

**Exercise 5-1:** Compute the following Laplace transforms by hand:

(a) Delta function:  $f(t) = \delta(t)$  (remember to integrate from  $0^-$  to  $\infty$ )

(b) Heaviside function:  $f(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

(c) Time-shifted delta function:  $f(t) = \delta(t - a)$ .

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**Exercise 5-2:** Use the Laplace transform to solve the following ODEs:

(a)  $\ddot{x} + 3\dot{x} + 2x = u(t)$

(b)  $\ddot{x} + 2\dot{x} + 2x = u(t)$

Solve each of these for the following initial conditions and forcing functions (hint: use the Laplace transforms from above so simplify the expression in the frequency domain. I would not recommend using convolution, if you can avoid it):

(i) Step response: A step input (i.e.,  $u(t)$  is a Heaviside function) with zero initial conditions

(ii) Impulse response: An impulsive input (i.e.,  $u(t)$  is a Delta function) with zero initial conditions

For each case, plot your solution and also plot Matlab's solution using the `step` and `impulse` commands.

(iii) Initial condition response:  $u(t) = 0$  with initial conditions  $x(0) = 1$  and  $\dot{x}(0) = 0$ .

(iv) Initial condition response:  $u(t) = 0$  with initial conditions  $x(0) = 0$  and  $\dot{x}(0) = 1$ .

Summary: Solve both equations (a) and (b) using the forcing and initial conditions from (i)-(iv).

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