ME565 - Winter 2020 **Due: 5:00 pm Friday January 17^{th}, 2020**
Homework 1

Exercise 1–1: Solve the following linear system of ODEs:

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x}, \quad \text{for} \quad \mathbf{A} = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

Please compute this by hand for a generic initial condition $\mathbf{x}(0) = \mathbf{x}_0$. Hint: you will want to compute the matrix exponential $e^{\mathbf{A}t}$. You may use MATLAB to check your work.

Exercise 1-2 Express the following in the form $a + bi$ (for real a and b) and also in the form $Re^{i\theta}$ (for real R and θ):

(a) $\frac{1}{3+2i}$

(b) $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3$

(c) $(-i)^2, (-i)^3, (-i)^4, (-i)^5, \ldots$

Exercise 1-3 Find all solutions of

(a) $e^z = 1$

(b) $e^z = -i$

(c) $e^z = -1 + i$

Exercise 1-4 Find all solutions of

(a) $z^3 = i$

(b) $z^3 = -i$

(c) $z^5 = 1$

(d) $z^2 = -1 - i$

Exercise 1-6 Find all analytic functions $f = u + iv$ with $u(x, y) = x^2 - y^2$. Simplify the expression $f(z)$ as much as possible.
