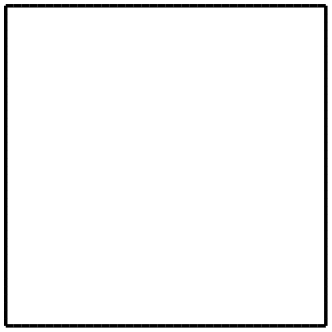
ME565 - Winter 2020 **Due: 5:00 pm Friday January 31st, 2020**
Homework 2

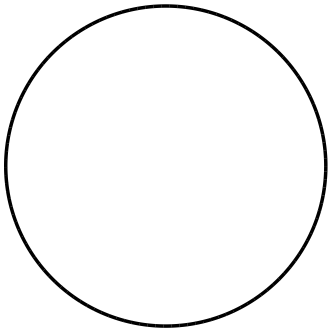
Exercise 2–1:

(a) Compute the area of a square of length 2 centered at the origin using Stokes' theorem:



Square of length 2

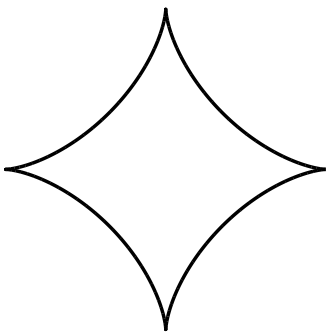
(b) Compute the area of a circle with radius 1 centered at the origin using Stokes' theorem:



Circle: $x^2 + y^2 = 1$

$$x = \cos(\theta)$$
$$y = \sin(\theta)$$

(c) Compute the area of a hypocycloid using Stokes' theorem (Hint: You can compare your answer to the derivation in the notes):



Hypocycloid: $x^{2/3} + y^{2/3} = 1$

$$x = \cos^3(\theta)$$
$$y = \sin^3(\theta)$$

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Exercise 2–2: Relate the surface area of a sphere ($4\pi r^2$) to the volume of a sphere ($\frac{4}{3}\pi r^3$) using Gauss's theorem. Hint: find a vector field that has a constant divergence, or that is constant on the surface. It may help to work in spherical coordinates, although it is not necessary. Note that the divergence has a different expression in spherical coordinates.

Exercise 2–3: Here we will compute using the directional derivative $D_{\mathbf{v}} f = \nabla f \cdot \mathbf{v}/\|\mathbf{v}\|$.

(a) Compute the directional derivative with $f(x, y) = x^3 \cos(xy)$ and $\mathbf{v} = \begin{bmatrix} 2 & 1 \end{bmatrix}^T$.

(b) You are goat climbing a mountain, where the altitude is given by the function $f(x, y) = 5 - x^2 - y^6$. To impress the humans below, you want to choose the steepest path up the mountain. You are at the point $(1, 1, 3)$. What direction should you go in to climb the steepest path?

(c) You are a cold mosquito flying around a room. The temperature is given by $T(x, y, z) = \cos(x^2)\sin(y)\cos(z)$, and you are at the point $(1, 2, 1)$. What direction should you fly to warm up?
