Self-Tuning Fiber Lasers

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***Abstract*—We demonstrate that the integration of data-driven machine learning strategies with adaptive control are capable of producing an efficient and optimal self-tuning algorithm for mode-locked fiber lasers. The adaptive controller, based upon a multi-parameter extremum-seeking control algorithm, is capable of obtaining and maintaining high-energy, single-pulse states in a mode-locked fiber laser while the machine learning characterizes the cavity itself for rapid state identification and improved optimization. The theory developed is demonstrated on a nonlinear polarization rotation (NPR) based laser using waveplate and polarizer angles to achieve optimal passive mode-locking despite large disturbances to the system. The physically realizable objective function introduced divides the energy output by the fourth moment of the pulse spectrum, thus balancing the total energy with the time duration of the mode-locked solution. Moreover, its peaks are high-energy mode-locked states that have a safety margin near parameter regimes where mode-locking breaks down or the multi-pulsing instability occurs. The methods demonstrated can be implemented broadly to optical systems, or more generally to any self-tuning complex systems.**

***Index Terms*– Mode locked laser, fiber laser, adaptive control, self-tuning**

I. Introduction

Adaptive, robust, and self-tuning mode-locked lasers have eluded practical implementation for more than two decades. The ability to achieve these goals has the potential to revolutionize both the commercial and research sectors associated with ultra-fast science. This has led to recent efforts of integrating state-of-the-art adaptive control algorithms [1] with newly developed servo-control of optical components [2] to demonstrate the first successful implementations of the long-envisioned goal of robust, fully self-tuning fiber lasers [3], [4]. In parallel, data-driven mathematical techniques, often falling under the aegis of *machine learning* [5], [6], [7], [8], are having a transformative impact across the engineering and physical sciences. By combining such machine learning methods with adaptive control, we demonstrate that robust and self-tuning mode-locked operation can be achieved and fiber laser performance efficiently optimized. To our knowledge, this is the first demonstration in the optical laser context of the integration of machine learning techniques with adaptive control, thus allowing for potentially transformative performance gains in fiber lasers.

Concurrently, it is anticipated that within the next decade, mode-locked fiber lasers will close the order-of-magnitude performance gap in comparison with their solid-state counterparts [9], a performance gap largely imposed by the multi-pulsing instability (MPI) [10], [11], [12]. Engineering design concepts and/or materials are critical to pushing fiber technology forward. They can help circumvent the performance limitations that are induced when laser cavities are pushed to their limits in producing high-power and/or ultra-short pulses. Interestingly, one of the most intriguing possibilities for designing around the deleterious MPI is the standard and well-known mode-locked fiber laser that relies on nonlinear polarization rotation (NPR) for achieving saturable absorption using a combination of waveplates and polarizer [13], [14], [15], [16] (See Fig. 1 (a)).

This NPR based laser concept is more than two decades old and is one of the most commercially successful mode-locked lasers due to its reliance on simple off-the-shelf telecom components, rendering it a highly cost-effective mode-locking source. More recently, tremendous performance advances have been made in power delivery by using all-normal dispersion fiber cavities [17], [18], [19] and/or self-similar pulse evolutions [20], [21], [22]. Indeed, a distinct and critical advantage of such lasers is that the transmission function generated by the NPR dynamics is periodic, thus allowing for engineering strategies whereby the MPI can be circumvented and significant, order-or-magnitude performance gains can be achieved [12], [23], [24]. In contrast, quantum-dot [25], carbon nanotube [26], [27], graphene [28], [29], [30], and/or waveguide array [31], [32] based lasers, for instance, have transmission functions (saturable absorption) that simply deteriorate for high powers due to, for example, three-photon absorption. Thus for high-powers, and from cost considerations, there is certainly a renewed interest in pushing the limits of NPR based mode-locked fiber lasers. However, such commercial lasers must enforce strict environmental control to maintain performance, i.e., the fiber is pinned into place and shielded from temperature fluctuations. Such system sensitivity has prevented performance advances, limiting power and pulsewidths. Our adaptive control strategy [3] overcomes this cavity sensitivity while optimizing, resulting in significant performance enhancements. It should be noted that passive techniques for performance stabilization have also been recently developed in SESAM based lasers using polarization maintaing and photonics crystal fibers [33], [34], [35]. In either case, the overarching goal is to remove the sensitivity to environmental effects.

The adaptive extremum-seeking controllers (ESC) advocated here, circumvent many limitations of standard feedback control methods. For example, it does not require an underlying model, and it works equally well for nonlinear systems

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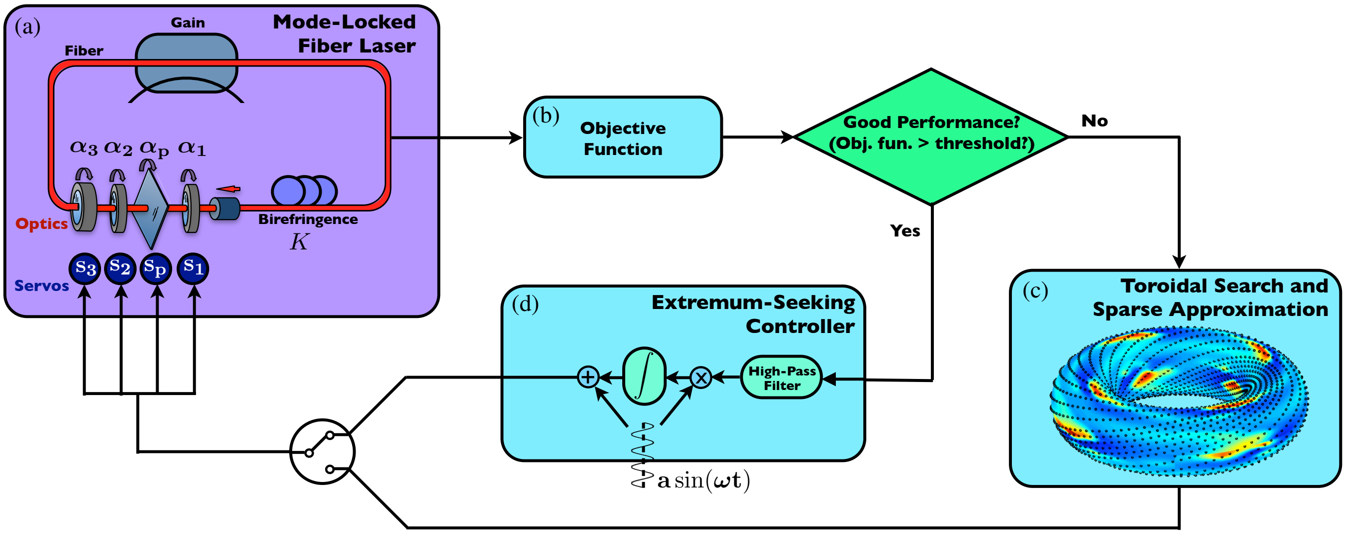

Fig. 1. Schematic of self-tuning fiber laser. The laser cavity and optic components (a) are discussed in Sec. III-A-III-B. The objective function (b) is discussed in Sec. III-C. The toroidal search and sparse approximation (c) are discussed in Sec. IV, and the extremum-seeking controller (d) is discussed in Sec. V.

such as mode-locked fiber lasers. Figure 1 shows the high-level integration of machine learning and control to close the loop. In ESC, sinusoidal perturbations are injected into servo-driven optics, providing an estimate of the objective-function gradient for fast tracking of the optimal mode-locked state (Note that Fig. 5, discussed later in the text, shows the basic operating principles for the ESC component). The objective function is modular and may be crafted specifically to yield optimal high-power, ultra-short, or fixed-bandwidth mode-locked solutions. The algorithm works with either digital or analog signals and is capable of rejecting significant disturbances. The demonstration of the ESC methodology represents a potentially disruptive technology in the fiber laser community, enabling self-tuning to optimal performance [3], while circumventing strict environmental controls. Such an adaptive controller presents the only feasible option for manipulating a multiple NPR system capable of suppressing multi-pulsing instability and achieving performance levels on par with solid-state designs [12], [23], [24].

In addition to adaptive control, which effectively becomes the expert-in-the-loop for optimizing cavity performance, we also incorporate a machine-learning architecture to characterize difficult-to-model system parameters, such as the fiber birefringence [8]. Indeed, the self-tuning adaptive controller developed here represents a significant technological advancement, allowing for the continued pursuit of performance comparable with solid-state lasers at a fraction of the cost. More broadly, these techniques apply to any tunable laser and/or optical system, promising superior performance by augmenting the system with adaptive control and machine learning algorithms.

The paper is outlined as follows: In Sec. II, a high-level overview of the three key mathematical strategies used in the laser system is given. Section III presents the governing equations of the laser cavity and highlights the tunable parameters available for adaptive control of the laser cavity. The expert-in-the-loop machine learning algorithm is highlighted in Sec. IV with the specific application of identifying the average cavity birefringence. The role of adaptive control is then demonstrated in Sec. V, showing that optimal performance can be achieved in an efficient manner. Section VI concludes with an outlook of the methods applied towards mode-locked fiber lasers and more broadly to generic complex systems.

II. Overview of Methods

Before proceeding to a detailed quantitative discussion of the modeling and computational efforts, we highlight some of the key mathematical strategies used in the self-tuning architectures. What is particularly attractive about this scheme, is that it is completely general, and the underlying paradigm for self-tuning can be applied more broadly to almost any optical system and/or complex system. A schematic of the self-tuning fiber laser is shown in Fig. 1.

A. Adaptive control

Feedback control makes it possible to take measurements of a system and synthesize the values of input variables to regulate the system behavior. Typically closed-loop feedback control will be used to stabilize unstable dynamics, improve time-domain tracking performance, reject unwanted disturbances, attenuate sensor noise, and compensate for unmodeled dynamics. Many powerful tools from control theory require linear dynamics and rely on an explicit model for the input-output dynamics. However, the mode-locked laser is strongly nonlinear, and it is infeasible to parameterize and model the effect of disturbances such as birefringence.

In contrast to classical linear feedback, adaptive controllers do not rely on a model, and they may be used on fully nonlinear systems with varying parameters. Extremum-seeking control is a method of injecting a sinusoidal input signal to estimate the gradient of an objective function. It is therefore a form of perturb-and-observe control, where the signal converges more rapidly when there is a large gradient in the objective function. Recent results establish guaranteed convergence bounds and stability [1].

B. Machine learning

The past decade has witnessed the transformative impact of data-driven methods applied to almost every area of the physical, biological, and engineering sciences. Often the goal of machine learning strategies is to reveal critical insight into underlying, and often dimensionally reduced, correlated patterns and/or clusters of activity in a given system [5], [6], [7]. Such underlying patterns are central in creating an expert-in-the-loop framework that can replace human expertise in decision making and control strategies. In the framework of mode-locked fiber lasers, the goal is to use such mathematical strategies to allow an algorithm to learn the underlying laser cavity characteristics and behavior, both good and bad. Thus the algorithm remembers the key operating regimes of the laser and how to manipulate a mis-aligned or sub-optimal laser cavity in an efficient manner. With such an algorithm, one can envision replacing a highly-experienced researcher with a fully automated and extremely rapid self-tuning system. Given the successful and highly transformative impact that machine learning is having across all disciplines, it is natural to also implement such strategies in fiber laser systems.

C. Genetic algorithm

Optimization of cavity performance is of primary importance in our advocated self-tuning architecture. However, for high dimensional parameter spaces, the objective function landscape often has numerous local minima and maxima, thus rendering many convex optimization strategies useless for finding global minima/maxima. For such problems, use of the so-called genetic algorithms, which are a subset of evolutionary algorithms, becomes instrumental [42]. The principle is quite simple and mirrors what is perceived to occur in evolution and/or genetic mutations. In particular, given a set of feasible trial solutions (either constrained or unconstrained), the objective function is evaluated. In the language of genetic algorithms, the objective function is now called the fitness function. The idea is to keep those solutions that give the maximal value of the objective function and mutate them in order to try and do even better. Thus beneficial mutations, in the sense of giving a better maximization, are kept while those that perform poorly are thrown away, i.e. survival of the fittest. This process is repeated through a prescribed number of iterations, or generations, with the idea that better and better fitness function values are generated via the mutation process. The advantage of such a strategy is that the maximization process does not get stuck in local maxima, allowing the algorithm to find the global maximizer. The genetic algorithm and machine learning strategy partner naturally to give an optimal way of informing an expert-in-the-loop strategy.

D. Combining methods

Synthesizing the methods in this paper provides an effective self-tuning mechanism for fiber lasers. The approach may be separated into two distinct components. First, laser cavity is characterized across a range of relevant parameter variation, and low-dimensional patterns are learned and concatenated into a library of signature behaviors. In addition, for each parameter value, the input angles corresponding to high-energy mode-locked solutions are recorded for quick future access. Once the machine learned library is assembled, adaptive control and parameter classification guarantee high-performance mode-locked solutions with minimal down-time. When the laser is turned on at the beginning of operation, the algorithm performs a short toroidal search of the input parameters and compares the instantaneous cavity signature against the library. Once the effective birefringence is estimated, the input angles are set to a known high-energy mode-locked configuration, and the adaptive extremum-seeking controller is turned on. The adaptive controller will compensate for moderate disturbances and slow environmental changes. However, if there is a large disturbance to the laser cavity, and the objective function performance drops below a threshold indicating that mode-locking has failed, the algorithm goes back and performs another short toroidal search to re-identify the birefringence.

III. Governing Equations: Cavity Dynamics

We model the laser cavity by evolving the intra-cavity pulse dynamics in a component by component manner. Thus the nonlinear propagation in the optical fiber is treated separately from the discretely applied waveplates and polarizer each round trip through the cavity. This treatment is discussed in detail in Refs. [36], [37], [38].

A. Coupled nonlinear Schrödinger equations

The propagation of the slowly-varying envelop of the electric field in the fiber is well-described by the coupled nonlinear Schrödinger equation (CNLS) [39], [40]:

$$\begin{aligned} i\frac{\partial u}{\partial z} + \frac{D}{2}\frac{\partial^2 u}{\partial t^2} - Ku + \left(|u|^2 + A|v|^2\right)u + Bv^2u^* &= iRu, \\ i\frac{\partial v}{\partial z} + \frac{D}{2}\frac{\partial^2 v}{\partial t^2} + Kv + \left(A|u|^2 + |v|^2\right)v + Bu^2v^* &= iRv. \end{aligned} \tag{1}$$

In the above equations, $u(z,t)$ and $v(z,t)$ are the two orthogonally polarized electric field envelopes in the optical fiber. The variable t is time in the frame of reference of the propagating pulse non-dimensionalized by the full-width at half-maximum of the pulse, and z is the propagation distance normalized by the cavity length. The functions u and v are often referred to as the fast and slow components, respectively. The parameter K is the average birefringence while D is the average group velocity dispersion of the cavity. The nonlinear coupling parameters A and B correspond to the cross-phase modulation and the four-wave mixing, respectively. They are determined by physical properties of the fiber, and $A+B=1$. For this case (a silica fiber), $A = 2/3$ and $B = 1/3$. The dissipative terms R_u and R_v account for the saturable, bandwidth-limited gain (for instance, from the Yb-doped fiber amplification) and attenuation. They satisfy the following equation:

$$R = \frac{2g_0\left(1+\tau\partial_t^2\right)}{1+(1/e_0)\int_{-\infty}^{\infty}\left(|u|^2+|v|^2\right)dt} - \Gamma$$

where g_0 is the non dimensional-pumping strength, and e_0 is the non-dimensional saturating energy of the gain medium. The pump bandwidth is τ and Γ quantifies losses due to output coupling and fiber attenuation.

B. Jones matrices for waveplates and polarizers

The application of the waveplates and passive polarizer after each round trip through the cavity may be modeled by the discrete application of Jones matrices [41].

$$W_{\lambda/4} = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}, W_{\lambda/2} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}, W_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Here, $W_{\lambda/4}$ is the quarter-waveplate (α_1 and α_2), $W_{\lambda/2}$ is the half-waveplate (α_3), and W_p is the polarizer (α_p). If the principle axes of these objects are not aligned with the fast field of the cavity, it is necessary to include the addition of a rotation matrix:

$$J_j = R(\alpha_j) W_j R(-\alpha_j), \quad R(\alpha_j) = \begin{bmatrix} \cos(\alpha_j) & -\sin(\alpha_j) \\ \sin(\alpha_j) & \cos(\alpha_j) \end{bmatrix}$$

where α_j is a waveplate or polarizer angle ($j = 1, 2, 3, p$). These rotation angles will be the control variables, allowing us to find mode-locked solutions. Recent experiments show that these control variables can be easily manipulated through electronic control [2].

C. Optimizing performance: Objective function

Given the governing equations, extensive numerical simulations can be performed in order to identify parameter regimes where mode-locking occurs. Each of these regimes can in turn be evaluated for their ability to produce high-energy, high-peak-power mode-locked states. In addition to being a costly exercise, such studies also rarely match the real cavity dynamics since, for instance, parameters like the fiber birefringence K are unknown. This motivates our use of machine learning, optimization and adaptive control strategies for characterizing the laser cavity. Interestingly, the integration of all three methods can be achieved without a detailed theoretical knowledge of the cavity equations, i.e. they are *equation-free* methods in the sense that learning the laser characteristics and applying adaptive control only relies on experimental measurements of the underlying system.

For any such data-driven strategy to be effective, we require an objective function, with local maxima that correspond to high-energy mode-locked solutions. Although we seek high-energy solutions, there are many chaotic waveforms that have significantly higher energy than mode-locked solutions. Therefore, energy alone is not a good objective function. Instead, we divide the energy function E by the fourth-moment (kurtosis) M_4 of the Fourier spectrum of the waveform

$$O = \frac{E}{M_4},$$

which is large for undesirable chaotic solutions. This objective function, which has been shown to be successful for applying adaptive control, is large when we have a large amount of energy in a tightly confined temporal wave packet [3].

IV. Learning the Cavity Dynamics

Exploring the input parameter space is a central part of the overall control strategy. There are a number of direct benefits to a simple, fast, and robust method of characterizing the cavity dynamics. First, it is necessary to identify a set of candidate high-energy mode locked solutions for use with the adaptive control algorithm. Ideally, these peaks would have the broadest support possible in parameter space. It is possible to use either a toroidal search [8] or a genetic algorithm [24] to find these high-energy candidate solutions. In addition to a set of candidate peaks, the toroidal search algorithm also provides a well-stereotyped pattern that changes with parameters (e.g., birefringence) that may be otherwise difficult to measure directly. Therefore, we use the library-building phase to construct a library of toroidal search patterns as we slowly vary the birefringence, as in [8].

Once the library is built, when we turn the laser cavity on, or when it suffers from a large disturbance and mode-locking is broken, we repeat a short toroidal search protocol and compare against our library to estimate the underlying parameter values. These parameters do not need to be numerical values, but may instead refer to proxy quantities, such as "birefringence A", "birefringence B", etc. Once the parameters are identified, it is possible to go directly to the pre-determined optimal input angles. At this point, the adaptive controller is applied to compensate for any uncertainty or error in the parameter estimation, and also to track slowly varying changes to parameters.

A. Toroidal search

We describe a toroidal search algorithm both to identify mode-locked states that may be used in conjunction with the adaptive controller, and also to identify and estimate the underlying parameters, which was advocated in [8]. All of the control inputs are periodic, so the input space forms a high-dimensional torus. It is possible to efficiently sample this toroid by increasing each of the control angles at incommensurate angular rates. This means that the ratio of each of the angular rates is an irrational number, and it is simple to show that after large enough time, the sampling scheme will approach arbitrarily close to every point in the input space. It turns out that for two rotating angles, the optimal incommensurate rates will be related by the golden ratio.

Because of the nature of some servomotors or stepper motors, it may be necessary to build in a mechanism for zeroing out the angles at the beginning of the search. This may be achieved by placing a small weight on each of the rotating optical components so that when power is cut to the motor, they all drop down to a zero-degree dead-center position. This is important when comparing two toroidal search profiles for the birefringence estimation.

Figure 2 shows the results from a typical toroidal search obtained by varying the parameters α_3 and α_p while holding α_1 and α_2 fixed. After less than two minutes, the algorithm has identified multiple candidate mode-locked states. The third figure panel shows examples of states with high and with low objective function. For multiple NPR cavities where a much

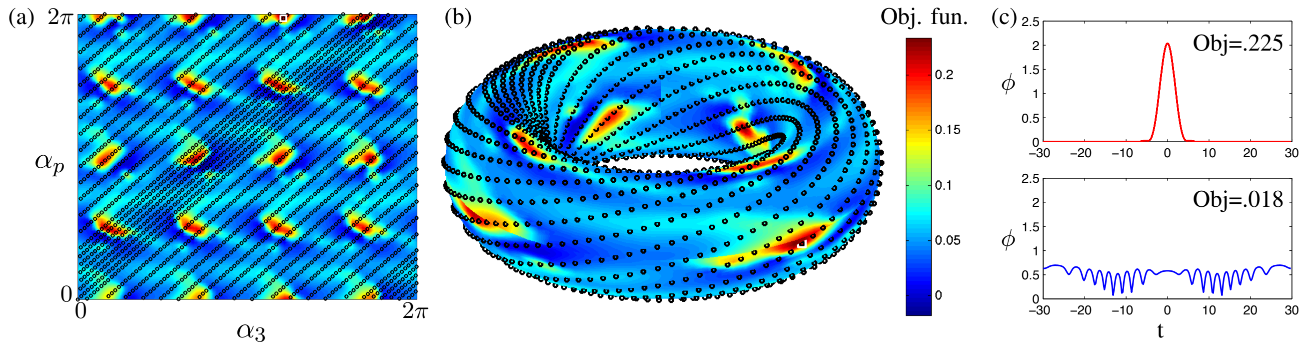

Fig. 2. Illustration of toroidal search for high-energy mode-locked states. Parameters α_3 and α_p are increased at an incommensurate rate $\alpha_3/\alpha_p = \sqrt{17}/\sqrt{19}$, while holding $\alpha_1 = -0.3755$ and $\alpha_2 = 0.0496$ fixed with $K = 0.1$ (values are chosen for consistency with ref. [8]). The two-dimensional parameter space is colored by objective function on both the universal cover (a) and the torus (b) obtained by identifying opposite sides of (a). Sampled points are shown as black circles. Pulse profiles with high and low objective function are shown in (c).

higher parameter space is required to manipulate, the toroidal search may be replaced with a genetic algorithm [24].

B. Sparse approximation in library for recognition

Practically speaking, one of the most challenging technological issues around the NPR-based mode-locked fiber laser is its sensitivity to birefringence changes. Indeed, temperature changes and/or small physical modifications of the fiber itself can easily compromise what was an ideal mode-locked state. This is why commercial versions of this laser must enforce strict environmental control to maintain performance, i.e., the fiber is pinned into place and shielded from temperature fluctuations. Such system sensitivity has prevented performance advances, limiting power and pulsewidths.

State-of-the-art theoretical models for characterizing birefringence have treated the birefringence as a stochastically varying parameter along the length of fiber. But even this is unsatisfactory since a user of a fiber laser needs to know precisely, and at all times, the birefringence in order for models to accurately predict the cavity dynamics. This is virtually an impossible task given the fact that temperature fluctuations and/or physical placement of the fiber can significantly alter the birefringence distribution. The machine learning algorithms advocated in [8] simply learn a relationship between the average birefringence and the objective function, thus recognizing uniquely the average cavity birefringence and adjusting the cavity parameters accordingly in order to optimize performance. Such a recognition task is significantly faster than re-executing the toroidal or genetic algorithm search as it allows one to move immediately to near the optimal solution where adaptive control can then maintain peak performance.

The recognition task developed in [8] is based upon producing a spectrogram of the objective function from the time-series output of the toroidal search in Fig. 2. For different average birefringence values, unique spectrograms are produced, thus providing an underlying pattern that can be used for recognition and evaluation of the K value. In practice then, what is required is a method for *classifying* the current state or the system with a birefringence library. The algorithm implemented here creates a large number of objective function spectrograms by using the toroidal search. These are then

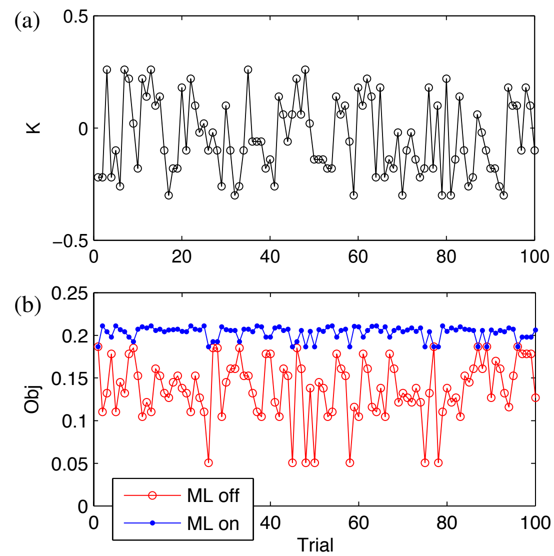

Fig. 3. Comparison of laser performance, measured by objective function (obj), with machine learning (ML) on and off (b). The birefringence is varying across trials (a). With ML off, the objective function value falls several times, indicating failure to mode-lock. With ML on, the laser maintains a high energy mode-locking state.

correlated with each other via a singular value decomposition (SVD). Only the dominant SVD modes (capturing 99.9% of the energy) are retained and used to characterize that particular birefringence. Thus for a large number of birefringence values, a library of dominant modes, Ψ is constructed. Once the library is built, it can be used for future evaluation of the state of the system using sparse approximation [43], [8].

The improvement of system performance by using sparse approximation to recognize the birefringence is shown in Fig. 3. As the birefringence is varied stochastically across 100 trials, the sparse approximation algorithm accurately predicts the birefringence K for over 90% of the trials. Even when the algorithm predicts the wrong value for K, it predict a neighboring K so that the system remains mode-locked. In contrast, without machine learning, the system performance is severely degraded by changing birefringence.

The machine learning algorithm is designed to re-identify a mode-locked solution when a large disturbance or environ-

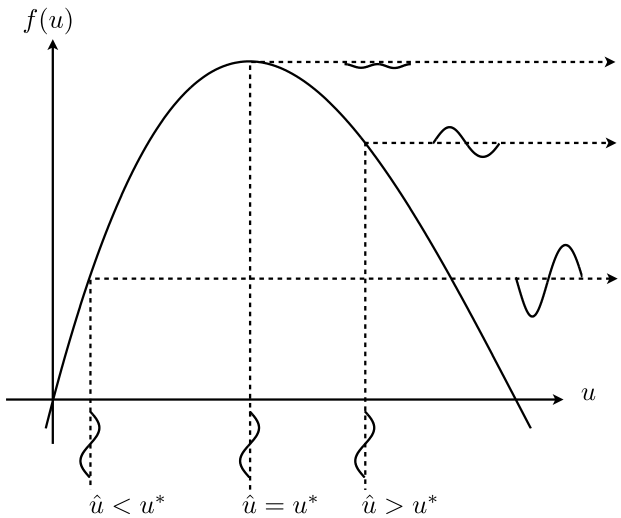

Fig. 4. Schematic of sinusoidal perturbations used for extremum-seeking control. When $\hat{u} < u^*$, the product of the output and input signals is purely positive, indicating that $\hat{u}$ must move right. Similarly, when $\hat{u} > u^*$, the demodulated signal is negative, so $\hat{u}$ moves left. Notice that the output sinusoid has larger magnitude when the function gradient is larger.

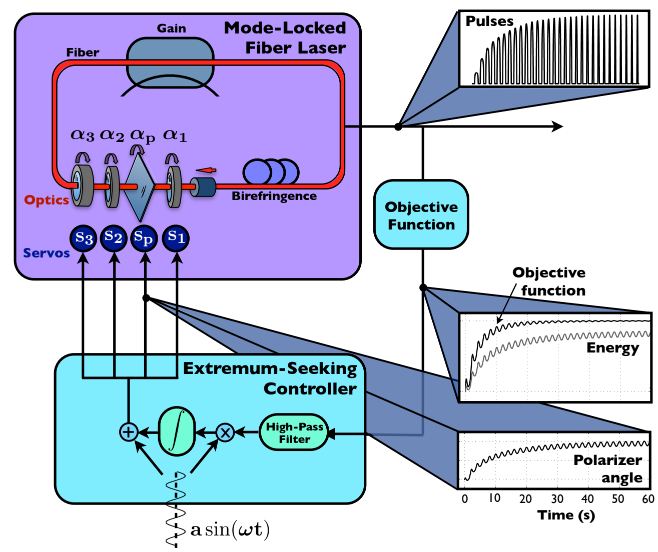

Fig. 5. Schematic of closed-loop extremum-seeking controller. The input to the controller is the objective function, and the output are reference angles for the servo-motor actuators for each optical component.

mental variation causes mode-locking to break down. Once significant objective function variation is detected, exceeding a threshold, the system undergoes performs the birefringence recognition algorithm described above, which involves a brief toroidal sample in parameter space followed by a sparse classification in a pre-learned library. After the birefringence value is identified, a high-energy mode-locked solution is chosen from the library, and the servomotors bring the laser into alignment. Finally, the adaptive controller from the next section is turned on, allowing robust tracking of the optimal mode-locked solution despite significant exogenous disturbances.

V. Adaptive Control

The adaptive control strategy advocated here does not rely on an underlying model and may be used on fully nonlinear systems with varying parameters. In this method, a sinusoidal input signal is injected into the system to estimate the gradient of our objective function. It is therefore a form of perturb-and-observe control, where the signal converges more rapidly when there is a large gradient in the objective function. Figure 4 illustrates the extremum-seeking algorithm on a quadratic cost function. A sinusoidal perturbation is added to the current estimate of the best control signal $\hat{u}$. This results in a perturbation on the output, which may be attenuated or phase shifted, but will have the same frequency as long as the perturbation is slow compared with the system dynamics. The product of the high-pass filtered output signal and input perturbation will be positive when the mean of the control signal is to the left of the optimum point u^* and will be negative when the mean is to the right of the optimum. This demodulated signal is then integrated to the mean and the controller faithfully tracks the optimum.

In many ways the laser cavity is ideal for the extremum-seeking control method since the transient dynamics operate on a timescale many orders of magnitude faster than the physical actuation of wave plate and polarizer angles [3]. This means that the only limitation on tracking bandwidth is imposed by the measurement and actuation hardware.

Figure 5 is a schematic illustrating the extremum-seeking controller in combination with the mode-locked laser. The input to the controller is the external perturbation $a\sin(\omega t)$ as well as a measurement of the objective function output of the laser cavity. The controller outputs a signal that goes to four servo motors connected to the optical components. We see that in addition to maximizing the objective function, the pulse energy increases.

Figure 6 shows the multiple-parameter extremum-seeking controller tracking the maximum of the objective function despite large variations in the birefringence occurring on the order of minutes. This presents a significant disturbance to the controller; however, performance is not effected, and instead, the controller adjusts the operating angles to compensate for this disturbance. In contrast, when no control is applied, the system performance is heavily degraded, and mode-locking fails at a number of instances in time. It is important to note that the slow drift in the mean of the oscillating input parameters suggests that the extremum-seeking controller will not track arbitrarily high frequency disturbances. For large enough disturbances, the controller will fail to track, and the birefringence identification from the previous section will be re-applied.

VI. Conclusions and Outlook

We have demonstrated the practical implementation of an adaptive, robust, and self-tuning algorithm that can be used in

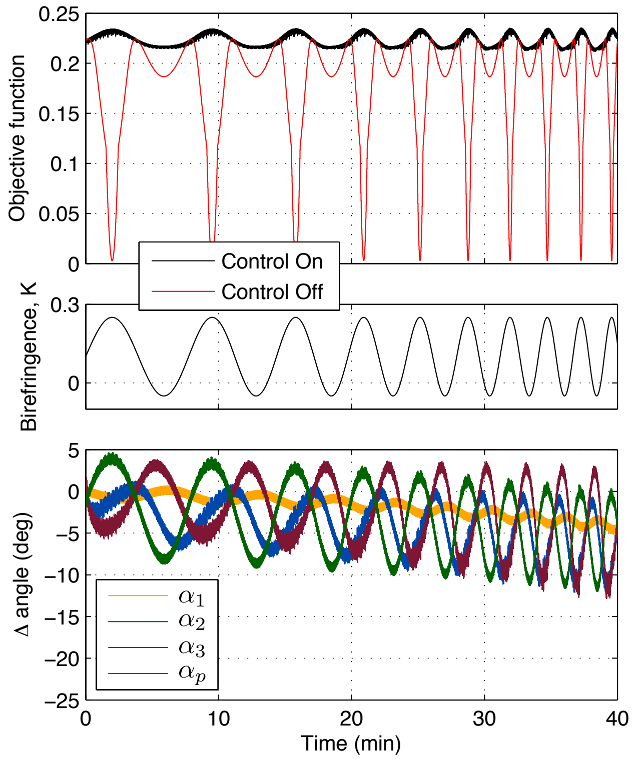

Fig. 6. Performance of extremum-seeking controller despite significant variance in birefringence over time. Without control, the objective function crashes several times, resulting in failure to mode-lock. With control, the system remains at a high-performance mode-locked state for the entire trial.

conjunction with mode-locked fiber lasers. Such a scheme has eluded practical implementation for more than two decades. But with state-of-the-art machine learning and state classification methods, along with adaptive control schemes that are model independent, the critical components are now in place to revolutionize both the commercial and research sectors associated with ultra-fast science. Although we demonstrate the methodology within the context of NPR-based fiber lasers, due to newly developed servo-control of optical components [2], [3], [4], the data-driven strategies are generic and capable of self-tuning almost any laser system. Here, by combining such machine learning strategies [8] with adaptive control [3], we demonstrate that robust and self-tuning mode-locked operation can be achieved and fiber laser performance efficiently optimized. To our knowledge, this is the first demonstration in the optical laser context of the integration of machine learning techniques with adaptive control, thus allowing for potentially transformative performance gains in fiber lasers.

The success of such self-tuning strategies hinges on two critical components: (i) identifying input (control) parameters and (ii) constructing an appropriate objective function that is feasible and serves well as a proxy for cavity performance. The algorithms for both learning the cavity behaviors and applying adaptive control are both *equation-free* methods [42]. Thus they do not rely on one's ability to construct accurate model equations. Rather, all characterization is done directly from experimental data and no reliance is made on a faithful model. Such strategies are highly advantageous since modeling often fails to provide accurate quantitative prediction of laser cavity dynamics. Even in the two decade old problem of NPR-based mode-locking, models have proved to be of value for qualitative modeling, but have had limited quantitative use since phenomenon such as the randomly varying birefringence simply are beyond our capabilities to model due to their extreme sensitivity and stochastic nature. The methods advocated here do not suffer from such sensitivity, they simply adapt to the changes and optimize for global performance.

Naturally, the methods presented have limitations and their own performance barriers. But we believe that many of these can be overcome with additional work in this area. Moreover, one may wish to use fully passive stabilization techniques, like those recently developed for SESAM based lasers [33], [34], [35], whenever possible over active techniques. But ultimately, it comes down to tradeoffs between performance gains and cost, something which would need to be evaluated on a laser by laser basis. For the NPR-based laser considered here, the potential for engineering the transmission curve for significant improvements in high-power performance [12], [23], [24] is the impetus for the implementation of an adaptive control strategy. In any case, intelligent systems and their design/implementation are ultimately desirable given their long-term potential impact in the field of mode-locked fiber lasers. The following subsections outline some broader issues to be pursued in future work as it relates directly to fiber laser performance and scalability of high-energy delivery.

A. Need for dimensionality reduction on input space

Extremum-seeking does not scale well with arbitrarily many input variables. This provides a complication for use with the multiple NPR case [12], [23], [24], regardless of the significant potential benefits in scaling the power delivery of mode-locked fiber lasers, i.e. making them competitive with their solid-state counterparts. There are two reasons for the difficulty. First, the controller convergence takes significantly longer with multiple parameters, and the design is more complicated. In addition, the radius of perturbation grows with the number of parameters, and for the mode-locked laser, this perturbation may kick the system into an unstable or chaotic regime. This is compounded by the need for a sufficiently large perturbation signal to achieve a reasonable signal to noise ratio.

The use of dimensionality reduction techniques such as principal components analysis (PCA) may allow us to determine low-dimensional control inputs that reduce the dimension of the input space. Just like the machine learning methods demonstrated, the key is to identify low-dimensional spaces that provide an accurate representation of the laser cavity dynamics. The low-dimensional input space has fewer degrees of freedom required to adjust and adapt in order to maintain the high-energy states.

B. Adaptive control and learning of complex systems

Every part of the algorithm advocated in this manuscript can be applied broadly to many complex systems. In particular, the specific advantage of the machine learning and adaptive control being *equation-free* allows it to be applied across the engineering, physical and biological sciences. What is critical is to identify the input space for control and a suitable objective function to optimize. The algorithm developed can simply be wrapped around a generic complex system with the goal of producing similar optimization results. In particular, Fig. 1 can be modified so that the mode-locked laser block in the purple box is replaced by another complex system. Of course, this does not guarantee you can control the complex system. But if the control inputs are indeed capable of significant manipulation of the system, then adaptive control in conjunction with machine learning may be used effectively to automate the expert-in-the-loop. In summary, the method is applicable in a much broader context than mode-locked fiber lasers.

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