ME565 - Winter 2020 **Due: 5:00 pm Friday February 14^{th}, 2020**
Homework 3

Exercise 3–1: Solve the one-dimensional Laplace equation for the equilibrium temperature distribution in a finite rod of length L for the following boundary conditions:

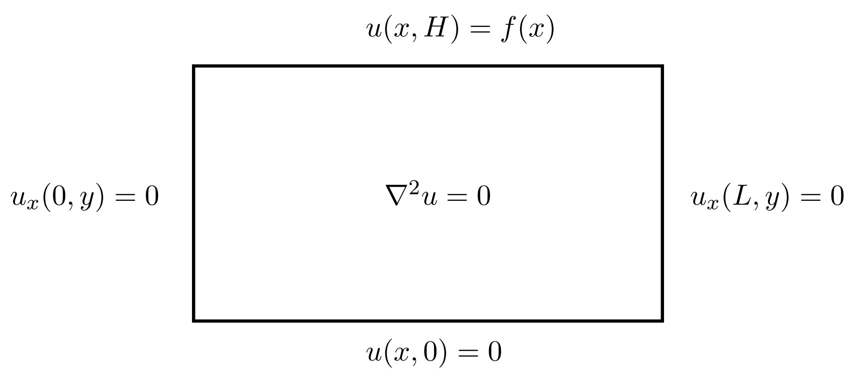
1. $u(0) = 1$ and $u(L) = 2$
2. $u_x(0) = 0$ and $u(L) = B$
3. $u(0) + u_x(0) = A$ and $u(L) + u_x(L) = B$

Please also draw a sketch for each boundary condition.

Exercise 3–2: Solve for the equilibrium temperature distribution using the 2D Laplace equation on an $L \times H$ sized rectangular domain with the following boundary conditions:

1. Left: $u_x(0, y) = 0$ (insulating)
2. Bottom $u(x, 0) = 0$ (fixed temperature)
3. Top: $u(x, H) = f(x)$ (zero temperature)
4. Right: $u_x(L, y) = 0$ (insulating)

Solve for a general boundary temperature $f(x)$. Also solve for a particular temperature distribution $f(x)$; you may choose any non-constant distribution you like.



How would this change if the left and right boundaries were fixed at zero temperature? (you don't have to solve this new problem, just explain in words what would change)

Exercise 3–3: Solve the modified Laplace equation with a source term (Poisson's equation!):

$$u_{xx} = -Q(x), \tag{1}$$

for $Q = x^2$. Use fixed temperature boundary conditions so that $u(0) = 0$ and $u(L) = 0$.

What can you say about the heat flux at $x = 0$ and $x = L$?

Plot the temperature distribution for $L = 10$.

Exercise 3–4: Heat conduction in a thin circular ring is described by the following PDE

$$u_t = u_{xx}, \text{ for } t \geq 0. \tag{2}$$

with periodic boundary conditions in space ($x \in [0, 2\pi]$):

$$u(0,t) = u(2\pi, t), u_x(0,t) = u_x(2\pi, t), \text{ etc.} \tag{3}$$

Solve using the separation of variables, i.e. $u(x,t) = X(x)T(t)$. Find a general solution (i.e. for arbitrary initial condition $u(x,0) = f(x)$) and describe the asymptotic behavior of *all* solutions as $t \to \infty$. Provide a physical interpretation of this behavior.