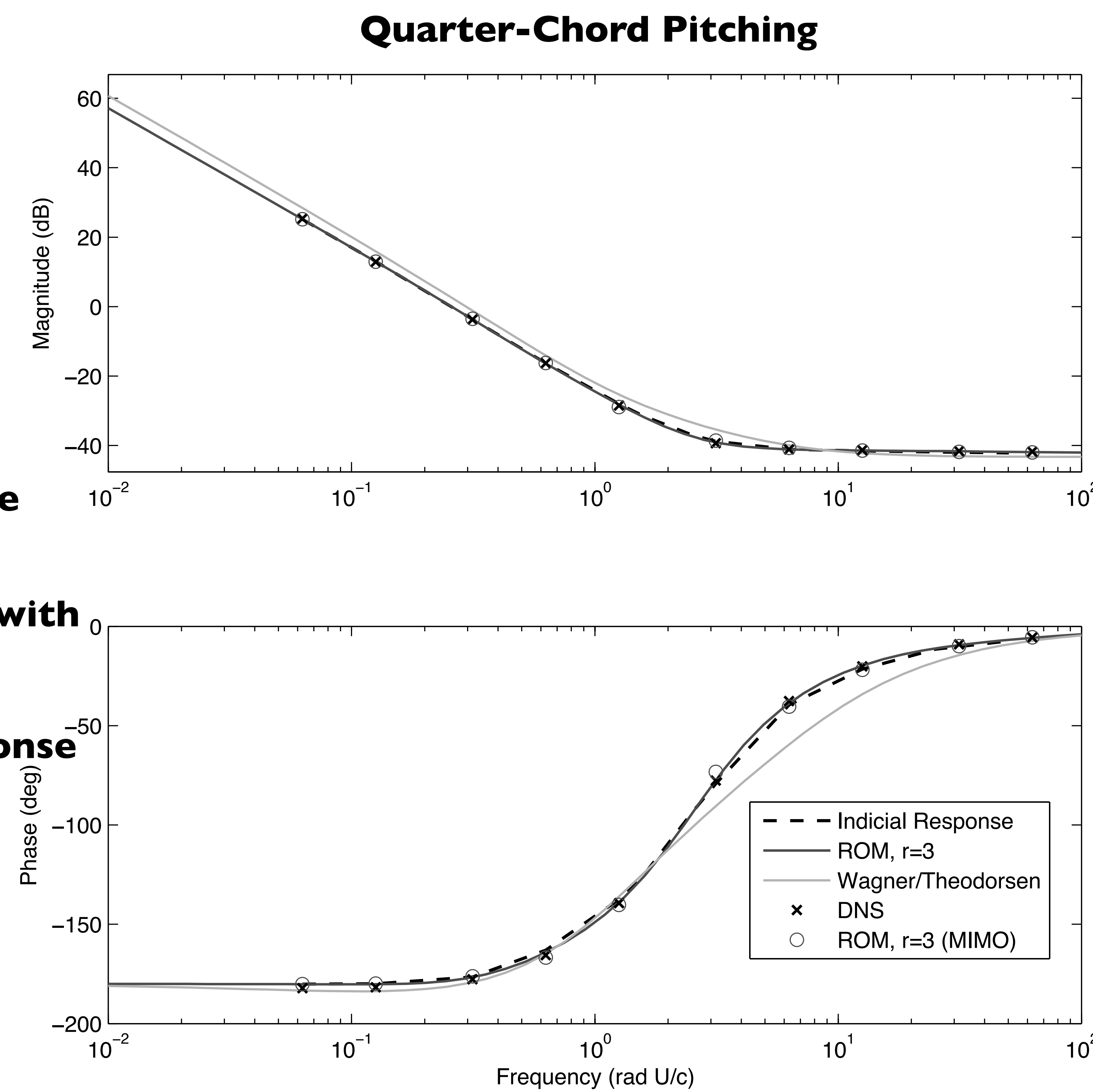
Pitching at quarter chord

Reduced order model with ERA r=3 accurately reproduces Indicial Response

Indicial Response and ROM agree better with DNS than Theodorsen's model.

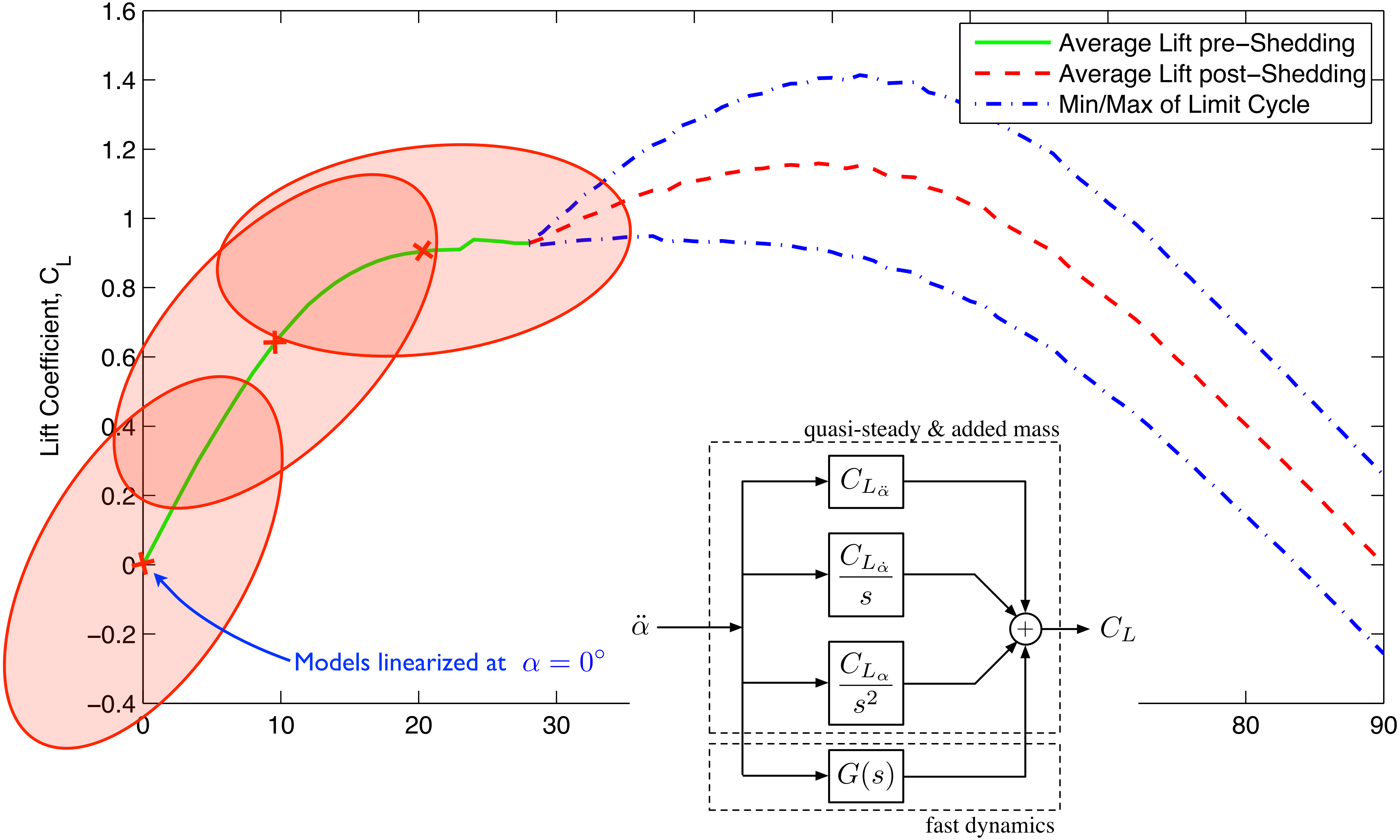
Asymptotes are correct for Indicial Response because it is based on experiment

Model for pitch/plunge dynamics [ERA, r=3 (MIMO)] works as well, for the same order model



Brunton and Rowley, *in preparation.*

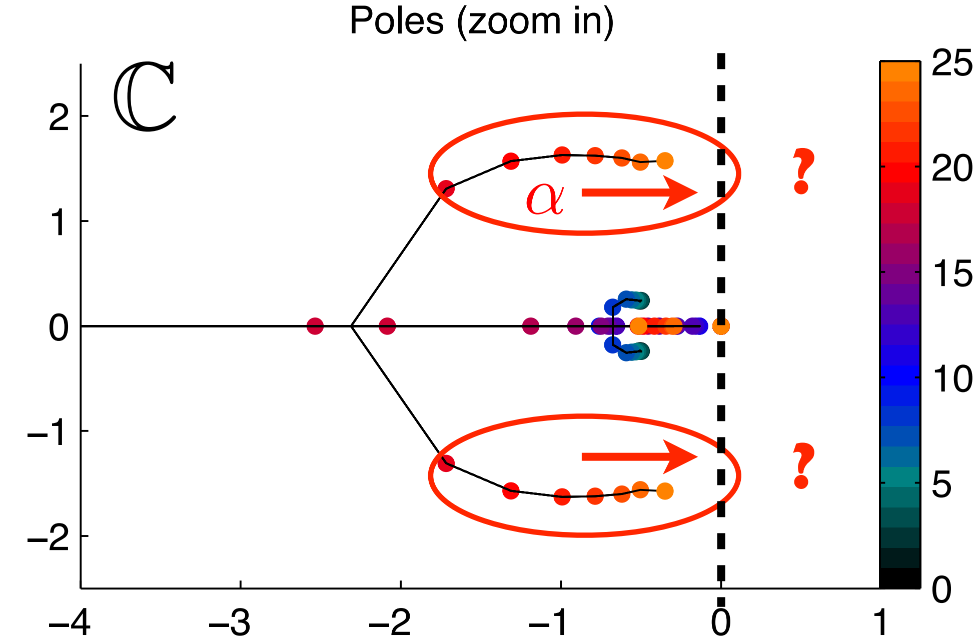
Lift vs. Angle of Attack

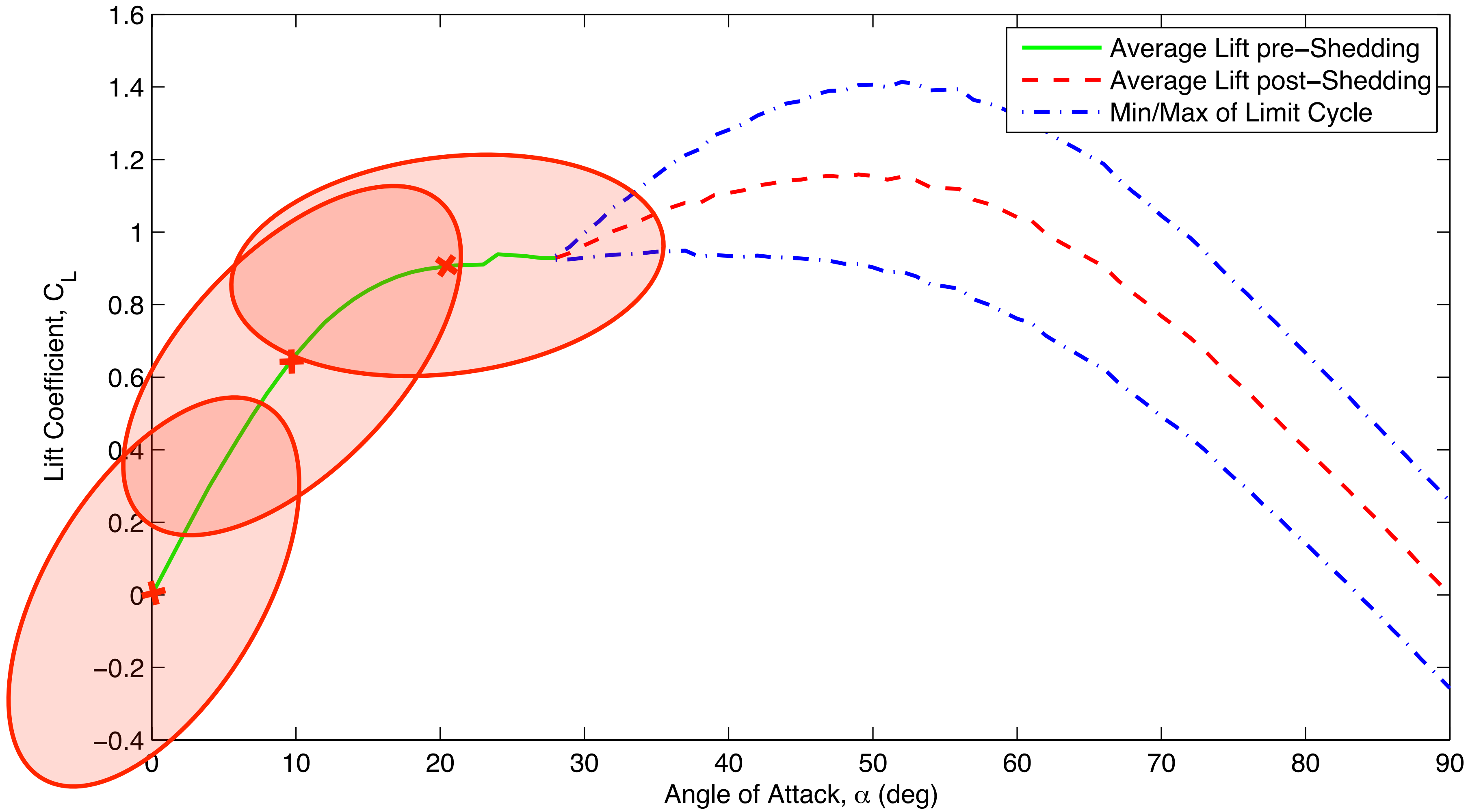


What we know

1. Hopf bifurcation at $\alpha = 28^\circ$
2. Linear models capture conjugate pair
3. Linear models based on overarching nonlinear model (Navier-Stokes)

Is it possible to obtain nonlinear reduced order model?





Nonlinear Unsteady Models

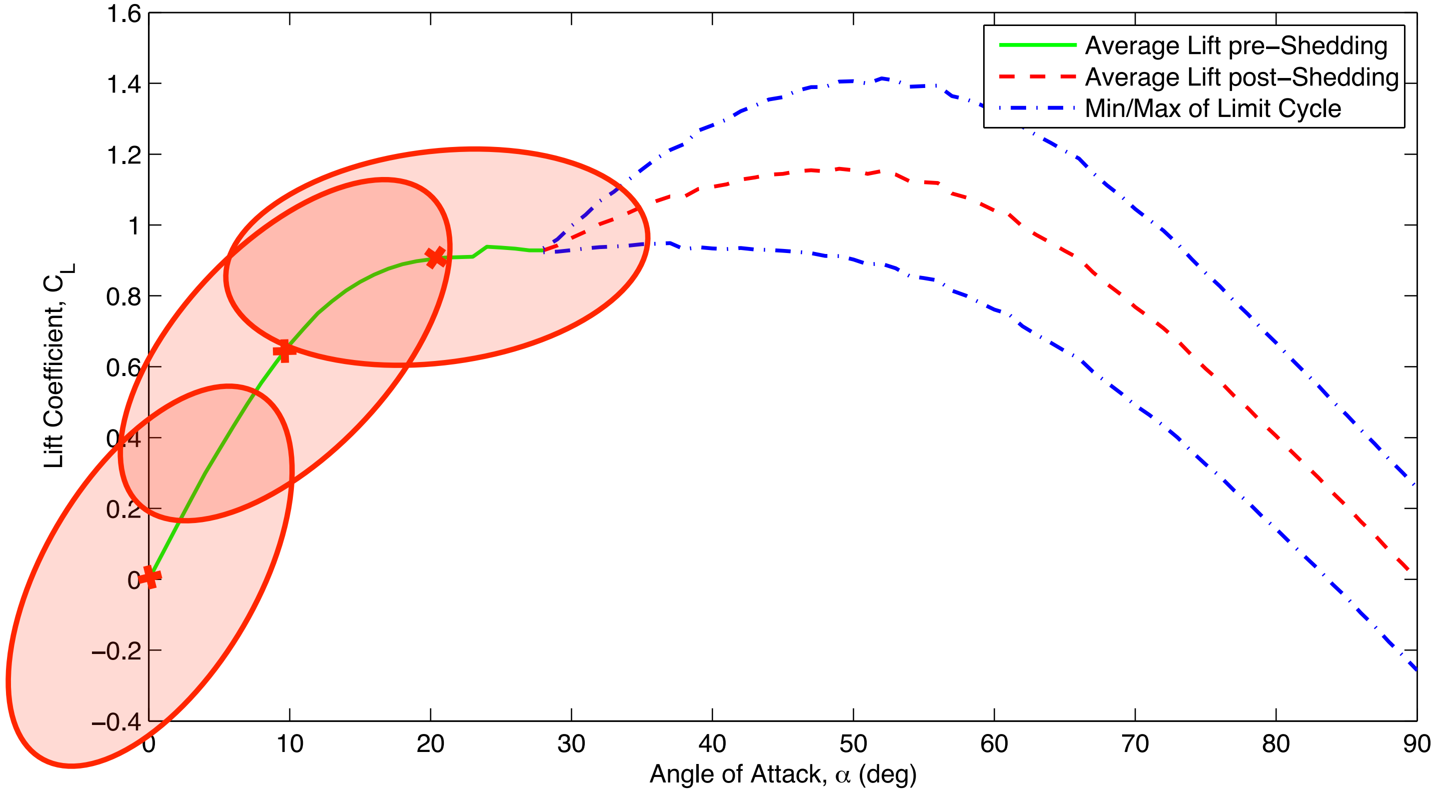
$$\dot{x} = f(x, u; \alpha)$$
$$y = g(x, u; \alpha)$$

u - input

y - output

x - state vector

α - bifurcation parameter

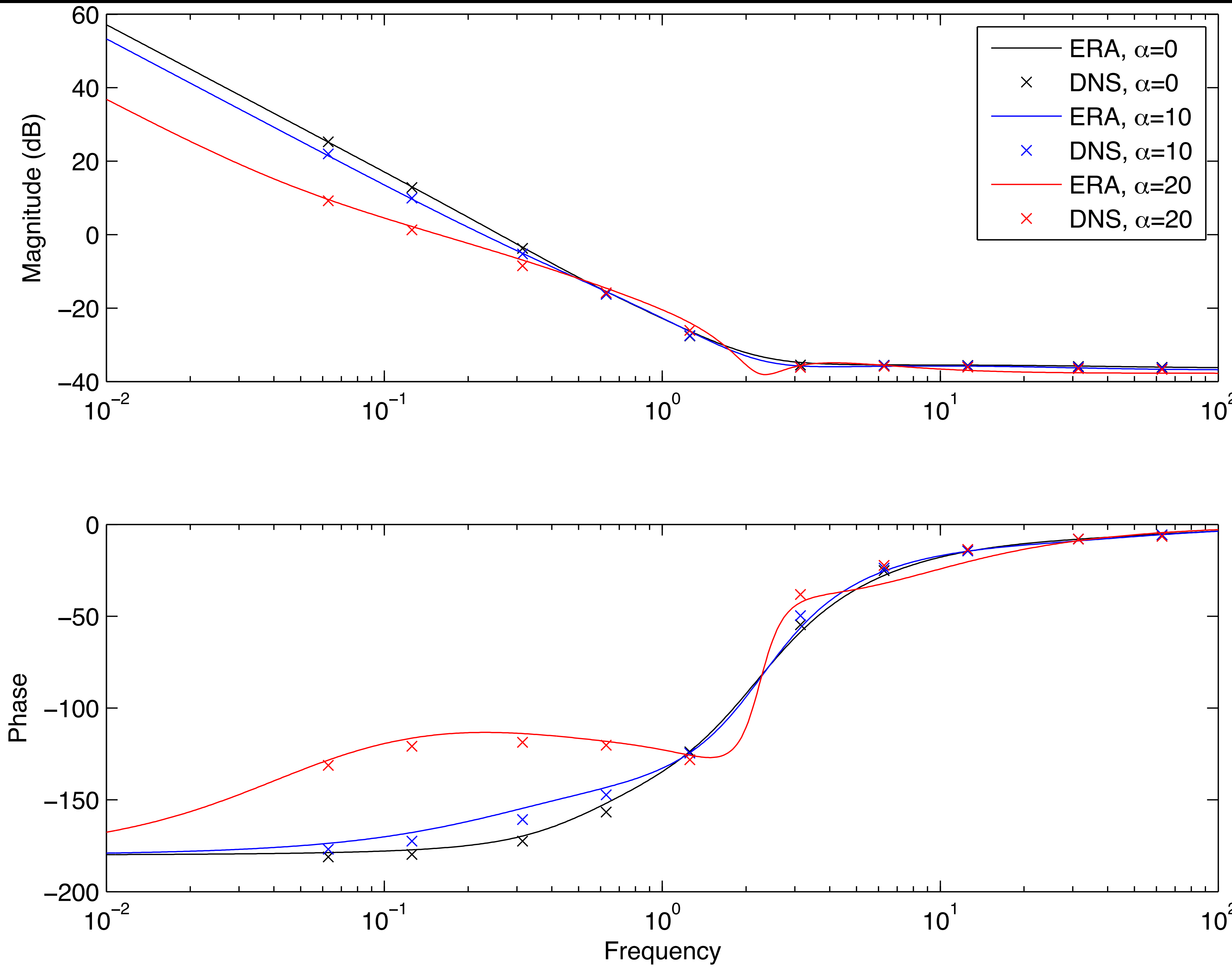


For $\alpha_0 < \alpha_{\text{crit}}$, equilibrium x=0 is stable, with linear dynamics given by:

$$\dot{x} = \underbrace{D_x f(0,0;\alpha_0)}_{A(\alpha_0)} \cdot x + \underbrace{D_u(0,0;\alpha_0)}_{B(\alpha_0)} \cdot u + \underbrace{D_\alpha(0,0;\alpha_0)}_{=0} \cdot (\alpha - \alpha_0)$$

$$y = \underbrace{g(0,0;\alpha_0)}_{C_L(\alpha_0)} + \underbrace{D_x g(0,0;\alpha_0)}_{C(\alpha_0)} \cdot x + \underbrace{D_u g(0,0;\alpha_0)}_{D(\alpha_0)} \cdot u + \underbrace{D_\alpha g(0,0;\alpha_0)}_{C_{L_\alpha}} \cdot (\alpha - \alpha_0)$$

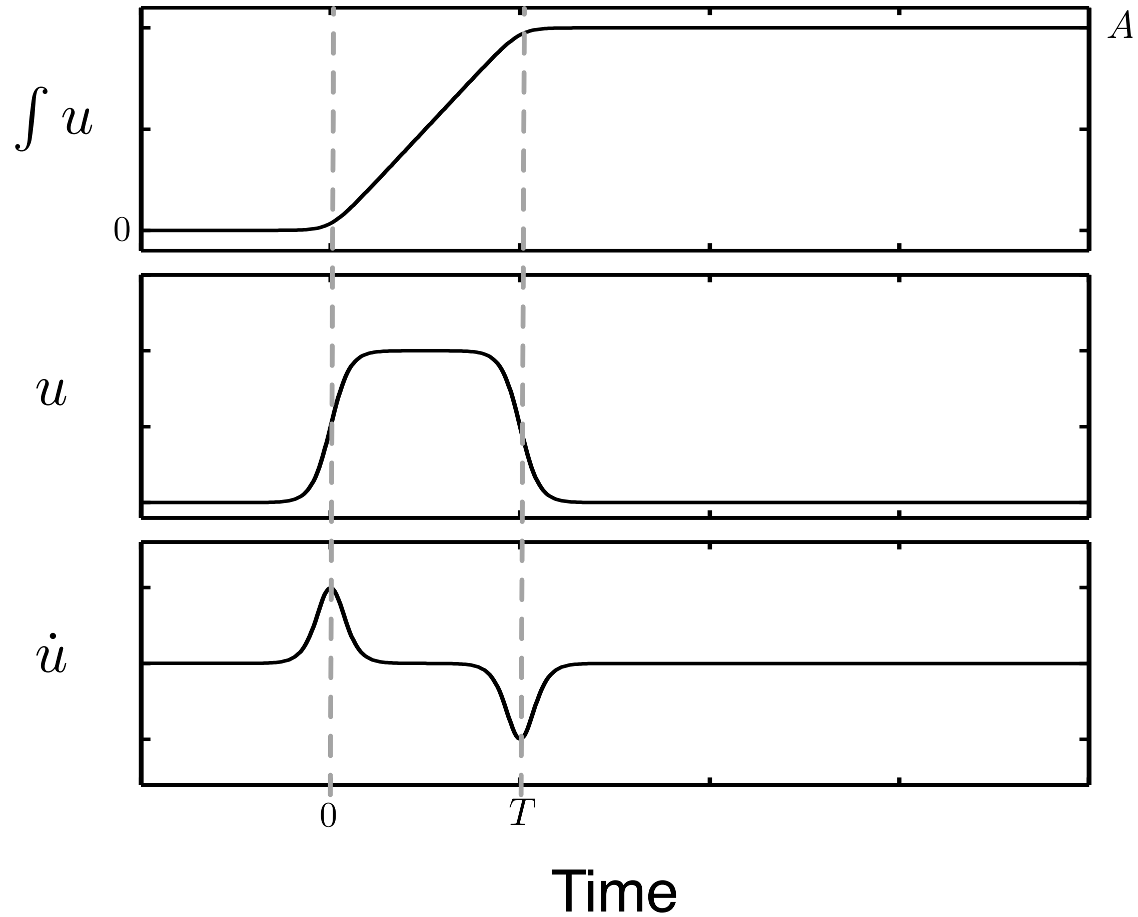
Bode Plot of Model (-) vs Data (x)

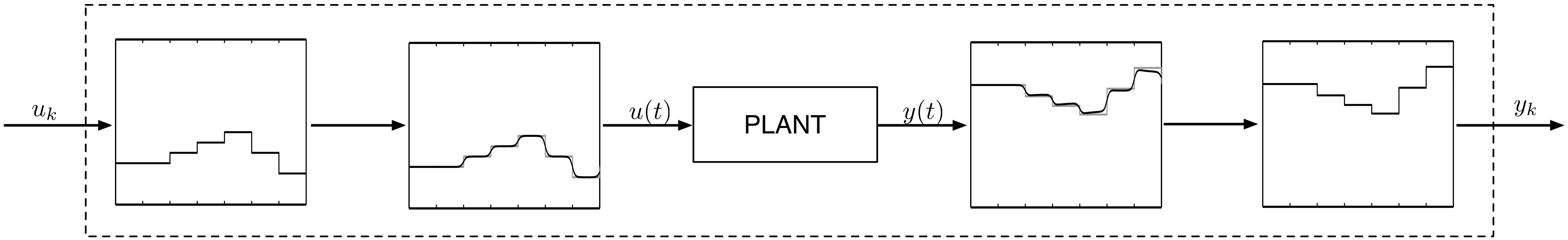


Direct numerical simulation confirms that local linearized models are accurate for small amplitude sinusoidal maneuvers

Brunton and Rowley, AIAA ASM 2011

(Indicial) Step Response

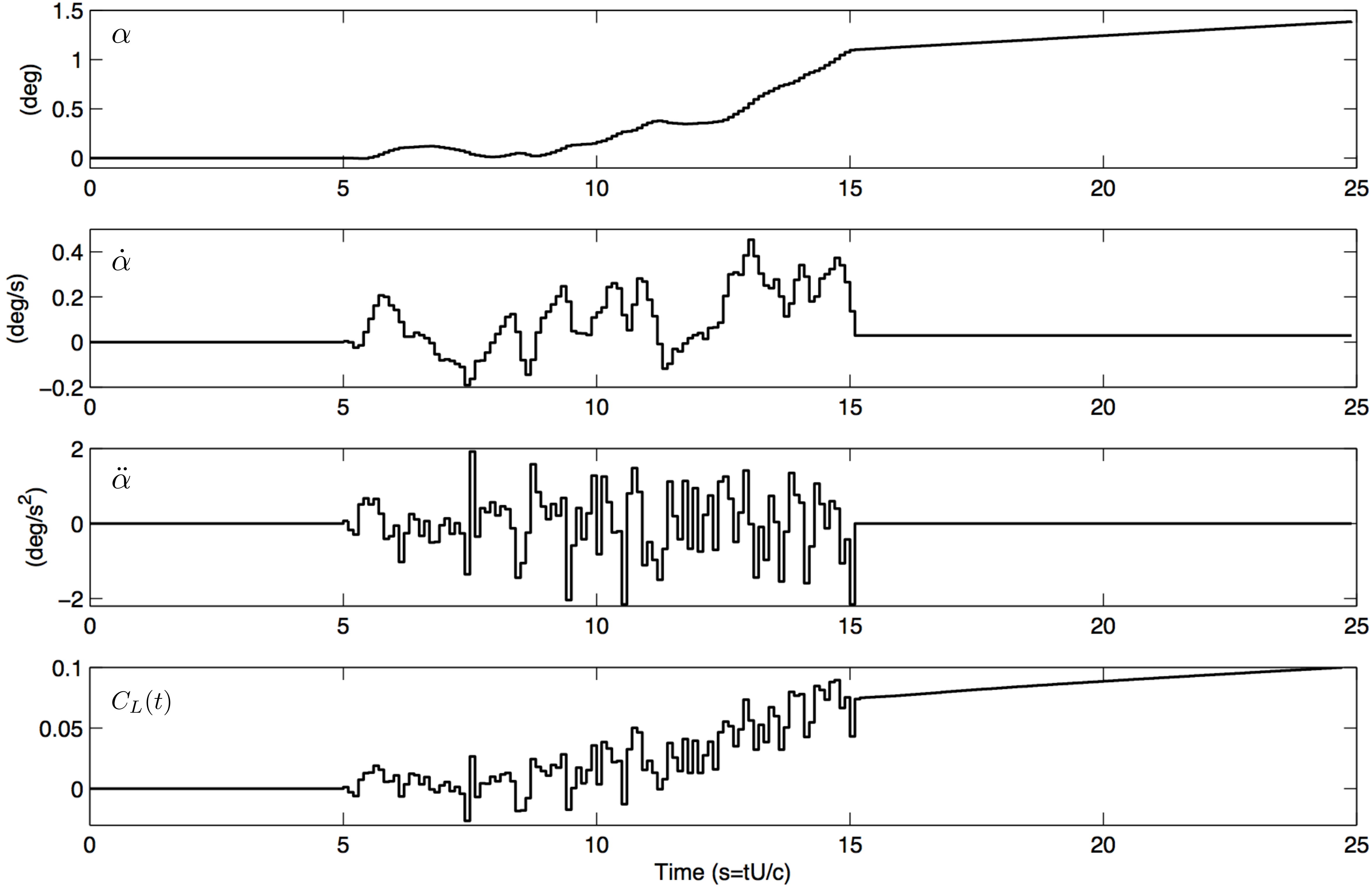




Previously, models are based on aerodynamic step response

Idea: Maneuver aircraft for 5-10 minutes, back out the Markov parameters, and construct ERA model.

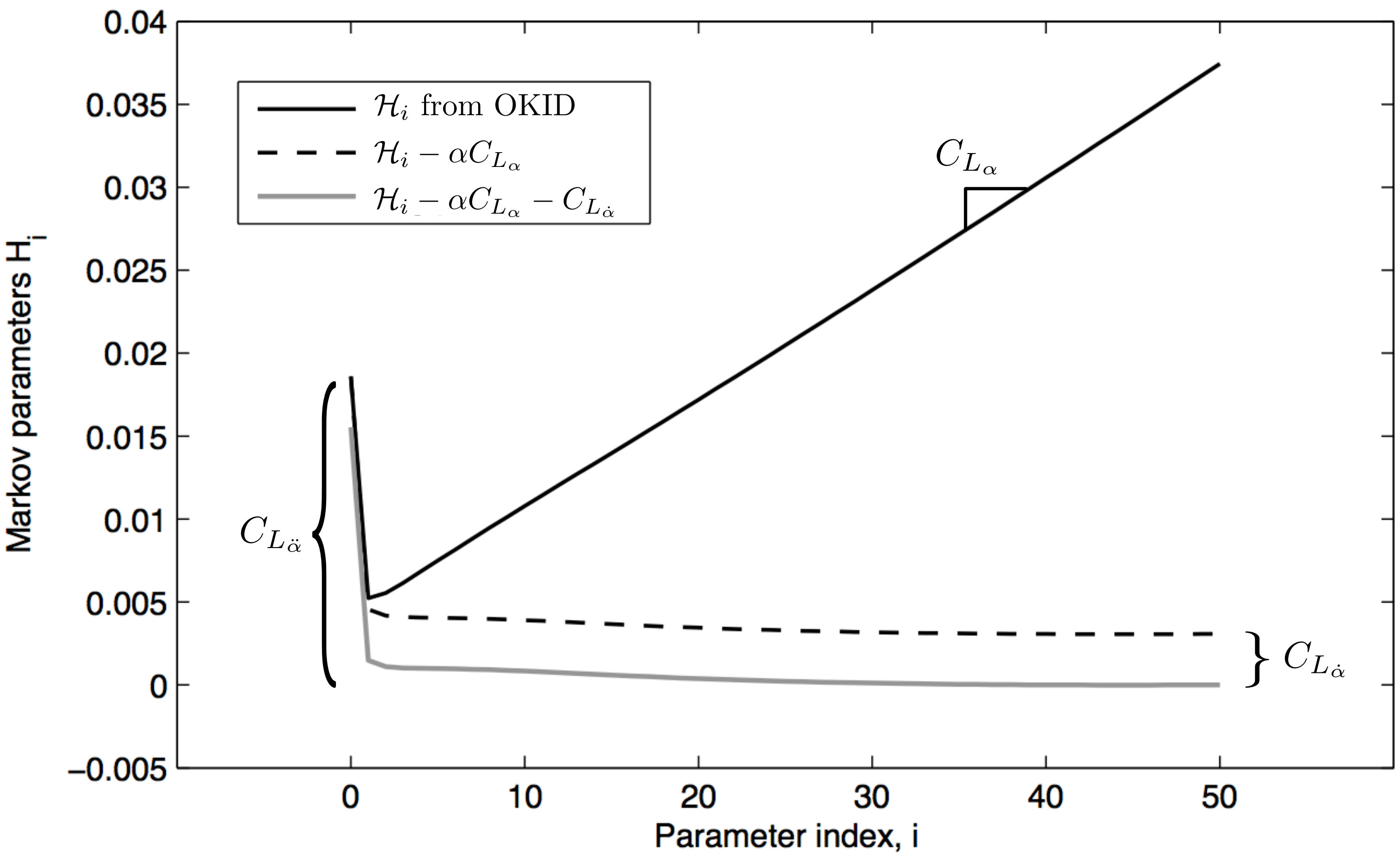
Random Input Maneuver



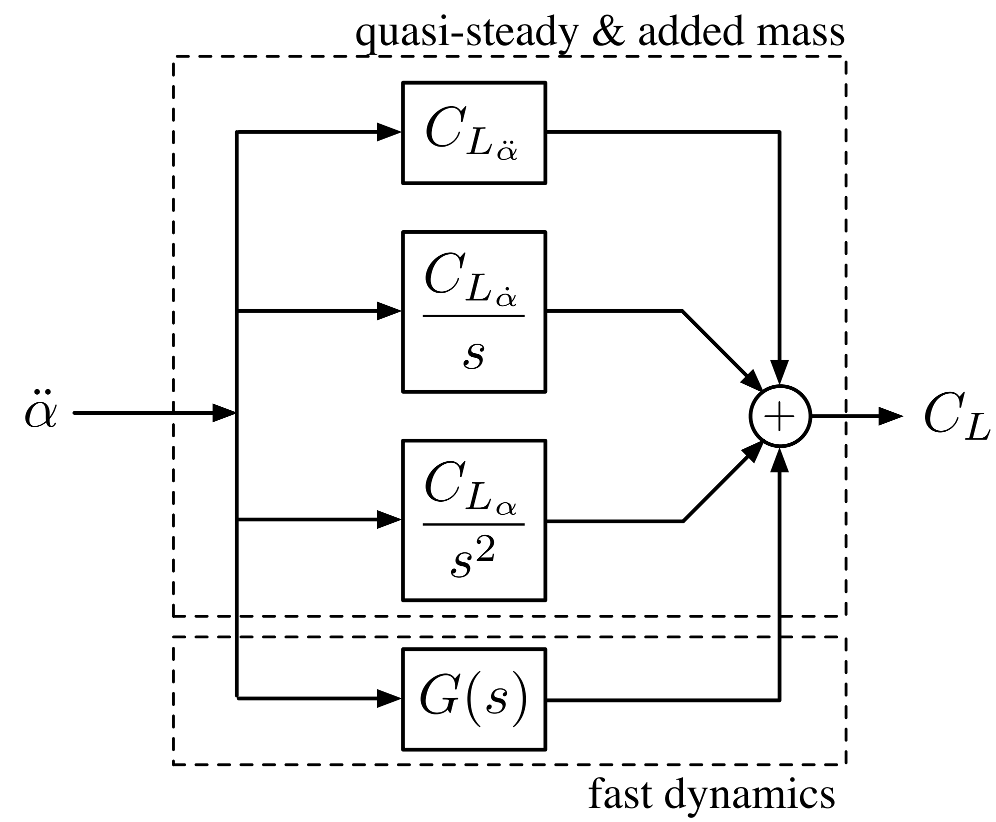
Idea: Maneuver aircraft for 5-10 minutes, back out the Markov parameters, and construct ERA model.

OKID Markov Parameters

Impulse response in $\ddot{\alpha}$



Absolutely need to use the correct form of the model!



$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} = \begin{bmatrix}A_r & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}B_r\\ 0\\ 1\end{bmatrix}\ddot\alpha$$

Observer/Kalman Filter Identification (OKID)

Juang, Phan, Horta, Longman, 1991.

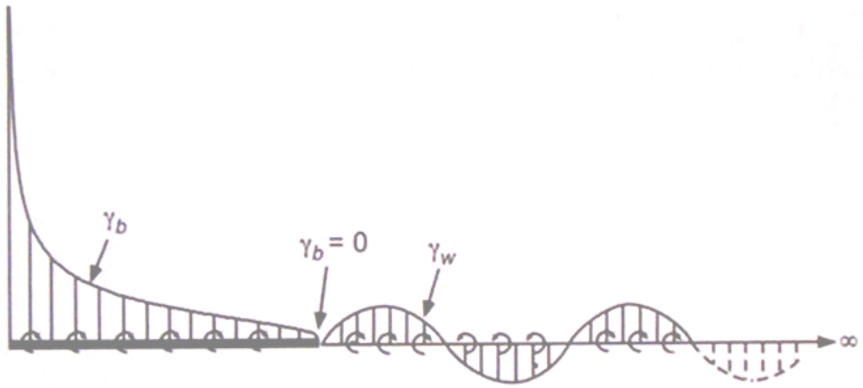
Brunton and Rowley, *submitted*.

$$C_L = \begin{bmatrix}C_r & C_{L_\alpha} & C_{L_{\dot\alpha}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + C_{L_{\ddot\alpha}}\ddot\alpha$$

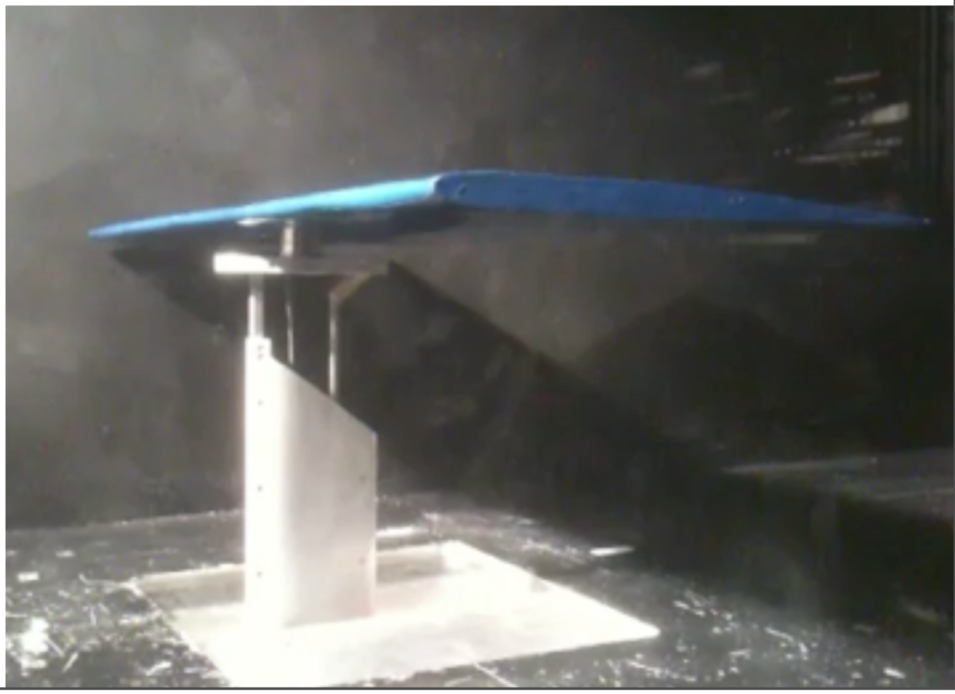
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



3. **Wind Tunnel Experiments**
   - **Physical setup and configuration**
   - **Aggressive system identification maneuver**
   - **Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$**

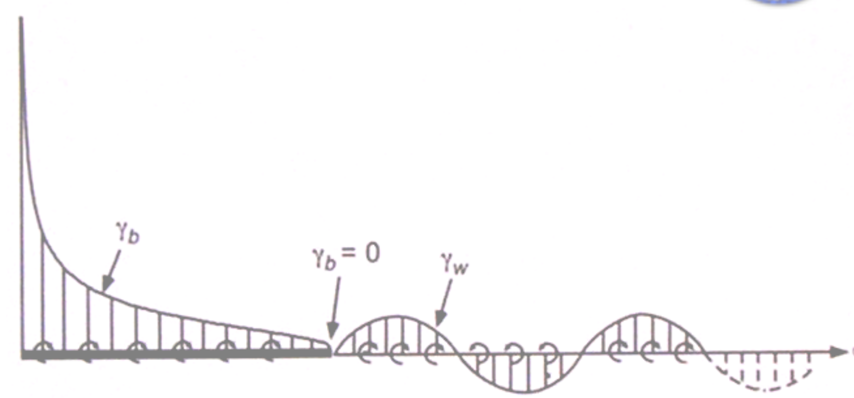


4. Conclusions and Future Work

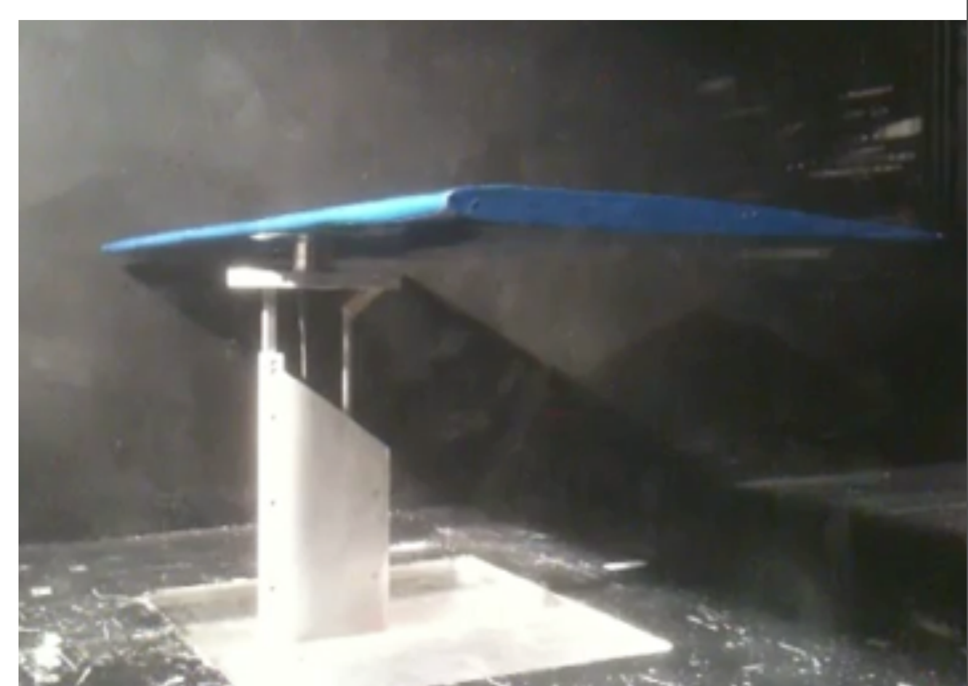
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID

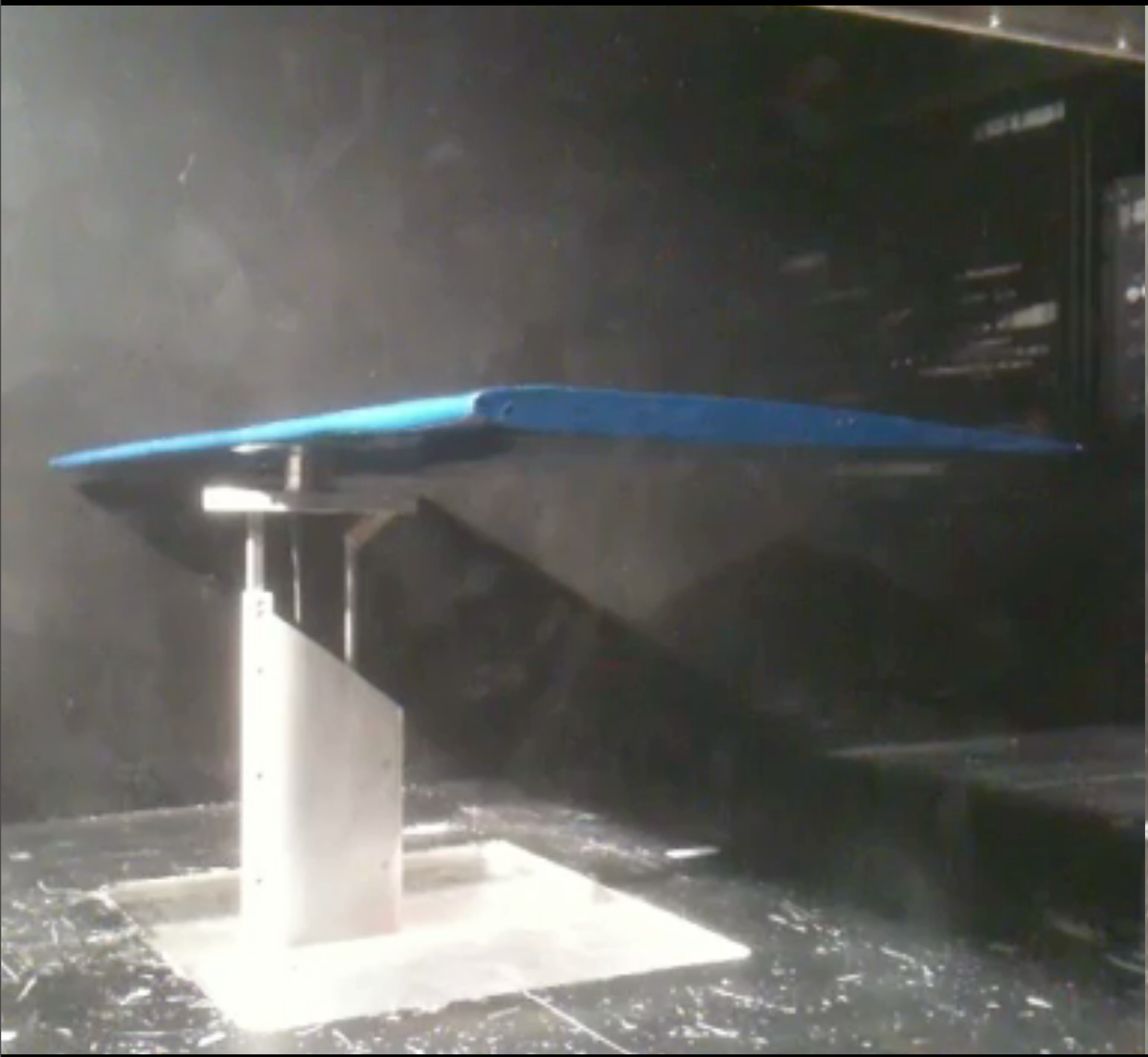
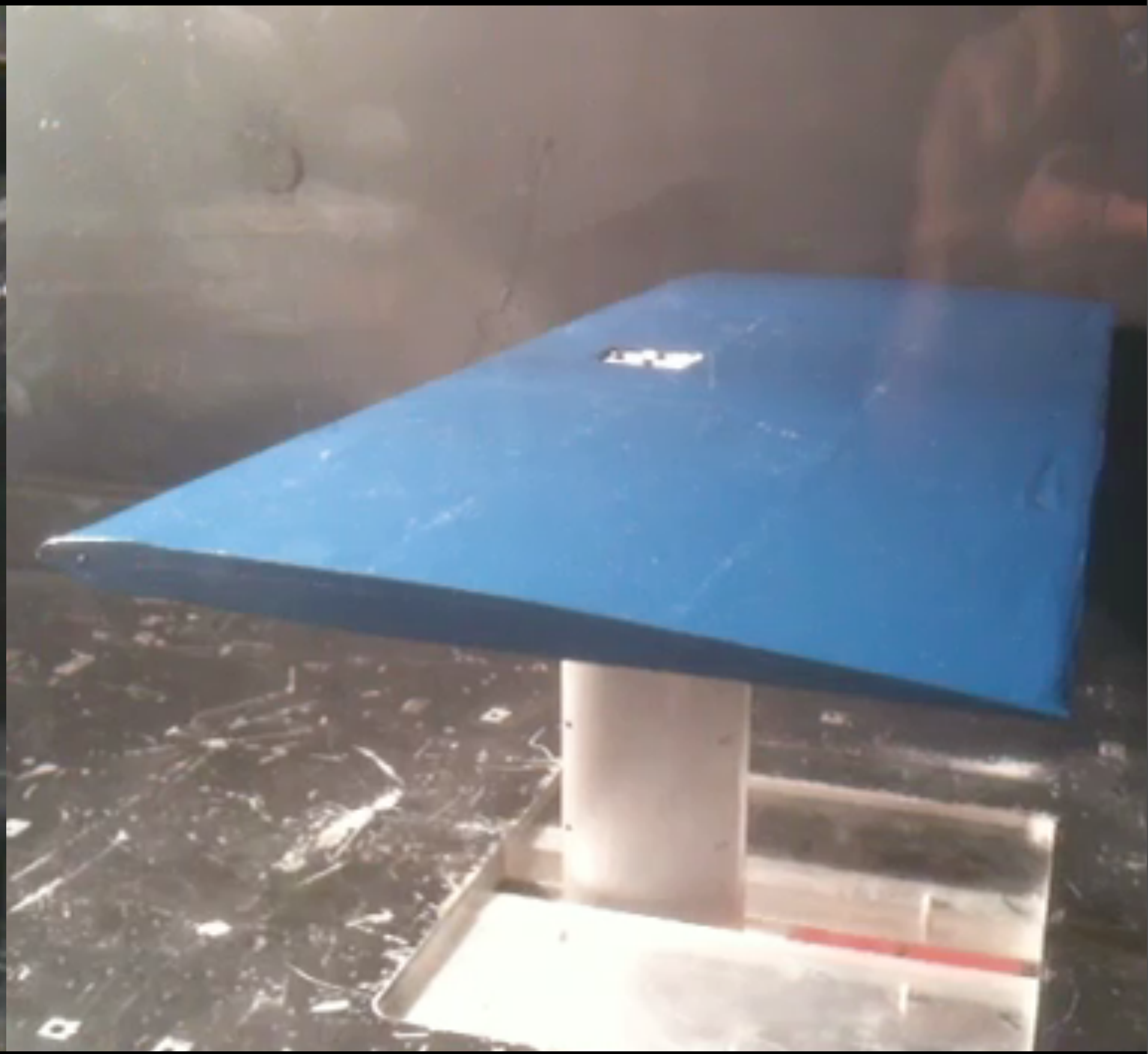


3. **Wind Tunnel Experiments**
   - **Physical setup and configuration**
   - **Aggressive system identification maneuver**
   - **Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$**



4. Conclusions and Future Work

Andrew Fejer Unsteady Flow Wind Tunnel
Principle Investigator - Dave Williams

Experimental Information

Andrew Fejer Unsteady Flow Wind Tunnel

Principle Investigator - Dave Williams

(.6m x .6m x 3.5m test section)

NACA 0006 Airfoil

Chord Length: 0.246 m

Free Stream Velocity: 4.00 m/s

1.0 Convection time = .06 seconds

Reynolds Number: 65,000

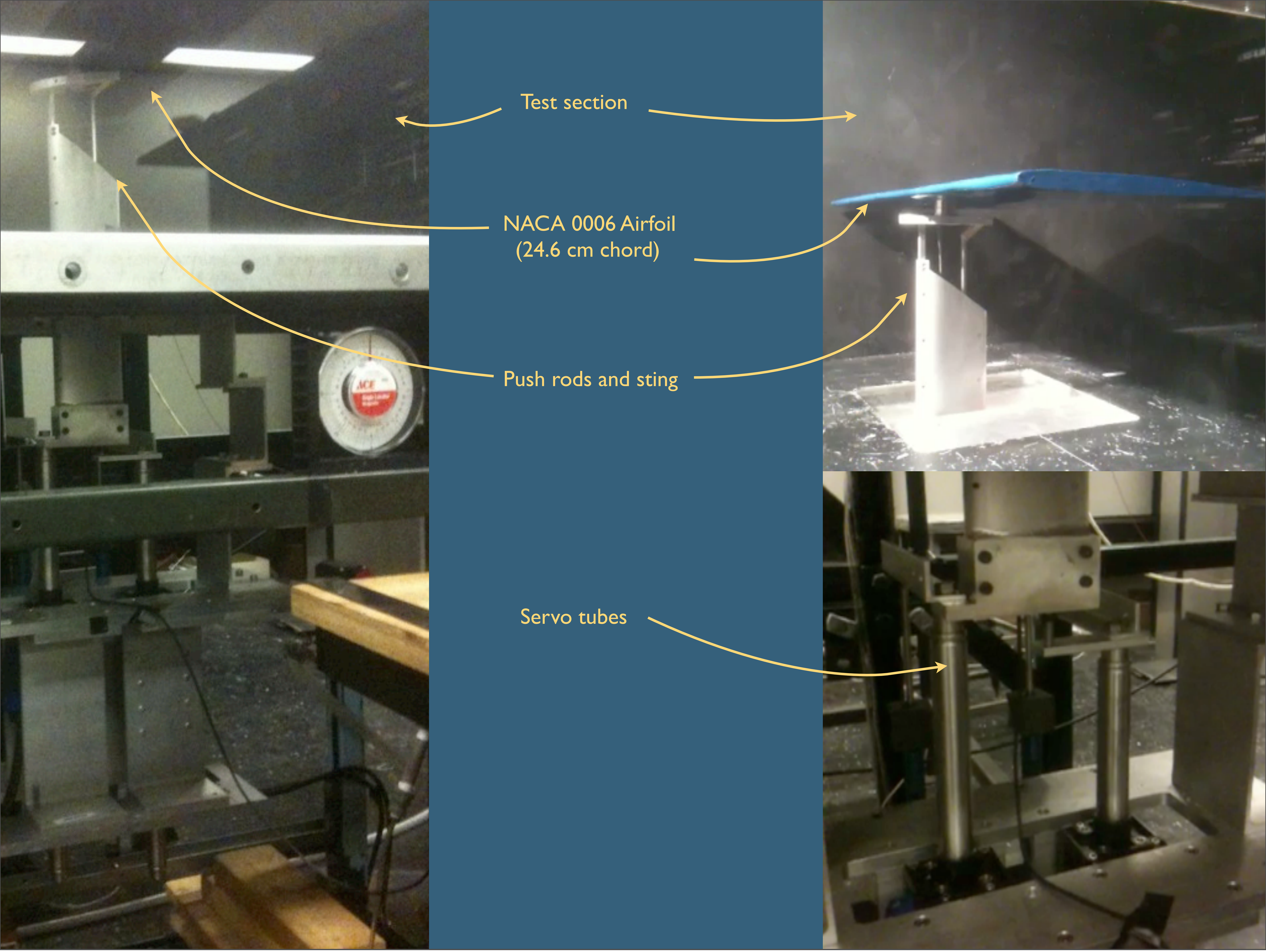
Pitch point x/c = .11 (11% chord)

Velocity measurement: Pitot tube,
Validyne DP-103 pressure transducer

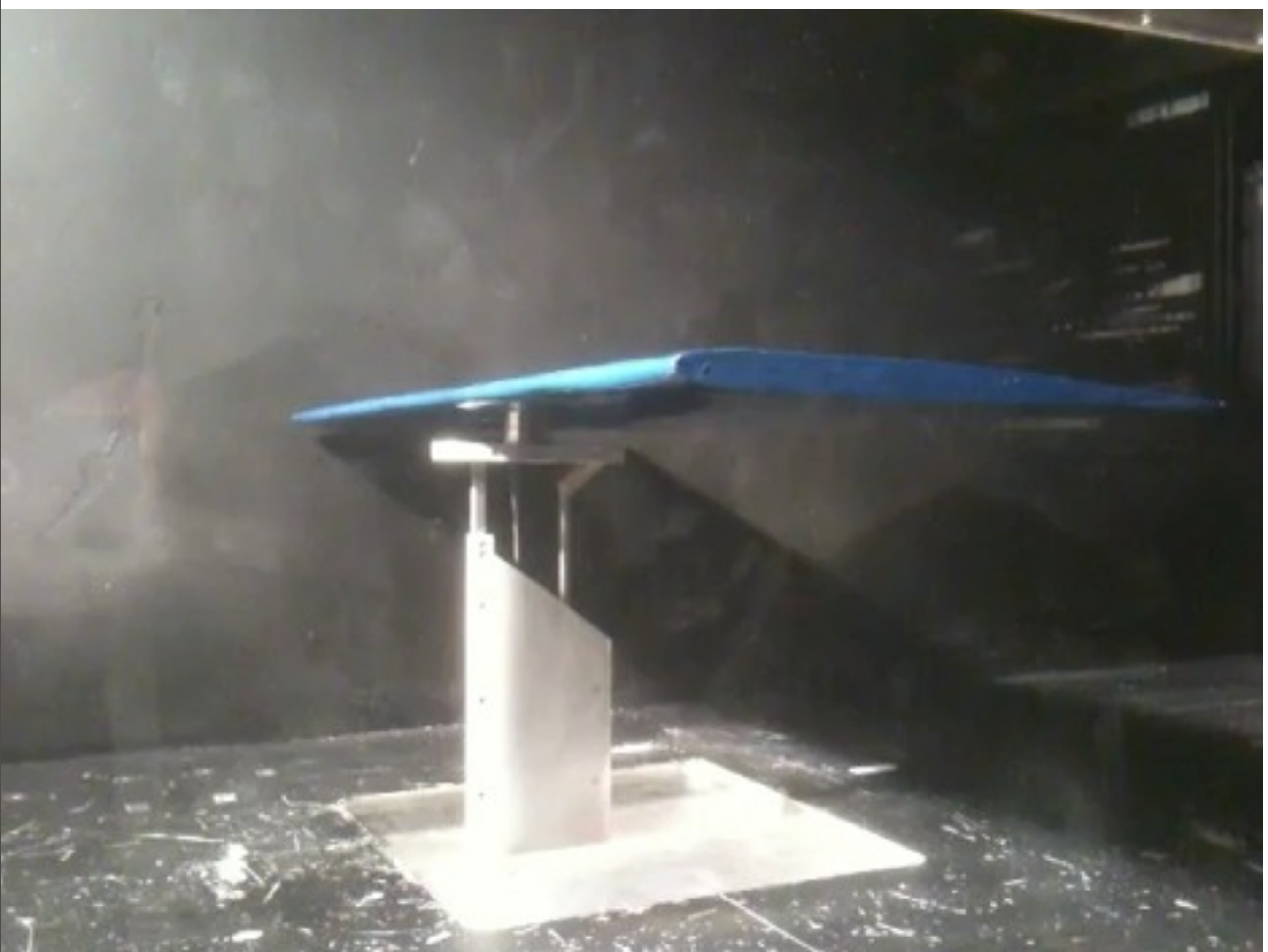
Force measurement: ATI Nano25 force transducer

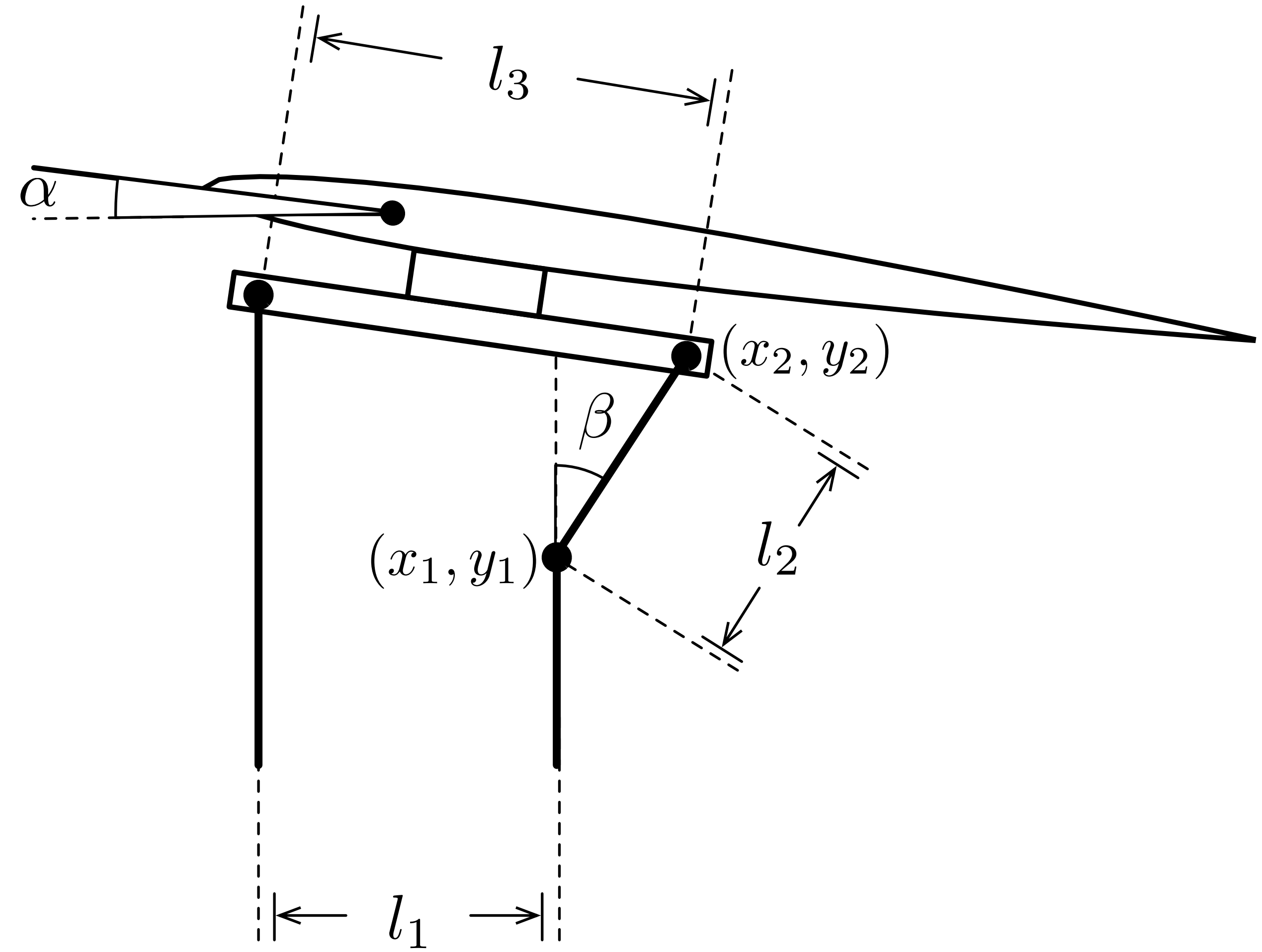
Pushrod position measurement: linear potentiometer

Pushrod actuation: Copley servo tubes


Test section
NACA 0006 Airfoil
(24.6 cm chord)
Push rods and sting
Servo tubes

NACA 0006 Model



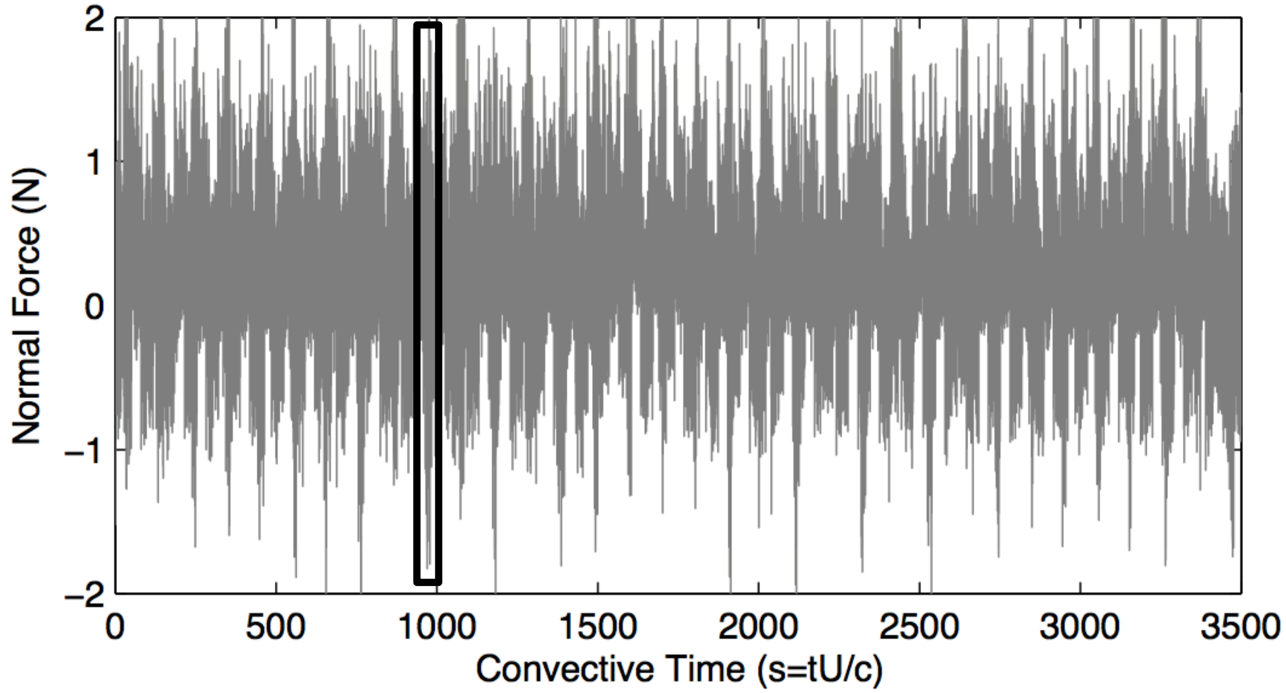


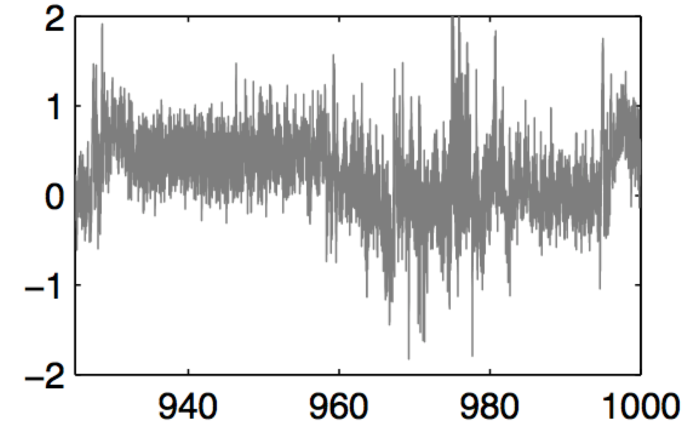
Summary

1. Account for hinge constraint nonlinearity
2. Rotate force vectors to obtain lift force
3. Subtract out point mass effects (mechanical)

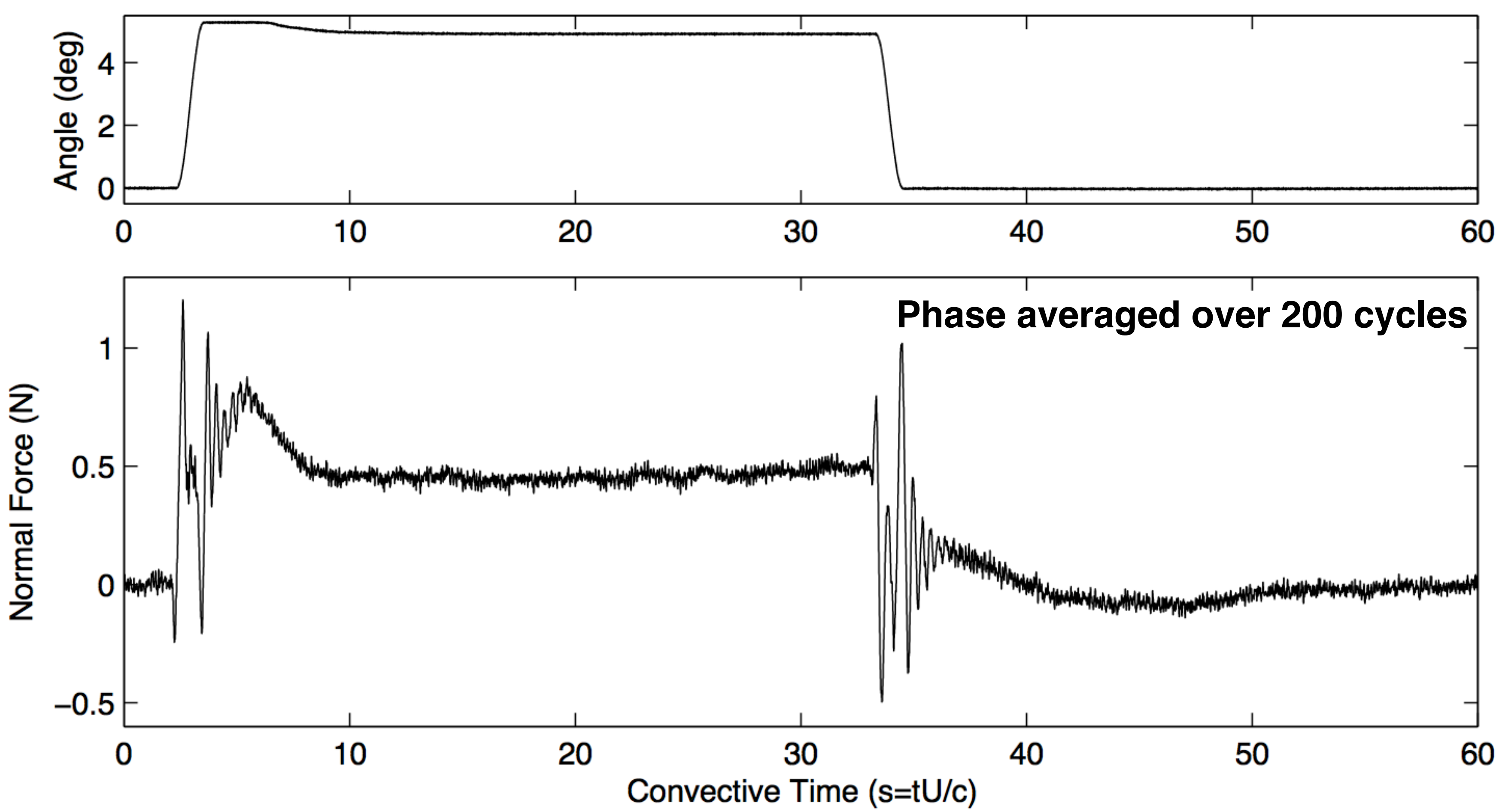
$$\begin{bmatrix} L \\ D \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}}_{R_\alpha} \begin{bmatrix} N \\ P \end{bmatrix}$$

Phase Averaged Data

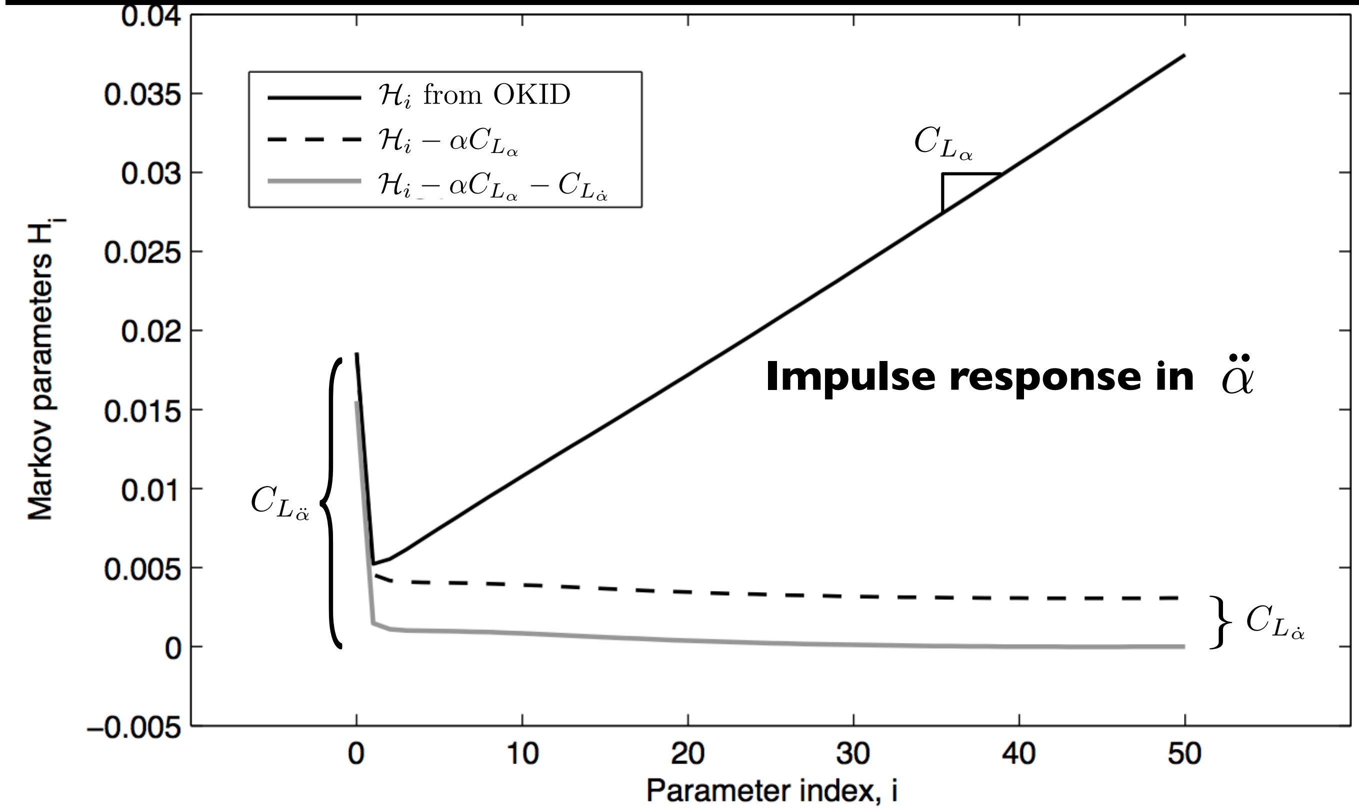




5 degree step-up, step-down maneuver



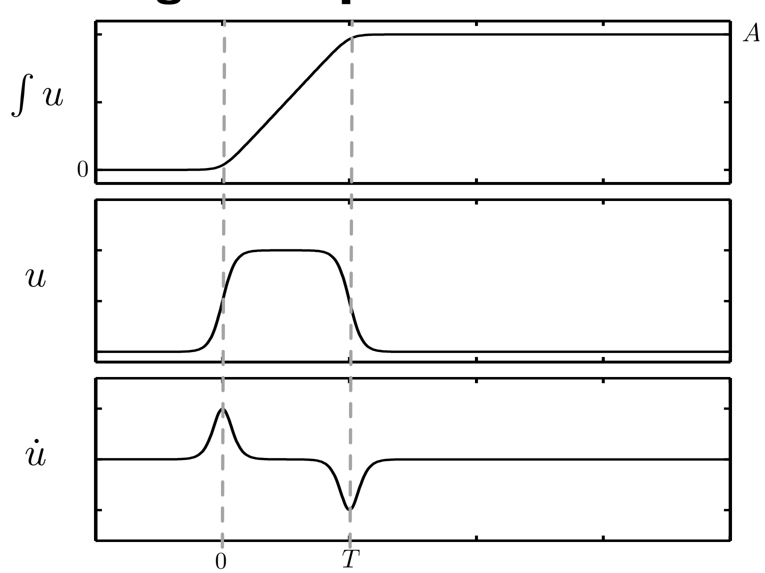
Wing Maneuver



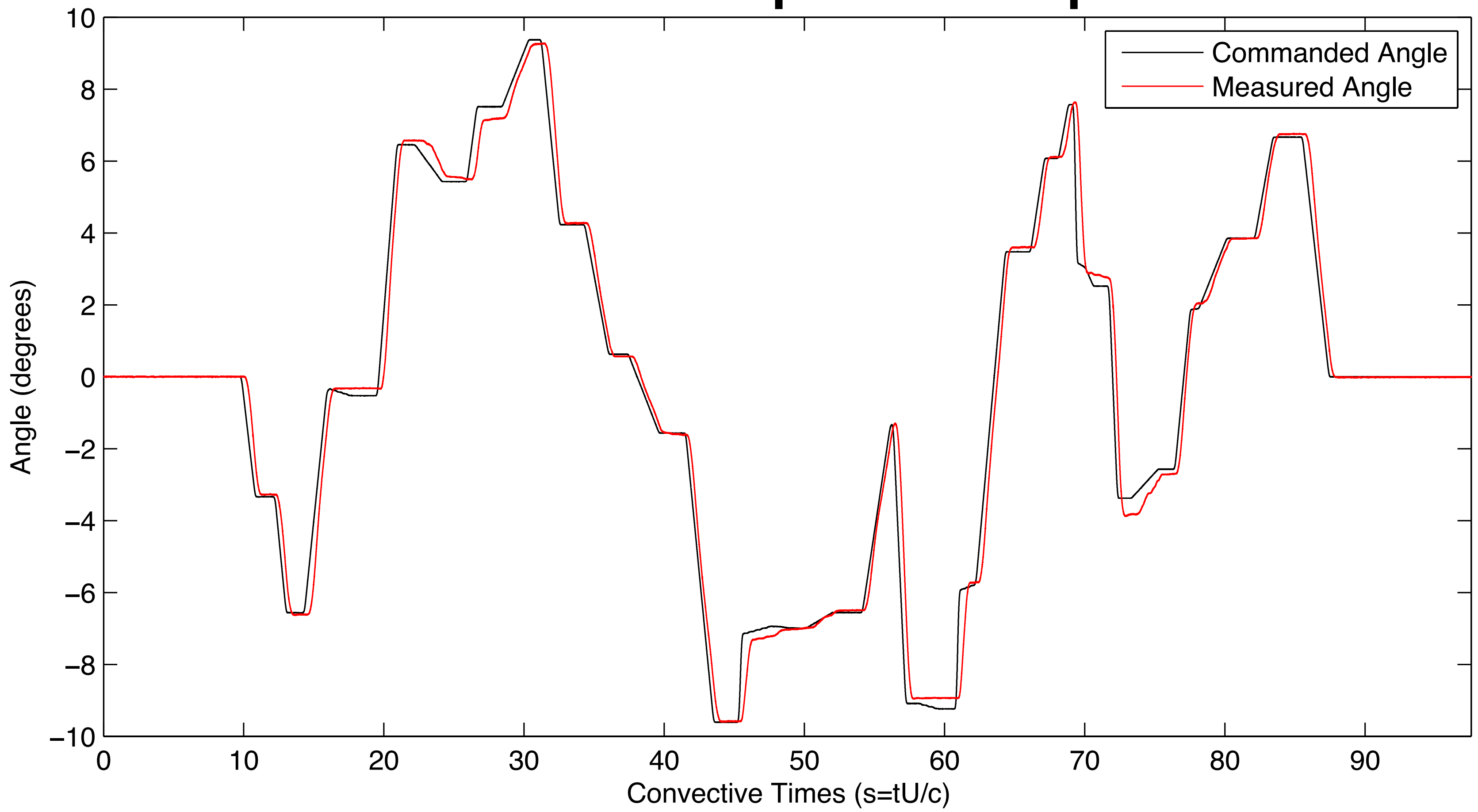
$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} = \begin{bmatrix}A_r & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}B_r\\ 0\\ 1\end{bmatrix}\ddot\alpha$$

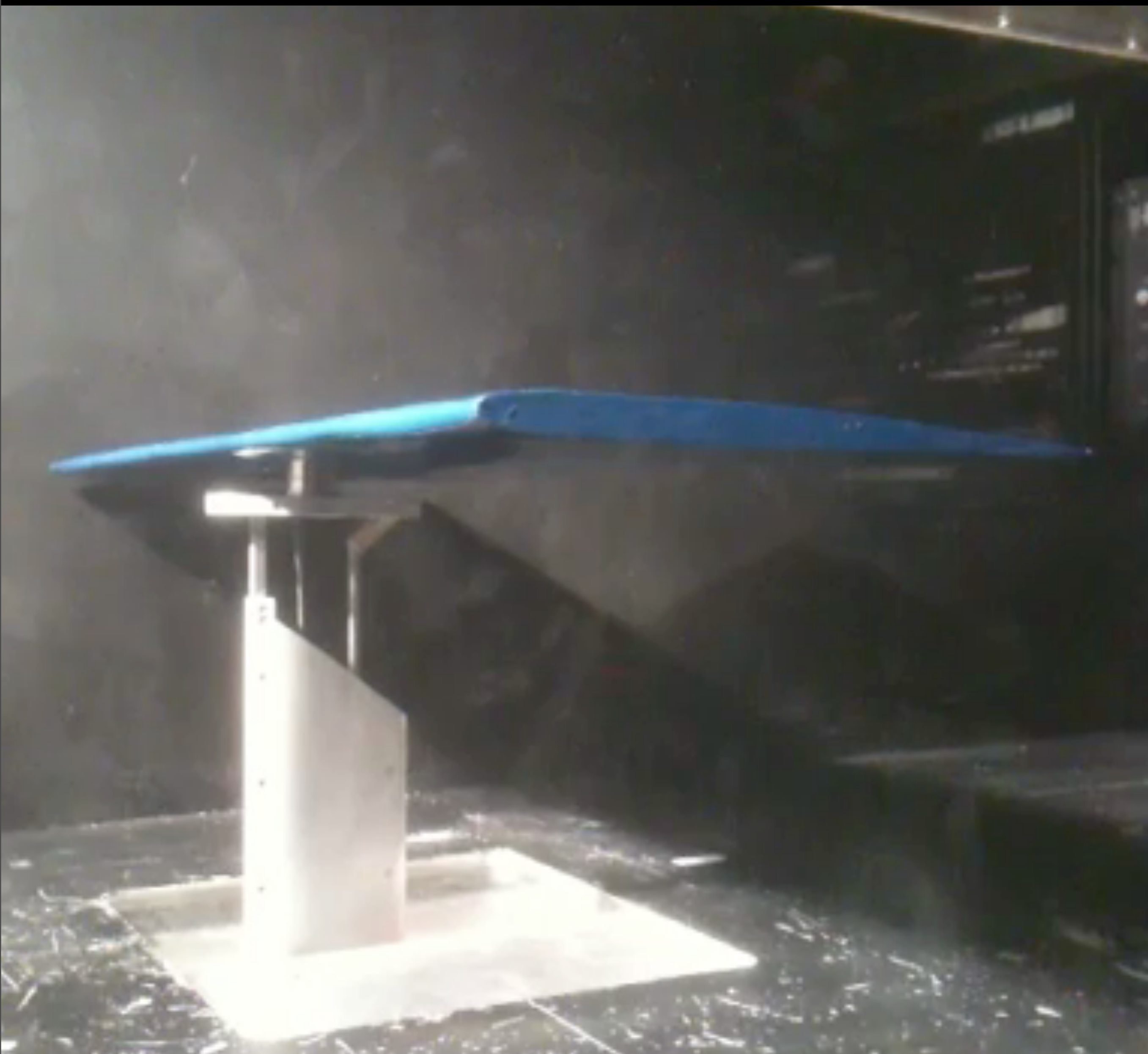
$$C_L = \begin{bmatrix}C_r & C_{L_\alpha} & C_{L_{\dot\alpha}}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \alpha\\ \dot\alpha\end{bmatrix} + C_{L_{\ddot\alpha}}\ddot\alpha$$

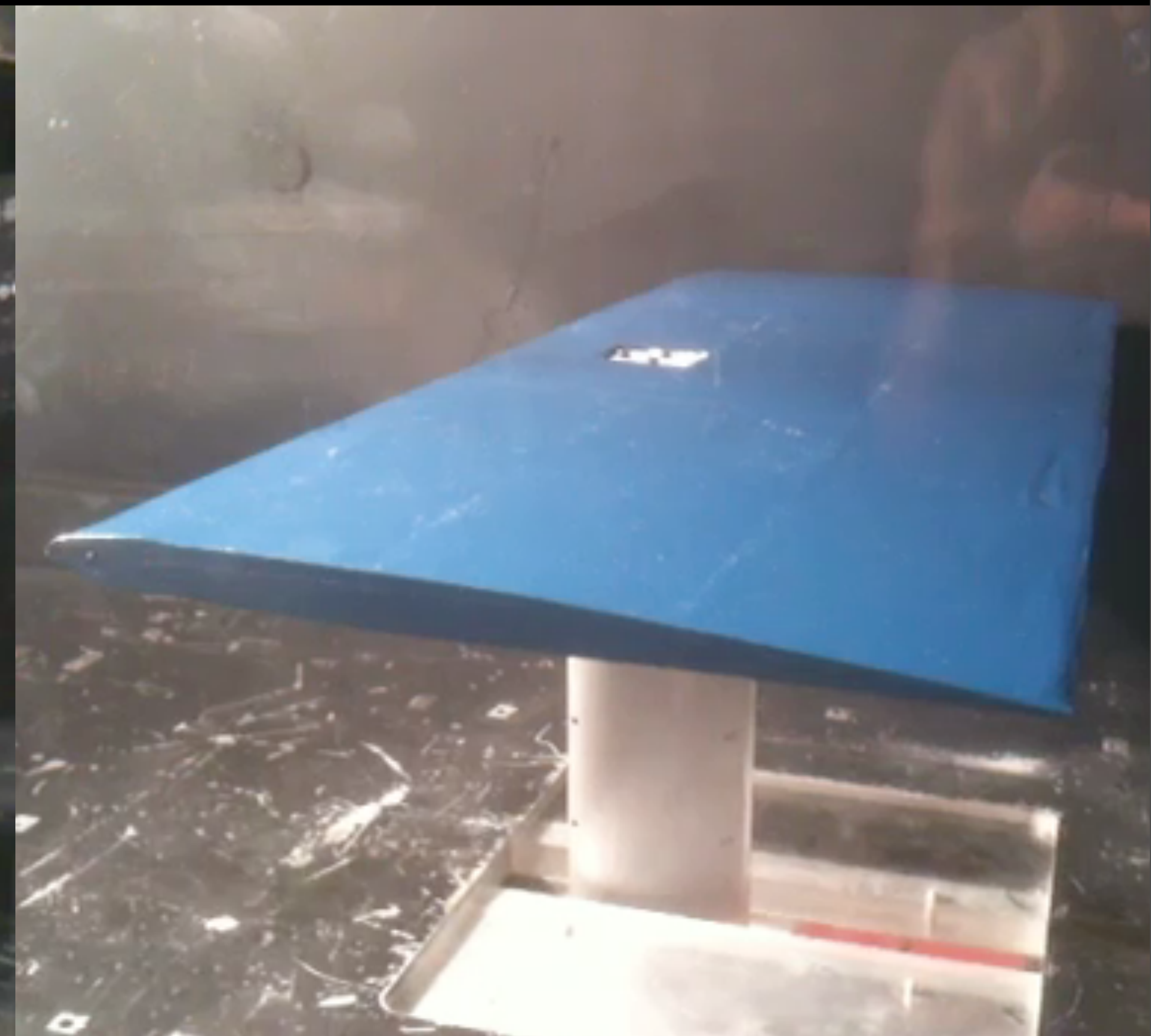
Single ramp-hold maneuver



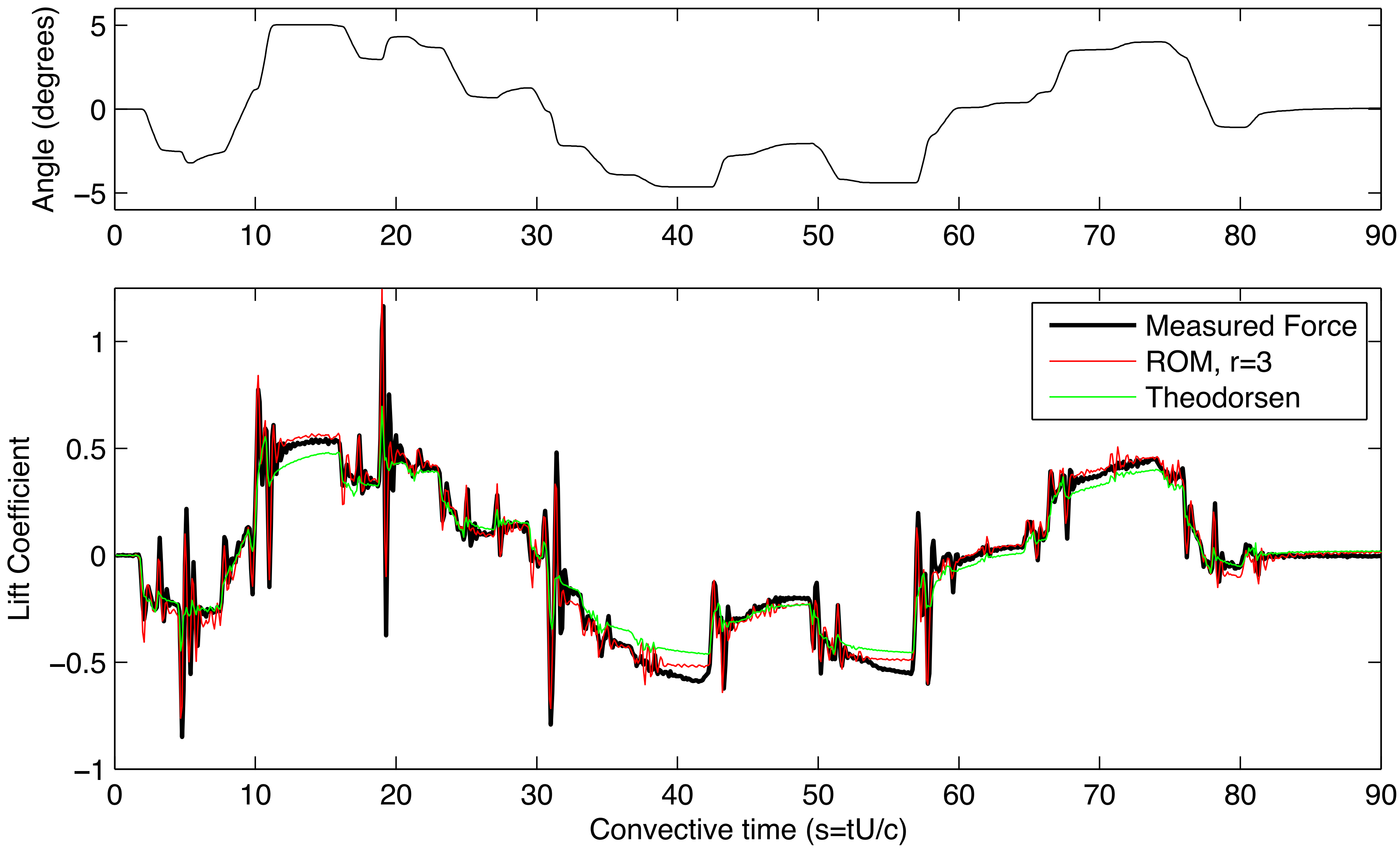
Pseudo-random sequence of ramp-hold maneuvers







System ID maneuver

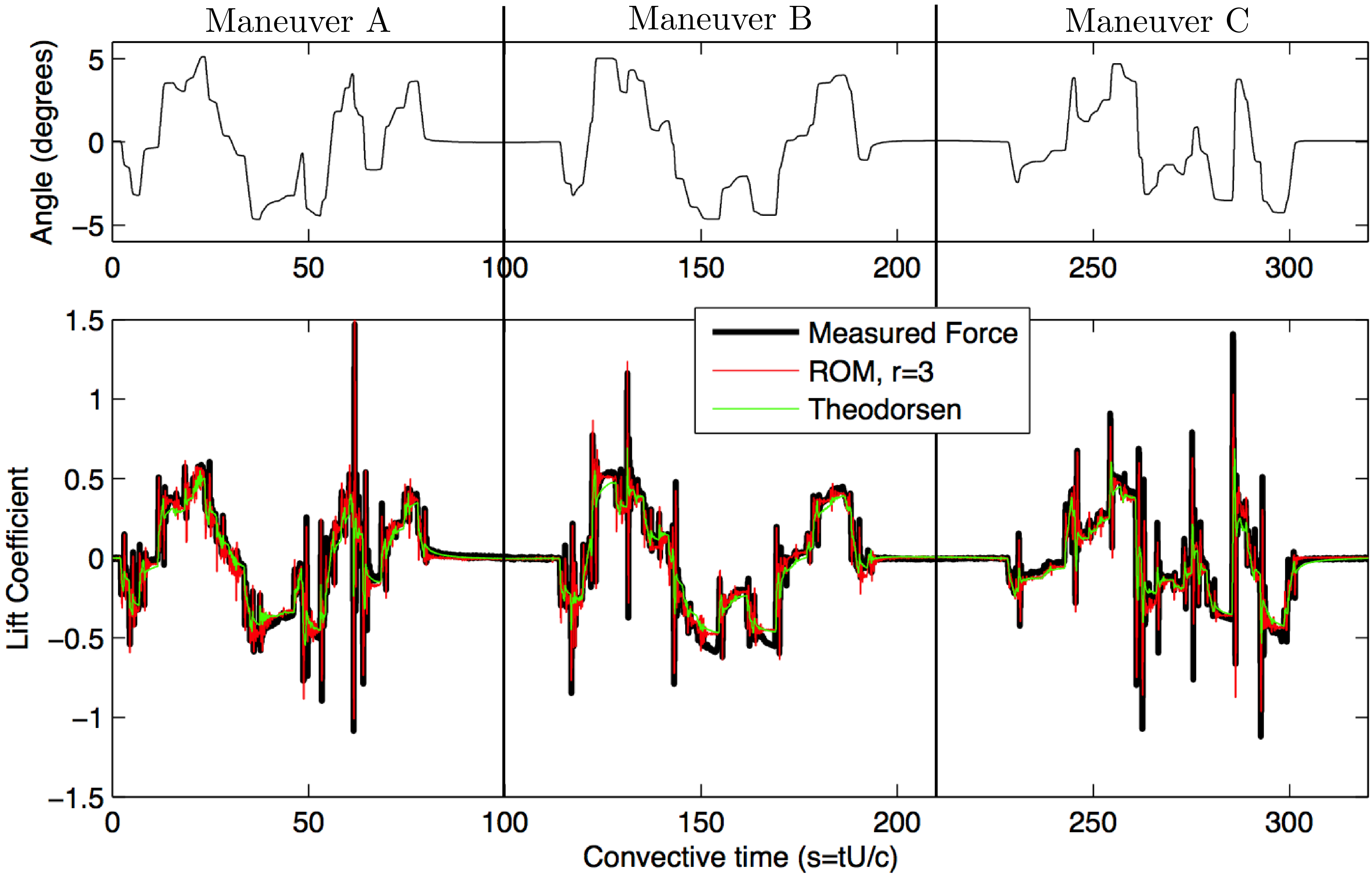


+/- 5 degree manuever, excites large range of frequencies

Reduced order model outperforms Theodorsen at low and high frequencies

AOA = 0 degrees

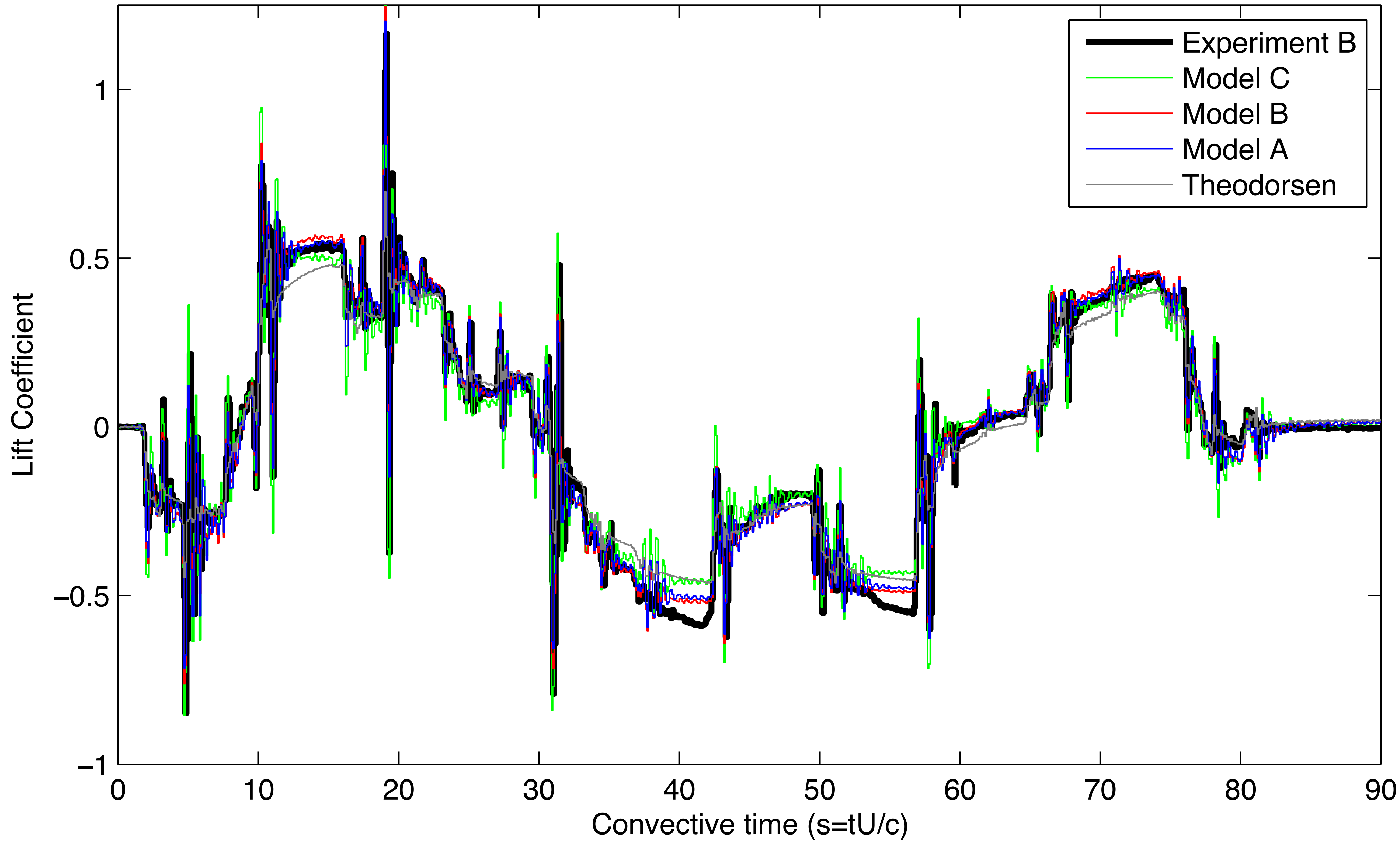
Three system ID maneuvers



AOA = 0 degrees

We tried three system ID maneuvers: A, B and C.

System ID maneuver

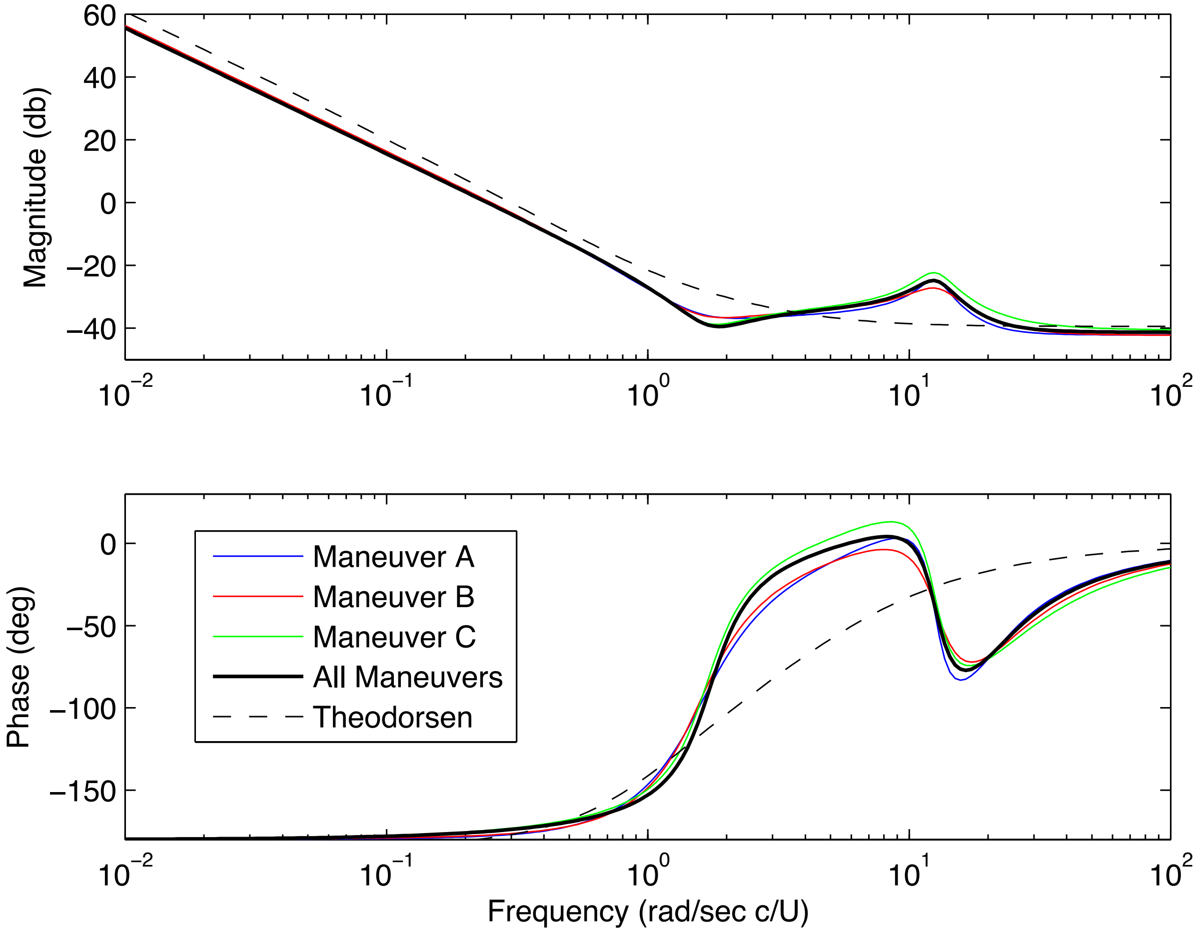


AOA = 0 degrees

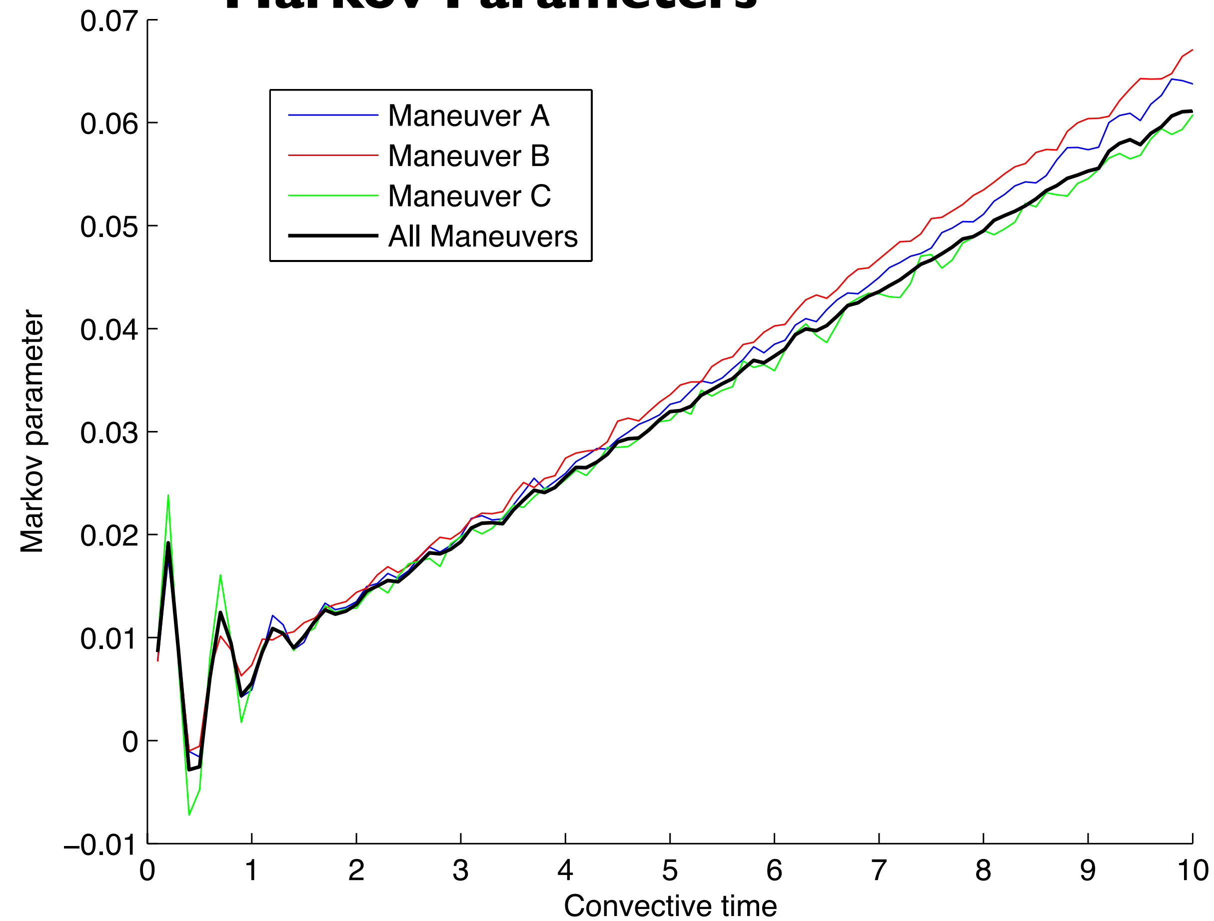
Bootstrap: It is important that models obtained from each ID maneuver accurately reproduce every other maneuver

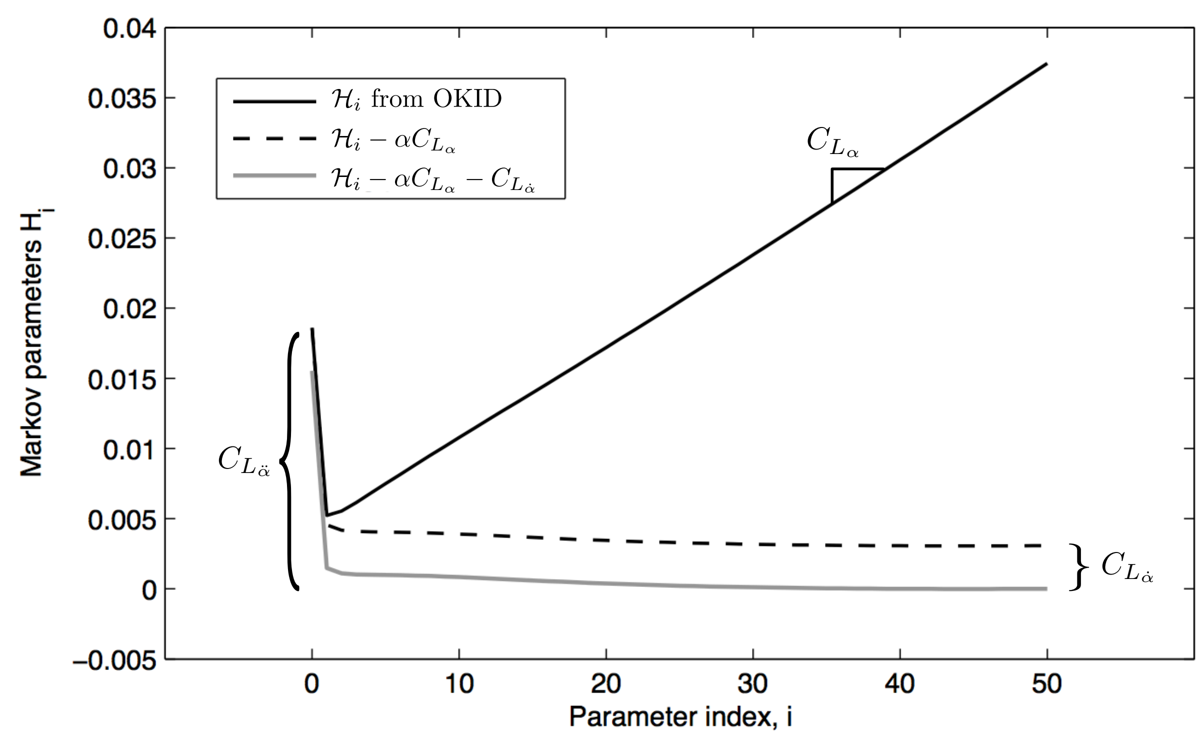
Bode plot and Markov parameters

Bode plot



Markov Parameters



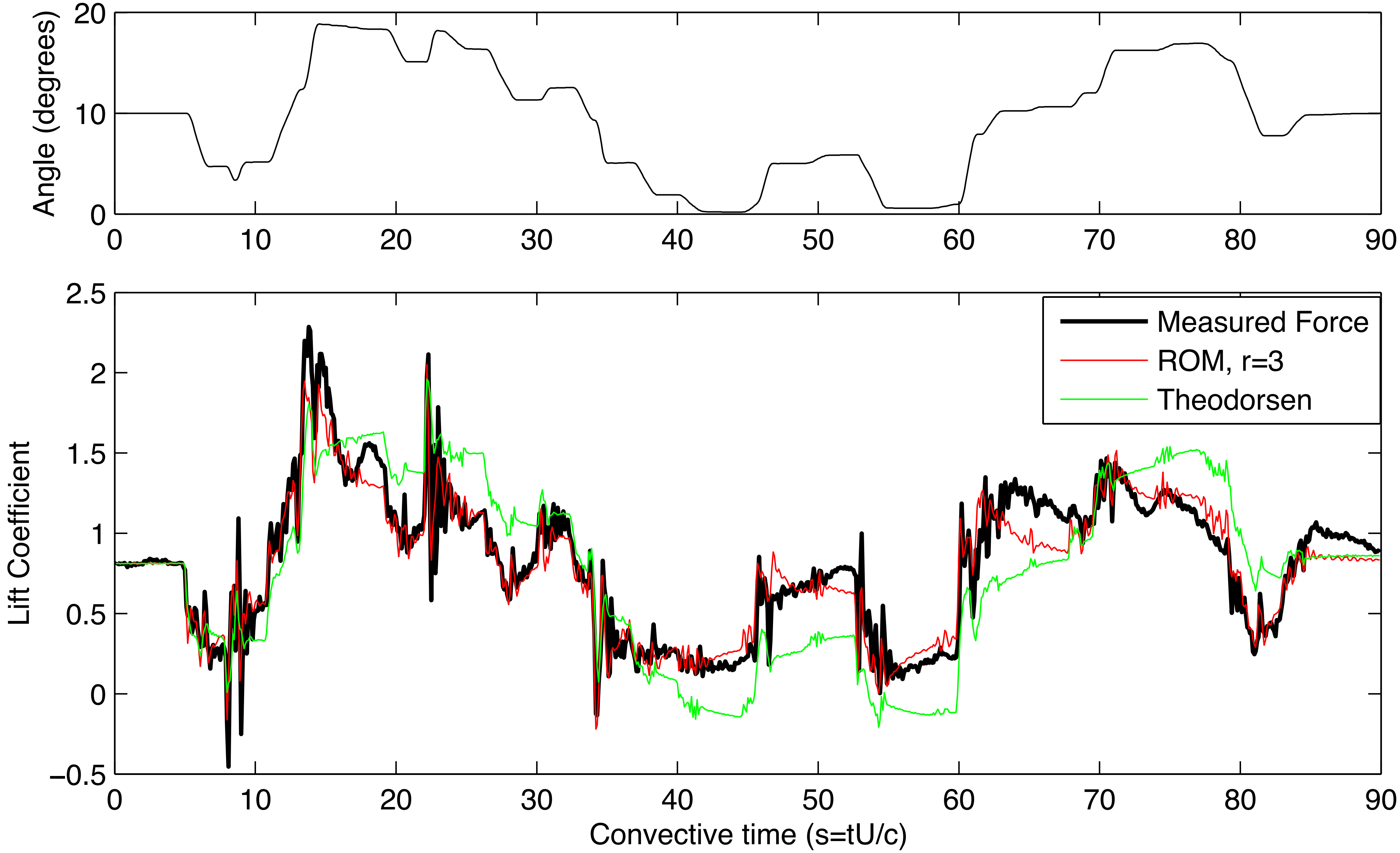


Combined maneuver effectively blends each of the three individual maneuvers

Added-mass is not exclusively in first Markov parameter, but is instead distributed in the first few, contributing to the added-mass "bump"

AOA = 0 degrees

System ID maneuver

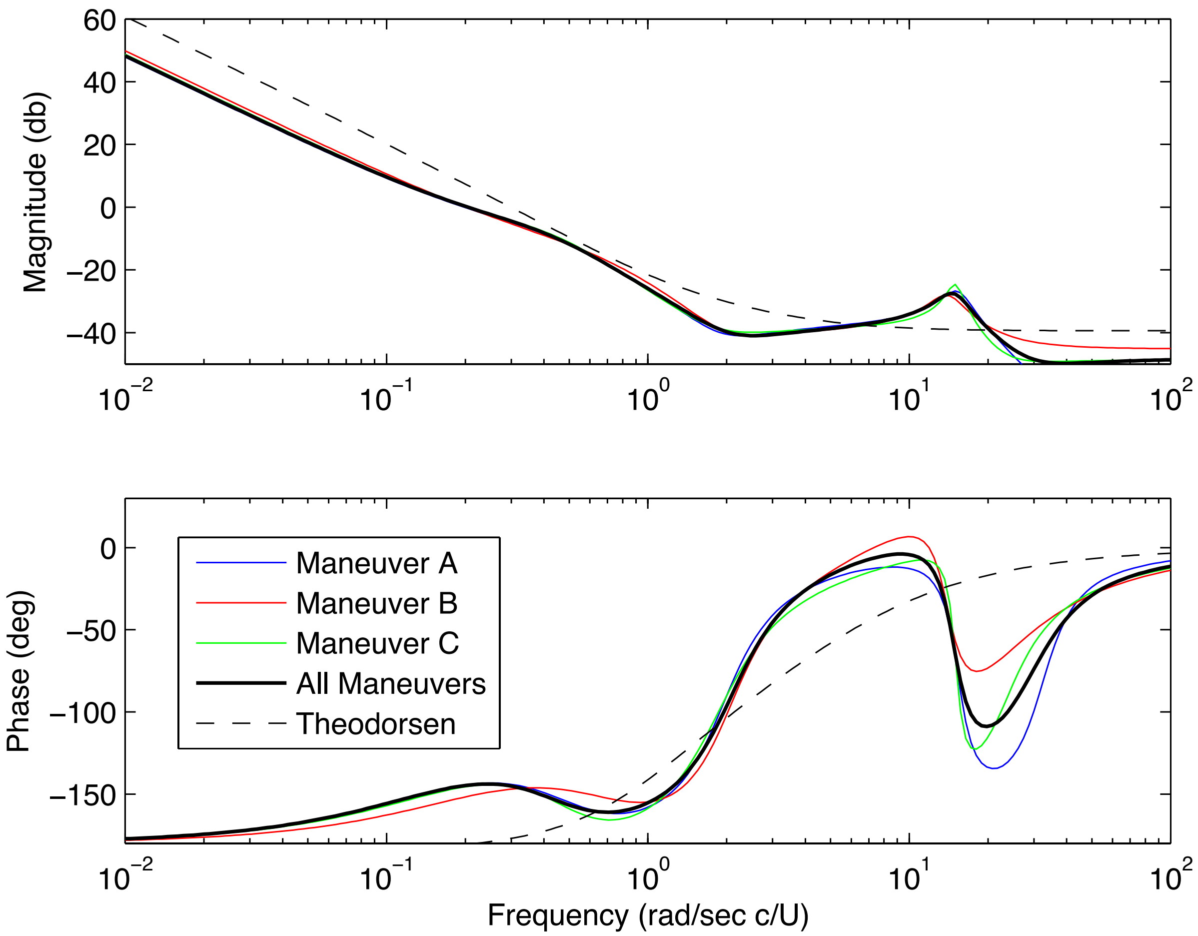


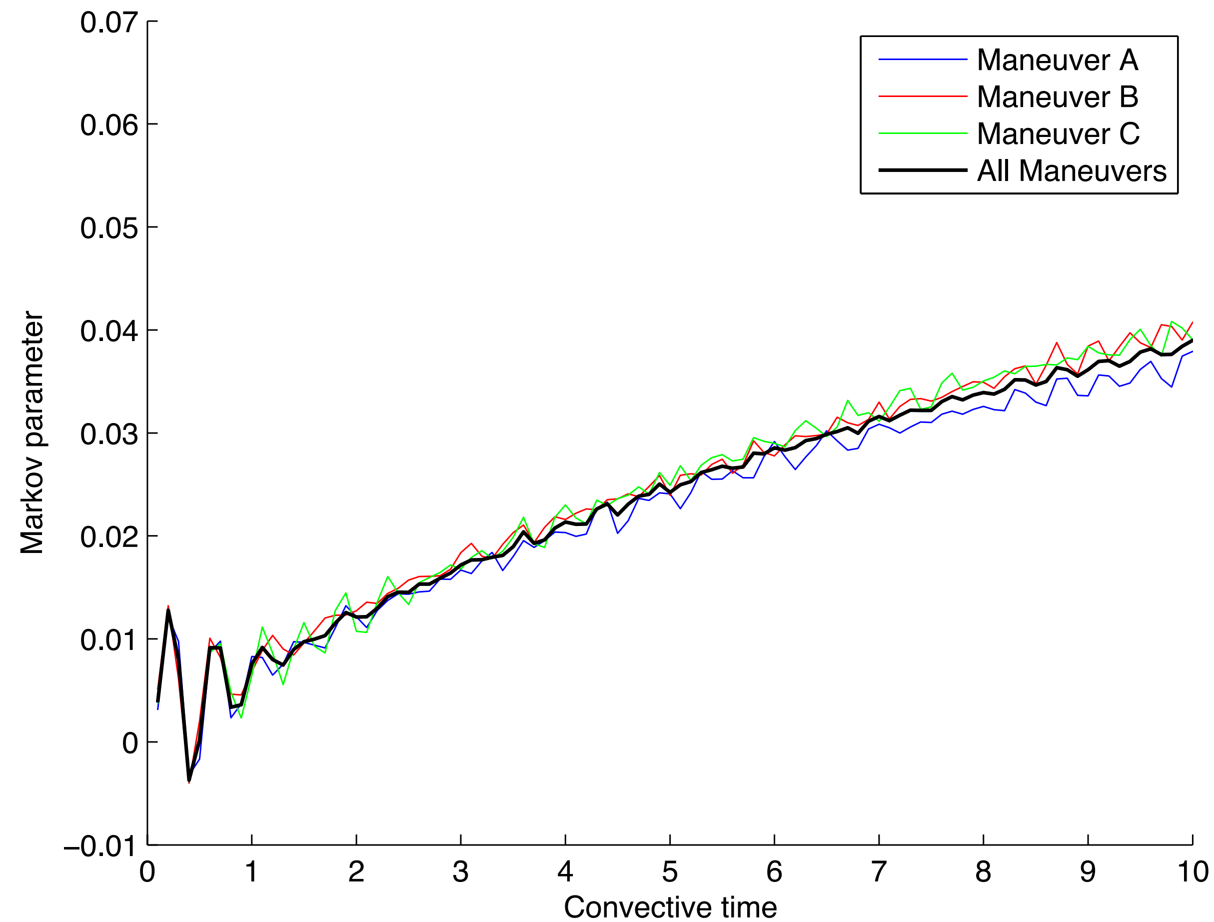
Theodorsen is significantly worse, due to large base angle of attack and flow separation effects.

+/- 10 degree manuever

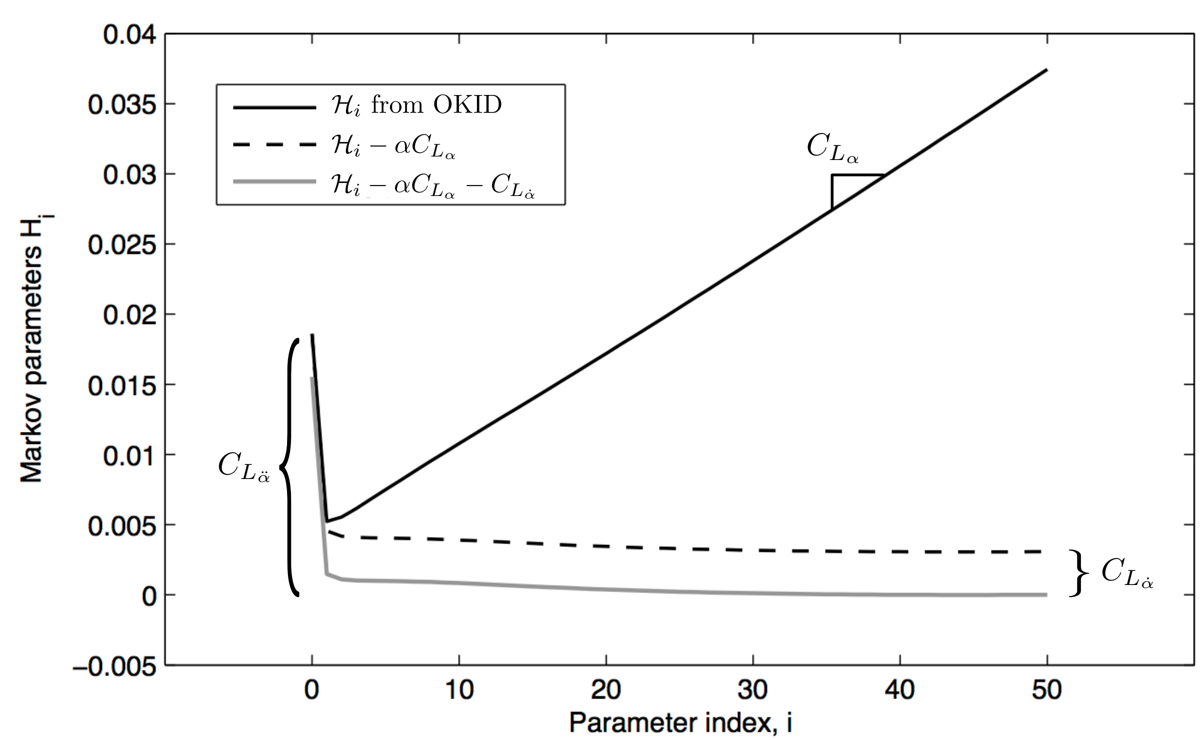
AOA = 10 degrees

Bode plot and Markov parameters





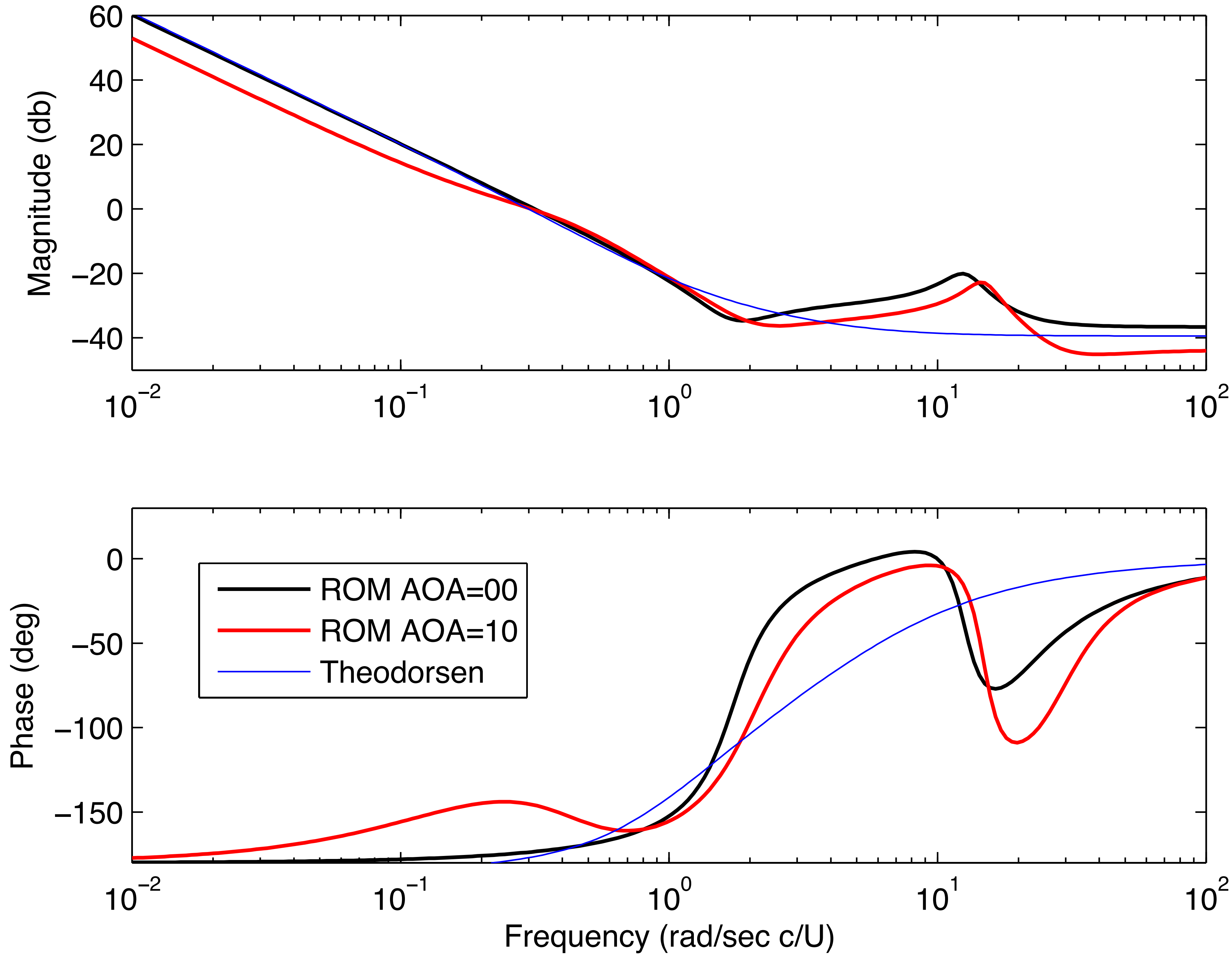
Flatter Markov parameters indicate smaller lift coefficient slope



Convergence to asymptote at lower frequency indicate longer transient decay to steady state (more separated flow)

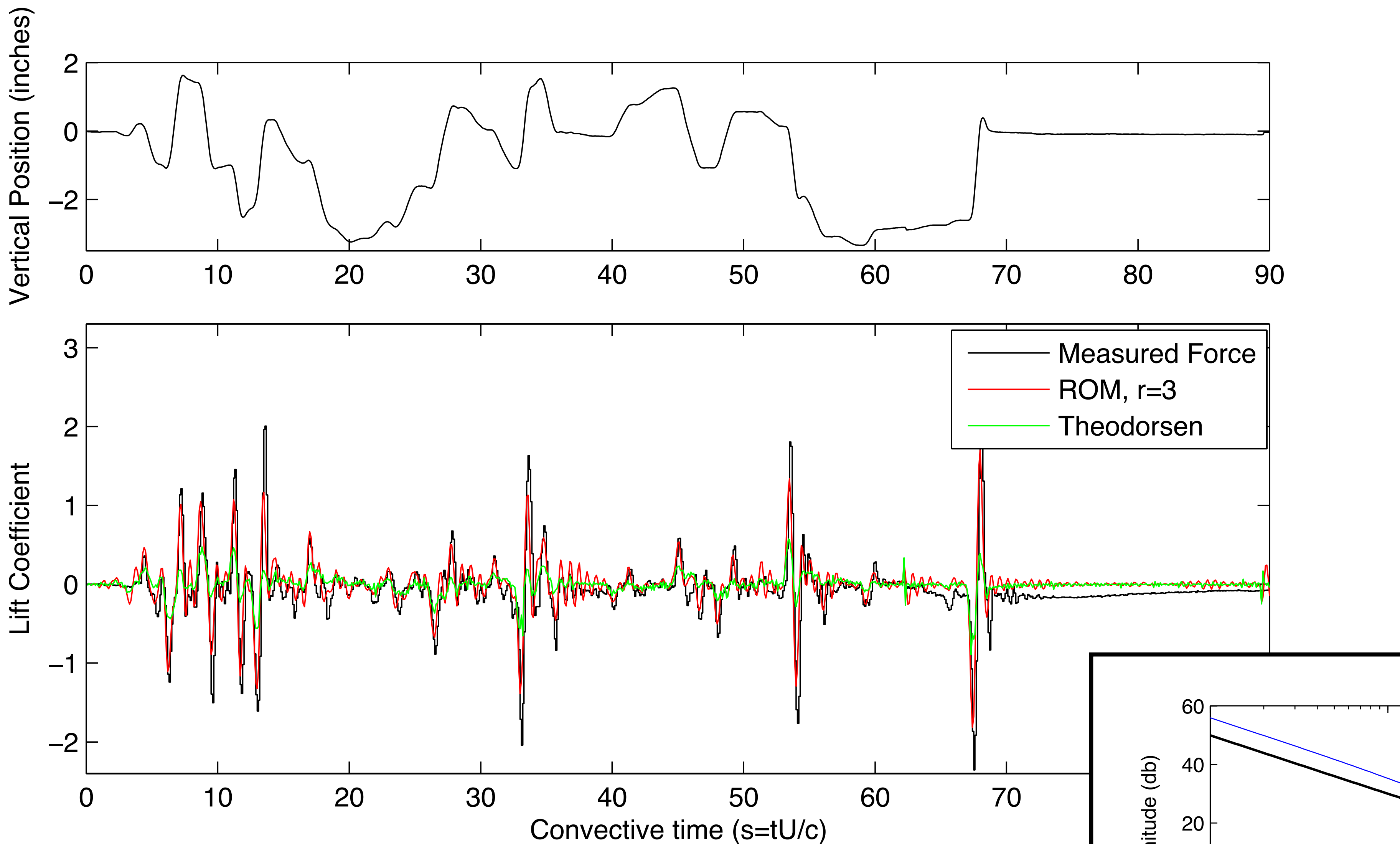
AOA = 10 degrees

AoA=00 vs. AoA=10

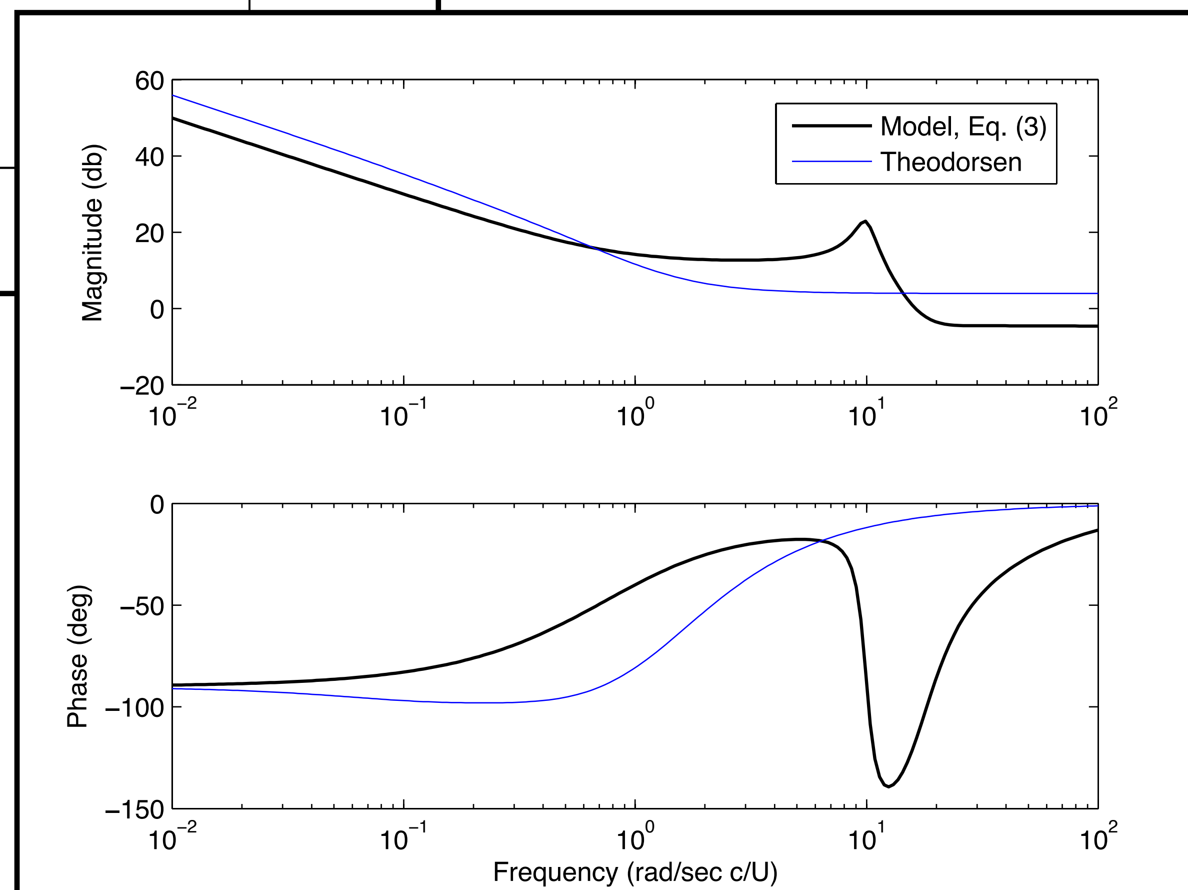


Trend is similar to DNS, where low frequency asymptote converges at lower frequency, for larger angle of attack.

Pure Plunge



Vertical Position (inches)
Lift Coefficient
Measured Force
ROM, r=3
Theodorsen
Convective time (s=tU/c)

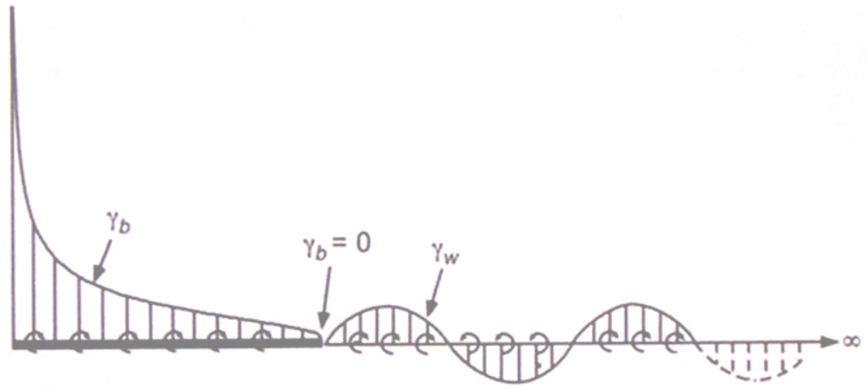


Magnitude (db)
Model, Eq. (3)
Theodorsen
Phase (deg)
Frequency (rad/sec c/U)

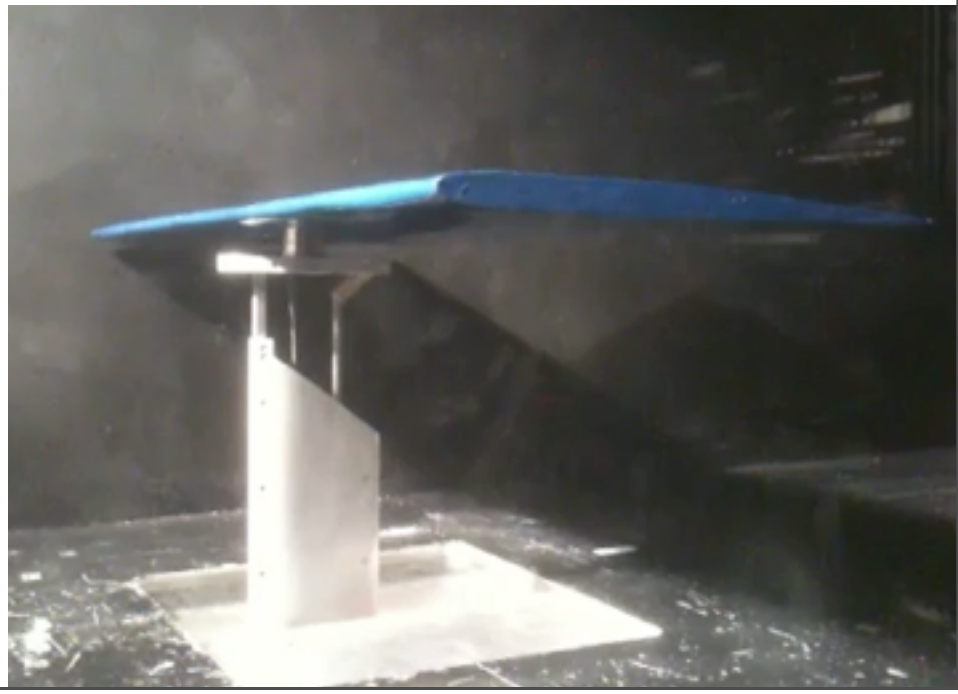
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$

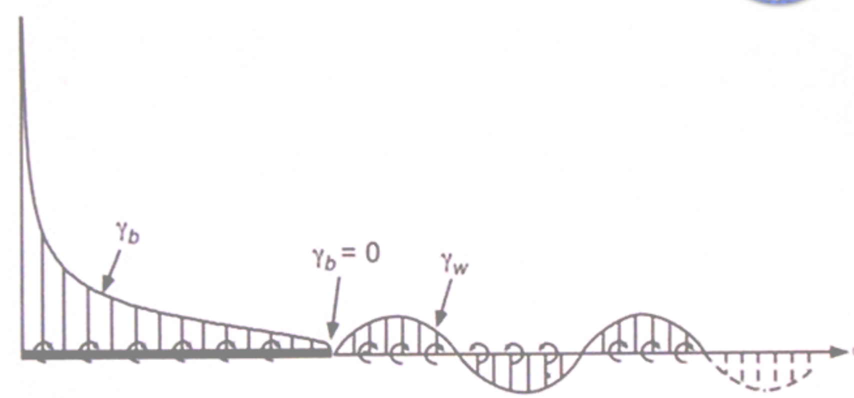


4. **Conclusions and Future Work**

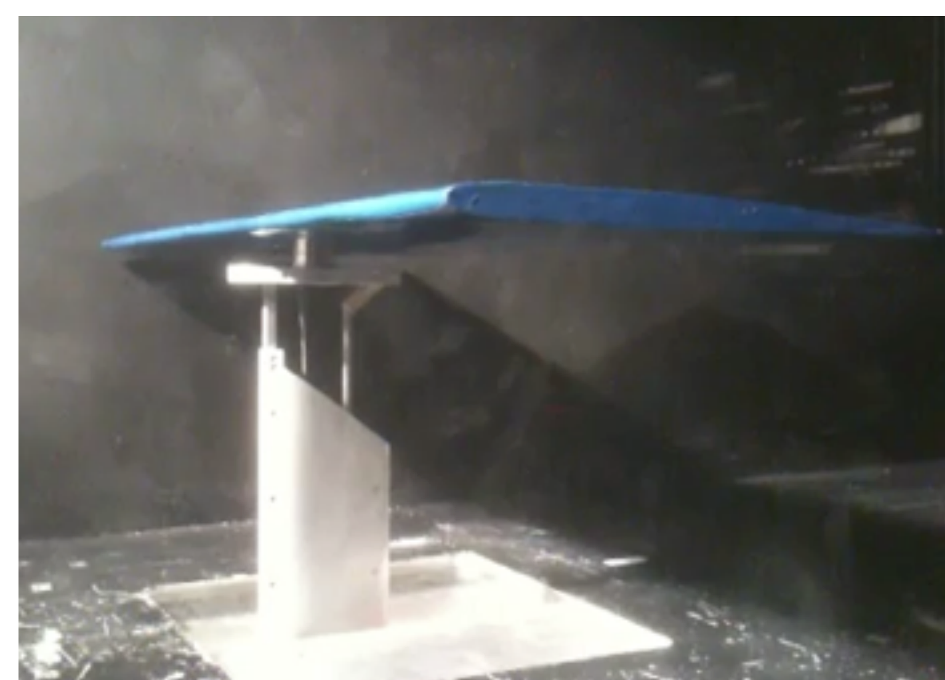
Outline

1. Motivation and Overview
   - Low Reynolds number aerodynamic models
   - Pitch, plunge and high angle-of-attack maneuvers

2. Review of Previous Work
   - State-space aerodynamic models
   - Indicial response and OKID



3. Wind Tunnel Experiments
   - Physical setup and configuration
   - Aggressive system identification maneuver
   - Models at $\alpha_0 = 0^\circ$ and $\alpha_0 = 10^\circ$



4. **Conclusions and Future Work**

Conclusions

Accurate, efficient linear reduced order models

- Models are linearization of full nonlinear model
- Constructed for specific geometry, Reynolds number
- Based on various input maneuvers

Modeling techniques applied to wind tunnel experiment at IIT

- Aggressive system ID maneuver developed, based on canonical maneuver
- Pitch and plunge dynamics investigated
- Reduced order model outperforms Theodorsen's model for all cases, especially at large angle of attack

Future Work:

- Use pitch/plunge models for optimal control (maneuver, lift stabilization)
- Combine linearized models into nonlinear model

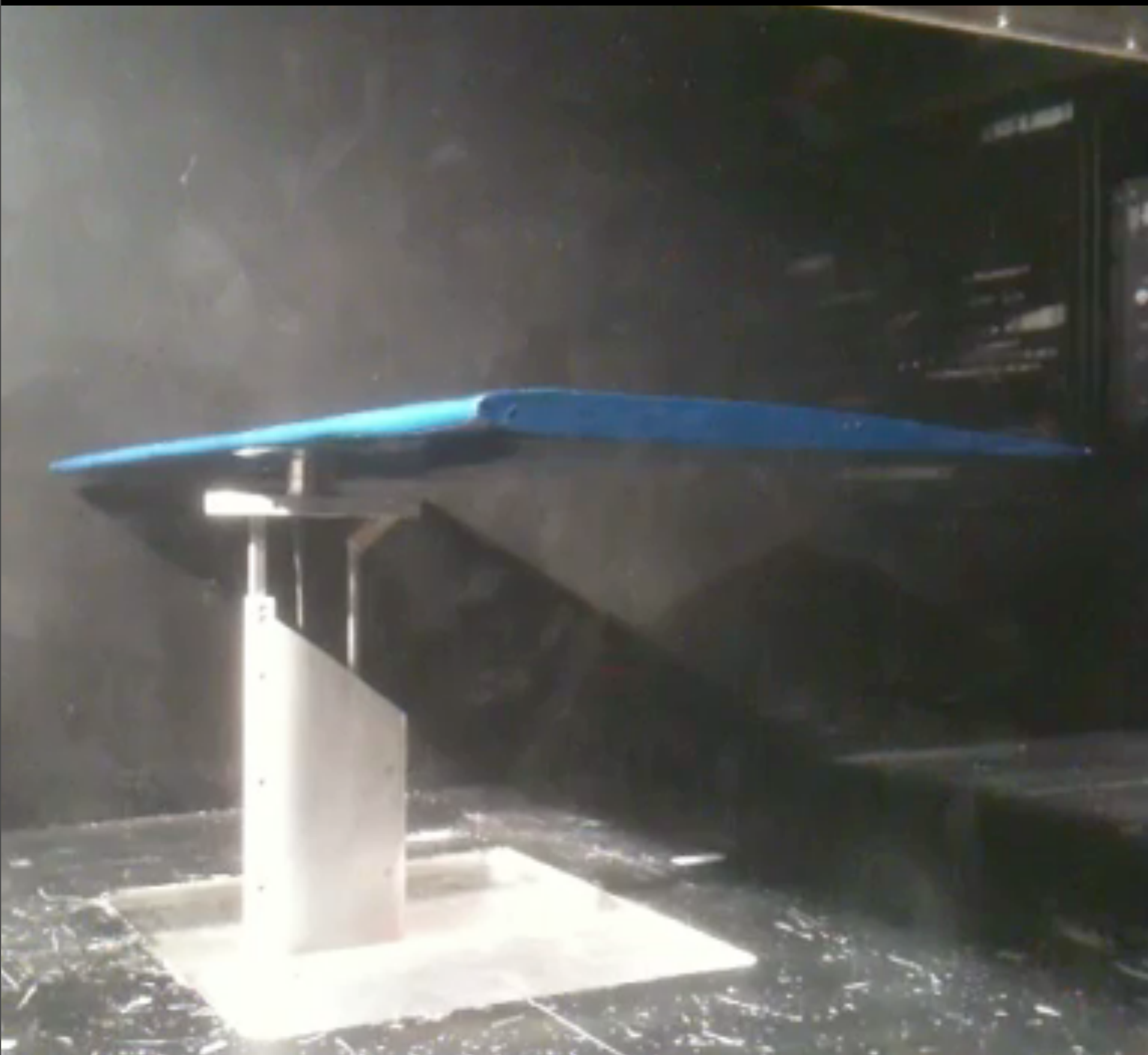
Wagner, 1925.

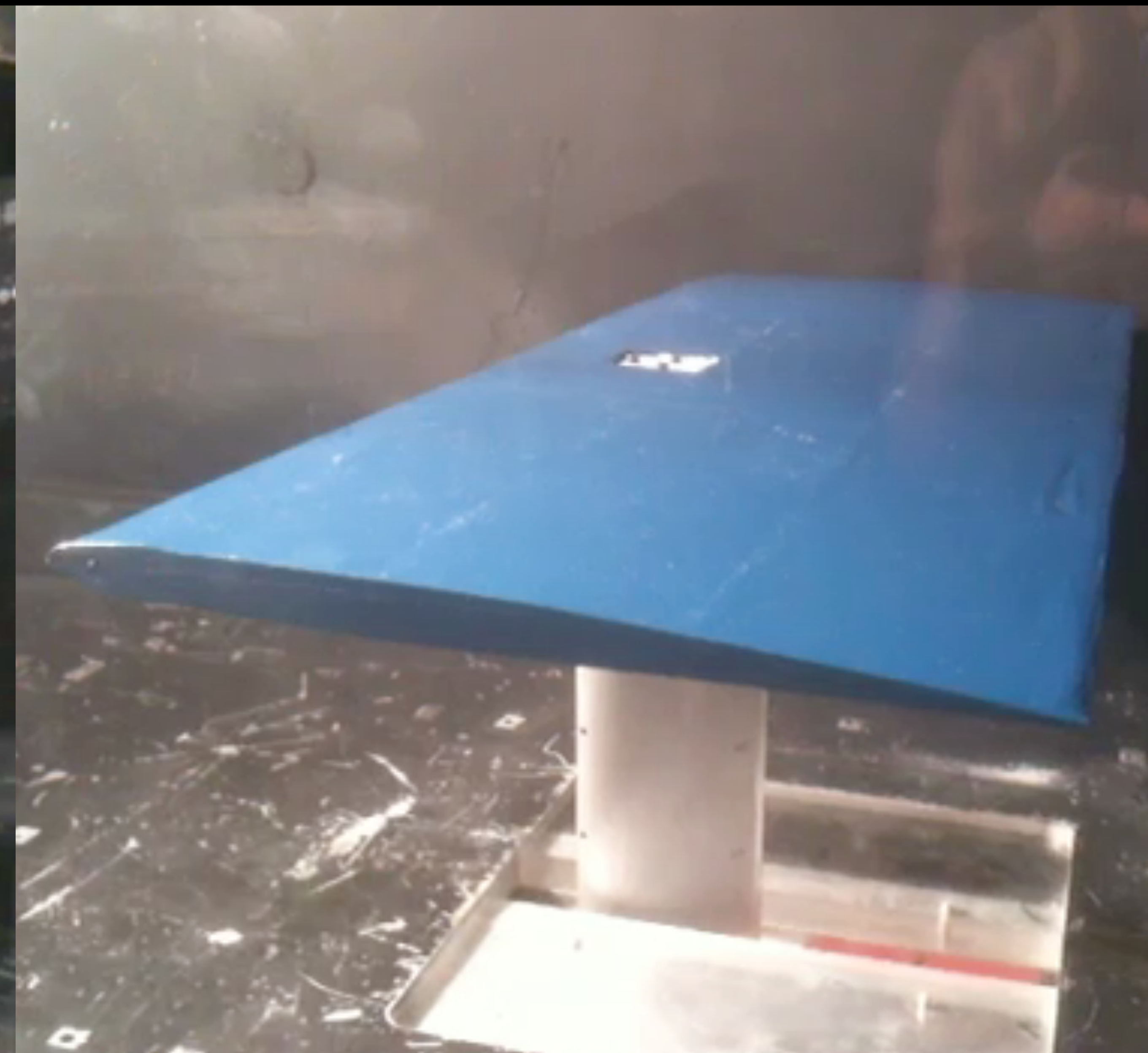
Theodorsen, 1935.

Brunton and Rowley, AIAA ASM 2009-2011

OL, Altman, Eldredge, Garmann, and Lian, 2010

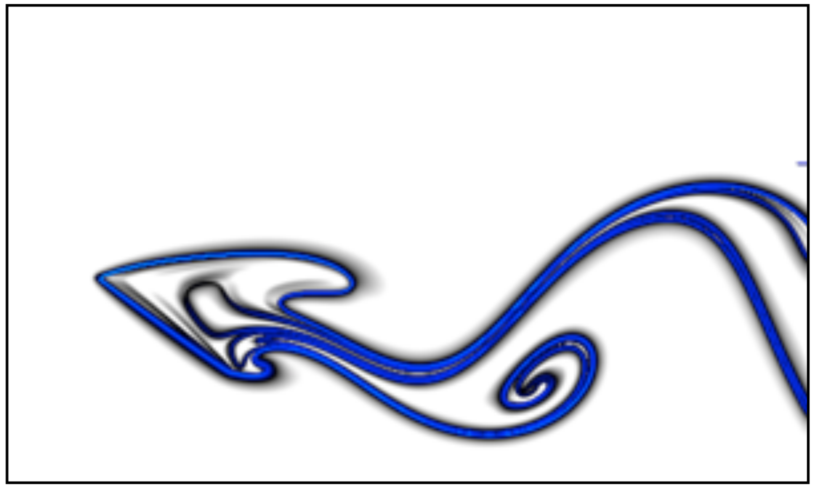
Questions?





3 Types of Unsteadiness

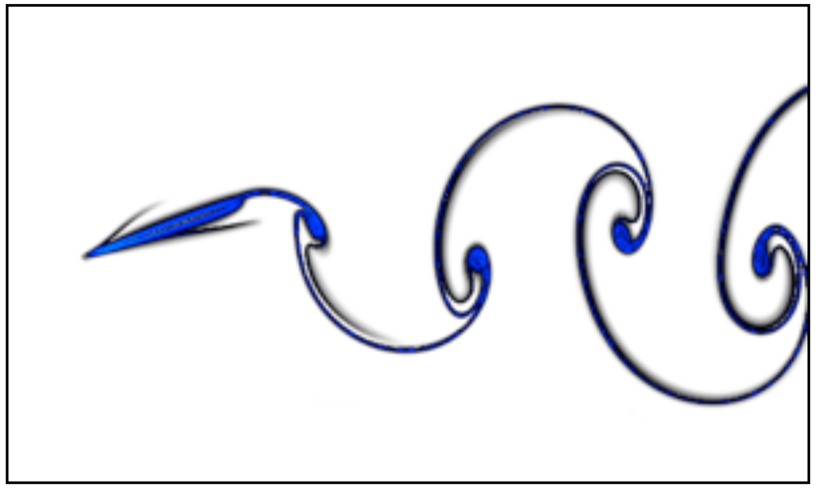
1. High angle-of-attack



$$\alpha > \alpha_{\text{stall}}$$

Large amplitude, slow

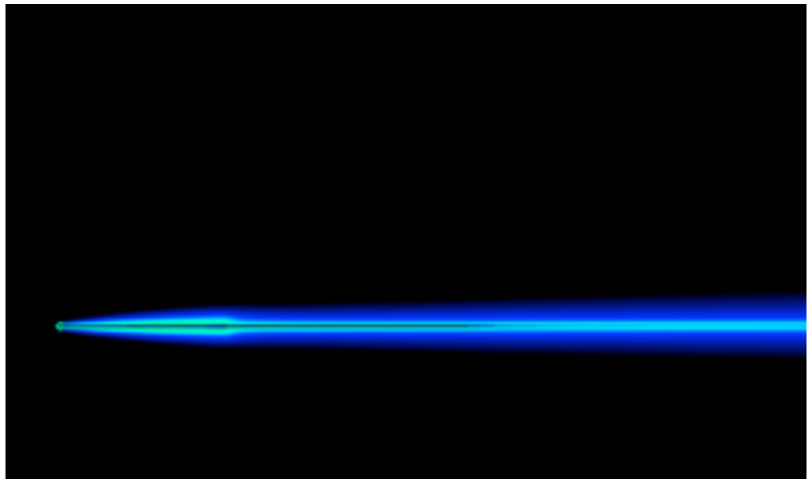
2. Strouhal number



$$St = \frac{Af}{U_\infty}$$

Moderate amplitude, fast

3. Reduced frequency



$$k = \frac{\pi f c}{U_\infty}$$

Small amplitude, very fast

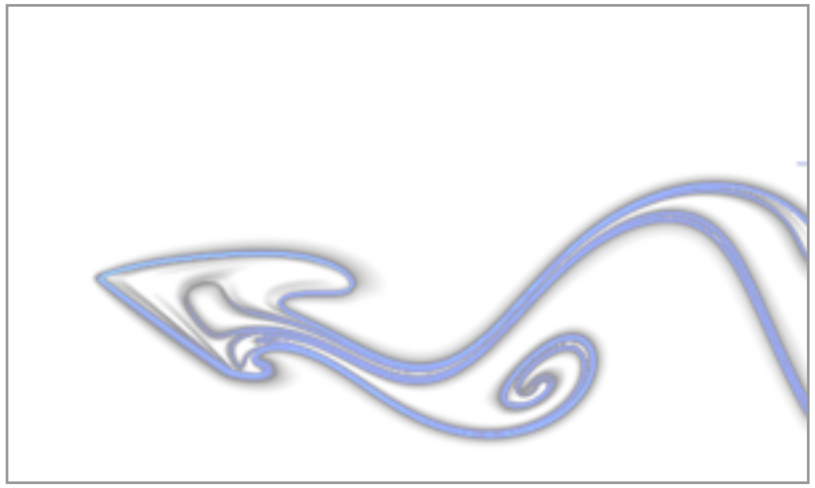
$$\underbrace{\hphantom{\text{Large amplitude, slow} \quad \text{Moderate amplitude, fast}}}_{\text{Closely related}}$$

$$\alpha_{\text{eff}} = \tan^{-1}(\pi St)$$

Brunton and Rowley, AIAA ASM 2009

3 Types of Unsteadiness

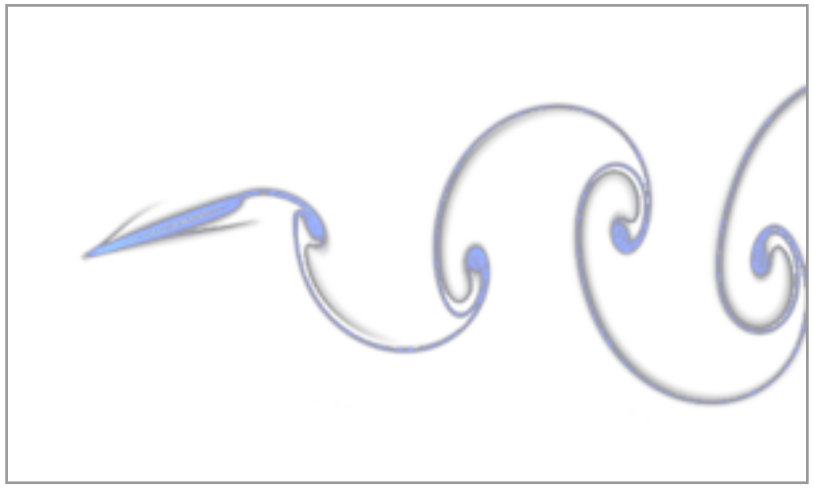
1. High angle-of-attack



$$\alpha > \alpha_{\text{stall}}$$

Large amplitude, slow

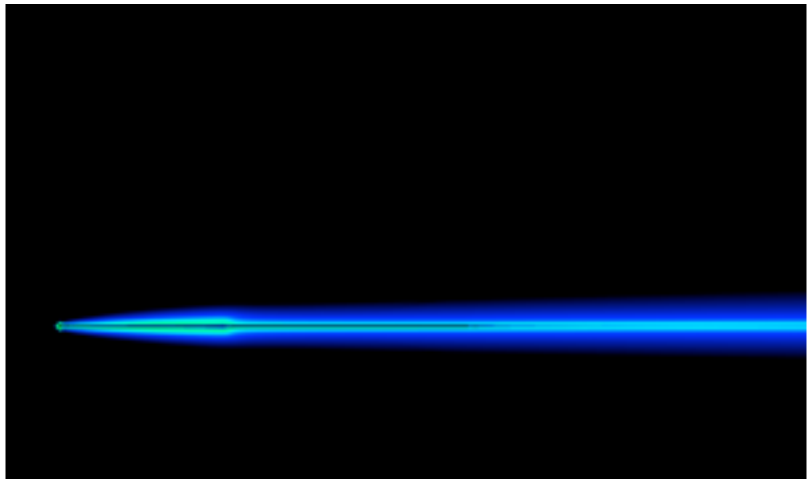
2. Strouhal number



$$St = \frac{Af}{U_\infty}$$

Moderate amplitude, fast

3. Reduced frequency



$$k = \frac{\pi f c}{U_\infty}$$

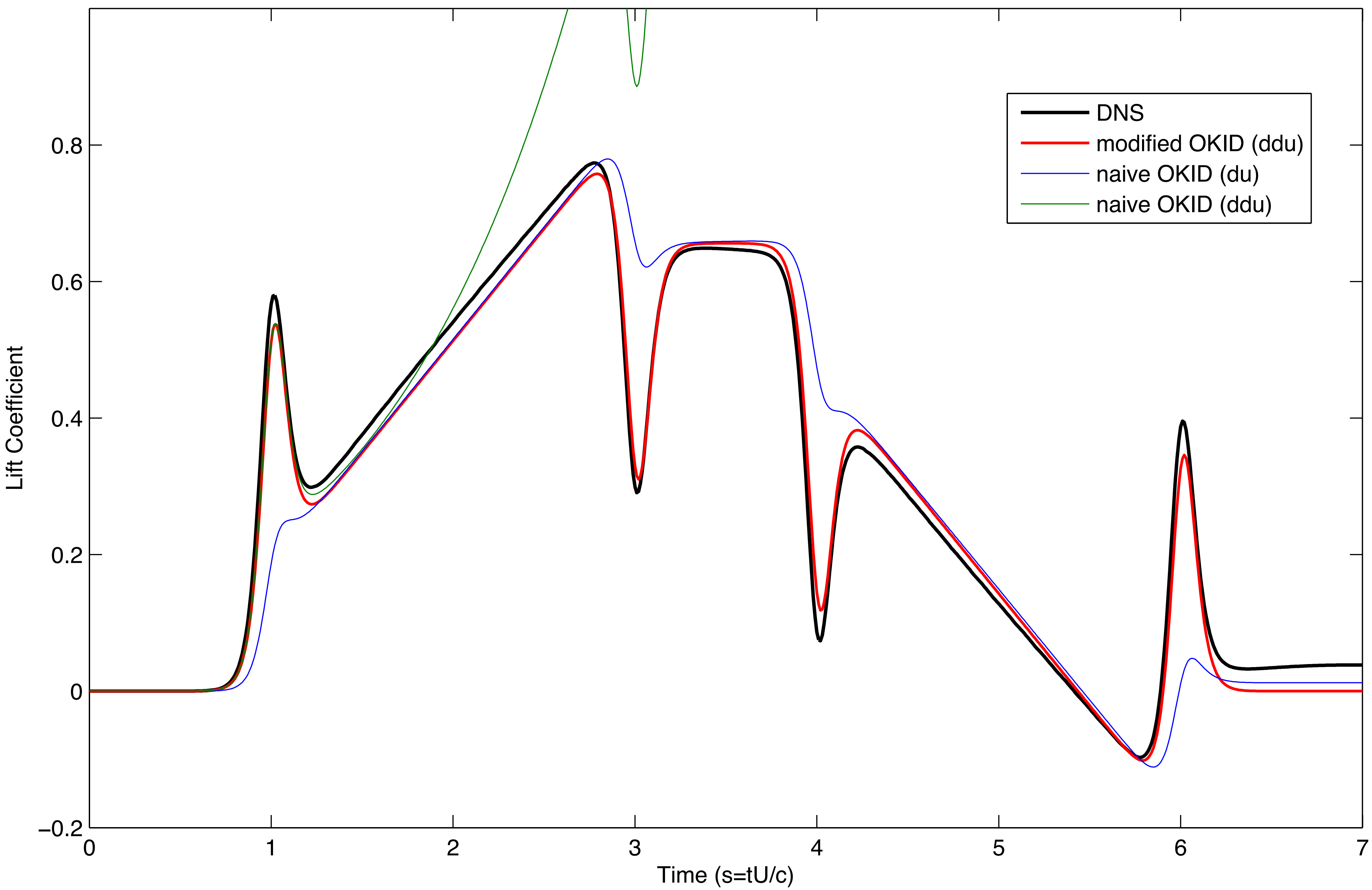
Small amplitude, very fast

$$\underbrace{\hspace{10em}}_{\text{Closely related}}$$

$$\alpha_{\text{eff}} = \tan^{-1}(\pi St)$$

Brunton and Rowley, AIAA ASM 2009

Caution Against Naive OKID



black - correct model

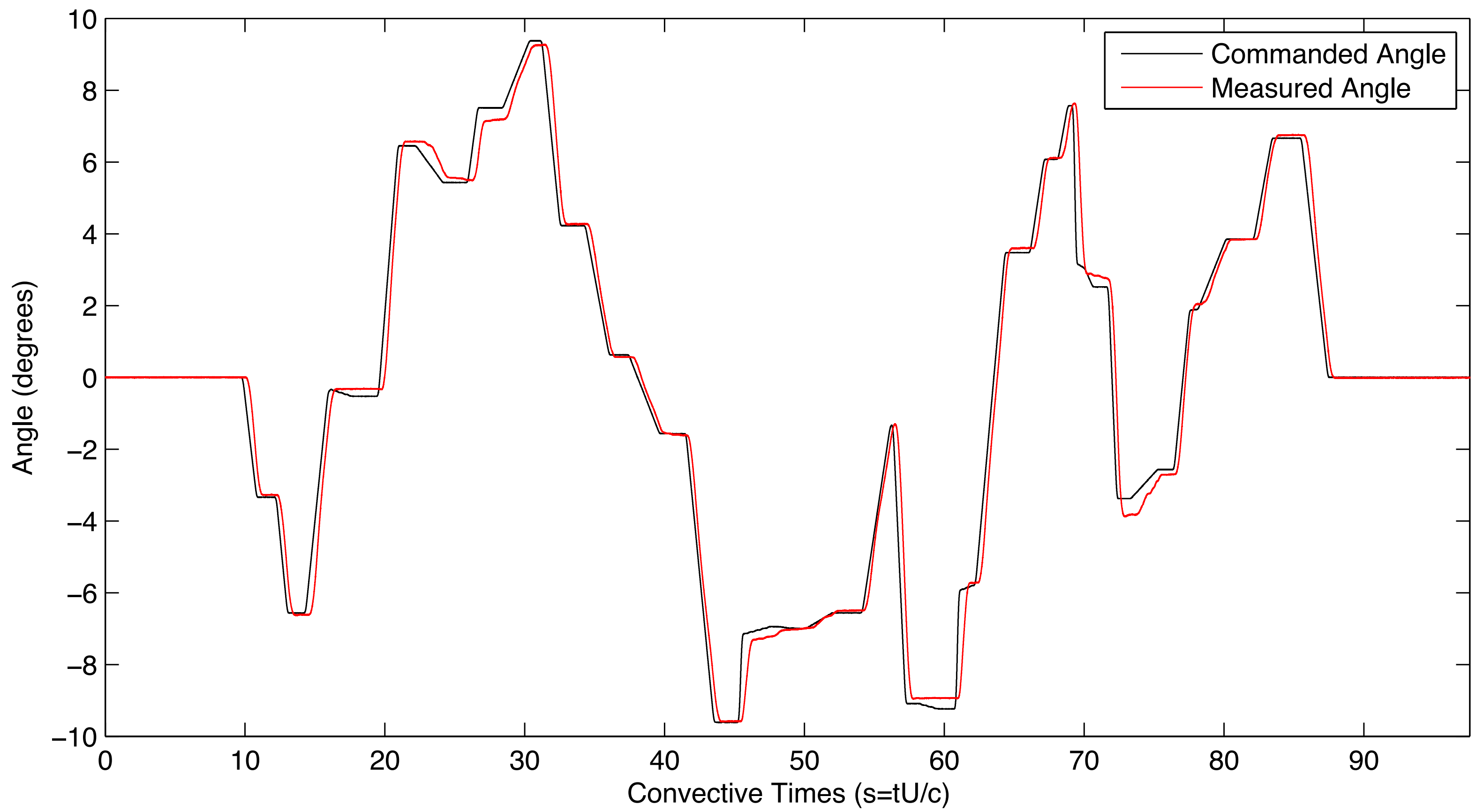
blue - using step response as input to OKID without modification

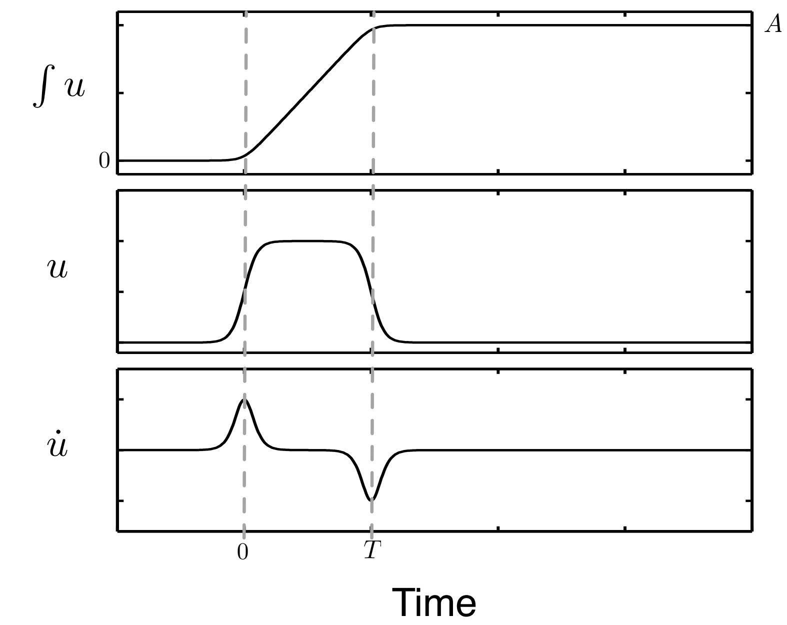
green - using step in pitch rate as input to OKID without modification

red - applying our model structure to OKID output

Brunton and Rowley, *submitted.*

Wing Maneuver





Pseudo-random sequence of ramp-hold maneuvers

Amplitude is constrained to be in +/- 10 degrees (or +/- 5)